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Abstract

This paper introduces a generalisation of the dynamic conditional correlation (DCC) multivariate GARCH model proposed by Engle (2002). In the multivariate GARCH literature one of the most relevant problems is represented by the elevate number of parameters. In order to solve this difficulty Bollerslev (1990) suggested to keep constant the correlations (he suggested the Constant Conditional Correlation model, CCC). Engle added to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constrained to be equal for all the correlations. In our view, this is an unnecessary restriction. In fact, we cannot impose that the correlations of, say, European sectorial stock indexes are identical to the correspondent US ones. We extend the DCC model introducing a block-diagonal structure that solves this problem. The dynamic is constrained to be equal only among groups of variables. Some possible applications are presented.

Keywords: Multivariate GARCH, Dynamic Correlation, Volatility, Asset Allocation, Risk Management.

1 Introduction

In today's global and highly volatile markets the efficient measurement and management of market risk has become a critical factor for the competitive-

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ness and even survival of financial institutions. One of the inputs required by risk managers, seeking to hold efficient portfolios, is the correlation between the securities to be included in the portfolio. Until recently, correlation was assumed to be constant and stable over time. However, all empirical studies that attempted to verify this finding, have failed to confirm the validity of this assumption. In fact, most experienced practitioners would attest that correlations increase in periods of high volatility and that both the magnitude and persistence of correlation is affected by volatility.

The asset allocation decision entails, inter alia, an assessment of the risks and returns of the various assets in the opportunity set. Optimal portfolio choice requires a forecast of the covariance matrix of the returns. Similarly, the calculation of the standard deviation of today's portfolio requires a covariance matrix of all the assets in the portfolio. For actual portfolios, with thousands of derivative and synthetic instruments, these functions require estimation and forecasting of very large covariance matrices.

Over the past 20 years, a tremendous literature has been developed where the dynamics of the covariance of assets has been explored, although the primarily focus has been on univariate volatilities and not on correlations (or covariances). In fact, in the multivariate GARCH literature one of the most relevant problems is represented by the elevate number of parameters. In order to solve this difficulty Bollerslev (1990) suggested to keep constant the correlations and suggested the Constant Conditional Correlation model (CCC). Only recently Engle (2002) proposes a new class of estimators that both preserves the ease of estimation of the Bollerslev's constant correlation model but allows the correlations to change over time. Engle adds to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constrained to be equal for all the correlations. In our view, this is an unnecessary restriction. In fact, we cannot impose that the correlations of, say, European sectorial stock indexes are identical to the correspondent US ones. We thus extend the DCC model introducing a block-diagonal structure that solves this problem. The dynamic is constrained to be equal only among groups of variables.

This block dynamic representation will also be useful in other fields, for example to investigate wether the formation of the EMU in Europe has increased the correlation among national assets and in more general terms to analyse the interdependence and contagion issues.

The outline of the paper is as follows. We start by surveying the multivariate GARCH models in Section 2. In Section 3 we discuss estimation and testing of the Block Dynamic Conditional Correlation Multivariate GARCH model. In Section 4 we describe an asset allocation problem, in which we analyse sectorial stock indexes. Section 5 concludes.

2 Multivariate GARCH models

We consider a n-dimensional process X_t, define by $I_t(X)$ the information set of X at time t and assume that:

$$X_t | I_{t-1}(X) \,\tilde{}\, iid\ D(\boldsymbol{\mu}_t, H_t) \tag{1}$$

where $D(\mu_t, H_t)$ is a non-specified multivariate distribution with time dependent mean μ_t and time dependent variance covariance matrix H_t. This formulation nests all multivariate GARCH representations that will be introduced in few steps, and allows also the specification of a multivariate ARMA process for the mean, as well as in-mean effects of the variances.

In the following, vector variables and matrices are denoted by bold or uppercase letters, while scalar are denoted by lowercase letters. The $Vech(\cdot)$ matrix operator will also be used: it stacks the lower triangular portion of a matrix. In addition, the residuals are defined as $E_t = X_t - \boldsymbol{\mu}_t$.

In a general framework, a multivariate GARCH process can be represented as

$$H_t = \boldsymbol{\omega} + C(L) H_t + D(L)[E_t E_t'] \tag{2}$$

$$C(L) = \sum_{i=1}^{p} C_i L^i \qquad D(L) = \sum_{i=1}^{q} D_i L^i \tag{3}$$

Given the symmetry of the variance covariance matrix and of the cross product of the mean residuals, this relation, even if useful for theoretical purposes, turns out to contain redundant relations. This observation, together with the need of a formulation that couples flexibility, limited number of parameters and easiness to be handled with software packages (consider the restrictions to be impose: positivity of conditional variances; invertibility of the variance-covariance matrix; stationarity of the process) leaded to specifications of different multivariate GARCH models. The next section presents some of the most known specifications: the Vech and BEKK multivariate GARCH of Engle and Kroner (1995), the CCC and DCC representations of Bollerslev (1990) and Engle and Sheppard (2001), together with a new extension of the DCC representation that modifies the dynamic of the correlations.

2.1 The Vech representations

These formulations derive from the work of Engle and Kroner (1995). The multivariate GARCH is represented as

$$Vech(H_t) = Vech(\boldsymbol{\omega}) + C(L) Vech(H_{t-1}) + D(L) Vech(E_t E_t') \tag{4}$$

where $\boldsymbol{\omega}$ is a positive definite matrix of dimension $n \times n$, $C(L) = \sum_{i=1}^{p} C_i L^i$, $D(L) = \sum_{i=1}^{q} D_i L^i$, C_i and D_i, are square matrices of dimension $r \times r$ and $r = n(n+1)/2$. The parameters in this formulation are $r \times [1 + r(p+q)]$ this is the main constraint on the estimation and application of this model. For example let we consider the bivariate model: the parameters will be 21. However, in this case Engle and Kroner (1995) show that the Vech-GARCH is stationary if and only if all the eigenvalues of $C(1) + D(1)$ are less than one in modulus.

A restricted case of the Vech-GARCH is its diagonal representation, the D-Vech-GARCH: it is defined diagonal as it assumes that all parameter matrices are diagonal. This boils down to a model that parameterizes all conditional variances and covariances as univariate GARCH(p,q) processes. The total number of parameters reduces to $3r$, for the bivariate case 9.

A much greater problem, that is not solved by restricting to the diagonal version, is the positive definiteness of the H_t matrix, which is very difficult to check and imposes controls in the optimization routines.

2.2 The BEKK representation

This formulation was suggested by Baba, Engle, Kraft and Kroner in a preliminary version of Engle and Kroner (1995). The main feature is that it does not need any restriction on parameters to get positive definiteness of the H_t matrix, given its quadratic structure. The fundamental BEKK-GARCH model is

$$H_t = \boldsymbol{\omega} + \sum_{i=1}^{p} C_i H_{t-i} C_i' + \sum_{j=1}^{q} D_j E_{t-j} E_{t-j}' D_j' \qquad (5)$$

where C_i and D_j are $n \times n$ matrices and $\boldsymbol{\omega}$ is a symmetric positive definite $n \times x$ matrix. In a general formulation the number of parameters are here $r + n \times n \times (p+q)$, for the bivariate case the number drops to only 11. Positive definiteness of the variance covariance matrix is controlled by the constant matrix $\boldsymbol{\omega}$, whose positive definiteness is often obtained assuming the factorization $\boldsymbol{\omega} = \boldsymbol{\Gamma\Gamma}'$, where Γ is a lower triangular matrix. The BEKK formulation and the Vech are strictly related as shown in Engle and Kroner (1995), in particular the stationarity condition of the BEKK model is very similar to the one of the Vech representation (for further details refer to the cited paper).

2.3 The Constant Conditional Correlation GARCH

This form was introduced by Bollerslev (1990) who tried to reduce the number of parameters of the Vech representations. He suggested the following structure:

$$\begin{aligned} H_t &= \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} & \cdots & \sigma_{1n,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 & & \sigma_{2n,t} \\ \vdots & & \ddots & \vdots \\ \sigma_{1n,t} & \sigma_{2n,t} & \cdots & \sigma_{n,t}^2 \end{bmatrix} \\ \sigma_{i,t}^2 &= \omega_i + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 \qquad i = 1...n \\ \sigma_{ij,t} &= \rho_{ij}\sigma_{i,t}\sigma_{j,t} \qquad i,j = 1...n\ ,\ i \neq j \end{aligned} \tag{6}$$

The main difference between this formulation and the previous one is in the assumption of constant correlation among variables. The total number of parameters is now $(p + q + 1)n + n(n + 1)/2$, i.e. 7 in the bivariate case. Positive definiteness of the variance covariance matrix is now controlled by the correlation matrix (for the conditional variances the usual GARCH constraints for positivity are required), since we can rewrite

$$H_t = diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n} \\ \rho_{1,2} & 1 & & \vdots \\ \vdots & & \ddots & \rho_{1,n-1} \\ \rho_{1,n} & \cdots & \rho_{1,n-1} & 1 \end{bmatrix} diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) \tag{7}$$

where $diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right)$ represents a diagonal matrix with the given elements. Moreover, the correlation matrix can be factorized similarly to the constant of the BEKK-GARCH to impose its positive definiteness and to ensure that it is a correlation matrix. It can be factorized as follows:

$$R = diag(\sqrt{\gamma_{1,1}}, \sqrt{\gamma_{2,2}}...\sqrt{\gamma_{n,n}})\mathbf{\Gamma\Gamma}' diag(\sqrt{\gamma_{1,1}}, \sqrt{\gamma_{2,2}}...\sqrt{\gamma_{n,n}})$$

where the internal matrices ensure positive definiteness, while the external ones ensure the boundness of correlations and the unit value of the main diagonal of R.

2.4 The Dynamic Conditional Correlation formulations

This alternative representation try to add some limited dynamics to the CCC-GARCH and has been introduced in recent papers by Engle and Sheppard

(2001) and Engle (2002). The idea behind this GARCH model derive from the CCC representation. The following factorization of the H_t matrix is assumed:

$$H_t = diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) R_t diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) \tag{8}$$

where the conditional variances are parameterized as in the CCC-GARCH and R_t is a dynamic correlation matrix satisfying

$$\begin{aligned}
R_t &= \left(Q_t^*\right)^{-1} Q_t \left(Q_t^*\right)^{-1} && (9)\\
Q_t &= \left[1-\alpha(1)-\beta(1)\right]\bar{Q} + \alpha(L)\,\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}' + \beta(L)\,Q_{t-1} && (10)\\
\boldsymbol{\varepsilon}_t &= E_t' diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right)\\
\bar{Q} &= T^{-1}\sum_{i=1}^{T}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' && (11)\\
\alpha(L) &= \sum_{i=1}^{\overline{q}}\alpha_i L^i \qquad \beta(L) = \sum_{i=1}^{\overline{p}}\beta_i L^i\\
Q_t^* &= diag(\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, ...\sqrt{q_{nn,t}})
\end{aligned}$$

that is just a particular kind of multivariate GARCH on the correlations. The Q_t^* diagonal matrix is introduced to ensure that R_t is a correlation matrix, while $\boldsymbol{\varepsilon}_t$ represents the vectors of standardised residuals of the univariate GARCH models. Positive definiteness of the DCC-GARCH is controlled by the correlation function and depends on a set of restrictions, namely the same positivity restriction of the univariate GARCH(p,q), $\alpha(1)+\beta(1)<1$. A final word on the matrix $\bar{Q}$: in Engle and Sheppard (2001) this matrix is defined as the unconditional covariance matrix of the standardized residuals, a definition in line with standard univariate GARCH result. In fact, it is well known that in a GARCH(p,q) the constant can be factorized as follows:

$$\omega = \left[1-\alpha(1)-\beta(1)\right]\sigma^2$$

i.e. the product of the characteristic polynome and of the unconditional variance. It should be therefore noted that the dynamic structure can be thought as a dynamic structure on the covariances of the standardized residuals. A deeper analysis of this point can be found in Engle (2002).

In this model the number of parameters is $(p+q+1)\times n + (\overline{p}+\overline{q})$, and in the bivariate case they are 8 if $\overline{p}=\overline{q}=1$ (the simplification introduced by the model are useful with larger scale models).

Sheppard (2002) provides a direct extension of this model by introducing asymmetry in the correlation dynamics and modifying the correlation equation in the following one:

$$Q_t = \left(\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G\right) + A'\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'A + B'Q_{t-1}B + G'\eta_{t-1}\eta_{t-1}'G$$

where $\eta_t = I(\boldsymbol{\varepsilon}_t < 0) \circ \boldsymbol{\varepsilon}_t$, $\circ$ being the Hadamar product (element by element), A, B, G are diagonal parameter matrices, $\bar{Q}$ is again the sample covariance matrix of the standardized residuals and $\bar{N}$ is the sample covariance matrix of η_t. This model adds flexibility to the previous one, however the number of parameters greatly increases.

2.5 Block-Dynamic Conditional Correlation

An additional and natural extension to this model is to allow for a block structure on the GARCH dynamic of the correlations, in fact the structure proposed by Engle and Sheppard (2001) presumes that all the correlations follow the same dynamics. This may not be the case: consider for example a stock market, with the assets grouped in homogeneous categories (energy, food, chemistry...) or think to a model for geographical areas, we may assume different patterns of correlation inside the groups and between the groups.

Such consideration is on the basis of this extension of the DCC-GARCH. We therefore introduce the Block-DCC-GARCH by reformulating the dynamic correlation equation in the following way:

$$\begin{aligned} Q_t &= [1 - \alpha(L) - \beta(L)] \odot \bar{Q} + \alpha(L) \odot \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta(L) \odot Q_t \\ \alpha(L) &= \sum_{i=1}^{\bar{q}} \boldsymbol{\alpha}_i L^i \qquad \beta(L) = \sum_{i=1}^{\bar{p}} \boldsymbol{\beta}_i L^i \end{aligned}$$

where $\boldsymbol{\alpha}_i$, $\boldsymbol{\beta}_i$ are square full matrices and $\odot$ indicates the Hadamar product. All matrices are of dimension $n \times n$ with the following structure: if we group the n variables in w sets of dimension $m_1, m_2...m_w$ and we indicate with $i(y)$ a column vector of ones if dimension y, then

$$\boldsymbol{\alpha}_i = \begin{bmatrix} \alpha_{i,11} i(m_1) i(m_1)' & \alpha_{i,12} i(m_1) i(m_2)' & \cdots & \alpha_{i,w1} i(m_1) i(m_w)' \\ \alpha_{i,12} i(m_2) i(m_1)' & \alpha_{i,22} i(m_2) i(m_2)' & & \alpha_{i,w2} i(m_2) i(m_w)' \\ \vdots & & \ddots & \vdots \\ \alpha_{i,w1} i(m_w) i(m_1)' & \alpha_{i,w2} i(m_w) i(m_2)' & \cdots & \alpha_{i,ww} i(m_w) i(m_w)' \end{bmatrix} \tag{12}$$

and in a similar way for $\boldsymbol{\beta}_i$. It is worth noting that the number of sets w and their dimensions $m_1, m_2...m_w$ must be constant between the $\boldsymbol{\alpha}_i$, and $\boldsymbol{\beta}_i$. Clearly, to be competitive this representation requires a small number of groups. In this new model the number of parameters are $(p + q + 1) \times n +$

$(\overline{p}+\overline{q}) \times w(w-1)/2$, and it is evidently not useful in the bivariate case. The Block-DCC model provides also a positive definite variance-covariance matrix H_t, since the Proposition 2 of Engle and Sheppard (2001) is still valid.

3 Estimation and testing

The estimation of the dynamic correlation models can be carried out by Quasi-Maximum Likelihood, following the approach suggested by Engle and Sheppard (2001). Let define by θ_1 the parameters of the univariate GARCH models and with θ_2 the parameters of the dynamic correlation structure. The likelihood of the model can be written as follows:

$$LogL(\theta_1,\theta_2|X_t) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi)+\log(|H_t|)+\mathbf{X}_t H_t^{-1}\mathbf{X}_t'\right]$$

or, exploiting the factorization of the variance-covariance matrix and defining $D_t = |diag(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t})|$, as

$$LogL(\theta_1,\theta_2|X_t) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi)+\log(R_t)+2\log(|D_t|)+\mathbf{X}_t' D_t^{-1}R_t^{-1}D_t^{-1}\mathbf{X}_t\right]$$

Engle and Sheppard suggest a first estimation stage where the correlation matrix is replaced by an identity matrix

$$LogL(\theta_1|X_t) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi)+\log(I_n)+2\log(|D_t|)+\mathbf{X}_t' D_t^{-1}I_n^{-1}D_t^{-1}\mathbf{X}_t\right]$$

which is equivalent to univariate estimation of GARCH models, and a second step conditional on the parameters estimated in the first one

$$LogL\left(\theta_2|\hat{\theta}_1,X_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi)+\log(R_t)+2\log(|D_t|)+\boldsymbol{\varepsilon}_t' R_t^{-1}\boldsymbol{\varepsilon}_t\right]$$

where $\boldsymbol{\varepsilon}_t = D_t^{-1}\mathbf{X}_t$ are the first stage standardized residuals. Under a set regularity conditions Engle and Sheppard (2001) provide proofs for the consistency and for asymptotic normality of the two-stage estimator. The proof extends directly to the Block-DCC model.

It is worth noting that such a procedure can be used also for the CCC-GARCH model, the constant correlation being estimated in the second step by simply $\boldsymbol{\varepsilon}_t'\boldsymbol{\varepsilon}_t/T$, which is exactly equal to $\bar{Q}$. This observation suggests a possible likelihood ratio test for constant correlation. Engle and Sheppard (2001) provide a testing framework for constant correlation, with an alternative hypothesis of a dynamic autoregressive structure for the correlation, moreover they evidence that standard likelihood ratio tests in a multiple step estimated model have an asymptotic distribution equal to a weighted sum of $r\ \chi^2$, r being the number of restrictions. It has to be noted that the weights are not constant and are complicate functions of the parameters.

We suggest an alternative approach. We are interested in testing the null hypothesis of constant correlation against the alternative of a given dynamic structure, namely the DCC(1,1), for the sake of exposition. We focus on the standardized residuals obtained with the first estimation stage. These variables clearly depend on the estimated parameters and an estimate of correlations or dynamic correlations on these data provide an estimate whose variance-covariance depends on the first stage estimates. However, the bias will equally affect both the constant and the dynamic specifications, we can thus think the standardized residuals as our new variables, and the aim is the estimation of the correlation structure of these data. A standard likelihood ratio test should not be used, unless we compute the asymptotic distribution shown in Engle and Sheppard (2001) and mentioned above. We follow an alternative approach: recalling that the standardised residuals are defined by $\boldsymbol{\varepsilon}_t = E_t' diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right)$, their Log-Likelihood can be represented by

$$LogL\left(\theta_2|\boldsymbol{\varepsilon}_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log\left(2\pi\right)+\log\left(R_t\right)+\boldsymbol{\varepsilon}_t' R_t^{-1}\boldsymbol{\varepsilon}_t\right]$$

We propose therefore to consider two different specifications of R_t the previous dynamic DCC structure $R_t = \left(Q_t^*\right)^{-1} Q_t \left(Q_t^*\right)^{-1}$ and a CCC-type structure where $R_t = R = \left(\bar{Q}^*\right)^{-1}\bar{Q}\left(\bar{Q}^*\right)^{-1}$. In this last equality the external square matrices are $\bar{Q}^* = diag(\sqrt{\bar{q}_{11}}, \sqrt{\bar{q}_{22}}, ...\sqrt{\bar{q}_{nn}})$. This last representation is motivated by two main reasons: at first, we are working with standardised residuals whose variance-covariance matrix should be equal to their correlation matrix, however, this is a theoretical equality and on a sample basis normally will not be satisfied, the quadratic structure ensure we obtain a correlation matrix. Second, the CCC-type representation is equivalent to the DCC when its parameters are equal to zero. Therefore, a combined significativity test of the parameters od the dynamic structure can be used to compare the CCC and the DCC models. Its distribution is a standard chi-square with two degrees of freedom, given the normal asymptotic distribution of the quasi maximum likelihood estimators. By the same reasoning such an approach can be used to compare constant correlation hypothesis to an alternative of block diagonal DCC model. In such a model the combined significativity test will have $(\overline{p}+\overline{q}) \times w\,(w+1)\,/2$ degrees of freedom. Differently, we can compare standard DCC models with our Block-DCC model with a set of parameter restrictions. In this case the asymptotic distribution has $(\overline{p}+\overline{q}) \times w\,(w+1)\,/2 - (\overline{p}+\overline{q})$ degrees of freedom.

4 An empirical application: sectorial allocation

To illustrate the model and to develop the examples we work with weekly data of Italian Stock market indices and analise the subdivision by sector of activity. There are three major sectors that compose the Italian Mibtel general index: Industrials, Services, Finance. Each of this three sub index is further divided in a several sub-sectors. The composition is summarised in Table 1.

INSERT TABLE 1

All the time series are provided by DataStream, are expressed in Euro and run from from January 1990 to April 2003, yielding 693 weekly observations. The problem of asynchronous data encountered by some authors (Corsetti G. et,) is not present, since the closing prices are determined at the same hour in the same market (Italian Stock market). The returns are calculated as usual through log difference transformation. In Table 2 we summarize the main statistics of each series.

INSERT TABLE 2

The average return of the general Mibtel index is positive (3.38% annually), but there are significant differences among the considered sector indexes. For example, the major Industrial sector return is negative (-0.83%) while the return of the major Service sector is greatly positive (13,16%), this is mainly due to the Public Utility Services sector. The sector analysis evidence great differences even for the annualised standard deviations, that vary between the 21,3 of the Real Estate sector and the 40,9 of the Finance Miscellaneous sector. The data presents also a skewness different from zero and a relevant excess kurtosis. The skewness is both positive (10 cases) and negative (10 cases) but with a prevalence, at the aggregate level (Mibtel general index), to be negative (-0,039). Finally, the excess kurtosis is always positive, evidencing the presence of fat tails in the empirical distributions. The Jarque-Bera test clearly rejects the null hypothesis of normality for all the series (not reported in Table 2). The main object of our analysis is to the study of the correlations behaviour between these series. In Table 3 the major sector indexes and the Mibtel general index are considered and there is a high positive correlation for all the indexes. The unconditional empirical correlations between sector indexes are summarised in Table 4 and vary between 0,17 for Industrial misc and Finance misc and 0,82 for Banks and Insurance. The average correlation among the major Industrial sectors is 0,51060, while are 0.48167 and 0.54499 for major Services and Finance sectors, respectively. It is also interesting to observe that within these three groups, there are sector indexes more correlated to each other.

INSERT TABLE 3 AND 4

If we consider a dynamic analysis of the time series of the sector indexes, the volatility is clearly far from being constant. The GARCH specifications can be useful to capture these features. Given the characteristics of the series an asymmetric GARCH specification is considered, in order to capture both excess kurtosis and asymmetric effects. A parsimonious specification of the EGARCH model proposed by Nelson (1991) is considered. The results are summarised in Table 5. The parameters are, with the exception of a few cases, significant at the 5% confidence level.

INSERT TABLE 5

The analysis of the residuals [1] evidences that the GARCH specifications are not able to explain a significant part of the non normality of the series, with the only exception of the Minerals Metals series.

It is also interesting to analyse the behaviour of the correlations over the time, evaluating if their values are stable or not. Considering rolling empirical correlations (with 52 weekly data) it is interesting to observe that almost all the correlations vary through time and also that they present different patterns. For sake of simplicity, the analysis is initially restricted to the three major sectors (Industrial, Service, Finance) compared to the Mibtel general index. Figure 1 evidences the dynamic correlation between the general index and each major sectors, while Figure 2 shows the dynamic correlation between the major sectors. In particular, Figure 1 points out that the correlation are positive and generally high during the sample period for all the major sector indexes while their dynamics is very different. Let us consider for example the first part of 2000: the Finance index correlation exhibits a sharp fall, the Service index correlation remains nearly constant

[1] Not presented but available from the authors.

and the Industrial index correlation increases. Even the correlations between the major sector indexes present very dissimilar patterns.

INSERT FIGURE 1 AND 2

Extending the analysis of the correlation dynamics to the sub-sectors, other considerations are possible. In general the correlation patterns are similar for series of the same major sector and different for series of different major sectors. For example, the dynamics of the correlation between the Food and Paper sector indexes is the same that the correlation between the Cars and Minerals Metals sector indexes while differ from that between Chemicals and Finance Holding sector indexes.

INSERT FIGURE 3

To describe these dynamics three models are estimated. The results for the CCC MV-GARCH proposed by Bollerslev (1990) and the DCC MV-GARCH proposed by Engle (2002) are summarized in Tables 6 and 7.

INSERT TABLES 6 and 7

Moreover, the estimates of the Block DCC MV-GARCH are proposed considering a different volatility and correlation behaviour for each block. The results are summarised in Table 8.

INSERT TABLE 8

The parameters slightly differ in the blocks and in the correlations between blocks. Likelihood ratio tests are used to verify the combined parameter significativity for DCC and Block-DCC models. Both parameterisations result highly significant. Moreover, a combined parameter restriction test is used to directly compare the two DCC models. The test statistic has a value of 143.697 and strongly reject the null hypothesis of parameter restriction (the test has a χ^2 distribution with 10 degrees of freedom). These differences are significative, then we measure the impact of such different specifications of the correlation matrix by considering some financial applications. In particular, we analyse the Value at Risk computation and an optimal portfolio composition. To perform these analysis the one step ahead forecast of the univariate GARCH and the forecast of the different estimates of the correlation matrix are considered. Table 9 reports the Value at Risk for an equally weighted portfolio composed by the 20 sector indexes. For each of the three considered confidence levels (0.1, 0.05, 0.01), the VaR obtained is different along with the chosen correlation model.

INSERT TABLE 9

To further assess the importance of the different specification of the variance-covariance matrix, the portfolio allocation problem is considered. In particular, the mean variance approach of Markovitz suggests that the optimal weights of a portfolio is function of the expected return vector and of the variance-covariance matrix. The expected return vector is estimated considering the historical mean for each sector. For a given level of the portfolio return, the optimal weight vector is calculated for the three different hypothesis about the variance matrix in both the cases of presence of the constraint on short selling (no negative weights) and possibility of short selling. In both cases the portfolio optimal composition is very different depending on the considered multivariate GARCH specification. These differences reflect in a different portfolio variance and point out that a better variance specification allow a more efficient portfolio composition. The last row of Table 10 contains the variances of the optimal portfolios and, both in the constrained and non constrained problem, the specification of the correlation matrix that allows lower variance given a level of expected return is the Block DCC MV-GARCH, followed by the DCC and the CCC specifications.

INSERT TABLE 10

5 Conclusions

We propose an extension of the new class of models recently proposed by Engle (2002), that both preserves the ease of estimation of the Bollerslev's constant correlation model but allows the correlations to change over time. Engle indeed added to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constraint to be equal for all the correlations. However, this is an unnecessary restriction, thus we extend the DCC model introducing a block-diagonal structure that solves this problem. The dynamic is constrained to be equal only among groups of variables. In fact, we cannot impose that the correlations of, for example European sectorial stock indexes are equal to the correspondent US ones. Keeping the ease of estimation of the Engle's model, the extension we propose allows richer dynamics of the correlations.

After discussing the estimation and testing issues, we consider an empirical application of the three models (CCC, DCC and Block DCC). The variables object of analysis are sectorial stock indexes representing the major disaggregation of the Italian general stock index. The estimates of the three models confirm, for the period of analysis, the presence of dynamics in the correlations, as well as for the volatility, but also evidence the presence of dissimilarities in these dynamics. Even if more rigorous investigation are needed, these preliminary results are very promising.

References

[1] Bollerslev T., 1990, Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH approach, *Review of Economic Studies*, 72, 498-505.

[2] Cappiello L., R.F. Engle and K. Sheppard, 2002, Asymmetric dynamics in the correlations of global equity and bond returns, preprint ECB.

[3] Corsetti G., M. Pericoli and M. Sbracia, 2002, 'Some contagion, some interdipendence' More pitfalls in tests of financial contagion, Working Paper.

[4] Ding, Z., C.W.J. Granger and R.F. Engle, 1993, A long memory property of stock market returns and a new model, *Journal of Finance*, 1, 83-106.

[5] Engle R.F. and K.F. Kroner, 1995, Multivariate simultaneous generalized ARCH, *Econometric Theory*, 11, 122-150.

[6] Engle R.F., 2002, Dynamic conditional correlation - a simple calss of multivaiate GARCH models, University of California, San Diego, forthcoming *Journal of Business and Economic Statistics.*

[7] Engle R.F. and K. Sheppard, 2001, Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH, University of California, San Diego, Discussion paper 2001-15.

[8] Glosten L., R. Jagannathan and D. Runkle, 1993, Relationship between the expected value and the volatility of the nominal excess returns on stocks, Journal of Finance, 48, 1779-1801.

[9] Morillo D. and L. Pohlman , 2002, Large scale multivariate GARCH risk modelling for long horizon international equity portfolios, Working Paper.

[10] Nelson D.B., 1991, Conditional heteroskedasticity in asset returns: a new approach, Econometrica, 59, 347-370.

<table>
<tr><td rowspan="20">MIBTEL (General)</td><td rowspan="10">INDUSTRIAL</td><td>FOOD</td></tr>
<tr><td>CARS</td></tr>
<tr><td>PAPER</td></tr>
<tr><td>CHEMICALS</td></tr>
<tr><td>CONSTRUCTION</td></tr>
<tr><td>ELECRONICS</td></tr>
<tr><td>PLANTS MACHINE</td></tr>
<tr><td>INDUSTRIALS MISC</td></tr>
<tr><td>MINERALS METALS</td></tr>
<tr><td>TEXTILE CLOTHING</td></tr>
<tr><td rowspan="4">SERVICE</td><td>DISTRIBUTION</td></tr>
<tr><td>MEDIA</td></tr>
<tr><td>PUBLIC UTILITY SERVICES</td></tr>
<tr><td>TRANSPORT TOURISM</td></tr>
<tr><td rowspan="6">FINANCE</td><td>INSURANCE</td></tr>
<tr><td>BANKS</td></tr>
<tr><td>FINANCE HOLDINGS</td></tr>
<tr><td>FINANCE MISC</td></tr>
<tr><td>REAL ESTATE</td></tr>
<tr><td>FINANCE SERVICES</td></tr>
</table>

Table 1: Italian indexes composition.

	Mean	Standard deviation	Asymmetry	Excess Kurtosis
MIBTEL	3,38	22,4	-0,039	2,17
INDUSTRIAL	-0,83	21,9	-0,285	1,61
FOOD	1,79	26,5	-0,063	1,79
CARS	-11,26	31,7	-0,269	3,07
PAPER	-11,34	30,2	1,278	13,83
CHEMICALS	-2,20	25,0	-0,368	1,67
CONSTRUCTION	-1,11	26,5	0,174	1,96
ELECRONICS	-4,25	29,3	0,009	2,22
PLANTS & MACHINE	5,10	24,9	-0,357	5,04
INDUSTRIALS MISC	-1,53	28,3	-0,227	6,83
MINERALS METALS	0,45	26,0	-0,029	0,66
TEXILE CLOTHING	3,97	23,4	-0,455	3,47
SERVICE	13,16	25,0	0,235	0,91
DISTRIBUTION	2,44	25,2	0,245	4,89
MEDIA	0,83	29,5	0,651	6,56
PUB. UTIL. SERV.	15,21	27,3	0,260	0,81
TRANS & TOURISM	7,65	23,1	0,094	3,20
FINANCE	1,14	24,2	-0,187	3,76
INSURANCE	2,44	25,7	-0,048	3,27
BANKS	2,14	26,0	-0,161	4,11
FINANCE HOLDINGS	-6,68	27,0	0,190	1,03
FINANCE MISC.	-2,82	40,9	0,937	6,12
REAL ESTATE	-0,86	21,3	0,700	4,46
FINANCE SERVICES	2,17	29,3	-0,335	5,88

Table 2: Summary statistics.

	GENERAL	**INDUSTRIALS**	**SERVICES**	**FINANCE**
GENERAL	1	0.925802	0.908596	0.948754
INDUSTRIALS	0.925802	1	0.783265	0.850153
SERVICES	0.908596	0.783265	1	0.768069
FINANCE	0.948754	0.850153	0.768069	1

Table 3: Empirical correlations.

FOOD	CARS	PAPER	CHEMICALS	CONSTRUCTION	ELECRONICS	PLANTS & MACHINE	INDUSTRIALS MISC	MIN	TEX	DISTRIBUTION	MEDIA	PUB. UTIL. SERV.	TRANS & TOURISM	INSURANCE	BANKS	FINANCE HOLDINGS	FINANCE MISC.	REAL ESTATE	FINANCE SERVICES
1	0,56	0,43	0,61	0,62	0,45	0,53	0,34	0,52	0,57	0,44	0,30	0,52	0,53	0,60	0,62	0,68	0,21	0,50	0,54
	1	0,52	0,66	0,64	0,56	0,57	0,36	0,50	0,59	0,47	0,42	0,56	0,57	0,67	0,66	0,72	0,26	0,52	0,57
		1	0,50	0,51	0,46	0,47	0,29	0,45	0,48	0,34	0,31	0,42	0,45	0,49	0,50	0,54	0,22	0,42	0,45
			1	0,68	0,64	0,57	0,42	0,54	0,63	0,55	0,42	0,65	0,60	0,68	0,70	0,75	0,31	0,54	0,59
				1	0,59	0,59	0,44	0,56	0,62	0,53	0,37	0,60	0,65	0,64	0,71	0,73	0,29	0,60	0,61
					1	0,46	0,35	0,40	0,55	0,51	0,56	0,71	0,56	0,63	0,64	0,74	0,36	0,55	0,57
						1	0,35	0,60	0,58	0,40	0,31	0,48	0,51	0,59	0,62	0,57	0,26	0,49	0,50
							1	0,34	0,37	0,35	0,21	0,35	0,38	0,38	0,39	0,42	0,17	0,36	0,34
								1	0,51	0,39	0,32	0,50	0,53	0,58	0,57	0,53	0,19	0,43	0,47
									1	0,50	0,44	0,58	0,52	0,64	0,67	0,65	0,28	0,48	0,54
										1	0,42	0,52	0,51	0,55	0,57	0,58	0,27	0,49	0,52
											1	0,50	0,39	0,48	0,50	0,56	0,34	0,46	0,43
												1	0,55	0,69	0,68	0,71	0,27	0,51	0,52
													1	0,60	0,63	0,66	0,28	0,60	0,56
														1	0,82	0,73	0,28	0,54	0,62
															1	0,76	0,31	0,59	0,70
																1	0,37	0,65	0,65
																	1	0,34	0,27
																		1	0,53
																			1

Table 4: Empirical correlations.

	ω	α	γ	β
FOOD	0.007667 *0.044489*	0.17429 *0.041231*	-0.042721 *0.016308*	0.945534 *0.019298*
CARS	-0.018458 *0.033559*	0.181820 *0.025871*	-0.049772 *0.014939*	0.960047 *0.012635*
PAPER	0.133547 *0.080686*	0.190099 *0.027615*	-0.115743 *0.021219*	0.903522 *0.031548*
CHEMICALS	0.06875 *0.052705*	0.210777 *0.048487*	-0.042927 *0.01917*	0.905615 *0.028169*
CONSTRUCTION	-0.017683 *0.040712*	0.206656 *0.035078*	-0.038557 *0.015415*	0.945024 *0.013988*
ELECRONICS	-0.105743 *0.047497*	0.273387 *0.043656*	-0.037565 *0.018623*	0.960144 *0.016466*
PLANTS & MACHINE	-0.001862 *0.052135*	0.257848 *0.038020*	-0.135494 *0.018234*	0.9174 *0.022963*
INDUSTRIALS MISC	0.043335 *0.049291*	0.200922 *0.031524*	-0.033979 *0.015484*	0.931596 *0.021698*
MINERALS METALS	0.136145 *0.112448*	0.254659 *0.051434*	-0.06675 *0.025988*	0.867494 *0.048532*
TEXILE CLOTHING	-0.006179 *0.067197*	0.304888 *0.055493*	-0.103177 *0.021758*	0.896652 *0.023522*
DISTRIBUTION	0.126355 *0.080918*	0.356644 *0.056348*	-0.104257 *0.021247*	0.840823 *0.041691*
MEDIA	-0.091437 *0.039332*	0.371298 *0.039381*	-0.004515 *0.018767*	0.924889 *0.017853*
PUB. UTIL. SERV.	0.056750 *0.088477*	0.164620 *0.045733*	-0.022391 *0.018623*	0.929796 *0.038658*
TRANS & TOURISM	0.034109 *0.086364*	0.341412 *0.041971*	-0.018215 *0.021881*	0.871284 *0.037845*
INSURANCE	0.145923 *0.110749*	0.267765 *0.041751*	-0.062281 *0.021792*	0.857678 *0.049524*
BANKS	-0.004814 *0.080188*	0.339686 *0.046219*	-0.046271 *0.026547*	0.892956 *0.036466*
FINANCE HOLDINGS	0.003824 *0.054225*	0.247872 *0.045694*	-0.025745 *0.020524*	0.923353 *0.025147*
FINANCE MISC.	-0.009788 *0.034614*	0.443871 *0.034797*	-0.086374 *0.023577*	0.906579 *0.014537*
REAL ESTATE	-0.067388 *0.035504*	0.312774 *0.026917*	0.014675 *0.016420*	0.923310 *0.016421*
FINANCE SERVICES	0.125354 *0.075030*	0.334182 *0.043659*	-0.055903 *0.023690*	0.859555 *0.036751*

Table 5: Univariate GARCH specifications and parameter estimates (standard deviation in italic).

$$\ln(\sigma_t^2) = \omega + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta\ln(\sigma_{t-1}^2)$$

FOOD	CARS	PAPER	CHEMICALS	CONSTRUCTION	ELECRONICS	PLANTS & MACHINE	INDUSTRIALS MISC	MIN	TEX	DISTRIBUTION	MEDIA	PUB. UTIL. SERV.	TRANS & TOURISM	INSURANCE	BANKS	FINANCE HOLDINGS	FINANCE MISC.	REAL ESTATE	FINANCE SERVICES
1	0,56	0,43	0,59	0,60	0,48	0,51	0,34	0,52	0,51	0,42	0,32	0,50	0,51	0,58	0,61	0,67	0,21	0,49	0,50
	1	0,49	0,64	0,61	0,57	0,55	0,34	0,48	0,54	0,45	0,40	0,56	0,55	0,64	0,62	0,71	0,24	0,51	0,53
		1	0,49	0,49	0,47	0,45	0,29	0,45	0,44	0,34	0,32	0,42	0,42	0,48	0,49	0,54	0,22	0,42	0,42
			1	0,65	0,63	0,55	0,39	0,53	0,59	0,54	0,39	0,64	0,57	0,66	0,68	0,74	0,29	0,51	0,55
				1	0,61	0,58	0,42	0,54	0,59	0,51	0,40	0,58	0,61	0,61	0,69	0,71	0,30	0,57	0,56
					1	0,47	0,35	0,42	0,53	0,50	0,48	0,69	0,57	0,63	0,66	0,75	0,32	0,54	0,55
						1	0,35	0,57	0,53	0,38	0,33	0,49	0,49	0,56	0,60	0,56	0,23	0,47	0,44
							1	0,33	0,34	0,33	0,21	0,32	0,35	0,35	0,36	0,40	0,14	0,36	0,32
								1	0,47	0,38	0,34	0,49	0,51	0,55	0,56	0,53	0,19	0,44	0,44
									1	0,46	0,43	0,56	0,49	0,59	0,62	0,62	0,28	0,46	0,48
										1	0,38	0,51	0,47	0,53	0,54	0,57	0,24	0,47	0,47
											1	0,46	0,37	0,46	0,48	0,53	0,26	0,41	0,37
												1	0,54	0,69	0,69	0,70	0,26	0,50	0,49
													1	0,57	0,61	0,65	0,27	0,57	0,52
														1	0,80	0,73	0,26	0,52	0,56
															1	0,75	0,31	0,58	0,63
																1	0,34	0,63	0,61
																	1	0,31	0,22
																		1	0,51
																			1

Table 6: CCC MV-GARCH estimated correlation matrix (after the univariate estimation).

Parameter	Estimate	Standard deviation	z-statistics
α	0,84223	0,0132	63,78584
β	0,03422	0,01483	2,30771
	Log Likelihood: -2926,59033		

Table 7: DCC MV-GARCH estimates.

	Industrial	Service	Finance
Industrial	0,83536 *0,01781* 0,05045 *0,02029*	0,76976 *0,02779* 0,18203 *0,02984*	0,80562 *0,02474* 0,12071 *0,02649*
Service	-	0,76213 *0,03203* 0,15305 *0,03715*	0,74346 *0,03330* 0,20796 *0,03623*
Finance	-	-	0,79144 *0,02777* 0,10341 *0,03120*

Table 8: Block DCC MV-GARCH: α and β parameters for each block (standard deviations in italic).

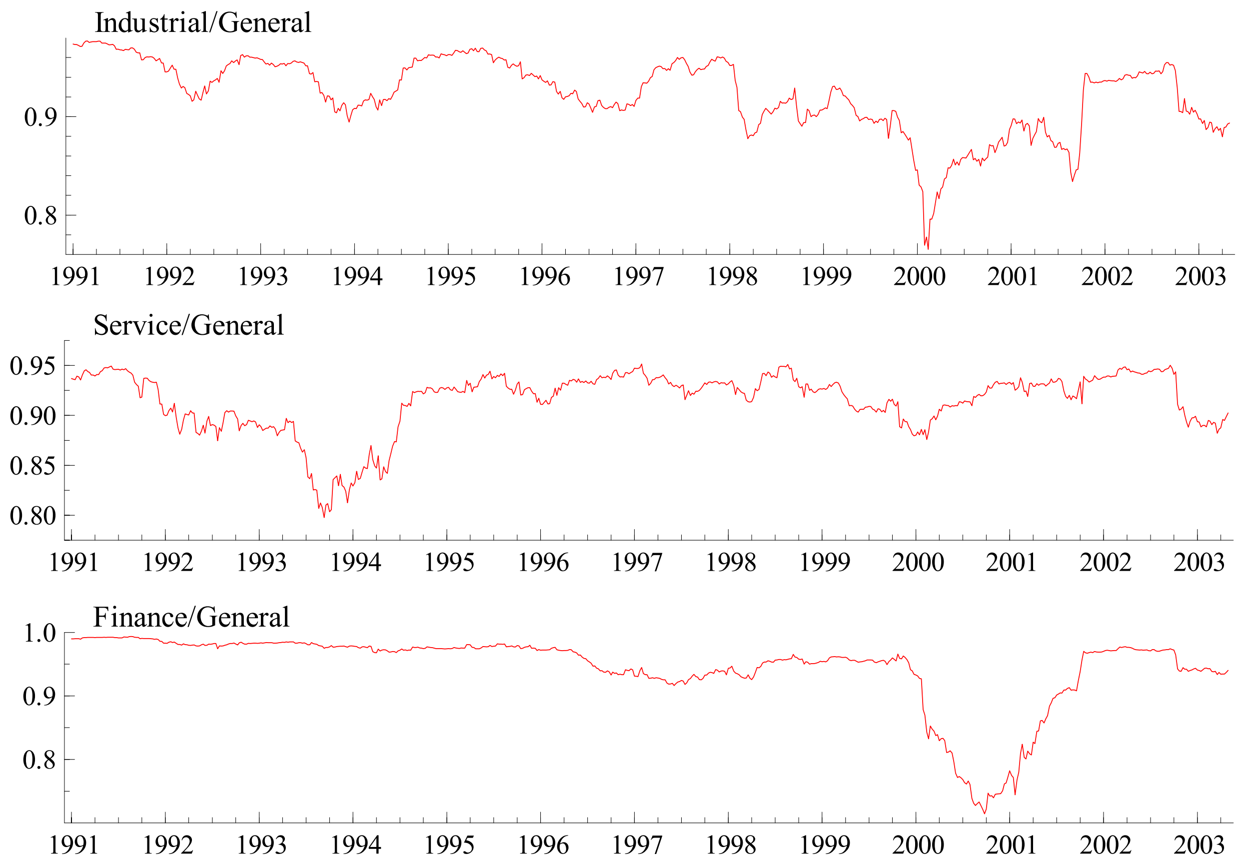

Figure 1: Correlation between Mibtel general index and major sectors indexes.

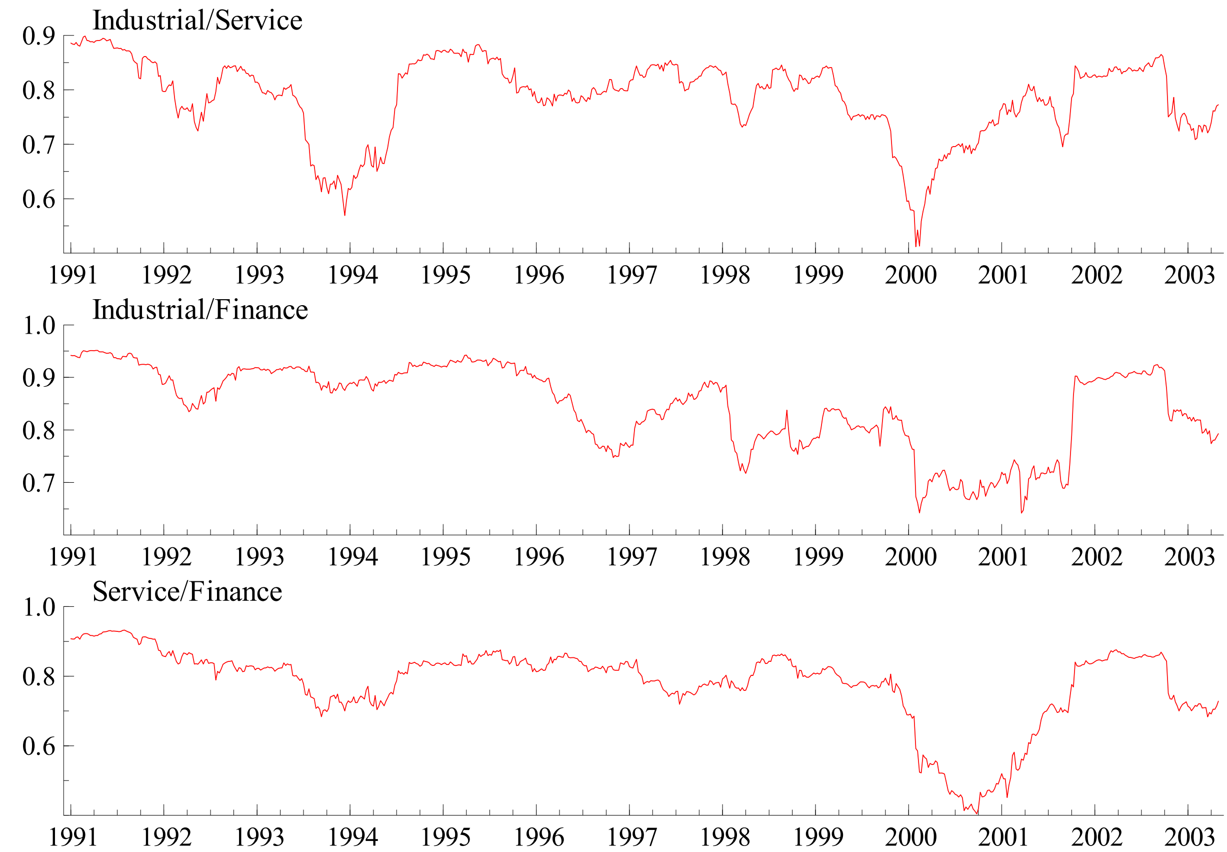

Figure 2: Correlation between major sector indexes.

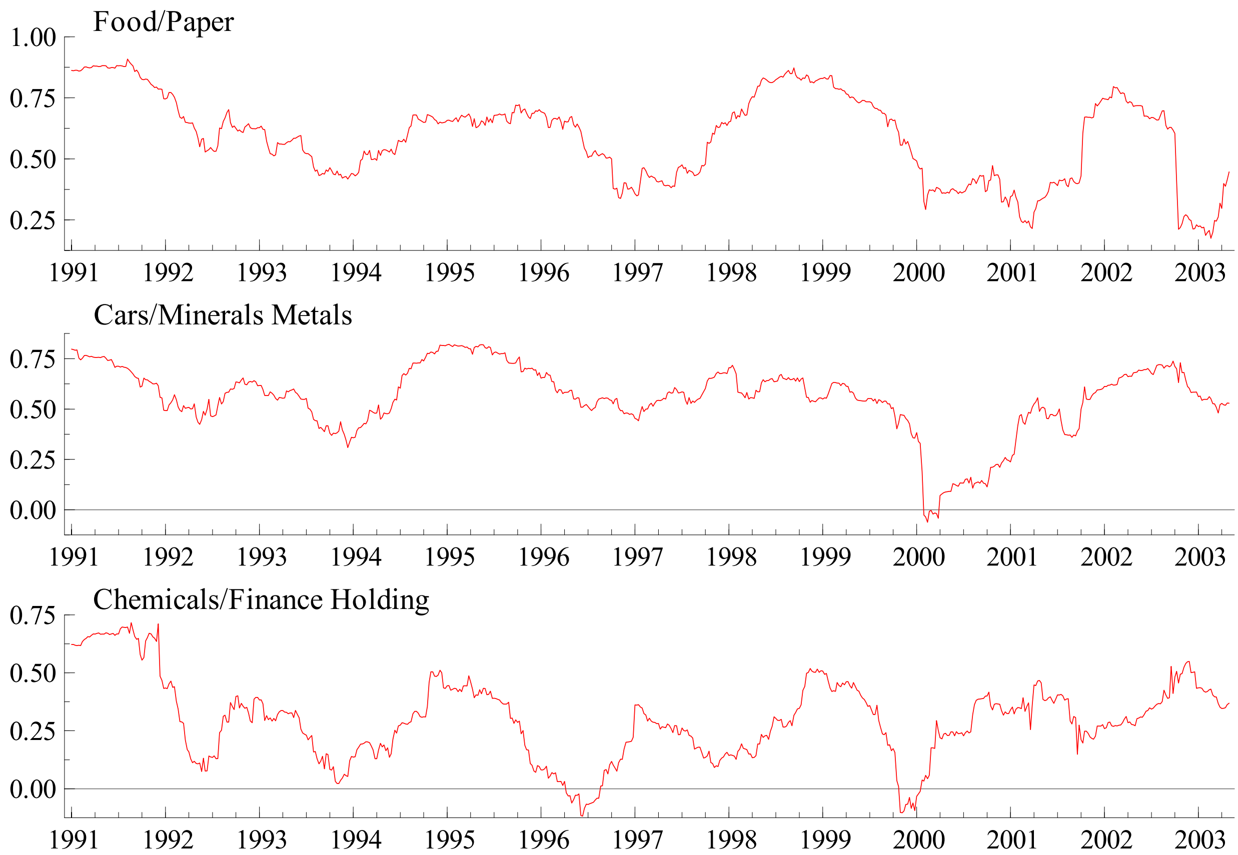

Figure 3: Correlation dynamics.

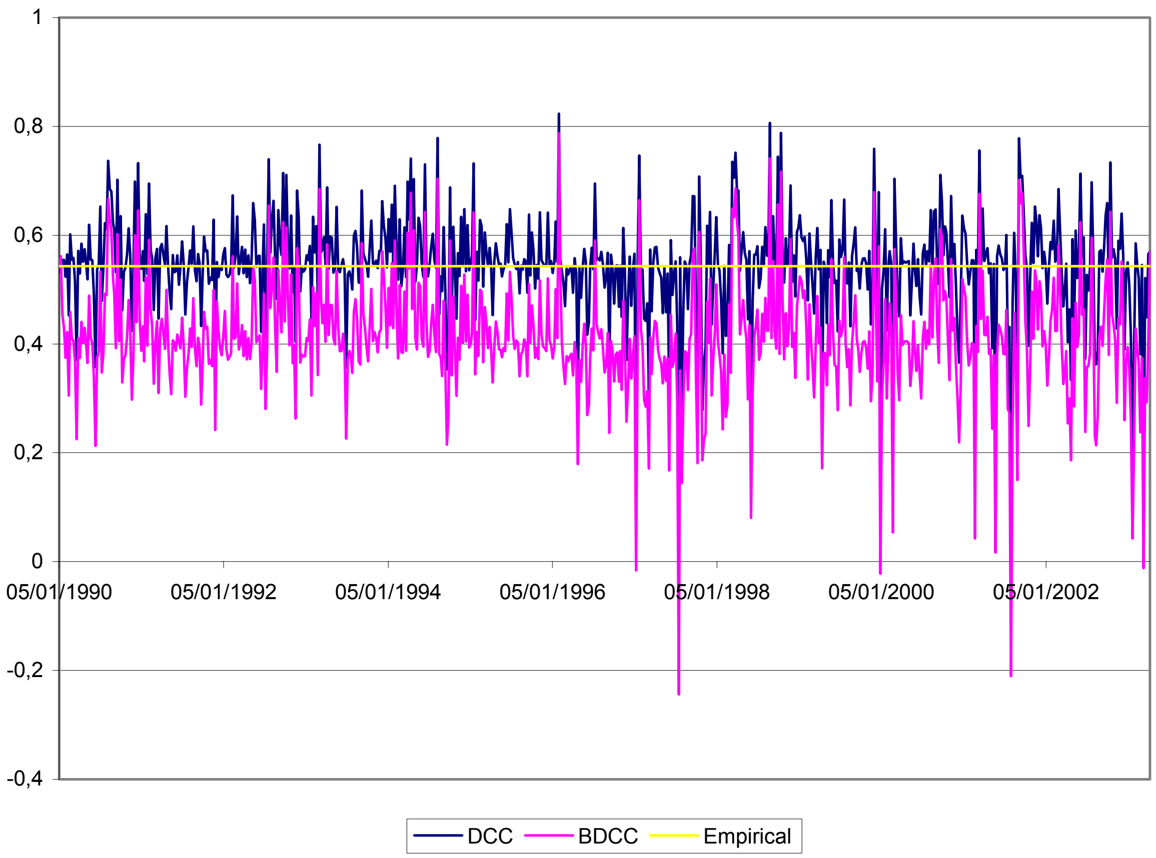

Figure 4: Correlation between Public Utility Service sector and Banks sector for the three models estimated.

Value at Risk	**CCC**	**DCC**	**BDCC**
0.1	3,58	3,23	3,72
0.05	4,59	4,15	4,77
0.01	6,47	5,81	6,72

Table 9: Value at Risk measures at different confidence level for the three models.

CCC	DCC	BDCC		CCC	DCC	BDCC
-	-	-	*FOOD*	-0,06	-0,03	-0,06
-	-	-	*CARS*	-0,08	-0,08	-0,12
-	-	-	*PAPER*	-0,01	-0,02	-0,01
-	0,02	-	*CHEMICALS*	0,14	0,15	0,22
-	-	-	*CONSTRUCTION*	-0,05	0,00	0,02
-	-	-	*ELECRONICS*	-0,07	-0,02	-0,08
-	-	-	*PLANTS MACHINE*	0,03	0,05	0,05
0,22	0,19	0,33	*INDUSTRIALS MISC*	0,20	0,18	0,33
-	-	-	*MINERALS METALS*	0,01	0,02	-0,02
-	-	-	*TEXTILE CLOTHING*	-0,05	-0,02	-0,04
0,21	0,22	-	*DISTRIBUTION*	0,25	0,22	-0,03
0,01	-	0,06	*MEDIA*	0,05	0,01	0,10
-	-	0,04	*PUBLIC UTILITY SERVICES*	-0,02	-0,05	0,05
0,46	0,46	0,47	*TRANSPORT TOURISM*	0,54	0,46	0,41
-	-	-	*INSURANCE*	-0,05	-0,01	-0,01
-	-	-	*BANKS*	-0,09	-0,06	-0,14
-	-	-	*FINANCE HOLDINGS*	0,14	0,03	0,09
0,03	0,02	0,03	*FINANCE MISC*	0,03	0,03	0,04
0,06	0,09	0,07	*REAL ESTATE*	0,11	0,13	0,16
-	-	-	*FINANCE SERVICES*	-0,02	0,01	0,04
4,25	2,91	2,31	***Optimal Portfolio Variances***	3,40	2,25	2,03

Table 10: Portfolio allocation in the Markovitz approach considering an annual return level of 0.03 in a constrained and a non constrained problem.