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A generalized performance attribution technique for mutual funds

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Abstract

In this contribution we propose a model which can be used to define a measure of the relative performance of mutual funds that takes into account all the different aspects considered by the traditional performance indexes. In addition, the model proposed can take into account also the subscription and redemption costs.

This model adopts a data envelopment analysis approach. In particular, among the inputs considered by the model we have different risk measures of the portfolio and the subscription costs and redemption fees. The set of outputs taken into account comprise the portfolio expected return, the traditional performance indexes and a stochastic dominance indicator.

An average cross efficiency measure is also suggested, in order to give a measure of the rating of a fund from the point of view of the styles of all the funds under consideration.

The generalized performance attribution techniques proposed are tested using data of the Italian mutual funds market.

Keywords: Performance attribution; Mutual fund performance; Data envelopment analysis

1. Introduction

It is a common practice to evaluate mutual funds by measuring their past performances, even if nothing ensures that the same return paths will occur again in the future.

To this aim, some numerical indexes are widely used that take into account an expected return indicator and a risk measure and synthesize them in a unique numerical value. Among them, we find the well-known Sharpe, Treynor and Jensen indexes.

As it is known, the *Sharpe index* (Sharpe, 1966) is defined as the ratio between the expected excess return and the standard deviation of the returns; in other

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terms, it measures the portfolio performance by means of the expected excess return per unit of risk.

Let us consider a set of n mutual funds $j = 1, 2, \ldots, n$ with risky returns R_j and assume to have to compare their performances. We denote by $E(R_j)$ the expected return and by $\sigma_j = \sqrt{Var(R_j)}$ the standard deviation of the return. Moreover, let us define the expected excess return $E(R_j) - \delta$ as the difference between the portfolio expected return and the riskless rate of return δ. The Sharpe ratio is computed as

$$I_{j,Sharpe} = \frac{E(R_j) - \delta}{\sigma_j}. \tag{1.1}$$

However, the standard deviation of the returns may be a proper risk measure when the investor holds only one risky asset and the returns probability distribution is symmetric.

In order to better define the portfolio risk, it is possible to take into account only the (undesirable) negative deviations from the mean or from a threshold value. In this way, different performance indexes can be obtained which have the same ratio structure of Sharpe index. The *reward to half-variance index* (Ang and Chua, 1979)

$$I_{j,half-var} = \frac{E(R_j) - \delta}{\sqrt{HV_j}} \tag{1.2}$$

measures the risk using the square root of the *half-variance* indicator

$$HV_j = E\left(\min\left[R_j - E(R_j), 0\right]\right)^2, \tag{1.3}$$

i.e. with the average of the squared negative deviations from the mean. The *reward to semivariance index* (Ang and Chua, 1979) measures the portfolio risk with the square root of the *semivariance*, which is the average of the squared negative deviations from a fixed threshold value; the threshold value represents the minimum target for the returns to be considered desirable by the investor.

The *Treynor index* (Treynor, 1965) measures the risk with the β of the portfolio, under the implicit assumption that investors have diversified their investments so that they are equivalent to a quota of the market portfolio. We have

$$I_{j,Treynor} = \frac{E(R_j) - \delta}{\beta_j}, \tag{1.4}$$

where $\beta_j = Cov(R_j, R_m)/Var(R_m)$ represents the ratio of the covariance between the portfolio return R_j and the market portfolio return R_m to the variance of the market portfolio return.

Jensen index (Jensen, 1968), instead, is not defined as a ratio but finds inspiration in the volatility estimation of a risky portfolio obtained in the C.A.P.M. framework and measures the portfolio performance by means of the intercept of the linear regression

$$E(R_j) - \delta = I_{j,Jensen} + \beta\left[E(R_m) - \delta\right]. \tag{1.5}$$

A significantly positive value for this intercept means that the mutual fund management has obtained positive results that overcome those obtained by the market portfolio.

It is not clear which of these performance indicators represents the best performance measure, as each of them may be valid under some assumptions but may be overcome by one of the other indicators in a different context, or even for different investors. Moreover, the traditional performance indexes do not consider the subscription and redemption costs required by the investment, even if the overall return on the investment is indeed affected by these costs.

In this contribution we propose a model which can be used to define a measure of the relative performance of mutual funds that takes into account all the different aspects considered by the traditional indexes. In addition, the model proposed can take into account also the subscription and redemption costs, and allows therefore to compute an indicator of the overall performance of the mutual fund investment.

This model adopts an approach suggested by the data envelopment analysis (DEA) methodology, which is an operational research technique that enables to measure the relative efficiency of a set of "decision making units". This technique can be applied to the measurement of mutual fund performance since each fund may be seen as a decision making unit which requires a set of "inputs" and supplies some "outputs". Applications of DEA approaches to the appraisal of mutual fund efficiency can be found in Murthi, Choi and Desai (1997), Morey and Morey (1999) and Basso and Funari (2001).

In particular, among the inputs simultaneously considered by the model we may include different risk measures of the portfolio and, above all, the subscription costs and redemption fees. The set of outputs taken into account may comprise, besides the portfolio expected return, all the traditional indicators. In addition, we have included among the outputs also a stochastic dominance indicator which reflects both the investors' preference structure and the time occurrence of the returns. This indicator is computed by assigning a higher score to the mutual funds which are not dominated by other funds in the higher number of subperiods.

The aim is to define a generalized performance indicator which allows to take different points of view into consideration in the performance measurement process.

The generalized DEA efficiency measure proposed is computed by solving a fractional linear programming problem, where an efficiency ratio, made up of a ratio of a weighted sum of outputs to a weighted sum of inputs, is maximized. This optimization problem, which can be converted into an equivalent linear programming problem, comes from the idea to define the DEA efficiency measure by assigning to each decision making unit the most favourable weights (Charnes, Cooper and Rhodes, 1978).

Moreover, an average cross efficiency measure is also suggested, in order to give a measure of the rating of a fund from the point of view of the styles of all the funds under consideration. This efficiency measure is obtained by averaging the efficiency scores calculated with the optimal weights of all funds.

The generalized performance attribution techniques proposed are tested using data of the Italian mutual funds market.

The structure of the paper is the following. Section 2 briefly introduces the DEA methodology. Section 3 summarizes the DEA measures for the evaluation of the mutual fund performance in the presence of investment costs proposed in the literature by Murthi, Choi and Desai (1997) and Basso and Funari (2001); these approaches are generalized by our performance indicators. In Section 4 we present our generalized DEA performance indicator for the appraisal of mutual funds. Section 5 illustrates the results of the empirical research on Italian market data carried out to compare the performance results provided by the generalized DEA performance indicator, the traditional performance indexes and the DEA efficiency measures previously proposed in the literature. Section 6 discusses the average cross efficiency measure for the mutual fund evaluation. Finally, Section 7 presents some concluding remarks.

2. The data envelopment analysis methodology

The data envelopment analysis (DEA) is an optimization based technique proposed by Charnes, Cooper and Rhodes (1978) to evaluate the relative performance of decision making units which are characterized by a multiple objectives and/or multiple inputs structure.

Operational units of this kind, for example, typically include no profit and governmental units such as schools, hospitals, universities. In these units, the presence of a multiple output–multiple input situation makes difficult to identify an evident efficiency indicator such as profit and complicates the search for a satisfactory measure of efficiency.

Charnes, Cooper and Rhodes (1978) propose a measure of efficiency which is essentially defined as a ratio of a weighted sum of outputs to a weighted sum of inputs. In a sense, the weighted sums allow to reduce the multiple input–multiple output situation to a single "virtual" input–"virtual" output case; the efficiency measure is then taken as the ratio of the virtual output to the virtual input. Of course, the higher the efficiency ratio is, the more efficient the unit is.

Such a weighted ratio requires a set of weights to be defined and this can be not easy. Charnes, Cooper and Rhodes suggest to define the efficiency measure by choosing for each unit the most favourable weights. With such a choice the weights will generally differ for the various units. However, a unit which proves to be inefficient with respect to other ones even with the most favourable weights cannot call upon the fact that this depends on the choice of the weights.

The most favourable weights are chosen as the ones which maximize the efficiency ratio of the unit considered, subject to the constraints that the efficiency ratios of all units, computed with the same weights, have an upper bound of 1. Therefore, an efficiency measure equal to 1 characterizes the efficient units: at least with the most favourable weights, these units cannot be dominated by the other ones in the set. We obtain a Pareto efficiency measure in which the efficient units lie on the efficient frontier (see Charnes, Cooper, Lewin and Seiford, 1994).

Let us define:

$j = 1, 2, \ldots, n$	decision making units
$r = 1, 2, \ldots, t$	outputs
$i = 1, 2, \ldots, m$	inputs
y_{rj}	amount of output r for unit j
x_{ij}	amount of input i for unit j
u_r	weight given to output r
v_i	weight given to input i

The DEA efficiency measure for the decision making unit j_0 ($j_0 = 1, 2, \ldots, n$) is computed by solving the following fractional linear programming model

$$\max_{\{v_i, u_r\}} \quad h_0 = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \tag{2.1}$$

subject to

$$\begin{aligned} &\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad && j = 1, 2, \ldots, n \\ &u_r \geq \varepsilon && r = 1, 2, \ldots, t \\ &v_i \geq \varepsilon && i = 1, 2, \ldots, m \end{aligned} \tag{2.2}$$

where ε is a convenient small positive number that prevents the weights from being zero (for the strict positivity constraints on the weights see Charnes, Cooper and Rhodes, 1979, and Charnes, Cooper Lewin and Seiford, 1994).

The above ratio form has an infinite number of optimal solutions: in fact, if $(v_1, \ldots, v_m, u_1, \ldots, u_t)$ is optimal, then $\beta(v_1, \ldots, v_m, u_1, \ldots, u_t)$ is also optimal for all $\beta > 0$. One can define an equivalence relation that partitions the set of feasible solutions of problem (2.1)-(2.2) into equivalence classes. Charnes and Cooper (1962) and Charnes, Cooper Lewin and Seiford (1994) propose to select a representative solution from each equivalence class. The representative solution that is usually chosen in DEA modelling is that for which $\sum_{i=1}^{m} v_i x_{ij_0} = 1$ in the input-oriented forms and that with $\sum_{r=1}^{t} u_r y_{rj_0} = 1$ in the output-oriented models.

In this way the fractional problem (2.1)-(2.2) can be converted into an equivalent linear programming problem which can be easily solved. Using the input-oriented form we thus obtain the so called input-oriented CCR (Charnes, Cooper and Rhodes) linear model

$$\max \quad \sum_{r=1}^{t} u_r y_{rj_0} \tag{2.3}$$

subject to

$$\begin{aligned} &\sum_{i=1}^{m} v_i x_{ij_0} = 1 \\ &\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad && j = 1, 2, \ldots, n \\ &- u_r \leq -\varepsilon && r = 1, 2, \ldots, t \\ &- v_i \leq -\varepsilon && i = 1, 2, \ldots, m. \end{aligned} \tag{2.4}$$

Its dual problem is also useful, both for computational convenience, as it has usually less constraints than the primal problem, and for its significance

$$\min \quad z_0 - \varepsilon \sum_{r=1}^{t} s_r^+ - \varepsilon \sum_{i=1}^{m} s_i^- \tag{2.5}$$

subject to

$$\begin{aligned} & x_{ij_0} z_0 - s_i^- - \sum_{j=1}^{n} x_{ij}\lambda_j = 0 \quad i = 1, 2, \ldots, m \\ & - s_r^+ + \sum_{j=1}^{n} y_{rj}\lambda_j = y_{rj_0} \quad r = 1, 2, \ldots, t \\ & \lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \\ & s_i^- \geq 0 \qquad i = 1, 2, \ldots, m \\ & s_r^+ \geq 0 \qquad r = 1, 2, \ldots, t \end{aligned} \tag{2.6}$$

The CRR primal problem has $t + m$ variables (the weights u_r and v_i which have to be chosen so as to maximize the efficiency of the targeted unit j_0) and $n + t + m + 1$ constraints.

The CCR model gives a piecewise linear production surface which, from an economic point of view, is a production frontier: it represents the maximum output empirically obtainable from a decision making unit given its level of inputs; at the same time, it represents the minimum amount of input required to achieve the given output levels. More precisely, the input-oriented models focus on the maximal movement toward the frontier through a reduction of inputs, whereas the output-oriented ones consider the maximal movement via an augmentation of outputs.

DEA model (2.1)-(2.2) is the simplest and most used DEA technique. In addition, a number of extensions and variants have been proposed in the literature to better cope with special purposes; for a review see for example Cooper, Seiford and Tone (2000).

Though born to evaluate the efficiency of no profit institutions, soon afterwards the DEA technique has been applied to measure the efficiency of any organizational unit; for example it has largely been used to compare the performance of different bank branches.

An important feature of the DEA approach is its ability not only to verify if a decision making unit is efficient, relative to the other units, but also to suggest to the inefficient units a "virtual unit" that they could imitate in order to improve their efficiency. In fact, for each inefficient unit the solution of the input-oriented CCR dual model (2.5)-(2.6) allows to detect a set of units, called *peer units*, which are efficient with the inefficient unit's weights. The peer units are associated with the (strictly) positive basic multipliers λ_i, that is the non null dual variables. Therefore, for each inefficient unit j_0 it is possible to build a composite unit with outputs $\sum_{j=1}^{n} \lambda_j y_{rj}$ $(r = 1, 2, \ldots, t)$ and inputs $\sum_{j=1}^{n} \lambda_j x_{ij}$ $(i = 1, 2, \ldots, m)$ that outperforms unit j_0 and lies on the efficient frontier.

3. Mutual fund performance evaluation with investment costs

The DEA technique can be used in order to include the initial and final costs of the mutual fund investment in the performance analysis: it suffices to exploit the capability of DEA to consider many input variables. In particular, the first applications of the DEA methodology to the appraisal of mutual fund performance formulate a DEA model which provides a performance indicator that generalizes the traditional performance indexes by considering the expected return (or the excess return) as output and several input variables encompassing a risk measure and the investment costs.

The first attempt to apply the DEA methodology in order to obtain a mutual fund efficiency indicator is the DPEI index developed by Murthi, Choi and Desai (1997). The DPEI index considers the excess return as output and the standard deviation of the return and various transaction costs as inputs. Besides subscription and redemption fees, among the transaction costs DPEI considers operational expenses, management fees and purchase and sale costs incurred by the management, which are costs that have already been deduced from the net return; in the DPEI index these transaction costs are lumped together in the turnover, loads and expense ratio indicators.

A second DEA indicator for mutual fund performance is the I_{DEA-1} index proposed by Basso and Funari (2001), who include in the analysis only the investment costs which directly weigh on the investors, i.e. subscription and redemption fees, but not the operational expenses that have already been deducted from the fund quotations. More precisely, the I_{DEA-1} is defined as the maximum value of the objective function, computed with respect to the output weight u and the input weights v_i, $i = 1, 2, \dots, m$, and w_i, $i = 1, 2, \dots, k$, of the following fractional linear problem

$$\max_{\{u,v_i,w_i\}} \quad \frac{u o_{j_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{3.1}$$

subject to

$$\begin{aligned} & \frac{u o_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 && j = 1, 2, \dots, n \\ & u \geq \varepsilon \\ & v_i \geq \varepsilon && i = 1, 2, \dots, h \\ & w_i \geq \varepsilon && i = 1, 2, \dots, k, \end{aligned} \tag{3.2}$$

where the output variable o is either the expected excess return ($o_j = E(R_j) - \delta$) or the expected return ($o_j = E(R_j)$), while the input variables are made up by h risk measures $q_1, \dots, q_h$ and k investment costs $c_1, \dots, c_k$. Note that in this way we may take into account not only the subscription and redemption costs, but also several risk measures together. For example, we can measure the performance by considering the two risk measures σ_j and β_j, in order to include in the analysis both a risk measure suitable for one risky asset and another one valid when the investments are diversified; we could even include σ_j, $\sqrt{HV_j}$ and β_j together.

The I_{DEA-1} index is therefore defined as a ratio between an expected return indicator and a weighted sum of risk measures and investment costs. For example, with the choices $o_j = E(R_j) - \delta$, $q_{1j} = \sigma_j$, $q_{2j} = \beta_j$ and denoting by c_1, c_2 the per cent subscription and redemption costs, respectively, the performance index I_{DEA-1} for the mutual fund j_0 is computed as follows

$$I_{j_0,DEA-1} = \frac{u^*(E(R_{j_0}) - \delta)}{v_1^*\sigma_{j_0} + v_2^*\beta_{j_0} + w_1^*c_{1j_0} + w_2^*c_{2j_0}}, \tag{3.3}$$

where the asterisk marks the optimal weights found solving the optimization problem (3.1)-(3.2).

It can be proved (see Basso and Funari, 2001) that the I_{DEA-1} index actually generalizes the traditional Sharpe, Treynor and reward-to-half-variance indexes. For example, if we consider the standard deviation of the return as the only input, thus omitting the subscription and redemption costs, then the I_{DEA-1} efficiency index (3.3) turns out to be equal to the Sharpe ratio multiplied by a normalization constant which scales it off in the interval [0,1]

$$I_{j_0,DEA-1} = \frac{I_{j_0,Sharpe}}{\max_j \, I_{j,Sharpe}}. \tag{3.4}$$

By taking as risk measure $\sqrt{HV_j}$ or β_j, in place of σ_j, we obtain a scaling of the reward-to-half-variance and the Treynor indexes, respectively.

We have seen that the solution of the CCR dual model (2.5)-(2.6) allows to identify a set of peer units, which form a composite unit with outputs $\sum_{j=1}^n \lambda_j y_{rj}$ and inputs $\sum_{j=1}^n \lambda_j x_{ij}$ that outperforms unit j_0 and lies on the efficient frontier.

From a financial point of view, this composite unit can be considered as a benchmark portfolio for the inefficient fund j_0, with a similar input/output orientation. The normalized multipliers $\overline{\lambda}_j = \lambda_j / \sum_{k=1}^n \lambda_k$ indicate the relative composition of the benchmark portfolio.

4. Generalized DEA performance indicators for mutual funds

We have seen in Section 2 that DEA takes into account a multiple input-multiple output situation by computing a performance measure that is based on a virtual output/input ratio. The DEA methodology may be used as a multiple criteria decision tool in which the weights that allow to aggregate the objectives are not fixed in advance in a subjective manner but are determined as the most favourable weights for each unit; on the application of DEA as a discrete alternative multiple criteria decision making tool see for example Sarkis (2000).

In particular, for the appraisal of the mutual fund performance the DEA methodology could be used in order to include in the analysis not only many input variables, as the DPEI and I_{DEA-1} indexes do, but also different output variables, which may take account of different aspects of performance evaluation.

In this connection, let us recall that each traditional index (think, for example, to the Sharpe, reward to half-variance, Treynor or Jensen indexes) may be applicable under some conditions while under different conditions other indicators may be more appropriate.

As a matter of fact, every performance index may help to shed light on a particular aspect of the link between risk and return. Therefore, why not employ a DEA procedure to combine the information contained in all the main traditional indexes? With regard to that, we can formulate a DEA model with both multiple inputs and multiple outputs. As regards the input variables, we may safely keep the inputs defined for I_{DEA-1}, namely one or more risk measures and the subscription and redemption costs. The output variables, on the contrary, may be extended to include, besides an expected return indicator, a few traditional performance indexes.

In effect, there is still another point that it would be useful to take into account in the performance measurement, namely the presence of eventual dominance relations among the mutual funds analyzed.

Let us recall that the stochastic dominance criteria allow to get a partial ranking of random variables by assuming that investors act according to the expected utility paradigm, that is they prefer those alternatives that maximize their expected utility. The first order stochastic dominance relation between two random portfolios X and Y follows from the principle of non satiety and is defined as follows: X is said to dominate Y according to the first order stochastic dominance criterion if for all non decreasing utility functions U we have $E(U(X)) \geq E(U(Y))$ and there exists a non decreasing utility function U^* such that $E(U^*(X)) > E(U^*(Y))$.

The second order stochastic dominance between the random portfolios X and Y arises from the principles of non satiety and risk aversion; X is said to dominate Y according to the second order stochastic dominance criterion if for all non decreasing and concave utility functions U we have $E(U(X)) \geq E(U(Y))$ and there exists a non decreasing and concave utility function U^* such that $E(U^*(X)) > E(U^*(Y))$.

Another widely accepted stochastic dominance criterion concerns the assumption of decreasing absolute risk aversion (DARA) of the investors' utility functions; X is said to dominate Y according to the DARA dominance criterion if for all utility functions U satisfying the DARA assumption we have $E(U(X)) \geq E(U(Y))$ and there exists a DARA utility function U^* such that $E(U^*(X)) > E(U^*(Y))$.

It is clear that a highly desirable property for a mutual fund is that it is not dominated by other funds: only the non dominated portfolios can be considered efficient. On the other hand, it can well happen that, in an analysis of mutual fund returns during a long period, a fund turns out to be dominated in some years but not in other ones.

Hence, a stochastic dominance indicator which could be usefully included in the DEA mutual fund performance evaluation can be defined by determining in how many periods (over the overall number of periods considered in the analysis of the historical data) a fund is efficient according to a given stochastic dominance criterion.

Formally, let us assume to analyze the past returns of mutual fund j over a given period and let us divide this period into convenient subperiods (for example, years). Then let us define the stochastic dominance indicator d as the relative number of subperiods in which the fund is not dominated by any other fund according to a chosen stochastic dominance criterion

$$d_j = \frac{\text{number of non dominated subperiods for fund } j}{\text{total number of subperiods}}. \tag{4.1}$$

A generalized DEA performance indicator for mutual funds can be computed as the optimal value of the objective function of the following fractional linear programming problem with multiple inputs and outputs

$$\max_{\{u_r,\omega_r,v_i,w_i\}} \quad \frac{u_1 o_{j_0} + u_2 d_{j_0} + \sum_{r=1}^{p} \omega_r I_{rj_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{4.2}$$

subject to

$$\begin{aligned} &\frac{u_1 o_j + u_2 d_j + \sum_{r=1}^{p} \omega_r I_{rj}}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \le 1 \qquad && j = 1, 2, \ldots, n \\ &u_r \ge \varepsilon \qquad && r = 1, 2 \\ &\omega_r \ge \varepsilon \qquad && r = 1, 2, \ldots, p \\ &v_i \ge \varepsilon \qquad && i = 1, 2, \ldots, h \\ &w_i \ge \varepsilon \qquad && i = 1, 2, \ldots, k, \end{aligned} \tag{4.3}$$

where $I_1, 2, \ldots, I_p$ are the traditional performance indexes included in the DEA model and the maximum is calculated with respect to the output weights u_r, ω_r and the input weights v_i, w_i.

If we set $I_1 = I_{Sharpe}$, $I_2 = I_{half-var}$, $I_3 = I_{Treynor}$, $I_4 = I_{Jensen}$ we obtain the generalized DEA performance index

$$I_{j_0,DEA-g} = \Big(u_1^* o_{j_0} + u_2^* d_{j_0} + \omega_1^* I_{j_0,Sharpe} + \omega_2^* I_{j_0,half-var} + \omega_3^* I_{j_0,Treynor} + \\ + \omega_4^* I_{j_0,Jensen}\Big) \Big/ \left(\sum_{i=1}^{h} v_i^* q_{ij_0} + \sum_{i=1}^{k} w_i^* c_{ij_0}\right). \tag{4.4}$$

where the asterisk denotes the optimal weights.

A two outputs DEA performance measure for mutual funds which also employs the stochastic dominance indicator d_j is the I_{DEA-2} index proposed by Basso and Funari (2001). The I_{DEA-2} index can be considered as a particular case of the generalized DEA performance indicator for mutual funds computed solving the fractional problem (4.2)-(4.3), in which the traditional indicators are left out from the outputs ($p = 0$)

$$I_{j_0,DEA-2} = \frac{u_1^* o_{j_0} + u_2^* d_{j_0}}{\sum_{i=1}^{h} v_i^* q_{ij_0} + \sum_{i=1}^{k} w_i^* c_{ij_0}}, \tag{4.5}$$

where the (starred) optimal weights are found solving the DEA problem (4.2)-(4.3) with $p = 0$.

5. An empirical analysis on market data

We have tested the DEA performance indexes for mutual fund investments discussed in the previous sections on data of the Italian financial market. We have considered the weekly logarithmic returns of 50 mutual funds, for which homogeneous information were available; in addition, for comparison purposes we have examined the weekly logarithmic returns of the Milan Stock Exchange Mibtel index (closing price) and the weekly instantaneous rate of return of the 12 months Italian Treasury bill (B.O.T.). The data regard the Monday net prices in the period 9/6/1997 to 11/6/2001.

The mutual funds belong to different categories: 23 are equity funds, 18 are bond funds and 9 are balanced funds. The empirical analysis has been carried out separately for these three categories of funds.

In the DEA models, as first output variable we have employed the mean return (computed on the historical values analyzed) instead of the mean excess return, as would have been suggested by a direct generalization of the Sharpe index; this choice allows to limit the presence of negative values among the outputs.

With regard to that, it has to be pointed out that, in the time interval considered for the data, two bond funds (Bpb Tiepolo and Fideuram Security) exhibit a negative mean excess return; this entails that the values of the Sharpe, reward to half-variance and Treynor indexes for these funds are negative and, above all, meaningless. In fact, when the excess return is negative these indexes can be misleading, since in this case the index with the higher value is sometimes related to the worse return-to-risk ratio: if we consider two mutual funds having a same negative value of the excess return but different (positive) values of the risk measure (either σ_j, $\sqrt{HV_j}$ or β_j), the return-to-risk ratio has a higher value for the fund with the higher value of the risk indicator.

In order to compute the stochastic dominance indicator (4.1), we have divided the 4 years period taken into account in the empirical analysis into 4 one-year-long subperiods and have tested the DARA dominance relations among the funds in each category in each subperiod.

Among the inputs, we have considered two classic risk measures: the standard deviation of the returns σ_j, as a proper risk measure for the investors who hold only one risky asset, and the β coefficient, for the investors who have diversified their investments. As market portfolio in the calculation of β_j we have used the Mibtel index. As a matter of fact, the correlation between the standard deviation σ_j and the square root of the half-variance $\sqrt{HV_j}$ on the data considered is so high (the correlation coefficient is equal to 0.999) that it was unnecessary to take into account both of them.

In addition, we have considered 6 input variables related to the entrance and exit investment costs: the per cent subscription costs per different amounts of initial investment (5 000, 25 000 and 50 000 Euros) and the per cent redemption costs per year of disinvestment (after 1, 2 and 3 years).

To allow the comparison of the mutual fund performance with the behaviour of a market benchmark, we have included also the Mibtel index in the analysis of the equity and balanced funds, and the B.O.T. in the analysis of the bond and balanced ones.

Table 1 presents the values of the performance indexes obtained for the different categories of mutual funds analyzed with the different indicators discussed in the previous sections, namely the Sharpe, reward to half-variance, Treynor and Jensen indexes and the DEA performance indicators I_{DEA-1}, I_{DEA-2} and I_{DEA-g}. The values of the Sharpe, reward to half-variance and Treynor indexes have been normalized by dividing the values by the highest value, so that they fall in the interval $[0, 1]$ and are directly comparable with the values of the DEA indicators. A "—" sign takes the place of the (negative) meaningless values of Sharpe, reward to half-variance and Treynor indexes. Table 1 shows that there exist significant differences among the performance results obtained with the different indexes. We may observe that often, but not always, the performance scores provided by the DEA models are higher than the normalized values computed with the traditional indexes. This is not surprising, since the DEA idea is to look for the most favourable weights for each fund, thus maximizing the efficiency measure.

Table 2 displays the relative ranking of the mutual funds obtained with the various performance indexes applied. These rankings, too, indicate that the efficiency results may differ markedly according to the performance measure used.

In order to analyze the analogies and the differences of the performance results achieved with the different performance indexes, Table 3 shows the correlation matrix computed between the performance results presented in columns 2 to 8 of Table 1, separately for the three categories of funds. From this table we can see that the correlation among the performance results tends to be higher for the equity funds and lower for the bond and balanced funds; hence, for the bond and balanced funds the differences tend to be more significant.

6. An average cross efficiency measure

By carrying out a performance evaluation with a DEA model, we compare the mutual fund efficiency by allowing each fund to achieve the maximum efficiency ratio between a weighted sum of outputs and a weighted sum of inputs. For this purpose, the DEA performance ratio is computed by defining the weights so that the weighting criterion which puts the fund under the best possible light is applied.

Nevertheless, the assignment of the weights may sometimes be such that some inputs and outputs of secondary importance are given a high weight while other important variables are neglected by receiving a much lower weight. For example, from Table 1 we can see that two bond funds (Fideuram Security and Bpb Tiepolo) have a negative mean excess return but all the same the values of their DEA performance measures are fairly good (the first one is even considered efficient), since they have low costs and low risk measures.

Table 1. Values of the performance indexes obtained for the Italian mutual funds analyzed with the different indicators: Sharpe, reward to half-variance, Treynor and Jensen indexes, and the DEA performance indicators I_{DEA-1}, I_{DEA-2} and I_{DEA-g}. The analysis has been carried out separately for the different categories of funds (equity, bond and balanced funds). The values of the Sharpe, reward to half-variance and Treynor indexes have been normalized by dividing the values by the highest value.

Fund	I_{Sharpe}	$I_{half-var}$	$I_{Treynor}$	I_{Jensen}	I_{DEA-1}	I_{DEA-2}	I_{DEA-g}
Equity funds							
Arca 27 az. estere	0.387	0.389	0.616	-0.0001	0.957	1.000	1.000
Azimut Borse Int.	0.507	0.499	0.693	0.0000	0.845	0.845	0.890
Centrale Global	0.321	0.310	0.427	-0.0005	0.642	0.642	0.670
Epta-International	0.200	0.194	0.268	-0.0010	0.473	0.473	0.473
Fideuram Azione	0.415	0.406	0.603	-0.0002	0.773	0.773	0.829
Fondicri Int.	0.460	0.453	0.686	0.0000	0.891	0.904	1.000
Comit Int.	0.392	0.380	0.586	-0.0002	0.801	0.801	0.879
Investire Int.	0.288	0.282	0.395	-0.0006	0.633	0.633	0.638
Prime Global	0.348	0.339	0.513	-0.0004	0.692	0.695	0.709
Sanpaolo Internazion.	0.351	0.347	0.467	-0.0006	0.624	0.624	0.637
Centrale Italia	0.843	0.840	0.854	0.0006	0.878	0.878	0.878
Epta Azioni Italia	0.729	0.732	0.734	0.0002	0.812	0.812	0.812
Fondicri Sel. Italia	1.000	1.000	1.000	0.0012	1.000	1.000	1.000
Comit Azioni Italia	0.917	0.901	0.917	0.0008	1.000	1.000	1.000
Gesticredit Borsit	0.867	0.876	0.866	0.0007	0.893	0.893	0.902
Imi Italy	0.833	0.821	0.820	0.0005	0.857	0.857	0.857
Investire Azion.	0.939	0.935	0.933	0.0009	1.000	1.000	1.000
Oasi Azionario Italia	0.861	0.858	0.857	0.0006	1.000	1.000	1.000
Azimut Europa	0.699	0.695	0.813	0.0004	0.908	0.908	0.913
Gesticredit Euro Az.	0.416	0.414	0.471	-0.0006	0.613	0.622	0.623
Imi Europe	0.529	0.519	0.618	-0.0002	0.734	0.734	0.758
Investire Europa	0.247	0.245	0.282	-0.0011	0.485	0.485	0.485
Sanpaolo Europe	0.376	0.386	0.448	-0.0007	0.585	0.595	0.595
Indice Mibtel	0.736	0.718	0.683	0.0000	0.821	0.821	0.821
Bond funds							
Arca Bond	0.964	0.945	0.325	0.0006	1.000	1.000	1.000
Azimut Rend. Int.	0.482	0.490	0.193	0.0004	0.205	0.205	0.441
Epta 92	0.609	0.587	0.163	0.0004	0.223	0.223	0.594
Comit Obbligaz. Estero	0.383	0.361	0.102	0.0002	0.545	0.545	0.545
Imi Bond	0.713	0.656	0.383	0.0006	0.192	0.192	0.724
Investire global Bond	0.996	0.969	0.225	0.0006	0.797	0.797	1.000
Oasi Bond Risk	1.000	1.000	0.498	0.0008	1.000	1.000	1.000
Primebond Int.	0.545	0.522	0.203	0.0003	0.304	0.304	0.546
Sanpaolo Bonds	0.474	0.449	0.148	0.0003	0.131	0.131	0.475
Fondicri Primary Bond	0.430	0.404	0.131	0.0002	0.243	0.243	0.453
Centrale Money	0.150	0.149	0.046	0.0001	0.107	0.107	0.107
Eurom. Int. Bond	0.694	0.669	0.243	0.0004	0.226	0.226	0.765
Bpb Tiepolo	—	—	—	-0.0001	0.869	0.901	0.901
IAM Obiettivo Reddito	0.488	0.448	0.315	0.0002	0.172	0.172	1.000
Eptabond	0.383	0.354	0.205	0.0001	0.392	0.392	1.000
Fideuram Security	—	—	—	-0.0001	1.000	1.000	1.000
Oasi Euro Risk	0.631	0.665	1.000	0.0005	1.000	1.000	1.000
Prime Bond Euro	0.041	0.038	0.029	0.0000	0.247	0.247	0.247
B.O.T.	0.000	0.000	0.000	0.0000	1.000	1.000	1.000
Balanced funds							
Arca BB	0.774	0.783	0.650	0.0004	1.000	1.000	1.000
Azimut Bil.	0.658	0.672	0.551	0.0002	0.340	0.340	0.701
Eptacapital	0.552	0.582	0.469	-0.0001	0.310	0.310	0.588
Comit Bil.	0.807	0.810	0.678	0.0005	0.353	0.353	0.998
Investire Bil.	0.714	0.730	0.594	0.0003	0.849	0.849	0.868
Arca TE	0.603	0.634	0.797	0.0004	1.000	1.000	1.000
Fideuram Performance	1.000	1.000	1.000	0.0013	1.000	1.000	1.000
Fondo Centrale	0.391	0.390	0.411	-0.0002	0.190	0.190	0.513
Comit Espansione	0.065	0.065	0.084	-0.0007	0.124	0.124	0.124
Indice Mibtel	0.656	0.657	0.490	0.0000	0.539	0.539	0.539
B.O.T.	0.000	0.000	0.000	0.0000	1.000	1.000	1.000

Table 2. Relative ranking of the Italian mutual funds analyzed according to the following performance indexes: Sharpe, reward to half-variance, Treynor and Jensen indexes, and the DEA performance indicators I_{DEA-1}, I_{DEA-2} and I_{DEA-g}. The analysis has been carried out separately for the different categories of funds (equity, bond and balanced funds).

Fund	I_{Sharpe}	$I_{half-var}$	$I_{Treynor}$	I_{Jensen}	I_{DEA-1}	I_{DEA-2}	I_{DEA-g}
Equity funds							
Arca 27 az. estere	17	16	14	13	5	1	1
Azimut Borse Int.	12	12	10	10	11	11	9
Centrale Global	21	21	21	18	18	18	18
Epta-International	24	24	24	23	24	24	24
Fideuram Azione	15	15	15	14	15	15	13
Fondicri Int.	13	13	11	11	8	7	1
Comit Int.	16	18	16	16	14	14	10
Investire Int.	22	22	22	21	19	19	19
Prime Global	20	20	17	17	17	17	17
Sanpaolo Internazion.	19	19	19	19	20	20	20
Centrale Italia	6	6	6	5	9	9	11
Epta Azioni Italia	9	8	9	9	13	13	15
Fondicri Sel. Italia	1	1	1	1	1	1	1
Comit Azioni Italia	3	3	3	3	1	1	1
Gesticredit Borsit	4	4	4	4	7	8	8
Imi Italy	7	7	7	7	10	10	12
Investire Azion.	2	2	2	2	1	1	1
Oasi Azionario Italia	5	5	5	6	1	1	1
Azimut Europa	10	10	8	8	6	6	7
Gesticredit Euro Az.	14	14	18	20	21	21	21
Imi Europe	11	11	13	15	16	16	16
Investire Europa	23	23	23	24	23	23	23
Sanpaolo Europe	18	17	20	22	22	22	22
Indice Mibtel	8	9	12	12	12	12	14
Bond funds							
Arca Bond	3	3	4	3	1	1	1
Azimut Rend. Int.	10	9	10	7	15	15	17
Epta 92	7	7	11	8	14	14	12
Comit Obbligaz. Estero	13	13	14	12	8	8	14
Imi Bond	4	6	3	2	16	16	11
Investire global Bond	2	2	7	4	7	7	1
Oasi Bond Risk	1	1	2	1	1	1	1
Primebond Int.	8	8	9	9	10	10	13
Sanpaolo Bonds	11	10	12	10	18	18	15
Fondicri Primary Bond	12	12	13	11	12	12	16
Centrale Money	15	15	15	15	19	19	19
Eurom. Int. Bond	5	4	6	6	13	13	10
Bpb Tiepolo	—	—	—	19	6	6	9
IAM Obiettivo Reddito	9	11	5	13	17	17	1
Eptabond	14	14	8	14	9	9	1
Fideuram Security	—	—	—	18	1	1	1
Oasi Euro Risk	6	5	1	5	1	1	1
Prime Bond Euro	16	16	16	17	11	11	18
B.O.T.	17	17	17	16	1	1	1
Balanced funds							
Arca BB	3	3	4	4	1	1	1
Azimut Bil.	5	5	6	6	8	8	7
Eptacapital	8	8	8	9	9	9	8
Comit Bil.	2	2	3	2	7	7	5
Investire Bil.	4	4	5	5	5	5	6
Arca TE	7	7	2	3	1	1	1
Fideuram Performance	1	1	1	1	1	1	1
Fondo Centrale	9	9	9	10	10	10	10
Comit Espansione	10	10	10	11	11	11	11
Indice Mibtel	6	6	7	7	6	6	9
B.O.T.	11	11	11	8	1	1	1

Table 3. Correlation coefficients between the performance measures obtained for the Italian mutual funds analyzed with the following performance indexes: Sharpe, reward to half-variance, Treynor and Jensen indexes, and the DEA performance indicators I_{DEA-1}, I_{DEA-2} and I_{DEA-g}. The analysis has been carried out separately for the different categories of funds (equity, bond and balanced funds).

	I_{Sharpe}	$I_{half-var}$	$I_{Treynor}$	I_{Jensen}	I_{DEA-1}	I_{DEA-2}	I_{DEA-g}
Equity funds							
I_{Sharpe}	1.000	1.000	0.951	0.956	0.820	0.798	0.734
$I_{half-var}$		1.000	0.951	0.956	0.818	0.797	0.731
$I_{Treynor}$			1.000	0.993	0.937	0.922	0.889
I_{Jensen}				1.000	0.924	0.908	0.870
I_{DEA-1}					1.000	0.998	0.986
I_{DEA-2}						1.000	0.987
I_{DEA-g}							1.000
Bond funds							
I_{Sharpe}	1.000	0.998	0.533	0.932	0.361	0.361	0.524
$I_{half-var}$		1.000	0.565	0.940	0.397	0.397	0.526
$I_{Treynor}$			1.000	0.605	0.455	0.455	0.518
I_{Jensen}				1.000	0.100	0.092	0.208
I_{DEA-1}					1.000	1.000	0.716
I_{DEA-2}						1.000	0.716
I_{DEA-g}							1.000
Balanced funds							
I_{Sharpe}	1.000	0.999	0.940	0.799	0.278	0.278	0.494
$I_{half-var}$		1.000	0.944	0.794	0.282	0.282	0.498
$I_{Treynor}$			1.000	0.845	0.370	0.370	0.538
I_{Jensen}				1.000	0.621	0.621	0.780
I_{DEA-1}					1.000	1.000	0.793
I_{DEA-2}						1.000	0.793
I_{DEA-g}							1.000

Table 4. Cross efficiency matrix for the Italian balanced funds analyzed, computed with the optimal weights of the generalized DEA performance indicator I_{DEA-g}.

Funds	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11
F1 : Arca BB	1.00	0.33	0.30	0.35	0.79	1.00	1.00	0.19	0.10	0.46	1.00
F2 : Azimut Bil.	1.00	0.70	0.59	0.91	0.85	0.94	0.96	0.48	0.08	0.35	1.00
F3 : Eptacapital	1.00	0.70	0.59	0.91	0.85	1.00	0.99	0.50	0.09	0.34	1.00
F4 : Comit Bil.	1.00	0.60	0.39	1.00	0.55	1.00	1.00	0.34	0.06	0.23	1.00
F5 : Investire Bil.	1.00	0.52	0.43	0.49	0.87	0.93	1.00	0.27	0.08	0.40	1.00
F6 : Arca TE	1.00	0.30	0.25	0.21	0.78	1.00	1.00	0.13	0.06	0.46	1.00
F7 : Fideuram Performance	0.52	0.20	0.00	0.61	0.22	1.00	1.00	0.00	0.00	0.00	1.00
F8 : Fondo Centrale	1.00	0.69	0.56	0.92	0.83	1.00	1.00	0.51	0.09	0.34	1.00
F9 : Comit Espansione	1.00	0.34	0.31	0.35	0.78	1.00	1.00	0.19	0.12	0.52	1.00
F10: Indice Mibtel	1.00	0.34	0.31	0.35	0.82	0.90	1.00	0.19	0.12	0.54	1.00
F11: B.O.T.	1.00	0.30	0.25	0.21	0.79	0.96	1.00	0.13	0.06	0.47	1.00

Table 5. Minimum, maximum and average scores achieved by each fund in the cross efficiency matrix computed with the optimal weights of the generalized DEA performance indicator I_{DEA-g} (the maximum score gives the I_{DEA-g} index). The last two columns (in italics) report the relative ranking obtained with I_{DEA-g} and the average cross efficiency measure.

Fund	minimum score	I_{DEA-g} max. eff. score	average cross eff. score	I_{DEA-g} ranking	average cross eff. ranking
Equity funds					
Arca 27 az. estere	0.504	1.000	0.850	*1*	*7*
Azimut Borse Int.	0.546	0.890	0.779	*9*	*11*
Centrale Global	0.416	0.670	0.577	*18*	*19*
Epta-International	0.217	0.473	0.415	*24*	*24*
Fideuram Azione	0.470	0.829	0.687	*13*	*15*
Fondicri Int.	0.574	1.000	0.832	*1*	*9*
Comit Int.	0.494	0.879	0.728	*10*	*13*
Investire Int.	0.343	0.638	0.561	*19*	*21*
Prime Global	0.350	0.709	0.624	*17*	*17*
Sanpaolo Internazion.	0.381	0.637	0.564	*20*	*20*
Centrale Italia	0.831	0.878	0.860	*11*	*6*
Epta Azioni Italia	0.679	0.812	0.765	*15*	*12*
Fondicri Sel. Italia	0.997	1.000	1.000	*1*	*1*
Comit Azioni Italia	0.949	1.000	0.987	*1*	*2*
Gesticredit Borsit	0.844	0.902	0.886	*8*	*5*
Imi Italy	0.762	0.857	0.815	*12*	*10*
Investire Azion.	0.858	1.000	0.965	*1*	*3*
Oasi Azionario Italia	0.837	1.000	0.928	*1*	*4*
Azimut Europa	0.670	0.913	0.843	*7*	*8*
Gesticredit Euro Az.	0.499	0.623	0.590	*21*	*18*
Imi Europe	0.572	0.758	0.682	*16*	*16*
Investire Europa	0.298	0.485	0.438	*23*	*23*
Sanpaolo Europe	0.427	0.595	0.547	*22*	*22*
Indice Mibtel	0.623	0.821	0.715	*14*	*14*
Bond funds					
Arca Bond	0.657	1.000	0.968	*1*	*2*
Azimut Rend. Int.	0.081	0.441	0.291	*17*	*16*
Epta 92	0.055	0.594	0.373	*12*	*12*
Comit Obbligaz. Estero	0.189	0.545	0.396	*14*	*10*
Imi Bond	0.063	0.724	0.386	*11*	*11*
Investire global Bond	0.480	1.000	0.836	*1*	*4*
Oasi Bond Risk	0.732	1.000	0.907	*1*	*3*
Primebond Int.	0.057	0.546	0.369	*13*	*13*
Sanpaolo Bonds	0.029	0.475	0.277	*15*	*17*
Fondicri Primary Bond	0.052	0.453	0.299	*16*	*15*
Centrale Money	0.017	0.107	0.051	*19*	*19*
Eurom. Int. Bond	0.077	0.765	0.457	*10*	*9*
Bpb Tiepolo	0.007	0.901	0.327	*9*	*14*
IAM Obiettivo Reddito	0.050	1.000	0.552	*1*	*8*
Eptabond	0.201	1.000	0.620	*1*	*6*
Fideuram Security	0.052	1.000	0.569	*1*	*7*
Oasi Euro Risk	0.594	1.000	0.784	*1*	*5*
Prime Bond Euro	0.038	0.247	0.117	*18*	*18*
B.O.T.	0.435	1.000	0.970	*1*	*1*
Balanced funds					
Arca BB	0.521	1.000	0.956	*1*	*4*
Azimut Bil.	0.199	0.701	0.457	*7*	*7*
Eptacapital	0.001	0.588	0.362	*8*	*9*
Comit Bil.	0.210	0.998	0.575	*5*	*6*
Investire Bil.	0.217	0.868	0.738	*6*	*5*
Arca TE	0.901	1.000	0.975	*1*	*3*
Fideuram Performance	0.962	1.000	0.995	*1*	*2*
Fondo Centrale	0.001	0.513	0.265	*10*	*10*
Comit Espansione	0.000	0.124	0.079	*11*	*11*
Indice Mibtel	0.001	0.539	0.374	*9*	*8*
B.O.T.	1.000	1.000	1.000	*1*	*1*

In order to overcome this drawback, it is possible to define the so called *cross efficiency matrix*, which provides information on the efficiency of each mutual funds computed with the optimal weights of all other funds. The element in row i and column j of the cross efficiency matrix represents the efficiency ratio of mutual fund j obtained with the weights that are optimal for alternative i (see e.g. Sarkis, 2000).

Table 4 shows the cross efficiency matrix for the Italian balanced funds analyzed, computed with the optimal weights of the generalized DEA performance indicator I_{DEA-g}. We may observe, for example, that fund Comit Bilanciato is efficient according to the generalized DEA performance indicator I_{DEA-g} (it has an efficiency score equal to 1 if its optimal weights are used), whereas its efficient score is equal to 0.35 if the weights optimal for fund Arca BB are used.

The average of the elements in column j of the cross efficiency matrix gives the average of the cross efficiencies of fund j obtained with the optimal weighting criteria of all funds. This average cross efficiency measure provides a measure of the appraisal of fund j from the point of view of the styles of all the funds under consideration. In particular, this measure may help to discriminate among efficient funds but, above all, it will take into account ratings obtained with different weights. Of course, the average cannot be greater than the maximum, so the average cross efficiency measure will be lower than the DEA measure: therefore, the efficiency measure will not be so favourable.

Table 5 reports the minimum and maximum values of the scores achieved by each fund in the cross efficiency matrix computed with the optimal weights of the generalized DEA performance indicator I_{DEA-g}; of course, the maximum efficiency for mutual fund j will be given by the entry jj of the matrix and represents the DEA index. The last two columns (in italics) report the relative ranking obtained with I_{DEA-g} and the average cross efficiencies performance measures, respectively. As can be seen from Table 5, the average cross efficiency measure lowers the efficiency score of most funds, by averaging on the results obtained with different choices of the weights; since this is not done in the same ratio for all funds, the relative ranking of the funds changes.

The DEA literature has pointed out a problem in determining the cross efficiency matrix: actually, the optimal weights computed looking for the DEA efficiency measure, and used to calculate the cross efficiencies, may not be unique; this does not affect the DEA measure but may influence the cross efficiency values. This problem can be overcome by defining a model which chooses the optimal weights that satisfy a secondary goal; on this subject, see Doyle and Green (1994) and Sarkis (2000).

7. Concluding remarks

In this contribution we propose a multiple inputs-multiple outputs DEA model to evaluate the performance of mutual funds. This model takes into account different risk measures and the investment costs, on the input side, and the expected return as well as the value of the traditional performance indexes, on the output side. In this way the resulting performance measure, the generalized DEA performance index, can take account of various points of view together which may be significant in the performance measurement process.

In addition, an average cross efficiency measure is presented which tries to consider different weights and then averages the resulting efficiency score.

The methods proposed, together with other DEA performance indicators for mutual fund appraisal, is tested on empirical data of the Italian financial market.

As a suggestion for future research, the average cross efficiency measure could be improved by implementing some of the methods suggested in the DEA literature to choose among the optimal weights the ones that achieve a secondary goal.

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