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Option Smile and Switching Volatility

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OPTION SMILE AND SWITCHING VOLATILITY[1]

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Abstract

In this paper, we examine the *smile* effect on European stock options by modelling stock prices with a mixture of distributions characterised by different volatilities. The mixture weights depend on a hidden Markov chain, which represents possible different regimes of the stock market. The asymmetric pattern of implied volatility smile is captured in our model taking into account the negative correlation between volatility and returns. In our work we carried out an empirical analysis on the FTSE100 index. We demonstrate that the asymmetric smile effect, that particularly characterises put options, could be accounted for by our approach and the comparison with some observed smile on FTSE100 future options and it gives interesting results. Moreover, we demonstrate that switching regime option models better capture the smile effect observed in the FTSE100 future option market with respect to the GARCH option pricing model proposed by Duan (1995).

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1 Introduction

The empirical evidence shows that the assumption of constant volatility, imposed in the Black and Scholes (1973) model (hereafter B&S), is almost constantly violated.

The volatility parameters implied from the market option prices, according to the Black and Scholes formula, vary across different maturities (T) and different strike prices (K). Therefore, implied volatility can instantaneously be represented as a surface whose shape is determined by a combination of different maturities and strike prices. Financial economists often refer to this phenomenon as the *volatility smile*. Empirical analysis, summarized in Bollerslev, Chou and Kroner (1992), is at odds with the constant volatility hypothesis underlying the B&S model. As a matter of fact, deep in-the money or out-the money options show an higher implied volatility level than at-the money options. After the '87 crash a *sneer*[4] appears: there is a negative relation between implied volatilities and strike prices. In particular, out-the money puts are traded at a higher implied volatility than out-of the money calls.

In order to solve this problem, a substantial literature developed models for pricing options under stochastic volatility processes. The main problem in extending the B&S formula to a stochastic volatility framework is that the solution of the option's partial differential equation (PDE) depends on risk preferences. In this case, a riskless investment strategy cannot be constructed using only the option and the underlying asset as volatility's volatility generates extra-risk. Many authors have tried to generalized the B&S model to allow for stochastic volatility and propose a solution to the latter problem (Hull and White (1987), Wiggins (1987), Scott (1987), Bailey and Stulz (1989), Heston (1993), Amin and Ng (1993) and Duan (1995)[5]).

One problem with these models is that the estimation of the parameters characterizing the stochastic volatility is usually difficult. Moreover, as suggested by Taylor (1994), these models have been motivated by convenience and intuition rather than by studies on observed prices.

In order to overcome these drawbacks, we explore a new approach in looking at volatility. We model stock prices with a mixture of distributions characterised by different volatilities. Mixture weights depend on a hidden

[4] Webster (1994, p. 1100) defines a *sneer* as *a scornful facial expression market by the slight raising of one corner of the upper lip.*

[5] For a comparative review of these models see Pelizzon (1996).

Markov chain, which represents possible different regimes of the stock market. This approach accounts for an asymmetric behaviour in the stock price, depending on whether the price is increasing or falling. Empirical evidence supports this pattern. The economic intuition is that, in periods of large move down of the stock, stop-loss strategies and budget constraints generate this regime switch (volatility of stock price increases).

This paper presents a switching regime technique which combines Hull and White (1987) option pricing formula - that applies when there is no correlation between the underlying asset and the volatility jumps - and a discretization of a discrete state variable multinomial approach that accounts for correlation. Moreover, we present the smile effect observed in the FTSE100 future option market with respect to the GARCH option pricing model proposed by Duan (1995).

As our task is only to illustrate the working of our procedure, the robustness of this methodology is beyond the scope of this work.

The paper is organized as follows. In section 1, we describe our dataset of FTSE100 index and FTSE100 index option prices. We also document typical B&S implied volatility patterns, time-varying volatility of the FTSE100 index and the negative relation between implied volatilities and returns. In section 2, we describe switching regime models. In section 3, we present an estimation of these models applied to the FTSE100 index. In section 4, we describe our pricing models for options with switching regime volatility. In Section 5, we present the empirical evidence of the application of the two models to the FTSE100 future option and we compare these results with some implied volatility patterns observed in the FTSE100 option future market. In Section 6 we shortly present the GARCH option pricing model of Duan (1995) and show the implicit volatility smile determined by the GARCH model. Section 7 concludes the paper with a summary of the main results and directions for future research.

2 FTSE100 futures option implied volatility smiles

Our sample contains implied volatilities of FTSE100 future options traded on the London International Financial Future Exchange (LIFFE) during the period from March 1992 to February 1996. FTSE100 future options are

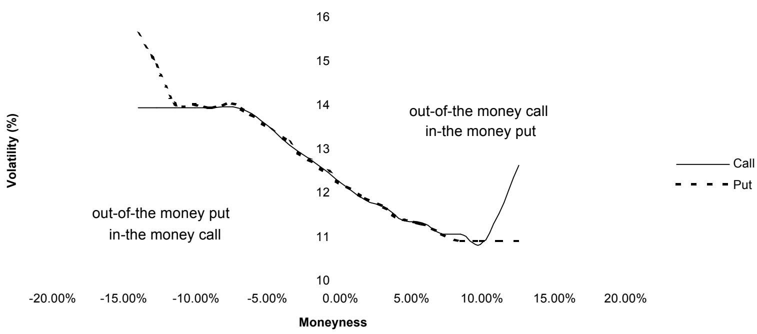

Figure 1: Black and Scholes implied volatilities on February 1, 1996, expire date April 19, 1996. Moneyness is defined as $K/F - 1$.

European style and expire on the third Friday of the contract month[6].

In order to show a typical pattern of B&S implied volatilities, we use call and put options (with expiry date April 19, 1996) observed on February 1. Figure 1 clearly depicts[7] the negative relationship between implied volatility and strike prices. Out-of the money puts trade at higher implied volatilities than out-of-the money calls. The smile has given way to a sneer. Historically, a smile pattern was observed prior to October 1987 crash, with higher B&S implied volatilities attached to both in-the money and out-of-the money compared to at-the money options. After the market crash, call and put option implied volatilities decrease monotonically with the strike price.

With regard to the underlying asset, we use monthly data of FTSE100 index traded on the LIFFE during the period February 1978 through February 1996. The empirical evidence shows that sudden market downturns occur more often and faster than upturns. For this reason volatility and implied volatility increase when return fall down. Evidence of this is documented by Franks and Schwartz (1991). This could be observed in Figure 2 which shows the level of the B&S three months at-the money implied volatility from June 14, 1994 to April 28, 1995 compared to the FTSE100 index level in the same period.

It is easy to observe from Figure 2 that increases in volatility usually

[6] The implied volatility is determined directly by LIFFE.

[7] *Moneyness* is determined by $K/F - 1$ where K is the strike price and F is the future index price.

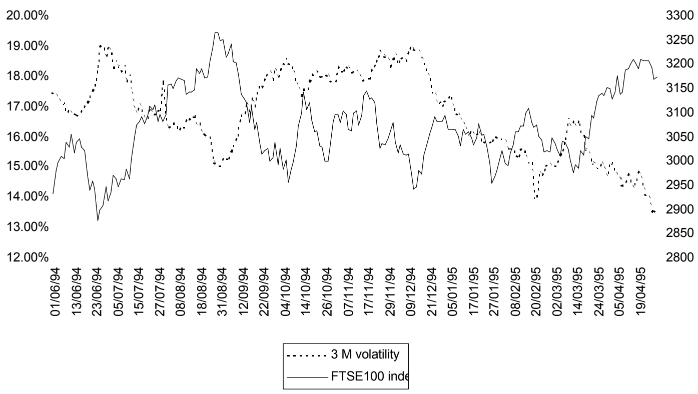

Figure 2: Comparison between implied volatility and index patterns. It is easy to observe the negative relation between volatility and levels index.

accompany a crash of the market that could be generated by aggregate supply or sells (stop-loss strategies, budget constraint), shifts in economic policies and political crisis. It is quite plausible that traders consider this different possibilities in forming an expectation about future volatility. Switching regime approach is a simple model to account for this patterns and their probabilities.

The observation of all these phenomena has suggested us to consider the application of switching regime models to option pricing.

3 Switching regime models

Switching regime models[8] are a simple way for taking into account random discontinuous shifts in returns and volatility. This approach permits to model

[8] Switching regime models is a methodology introduced by Hamilton (1989, 1990). See also Hamilton (1994).

the returns (R_t), for example in the simple two regime case[9], as follows[10]:

$$R_t = \mu + \sigma_{k_t}\varepsilon_t$$

where

$$\sigma_{k_t} = \begin{cases} \sigma_0 & \text{if } k_t = 0 \\ \sigma_1 & \text{if } k_t = 1 \end{cases}$$

$\varepsilon \sim \mathcal{N}(0,1)$ and k_t is a Markov chain which takes values {0,1} and is described by the following transition probabilities:

$$\begin{array}{ll} P(k_t = 0|k_{t-1} = 0) = p_{00} & P(k_t = 1|k_{t-1} = 0) = 1 - p_{00} \\ P(k_t = 1|k_{t-1} = 1) = p_{11} & P(k_t = 0|k_{t-1} = 1) = 1 - p_{11} \end{array}$$

The parameters p_{00} and p_{11} determine the probability of the volatility to remain in the same regime. This model allows for a change in the variance of returns only in response to occasional, discrete events.

An important implication of such a model is that returns follow a fat-tailed distribution, which is a well documented empirical observation (Mandelbrot (1963)). Moreover, the switching regime model characterizes the R_t distribution as a mixture of normals. These characteristics are important, since, as Johnson and Shanno (1987) observed:

> *The consensus is that the distribution [of returns] is fat-tailed and skewed, and hence, non normal; however, stationary stable distributions with infinite variance do not fit the data as well as mixtures of normals with different variances (i.e., subordinated processes).*

Markovian switching models, therefore, represent promising extensions of this observation.

Using switching regime models, returns time series could be modelled in the following way:

$$R_t = \mu_{k_t} + \sigma_{k_t}\varepsilon_t \tag{1}$$

[9] The analysis, however, generalizes naturally to the case of multiple regimes for the volatility process.

[10] Where return (R_t) is defined as:

$$R_t = \ln(F_t/F_{t-1})$$

and F_t is the stock price or the index price.

where $\varepsilon \sim \mathcal{N}(0,1)$ and k_t is a Markov chain which can take N values and is described by the transition $P = [p_{ij}] = [P(k_t = i|k_{t-1} = j)]$. This model presents a contemporaneous switching regime in mean and variance.

Moreover, given the evidence from previous studies (Tucker and Scott (1987)) that correlation exists between stock returns and volatility, we analysed also the following model:

$$R_t = \mu_{k_t} + \sigma_{k_t}\varepsilon_t \tag{2}$$

where $\varepsilon \sim \mathcal{N}(0,1)$ and k_t is a Markov chain which can takes N values, depending on R_{t-1}, that is, with transition probabilities:

$$\begin{aligned} P(k_t = i|k_{t-1} = j, R_{t-1}) &= \frac{\exp(a^j_{i0} + a_{i1}R_{t-1})}{1 + \sum_{i=1}^{N-1}\exp(a^j_{i0} + a^j_{i1}R_{t-1})} \qquad i = 1, ..., N-1 \\ P(k_t = i|k_{t-1} = j, R_{t-1}) &= 1 - \sum_{i=1}^{N-1} P(k_t = i|k_{t-1} = j, R_{t-1}) \quad i = N \end{aligned}$$

with $j = 1, ..., N$. This model is quite simple. It is the same as model (1), but with transition probabilities which depend on the previous value of returns. In particular, this model considers the possibility of correlation between shock and probability of changes in volatility. That is, this model accounts for the case when, given a previous large negative return, the probability to switch from a low volatility regime to a high volatility regime is greater than when the previous return is positive.

In order to assess the plausibility of pricing options with switching regime volatility, we need to determine the switching regime model that better characterizes the underlying asset, that in our example is the FTSE100 index, and then estimate it.

4 Estimation results

Using the monthly data of FTSE100 index traded on the LIFFE during the period February 1978 through February 1996, the switching regime models are estimated.

In our analysis we estimated the parameters with the Maximum Likelihood approach, considering the Hamilton's filter[11]. Table 1 presents the estimation results for the model (1) in the three switching regime case.

[11] See Hamilton (1994) for an analytic description of it.

Parameters	Value	Standard Error
μ_0	0.008	0.005
μ_1	0.019	0.003
μ_2	-0.005	0.055
σ_0	0.044	0.009
σ_1	0.025	0.006
σ_2	0.093	0.118
π_{00}	0.78	0.51
π_{01}	6 e-10	0.23
π_{10}	5 e-06	0.13
π_{11}	0.89	0.18
π_{20}	0.74	1.04
π_{21}	0.26	0.62

Table 1: Three switching regime model estimated parameters.

As shown by the values, we can observe that regime 2 presents a volatility three times higher than the regime 1, but the probability to remain in the regime 2 is very low. The probability to remain in regime 0 (high volatility) or regime 1 (low volatility) is high, this means that these two regimes are quite persistent and there is, therefore, evidence of the cluster effect. The probability that a crash will be followed by a low volatility regime is low. Instead, it is surprising that the probability to switch from a low volatility regime to a crash is greater than the probability to switch to a high volatility regime. Nevertheless, we have to consider that the probabilities to switch from one regime to the other have an high standard deviation.

In order to determine the future distribution and then the price of the option, we need to know the actual regime. The probabilistic inference of being in one of the three regimes can be calculated for each date t of the sample using the Hamilton filter and smoother algorithm (Hamilton (1994)). The probability of being in the low volatility regime (regime 1), conditionally on sample informations, on February 1996 for the FTSE100 is bigger than the others. As an illustrative example we show (Figure 3) the density of the mixture of the three distributions, representing the distribution probability of returns given that we are in low volatility regime and the probability: (i) to remain on it is equal 88.7%, (ii) to switch to regime 0 is almost zero and

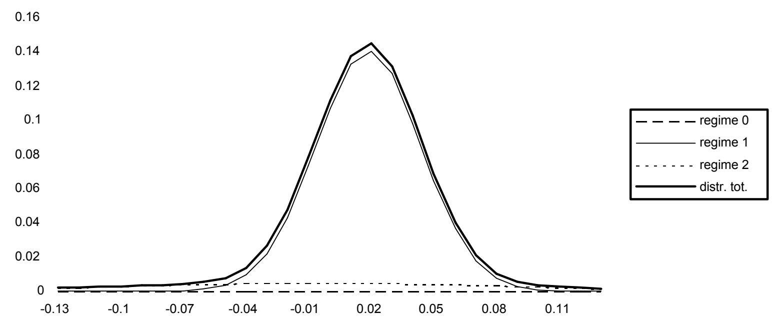

Figure 3: Mixture of three distributions density, representing the distribution probability of returns given that we are in regime 1 and the probability to remain on it is 88.7%.

(*iii*) to switch to regime 2 is 11.25%.

From this figure it is easy to understand why switching regime models generate fat-tail distribution with skewness and kurtosis, as discussed above.

The estimation of the model (2) is more complicated, because the correlation increases the number of model's parameters. In any case, given the correlation between Markov chain and lagged returns, the two regimes model is sufficient to describe the series under study (the high volatility regime can account also for crashes). In that case the parameters are eight: two means, two variances and four probability parameters. The estimation results for the model (2) are shown in Table 2.

The main result is that the parameter a_{11} is bigger than parameter a_{10} and has a negative sign. This means that the probability to remain in the regime with high volatility increases if a negative return has been previously observed.

This model allows us to estimate a well observed event, that is, the asymmetric behaviour in the stock price, that depends on whether the price is increasing or decreasing. The economic intuition - as stated above - is that, in periods of large decrease in the stock price, stop-loss strategies and budget constraints generate this regime switch (volatility of stock price increases).

Later in section 5, we show that this effect can explain the sneer pattern of implied volatility.

Parameters	Value	Standard Error
μ_0	0.032	0.001
μ_1	0.011	0.003
σ_0	0.003	0.001
σ_0	0.045	0.002
a_{00}	2.592	0.503
a_{10}	0.645	8.225
a_{01}	-1.88	0.390
a_{10}	-8.11	3.299

Table 2: Estimated parameters of model (2) with correlation between Markov chain and lagged returns.

5 Option pricing models with switching volatility

The key argument in the B&S model is that investors can replicate any option through continuous trading in the stock and in the riskless asset, so that option markets are redundant. But in the B&S model volatility is constant. Introducing stochastic volatility in the definition of the stochastic differential equation of the underlying asset creates several complications. A dynamic portfolio with only one option and one stock is not sufficient to create a riskless investment strategy.

The problem arises since the stochastic differential equation for the option contains two sources of uncertainty. Unfortunately, it is impossible to eliminate volatility market risk premium and correlation parameters from the PDE using only one option and one stock. Moreover, these parameters are difficult to estimate[12] and extensive use of numerical techniques is required to solve the two-dimensional PDE. Unfortunately, pricing options with switching regime in the volatility presents the same problems as others models.

Naik (1993) developped a model in which the volatility of risky assets

[12] An exception occurs where the volatility is a deterministic function of the asset price or time. In this case is possible to find easily solution to the PDE. Derman and Kani (1994), Rubinstein (1994) Dumas, Fleming and Whaley (1996) developed many deterministic volatility function's option valuation models.

is subject to random and discontinuous shifts over time. In his work Naik (1993) demonstrated that, under particular hypothesis, his formula is the same as the Hull and White (1987).

In the Hull and White (1987) formula, the option price is determined assuming that volatility market risk premium is zero and there is zero correlation between the two Wiener processes of the underlying asset and the volatility, i.e. the volatility is uncorrelated with the stock price. With these assumptions and using a risk-neutral valuation procedure, they show that the price of an option with stochastic volatility is the B&S price integrated over the distribution of the mean volatility:

$$C(F_t, \sigma_t^2) = \int C_{B\&S}(V) h(V|\sigma_t^2) dV \tag{3}$$

where:

$$V = \frac{1}{T-t} \int_t^T \sigma^2(\tau) d\tau \tag{4}$$

i.e. V is the mean variance over the interval $T-t$, T is the maturity and $C_{B\&S}$ is the B&S price of an option calculated with variance V.

This model can be easily applied to option pricing with switching regime in volatility, as Naik (1993) demonstrated. However, we have to assume: (i) risk-neutrality, (ii) volatility market risk premium equal zero and (iii) zero correlation between the Markov chain k_t and the White Noise process ε_t.

Since we characterize the volatility with a discrete stochastic process the integral of formula (4) becomes a simple sum. For each path that σ^2 may follow, we determine the mean variance over the interval $T-t$ as:

$$V_i = \frac{1}{L} \sum_{j=1}^{L} {}_i\sigma_j^2 \tag{5}$$

where $L = (T-t)/\ell$ is the number of possible regime changes, ℓ is the interval between two regime changes, ${}_i\sigma_j^2$ is the variance for the interval j in the i-th path.

Considering that, in each interval of time, the volatility can switch in regime, all the possible path that the variance can follow are N^L where N is the number of states of the Markov chain k_t. The probability of these mean variance values is determined by the transition probability of the Markov chain, given the current value of the variance σ_t^2.

The option pricing formula is:

$$C = \sum_{i=1}^{N^L} B(V_i) p(V_i | k_t) \tag{6}$$

where V_i is calculated as in equation (5).

Equation (6) states that the option price is the average of the B&S prices determined over the distribution of the mean volatility.

Unfortunately, the assumptions behind this formula are less acceptable because casual empiricism suggests that stock returns are negatively correlated with volatility. This correlation is relevant for the determination of the smile. Heston (1993) observed that, if the volatility of the asset is stochastic, the distribution of the return on this asset exhibits kurtosis which depends on the variance of σ_t, and skewness which depends on the correlation between the two Wiener processes.

These peculiarities in the distribution have an important effect on the price of the options and on the implied volatility re-calculated by the B&S formula. This could be easily deducted, considering that the price of the option is the discounted expected value of final payoffs and, with negative skewness, the payoff of the put presents greater probability than the payoff of the call. Moreover, as Heston (1993) shows, when there is negative correlation, i.e. negative skewness, implied volatility, determined by B&S, decreases monotonically with the option exercise price. This means that the sneer effect could be generated by the negative correlation between the asset returns and the volatility.

This evidence has led us to use, complementary to the Hull and White formula, a multinomial approach that could capture this effect. Unfortunately, the multinomial approach is a rough methodology, but in our opinion, it is sufficient for our analysis. The multinomial tree used for pricing options with two regimes in volatility and correlation is illustrated in Figure 4.

The up and down values for each regime j are defined using the usual risk-neutral approach, i.e.:

$$u_j = e^{\sigma_j \sqrt{\Delta t}} \quad d_j = \frac{1}{u_j} \tag{7}$$

and the up and down probabilities are given by:

$$p(u_j) = \frac{r - d_j}{u_j - d_j} \tag{8}$$

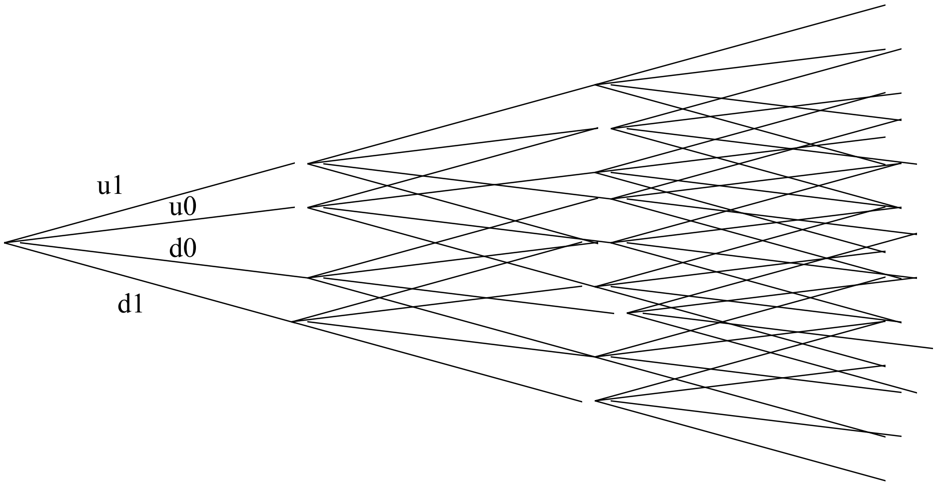

Figure 4: Multinomial Tree.

where r is the constant one-period risk free rate of return. The probability of a regime switch, conditional to a positive or negative return, are

$$\begin{aligned} p_{ij}^u &= P(k_t = i | k_{t-1} = j, R_{t-1} > 0) \\ p_{ij}^d &= P(k_t = i | k_{t-1} = j, R_{t-1} < 0) \end{aligned} \tag{9}$$

Therefore, the conditional probabilities of the multinomial tree for the two regimes model (2) will be calculated as in Table 3.

	u_0	d_0	u_1	d_1
u_0	$p_{00}^u * p(u_0)$	$p_{00}^u * (1 - p(u_0))$	$(1 - p_{00}^u) * p(u_1)$	$(1 - p_{00}^u) * (1 - p(u_1))$
d_0	$p_{00}^d * p(u_0)$	$p_{00}^d * (1 - p(u_0))$	$(1 - p_{00}^d) * p(u_1)$	$(1 - p_{00}^d) * (1 - p(u_1))$
u_1	$p_{11}^u * p(u_0)$	$p_{11}^u * (1 - p(u_0))$	$(1 - p_{11}^u) * p(u_1)$	$(1 - p_{11}^u) * (1 - p(u_1))$
d_1	$p_{11}^d * p(u_0)$	$p_{11}^d * (1 - p(u_0))$	$(1 - p_{11}^d) * p(u_1)$	$(1 - p_{11}^d) * (1 - p(u_1))$

Table 3: Markov chain probability metrics.

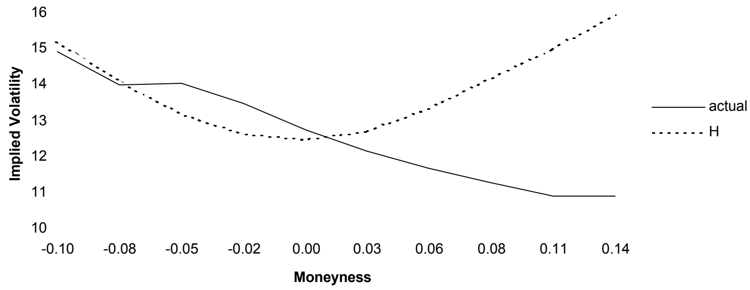

Figure 5: Call B&S implied volatilities calculated respect the Hull and White price formula (three regime model) and the actual price.

6 Empirical evidence

We would like to shed some light about the price of options with switching regime in the volatility using the Hull and White model and the multinomial approach. In order to do so, we compare them graphically with the actual market prices of options.

Theoretical option prices are calculated using the estimated parameters of models (1) and (2). We consider option prices as observed on February 1, 1996 on a contract expiring on April 19, 1996. The volatility is determined implicitly from these theoretical prices, using the B&S formula.

We applied the Hull and White formula to the three regime model. Results are shown in Figure 5.

It is easy to observe from Figure 5 that a classical smile and not a sneer is generated by the Hull and White formula. This derives from the hypothesis that there is no correlation between stock and volatility and the volatility risk premium is equal to zero (see Pelizzon (1996b) for the effect of volatility risk premium in the volatility smile).

However, it is surprising to see how good the fit is with respect to the actual out-of-the money put implied volatilities. The former, in fact, has factor in the possibility of these three different regimes, and, in particular, the possibility of a crash of the market.

As described before, in order to overcome the problems deriving from the hypothesis of Hull and White formula, we used the multinomial approach.

We used the estimated parameters of model (2)to determine the prob-

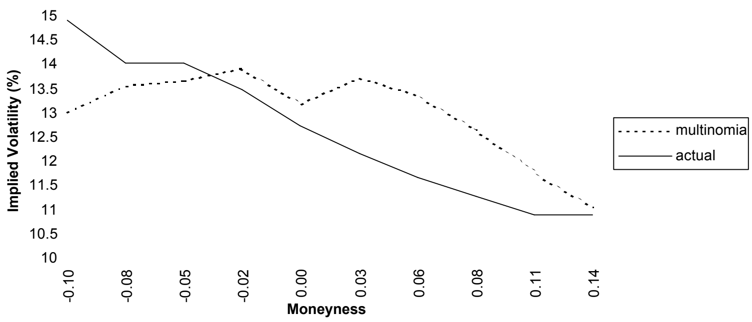

Figure 6: Put B&S implied volatilities calculated respect the multinomial approach and the actual price using the model with switch in mean and variance with correlation between volatility and lagged returns. Moneyness is defined as K/F-1.

ability values p_{ij}^u and p_{ij}^d from formula (9). The values are shown in Table 4.

p_{00}^u	p_{00}^d	p_{11}^u	p_{11}^d
0.93	0.93	0.57	0.73

Table 4: Estimated probabilities of model (2).

As the values of Table 4 show, correlation influences more the probability of remaining in the high regime rather than in the normal regime. For determining the values u_j and $p(u_j)$ we used formulas (7) and (8) and the values σ_j Table 2. Conditional probabilities are calculated as indicated in Table 3. The pattern of actual and theoretical implied volatility for puts - calculated using the multinomial approach illustrated above - are presented in Figure 7. The results reported in Figure 7 confirm our hypothesis. The patterns and the levels of implied and actual volatility are quite similar.

It is important to stress that we used a multinomial approach with only three steps, and then there is room for improvement in the estimation procedure. As a consequence the results are certainly interesting and they confirm the opportunity to apply switching regime volatility to option pricing.

Finally, in order to compare the ability of the switching regime model with respect to the GARCH option pricing model, we consider the GARCH-

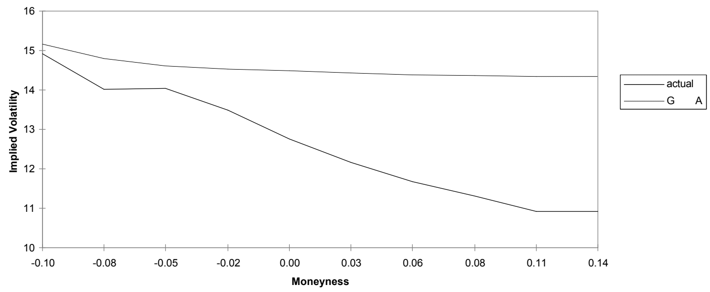

Figure 7: Put B&S implied volatilities calculated respect the GARCH option approach and the actual price. Moneyness is defined as $K/F-1$.

M model proposed by Duan (1995)[13]. We use the same methodology (Monte Carlo simulation) presented by Duan (1995) in order to determine the GARCH option price. Fifty thousand simulation runs are carried out for a given set of GARCH option prices corresponding to different moneyness ratio.

Implied volatilities calculated using the GARCH option model presented above, are shown in Figure 9. Moreover, in this figure we represent the pattern of actual implied volatility and theoretical implied volatility for puts calculated using the two switching regimes models presented above.

From Figure 9 it is easy to observe that the GARCH option pricing approach is able to generate an implied valatility that is grater for out-of-the money Put with respect to at-the money or in-the money Put. However,

[13]The model is:

$$\begin{aligned} R_t &= r + \lambda\sigma_t - \tfrac{1}{2}\sigma_t^2 + \varepsilon_t \\ \varepsilon_t &\sim \mathcal{N}(0, \sigma_t^2) \\ \sigma_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \end{aligned} \qquad (10)$$

where r is the constant one-period risk free rate of return and λ is the constant unit risk premium. The estimated parameters for the FTSE100 index series are:

Parameters	Value	Standard Error
α_0	0.002	2.49e-4
α_1	0.006	0.012
β_1	2.05e-8	4.64e-6
λ	-1.3834	0.526

the shape of the implied volatility is far from replicating the smile that we observe in the market.

This result is therefore not too surprising, since GARCH models are unable to properly take into account the crash phenomena of October 1987 nine years later. However, these results lend support to the switching volatility option pricing models, in particular the one with correlation between returns and volatility.

7 Conclusions

In this paper, we examine the pricing of European stock options with switching regime volatility. The idea of this study derived from: (*i*) smile behaviour analysis (sneer, negative correlation between volatility and stock price), (*ii*) economic reasons of smile, and (*iii*) analysis of the main problems on options with stochastic volatility models.

We have considered it useful to model stochastic volatility with a simple regime switching model instead of a continuos diffusion model, since it is simpler and - we think - potentially more appealing to practitioners.

We used models of switching regime with changes in mean and variance to characterize returns. We followed both the general approach proposed by Hull and White (1987) and the multinomial approach. We used the FTSE100 index time series to estimate the four models and the FTSE100 futures options implied volatility for a graphical comparison with our results.

As the tables, the figures and the comparison with the GARCH option pricing model illustrate, the approach we proposed is plausible and useful to describe the FTE100 option future smile behaviour more clearly. Moreover, this approach allows us to define expected volatility considering the probability of market large moves down.

However, considering that the purpose of the analysis was merely to assess the plausibility of this approach, our results suggest some future investigation. As a first step further, we suggest to use another pricing formula for switching regime with correlation and perform a proper statistical test. Furthermore, the hypothesis about price volatility risk parameter could be changed. Moreover, the estimation algorithm of the switching regime models could be improved and we suggest to use data with greater frequency. Additionally, it could be useful to analyze how to use the information deriving from switching regime for hedging. Finally, switching regime approach could be used to determine implicitly, from the option's market price, the probabilities of the switching regime, the correlation and the mean and variance parameters of the underlying asset.

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