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Flexible Dynamic Conditional Correlation Multivariate GARCH for Asset Allocation

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Abstract

This paper introduces a generalisation of the dynamic conditional correlation (DCC) multivariate GARCH model proposed by Engle (2002). In the multivariate GARCH literature one of the most relevant problems is represented by the elevate number of parameters. In order to solve this difficulty Bollerslev (1990) suggested to keep constant the correlations (he suggested the Constant Conditional Correlation model, CCC). Engle added to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constrained to be equal for all the correlations. In our view, this is an unnecessary restriction. In fact, we cannot impose that the correlations of, say, European sectorial stock indexes are identical to the correspondent US ones. We extend the DCC model introducing a block-diagonal structure that solves this problem. The dynamic is constrained to be equal only among groups of variables. We present an application to a sectorial asset allocation problem.

Keywords: Multivariate GARCH, Dynamic Correlation, Volatility, Asset Allocation, Risk Management.

1 Introduction

In today's global and highly volatile markets the efficient measurement and management of market risk has become a critical factor for the competitive-

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ness and even survival of financial institutions. One of the inputs required by risk managers, seeking to hold efficient portfolios, is the correlation between the securities to be included in the portfolio. Until recently, correlation was assumed to be constant and stable over time. However, all empirical studies that attempted to verify this finding, have failed to confirm the validity of this assumption. In fact, most experienced practitioners would attest that correlations increase in periods of high volatility and that both the magnitude and persistence of correlation is affected by volatility.

The asset allocation decision entails, inter alia, an assessment of the risks and returns of the various assets in the opportunity set. Optimal portfolio choice requires a forecast of the covariance matrix of the returns. Similarly, the calculation of the standard deviation of today's portfolio requires a covariance matrix of all the assets in the portfolio. For actual portfolios, with thousands of derivative and synthetic instruments, these functions require estimation and forecasting of very large covariance matrices.

Over the past 20 years, a tremendous literature has been developed where the dynamics of the covariance of assets has been explored, although the primarily focus has been on univariate volatilities and not on correlations (or covariances). In fact, in the multivariate GARCH literature one of the most relevant problems is represented by the elevate number of parameters. In order to solve this difficulty Bollerslev (1990) suggested to keep constant the correlations and suggested the Constant Conditional Correlation model (CCC). Only recently Engle (2002) proposes a new class of models that both preserves the ease of estimation of the Bollerslev's constant correlation model but allows the correlations to change over time. Engle adds to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constrained to be equal for all the correlations. In our view, this is an unnecessary restriction. In fact, we cannot impose that the correlations dynamic evolution of, say, European sectorial stock indexes is identical to the correspondent US ones. We thus extend the DCC model introducing a block-diagonal structure that solves this problem. The correlation dynamic is constrained to be equal only among groups of variables. The suggested model provide a much more flexible parameterisation of correlation dynamic mantaining at the same time the parameter number at a feasible level; we called this new model "Flexible Dynamic Conditional Correlation".

This block dynamic representation will also be useful in other fields, for example to investigate wether the formation of the EMU in Europe has increased the correlation among national assets and in more general terms to analyse the interdependence and contagion issues.

The outline of the paper is as follows. We start by surveying the multivariate GARCH models and introuce our Flexible Dynamic Conditional

Correlation model in Section 2. In Section 3 we discuss estimation and testing of constant and dynamic conditional correlation models. In Section 4 we describe an asset allocation problem, in which we analyse sectorial stock indexes and we empirically motivate the need for different dynamics. Section 5 concludes.

2 Multivariate GARCH models

We consider a n-dimensional process X_t, define by $I_t(X)$ the information set of X at time t and assume that:

$$X_t | I_{t-1}(X) \tilde{\ } id\ D(\boldsymbol{\mu}_t, H_t) \tag{1}$$

where $D(\mu_t, H_t)$ is a non-specified multivariate distribution with time dependent mean μ_t and time dependent variance covariance matrix H_t. This formulation nests all multivariate GARCH representations that will be introduced in few steps, and allows also the specification of a multivariate ARMA process for the mean, as well as in-mean effects of the variances.

In the following, vector variables and matrices are denoted by bold or uppercase letters, while scalars are denoted by lowercase letters. The $Vech(\cdot)$ matrix operator will also be used: it stacks the lower triangular portion of a matrix. In addition, the residuals are defined as $N_t = X_t - \boldsymbol{\mu}_t$.

In a general framework, a multivariate GARCH process can be represented via a dynamic equation for the variance-covariance matrix. Several formulaer has been suggested in the literature, two of the most known are the BEKK and the Vech representations.

2.1 The Vech representations

These formulations derive from the work of Engle and Kroner (1995). The multivariate GARCH is represented as

$$Vech(H_t) = \boldsymbol{\omega} + C(L) Vech(H_{t-1}) + D(L) Vech(N_t N_t') \tag{2}$$

where $\boldsymbol{\omega}$ is a vector of dimension $r \times 1$, $C(L) = \sum_{i=1}^{p} C_i L^i$, $D(L) = \sum_{i=1}^{q} D_i L^i$, C_i and D_i, are square matrices of dimension $r \times r$ and $r = n(n+1)/2$. This formulation requires $r \times [1 + r \times (p+q)]$ parameters; this is the main constraint on the estimation and application of this model. For example let we consider the bivariate model: the parameters will be 21. However, in this case Engle and Kroner (1995) show that the Vech-GARCH is stationary if and only if all the eigenvalues of $C(1) + D(1)$ are less than one in modulus.

A restricted case of the Vech-GARCH is its diagonal representation, the D-Vech-GARCH: it is defined diagonal as it assumes that all parameter matrices are diagonal. This boils down to a model that parameterizes all conditional variances and covariances as univariate GARCH(p,q) processes. The total number of parameters reduces to $3r$, for the bivariate case 9. A much greater problem, which is not solved by restricting to the diagonal version, is the positive definiteness of the H_t matrix, which is very difficult to check and requires several constraints in the optimization routines. A final comment concerns the outcome of the Vech model: we obtain a dynamic equation for the covariances while for practical needs (asset allocation, risk evaluation...) the focus is on the correlations. Therefore, the Vech is a 'wrong' model for estimating dynamic correlations.

2.2 The BEKK representation

This formulation was suggested by Baba, Engle, Kraft and Kroner in a preliminary version of Engle and Kroner (1995). The main feature is that it does not need any restriction on parameters to get positive definiteness of the H_t matrix, given its quadratic structure. One of the possible BEKK-GARCH model is

$$H_t = \boldsymbol{\omega} + \sum_{i=1}^{p} C_i H_{t-i} C_i' + \sum_{j=1}^{q} D_j N_{t-j} N_{t-j}' D_j' \tag{3}$$

where C_i and D_j are $n \times n$ matrices and $\boldsymbol{\omega}$ is a symmetric positive definite $n \times n$ matrix. In a general formulation the number of parameters are here $r + n \times n \times (p+q)$; for the bivariate case the number drops to only 11. Positive definiteness of the variance covariance matrix is controlled by the constant matrix $\boldsymbol{\omega}$, whose positive definiteness is often obtained assuming the factorization $\boldsymbol{\omega} = \boldsymbol{\Gamma}\boldsymbol{\Gamma}'$, where Γ is a lower triangular matrix. The BEKK formulation and the Vech are strictly related as shown in Engle and Kroner (1995); in particular, the stationarity condition of the BEKK model is very similar to the one of the Vech representation (for further details refer to the cited paper). The comment previously referred to the Vech applies on the BEKK: the model focuses on covariances while the practical attention is on correlations.

2.3 The Constant Conditional Correlation GARCH

This form was introduced by Bollerslev (1990) who tried to reduce the number of parameters of the Vech and BEKK representations. He suggested the following structure:

$$
\begin{aligned}
H_t &= \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} & \cdots & \sigma_{1n,t} \\ \sigma_{12,t} & \sigma_{2,t}^2 & & \sigma_{2n,t} \\ \vdots & & \ddots & \vdots \\ \sigma_{1n,t} & \sigma_{2n,t} & \cdots & \sigma_{n,t}^2 \end{bmatrix} \qquad (4) \\
\sigma_{i,t}^2 &= \omega_i + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 \qquad i = 1...n \\
\sigma_{ij,t} &= \rho_{ij}\sigma_{i,t}\sigma_{j,t} \qquad i,j = 1...n \ , \ i \neq j
\end{aligned}
$$

The main difference between this formulation and the previous one is in the assumption of constant correlation among variables. The total number of parameters is now $(p + q + 1)n + n(n + 1)/2$, i.e. 7 in the bivariate case. Positive definiteness of the variance covariance matrix is now controlled by the correlation matrix (for the conditional variances the usual GARCH constraints for positivity are required), since we can rewrite

$$
\begin{aligned}
H_t &= diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n} \\ \rho_{1,2} & 1 & & \vdots \\ \vdots & & \ddots & \rho_{1,n-1} \\ \rho_{1,n} & \cdots & \rho_{1,n-1} & 1 \end{bmatrix} diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right) \qquad (5) \\
&= D_t R D_t \qquad (6)
\end{aligned}
$$

where $diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right)$ represents a diagonal matrix with the given elements. Moreover, the correlation matrix can be factorized similarly to the constant of the BEKK-GARCH to impose its positive definiteness and to ensure that it is a correlation matrix. It can be factorized as follows:

$$
R = diag(\sqrt{\gamma_{1,1}}, \sqrt{\gamma_{2,2}}...\sqrt{\gamma_{n,n}})\mathbf{\Gamma\Gamma}' diag(\sqrt{\gamma_{1,1}}, \sqrt{\gamma_{2,2}}...\sqrt{\gamma_{n,n}}) \qquad (7)
$$

where the internal tringular matrices ensure positive definiteness, while the external ones ensure the boundness of correlations and the unit value of the main diagonal of R. Differently, a two step estimator can be used (see section 3).

2.4 The Dynamic Conditional Correlation formulations

This alternative representation try to add some limited dynamics to the CCC-GARCH and has been introduced in recent papers by Engle and Sheppard (2001) and Engle (2002). The idea behind this GARCH model derives from the CCC representation. The following factorization of the H_t matrix is assumed:

$$H_t = D_t R_t D_t \tag{8}$$

where the conditional variances are parameterized as in the CCC-GARCH and R_t is a dynamic correlation matrix satisfying

$$\begin{aligned} R_t &= \left(Q_t^*\right)^{-1} Q_t \left(Q_t^*\right)^{-1} && (9)\\ Q_t &= \left[1-\alpha(1)-\beta(1)\right]\bar{Q} + \alpha(L)\,\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}' + \beta(L)\,Q_{t-1} && (10)\\ \boldsymbol{\varepsilon}_t &= D_t^{-1} N_t \\ \bar{Q} &= T^{-1}\sum_{i=1}^{T} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' && (11)\\ \alpha(L) &= \sum_{i=1}^{\overline{q}} \alpha_i L^i \qquad \beta(L) = \sum_{i=1}^{\overline{p}} \beta_i L^i \\ Q_t^* &= diag(\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, \ldots \sqrt{q_{nn,t}}) \end{aligned}$$

which is just a particular kind of multivariate GARCH on the correlations. The Q_t^* diagonal matrix is introduced to ensure that R_t is a correlation matrix, while $\boldsymbol{\varepsilon}_t$ represents the vectors of standardised residuals of the univariate GARCH models. Given a suitable positive definite starting point (i.e. $Q_o = \bar{Q}$), the model is positive defines since Q_t is the sum of positive definite ($\left[1-\alpha(1)-\beta(1)\right]\bar{Q}$, $\beta(L)\,Q_{t-1}$) and semidefinite matrices ($\alpha(L)\,\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}'$). Explosive patterns of the correlations are ruled out by imposing a GARCH-type parameter restriction $\alpha(1)+\beta(1)<1$. Furthermore, the unconditional correlation of the DCC model is the sample correlation Γ. We will refer to this property as 'correlation targeting'. In this model the number of parameters is $(p+q+1)\times n + (\overline{p}+\overline{q})$.

The DCC model was generalised by Engle (2002), who suggested the following Generalised DCC

$$Q_t = \left[ii' - A - B\right]\circ \bar{Q} + A\circ \boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}' + B\circ Q_{t-1} \tag{12}$$

where $\circ$ is the Hadamard product (elementwise matrix multiplication), A and B are square matrices and positive definiteness is guaranteed by positive definiteness of A and B, see Ding and Engle (2001). Moreover, the number of parameters greatly increases and makes the model empirically inattractive.

Cappiello, Engle and Sheppard (2002) provide a different extension of the DCC model by introducing asymmetry in the correlation dynamics and modifying the correlation equation in the following way:

$$Q_t = \left(\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{F}G\right) + A'\varepsilon_t\varepsilon_t'A + B'Q_{t-1}B + G'\eta_{t-1}\eta_{t-1}'G \tag{13}$$

where $\eta_t = I(\varepsilon_t < 0) \circ \varepsilon_t$, $\circ$ being the Hadamar product (element by element), A, B, G are diagonal parameter matrices, $\bar{Q}$ is again the sample covariance matrix of the standardized residuals and $\bar{F}$ is the sample covariance matrix of η_t. This model adds flexibility to the previous one, however the number of parameters increases and positive definiteness is obtained by constraining the matrix $\bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{F}G$ to be positive definite, quite a complex task.

Furthermore, Franses and Hafner (2003) suggested another Generalised DCC:

$$Q_t = \left[1 - \sum_{i=1}^{n} \alpha_i - \beta\right]\bar{Q} + \alpha\alpha' \circ \varepsilon_{t-1}\varepsilon_{t-1}' + \beta Q_{t-1} \tag{14}$$

with α being a vector of dimension n. Positive definiteness is guaranteed but the correlation targeting property is no more valid.

Finally, Chan, Hoti and McAleer (2003) generalise the model providing a general representation in which all the dynamic correlations can have a different dynamic pattern. The main results provided by Chan et al.'s paper concern the regularity conditions for all the moments and the asymptotic properties of the Generalised Auto Regressive Conditional Correlation model. The paper provides consistency and asymptotic distribution of the Quasi Maximum Likelihood estimator. Moreover, it shows that the Engle's DCC is obtained as a special case.

2.5 Feasible Dynamic Conditional Correlation models

The previously reported models provide several extensions of the original DCC formulation. However, they have too many parameters or they require particular parameter constraints. Going back to the Engle's DCC, its structure presumes that all the correlations have the same dynamic (same parameters apply to all correlations). Unfortunately, this may not be the case.Consider as an example a portfolio that includes several assets grouped in homogeneous cathegories (energy, food, chemistry...): correlations within and between groups do not generally share the same dynamic evolution. A similar example can consider a geographical distribution (Europe, America, Asia...) or a distinction of assets by types (stocks, bonds, cash...). Given this consideration we suggest here two DCC-type models that allow for a block structure in the parameters. In this way we can, on the one side, generalise the model allowing for different dynamic between group of assets; on the other side, we match the needs of geographical or sectorial asset allocation.

As a starting point we can consider a particular case of (12):

$$\begin{aligned} Q_t &= [1-\alpha(L)-\beta(L)] \circ \bar{Q} + \alpha(L) \circ \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \beta(L) \circ Q_t \\ \alpha(L) &= \sum_{i=1}^{\overline{q}} \boldsymbol{\alpha}_i L^i \qquad \beta(L) = \sum_{i=1}^{\overline{p}} \boldsymbol{\beta}_i L^i \end{aligned} \tag{15}$$

where the parameter matrices have the following structure: if we group the n variables in w sets of dimension $m_1, m_2 ... m_w$ and we indicate with $i(y)$ a column vector of ones if dimension y, then

$$\boldsymbol{\alpha}_i = \begin{bmatrix} \alpha_{i,11} i(m_1) i(m_1)' & \alpha_{i,12} i(m_1) i(m_2)' & \cdots & \alpha_{i,w1} i(m_1) i(m_w)' \\ \alpha_{i,12} i(m_2) i(m_1)' & \alpha_{i,22} i(m_2) i(m_2)' & & \alpha_{i,w2} i(m_2) i(m_w)' \\ \vdots & & \ddots & \vdots \\ \alpha_{i,w1} i(m_w) i(m_1)' & \alpha_{i,w2} i(m_w) i(m_2)' & \cdots & \alpha_{i,ww} i(m_w) i(m_w)' \end{bmatrix} \tag{16}$$

similarly for $\boldsymbol{\beta}_i$. It is worth noting that the number of sets w and their dimensions $m_1, m_2 ... m_w$ must be constant between the $\boldsymbol{\alpha}_i$, and the $\boldsymbol{\beta}_i$. Clearly, to be competitive, this representation requires a small number of groups. This parameterisation has $(p+q+1) \times n + (\overline{p}+\overline{q}) \times w(w-1)/2$ parameters and it is evidently not useful in the bivariate case. However, this model requires several constraints to ensure positive definiteness of Q_t ($\boldsymbol{\alpha}_i$, and $\boldsymbol{\beta}_i$ are not positive definite). Moreover, these constraint cannot explicitly derived but must be imposed in the optimisation routines. As a result, the model is feasible with some additional restriction: $\boldsymbol{\alpha}_i$, and $\boldsymbol{\beta}_i$ must be block-diagonal matrices, with zeros in off-diagonal blocks. Let us call this model the Block Diagonal DCC. In that case, we must impose a constraint on the matrix $[1-\alpha(L)-\beta(L)] \circ \bar{Q}$ requiring its eigenvalues to be greater than zero. Furthermore, parameters included in the $\boldsymbol{\alpha}_i$ and in the $\boldsymbol{\beta}_i$ must be positive and satisfy the restriction $\boldsymbol{\alpha}_i + \boldsymbol{\beta}_i < 1$.

Our final purpose is the formulation of a model that preserves the parameter block structures and that does not require too complex parameter restrictions. Therefore, we suggest a further block structure model which can be viewed as a modified Franses and Hafner DCC:

$$Q_t = cc' \circ \bar{Q} + \alpha\alpha' \circ \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}_{t-1}' + \beta\beta' \circ Q_{t-1} \tag{17}$$

where α, β and c are partitioned vectors similar to

$$\alpha = \{\alpha_1, \alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_3, \alpha_3, \alpha_3, \}$$

ensuring the cross-product to be partitioned matrices. One of the new point is the constant. This DCC-type model requires the estimation of a constant term and therefore, the correlation targeting property is in general lost. The correlation targeting property can be obtained as a special case imposing a set of parameter restrictions: $\alpha_i\alpha_j + \beta_i\beta_j + c_i c_j = 1$ for $i, j = 1...n$. Furthermore, the parameters must be positive and satisfy the standard GARCH-type restrictions $\alpha_i\alpha_j + \beta_i\beta_j < 1$, needed to avoid explosive patterns in the correlations. Finally, positive definiteness is guaranteed: assume a suitable starting point is used, then Q_t is the sum of positive definite and semidefinite matrices. This model adds great flexibility compared to standard DCC type models allowing a feasible estimation of block-structures. Let us call this model the Flexible-DCC.

3 Estimation and testing

The estimation of the dynamic correlation models presented in this paper can be carried out by Quasi-Maximum Likelihood, following the approach suggested by Engle (2002). Let define by θ_1 the parameters of the univariate GARCH models and with θ_2 the parameters of the dynamic correlation structure. The likelihood of the model can be written as follows:

$$LogL\left(\theta_1, \theta_2 | X_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log\left(2\pi\right) + \log\left(|H_t|\right) + \mathbf{X}_t H_t^{-1}\mathbf{X}_t'\right] \tag{18}$$

or, exploiting the factorization of the variance-covariance matrix and defining $D_t = |diag\left(\sigma_{1,t}\sigma_{2,t}...\sigma_{n,t}\right)|$, as

$$LogL\left(\theta_1, \theta_2 | X_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log\left(2\pi\right) + \log\left(R_t\right) + 2\log\left(|D_t|\right) + \mathbf{X}_t' D_t^{-1} R_t^{-1} D_t^{-1}\mathbf{X}_t\right] \tag{19}$$

Engle suggests a first estimation stage where the correlation matrix is replaced by an identity matrix

$$LogL\left(\theta_1 | X_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log\left(2\pi\right) + \log\left(I_n\right) + 2\log\left(|D_t|\right) + \mathbf{X}_t' D_t^{-1} I_n^{-1} D_t^{-1}\mathbf{X}_t\right] \tag{20}$$

which is equivalent to univariate estimation of GARCH models, and a second step conditional on the parameters estimated in the first one

$$LogL\left(\theta_2 | \hat{\theta}_1, X_t\right) = -\frac{1}{2}\sum_{t=1}^{T}\left[k\log\left(2\pi\right) + \log\left(R_t\right) + 2\log\left(|D_t|\right) + \boldsymbol{\varepsilon}_t' R_t^{-1}\boldsymbol{\varepsilon}_t\right] \tag{21}$$

where $\varepsilon_t = D_t^{-1}\mathbf{X}_t$ are the first stage standardized residuals.

It is worth noting that such a procedure can be used also for the CCC-GARCH model, the constant correlation being estimated in the second step by simply $\varepsilon_t'\varepsilon_t/T$, which is exactly equal to $\bar{Q}$. This observation suggests a possible likelihood ratio test for constant correlation.

Standard likelihood ratio tests cannot eìbe used in comparing CCC and DCC to our BDDCC and FDCC models since they are not nested representations. Therefore, the comparison can be made only in terms of information criteria. Given that all models are estimated with the same two step approach, the information criteria to be compared will be derived on the basis of the second step likelihood (21).

4 An empirical application: sectorial asset allocation

In this section we consider an empirical application fucusing on a dataset of daily data from Italian Stock market indices and we tackle with a sectorial asset allocation problem. There are three major sectors that compose the Italian Mibtel general index: Industrials, Services, Finance. Each of this three sub index is further divided in several sub-sectors. The composition is summarised in Table 1.

INSERT TABLE 1

All time series were downloaded from DataStream and are expressed in Euro. They run from from January 1991 to September 2003, yielding more than 3000 daily observations. The problem of asynchronous data encountered by some authors (Corsetti et al. 2002) is not present in our case, since the closing prices are determined at the same hour in the same market (Italian Stock market). The returns are calculated, as usual, through a log difference transformation. In Table 2 we summarize the main statistics of each series (full sample daily statistics).

INSERT TABLE 2

The average return of the general Mibtel index is positive, but there are significant differences among the considered sector indexes. For example, the major Industrial sector return is negative while the return of the major Service sector is greatly positive, this is mainly due to the Public Utility Services sector. The sector analysis evidence great differences even for the annualised standard deviations, that vary between the 18.3 of the Real Estate sector and the 40,8 of the Finance Miscellaneous sector. The data presents also a skewness different from zero and a relevant excess kurtosis. The skewness report both positive (7 cases) and negative (13 cases) values with a prevalence, at the aggregate level, to be negative. Finally, the excess kurtosis is always positive, evidencing the presence of fat tails in the empirical distributions. The Jarque-Bera test clearly rejects the null hypothesis of normality for all the series (not reported in Table 2). The main object of our analysis is to the study of the correlations behaviour between these series. In Table 3 the major sector indexes and the Mibtel general index are considered and there is a high positive correlation for all the indexes. The unconditional empirical correlations between sector indexes are summarised in Table 4 and vary between 0.13 for Industrial misc and Finance misc and 0,83 for Banks and Insurance (not a surprising result). The average correlation among the indices composing major Industrial sectors is 0.48, while they are 0.45 and 0.51 for the major Services and Finance sectors, respectively. It is also interesting to observe that within these three groups, there are sector indexes more correlated to each other: Banks and Finance Holdings are the indices with the highest correlations, while Industrial Misc. and Finance Misc. report the lowest correlations (again not a surprising outcome).

INSERT TABLE 3 AND 4

If we consider a dynamic analysis of the time series of the sector indexes, the volatility is clearly far from being constant. The GARCH specifications can be useful to capture these features. Given the characteristics of the series, an asymmetric GARCH specification is considered in order to capture both excess kurtosis and asymmetric effects. The parsimonious GJR-GARCH specification is considered (Glosten et al. 1993). The results are summarised in Table 5. The parameters are, with the exception of a few cases, significant at the 5% confidence level.

INSERT TABLE 5

The analysis of the residuals (not included here) evidences that the GARCH specifications are not able to explain a significant part of the non normality of the series, with the only exception of the Minerals Metals series.

It is also interesting to analyse the behaviour of the correlations over time, evaluating if their values are stable or not. Considering rolling empirical correlations it is interesting to observe that almost all the correlations vary through time and also that they present different patterns. For the sake of simplicity, the analysis is initially restricted to the three major sectors

(Industrial, Service, Finance) compared to the Mibtel general index. Figure 1 evidences the dynamic correlation between the general index and each major sectors, while Figure 2 shows the dynamic correlation between the major sectors. The figures consider a larger sample ranging from 1991 to present. In particular, Figure 1 points out that the correlation are positive and generally high during the sample period for all the major sector indexes while their dynamics is very different. Let us consider for example the first part of 2000: the Finance index correlation exhibits a sharp fall, the Service index correlation remains nearly constant and the Industrial index correlation increases. Even the correlations between the major sector indexes present very dissimilar patterns.

INSERT FIGURE 1

INSERT FIGURE 2

Extending the analysis of the correlation dynamics to the sub-sectors, other considerations are possible. In general the correlation patterns are similar for series of the same major sector and different for series of different major sectors. For example, the dynamics of the correlation between the Food and Paper sector indexes is similar to the one between the Cars and Minerals Metals sector indexes while it differs from that between Chemicals and Finance Holding sector indexes.

INSERT FIGURE 3

In ordeer to describe these dynamic patterns dynamic correlation modesl are estimated. The results for the CCC-GARCH proposed by Bollerslev (1990) and the DCC-GARCH proposed by Engle (2002) are summarized in Tables 6 and 7. In both cases the correlations are estimated by the two step procedure described in Section 3 on the last 4 years of the sample.

INSERT TABLE 6

INSERT TABLE 7

Differently, Table 8 reports the DCC model estimated on the three macro sectors: it is evident that parameters are different between the sectors. Moreover, Figures (4) and (5) report some dynamic correlations obtained by the models of Table 8. The evident differences are not only in the parameters but also in the patterns.

INSERT TABLE 8

INSERT FIGURE 4

INSERT FIGURE 5

We move then to a FDCC type model leaving the empirical application of the BDDCC to future studies. The model considers three groups of assets (the three macro indices) and therefore requires the estimation of 9 parameters. The results are reported in Table 9 (for comparability, even in this case we restricted our attention on the last 4 years of the sample).

INSERT TABLE 9

The advantages of moving to a FDCC model are clearly evidenced by a standard likelihood ratio test which rejects the null hypothesis of DCC model (common parameters in all blocks and correlation targeting restriction - LR statistics is in the order of 975). Moreover, all parameters are highly significant (quasi maximum likelihood standard errors). CPU time needed for FDCC estimation is around 20 minutes (4 year daily data).

The comparison between correlation models cannot be restricted to a pure statistical analysis but should be combined with some empirical evidence. For this reason, we considered a simulated exercise. Within a Markovitz approach and in more a restricted sample (last 2 years data) we estimate mean variance portfolios with CCC, DCC and FDCC time varying correlations models. Portfolio weights are computed assuming no risk-free asset, with or without positivity restrictions (short selling), no transaction costs and two weeks revision (10 days). The models are estimated every 10 days, one-step-ahead correlations and variances forecasts are computed and stored. With the one-step-ahead variance-covariance matrix forecasts a mean-variance problem is considered. Additional assumptions refer to the index returns on which we base portfolio weights computation and on the portfolio required return. For the first we consider last two months returns to get a closer matching with maket movements. In portfolio weights computation we consider several cases of portfolio returns: last two months return of an equally weighted portfolio; last two month return of the general Mibtel index; the global optimal portfolio; a 20% annual return. Table 10 reports the optimal portfolio annualised standard deviations of the final estimation (last two months of the sample):

INSERT TABLE 10

Moving from the constant correlation assumption to Engle's DCC model the optimal portfolio variance marginally decreases; this result is stable over the whole sample considered. Differently, the variance reduction implied by our Flexible DCC is more evident. Table 11 reports the estimated portfolio weights for global optimal portfolios in the final estimation.

INSERT TABLE 11

In the following step we assume that parameters are stable for a two week horizon and we computed a 10-day sequence of one-step-ahead forecasts of the variance-covariance matrix. These forecasts have then been used to compute portfolio returns for the constrained and unconstrained cases and the several assumptions on portfolio returns. As a general result we can state that, under the same assumption for portfolio returns, the FDCC model provides the lowest portfolio variance and the highest portfolio return. Figure 6 reports the returns evolution over one year of a particular case: portfolio weights are the global optimal portfolio one's.

INSERT FIGURE 6

5 Conclusions

We propose an extension of the new class of models recently proposed by Engle (2002), that both preserves the ease of estimation of the Bollerslev's constant correlation model but allows the correlations to change over time. Engle indeed added to the CCC a limited dynamic in the correlations, introducing a GARCH-type structure. However, the dynamic is constraint to be equal for all the correlations. However, this is an unnecessary restriction, thus we extend the DCC model introducing a constant and a block-diagonal structure that solves this problem. The dynamic is constrained to be equal only among groups of variables. In fact, we cannot impose that the correlations of, for example European sectorial stock indexes are equal to the correspondent US ones. Keeping the ease of estimation of the Engle's model, the extension we propose allows richer dynamics of the correlations.

After discussing the estimation and testing issues, we consider an empirical application of the three models (CCC, DCC and Flexible DCC). The variables object of analysis are sectorial stock indexes representing the major disaggregation of the Italian general stock index. The estimates of the three models confirm, for the period of analysis, the presence of dynamics in the correlations, as well as for the volatility, but also evidence the presence of dissimilarities in these dynamics. A simulated portfolio allocation excercise (under a Markovitz approach) shows that the FDCC model provide the lowest optimal portfolio variance and the highest portfolio returns.

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<table>
<tr><td rowspan="20">MIBTEL (General)</td><td rowspan="10">INDUSTRIAL</td><td>FOOD</td></tr>
<tr><td>CARS</td></tr>
<tr><td>PAPER</td></tr>
<tr><td>CHEMICALS</td></tr>
<tr><td>CONSTRUCTION</td></tr>
<tr><td>ELECRONICS</td></tr>
<tr><td>PLANTS MACHINE</td></tr>
<tr><td>INDUSTRIALS MISC</td></tr>
<tr><td>MINERALS METALS</td></tr>
<tr><td>TEXTILE CLOTHING</td></tr>
<tr><td rowspan="4">SERVICE</td><td>DISTRIBUTION</td></tr>
<tr><td>MEDIA</td></tr>
<tr><td>PUBLIC UTILITY SERVICES</td></tr>
<tr><td>TRANSPORT TOURISM</td></tr>
<tr><td rowspan="6">FINANCE</td><td>INSURANCE</td></tr>
<tr><td>BANKS</td></tr>
<tr><td>FINANCE HOLDINGS</td></tr>
<tr><td>FINANCE MISC</td></tr>
<tr><td>REAL ESTATE</td></tr>
<tr><td>FINANCE SERVICES</td></tr>
</table>

Table 1: Italian indexes composition.

	Mean	Standard deviation	Asymmetry	Excess Kurtosis
MIBTEL	***0.015***	***20.346***	***-0.596***	***2.707***
INDUSTRIAL	**-0.001**	**19.848**	**-0.694**	**3.611**
FOOD	0.013	23.956	-0.193	4.097
CARS	-0.038	28.164	-0.387	2.419
PAPER	-0.043	24.914	0.175	3.677
CHEMICALS	-0.006	21.920	-0.832	4.586
CONSTRUCTION	0.000	21.318	-0.342	2.931
ELECRONICS	-0.013	25.212	-0.160	2.607
PLANTS & MACHINE	0.021	21.654	-0.265	4.421
INDUSTRIALS MISC	-0.001	27.743	0.694	6.266
MINERALS METALS	0.001	24.231	-0.378	2.880
TEXILE CLOTHING	0.022	20.475	-0.689	3.853
SERVICE	**0.051**	**23.108**	**-0.388**	**1.803**
DISTRIBUTION	0.014	22.090	1.392	17.425
MEDIA	0.006	26.337	1.550	14.404
PUB. UTIL. SERV.	0.059	25.471	-0.516	2.158
TRANS & TOURISM	0.032	21.411	0.071	8.992
FINANCE	**0.007**	**21.553**	**-0.612**	**3.083**
INSURANCE	0.011	23.439	-0.438	2.567
BANKS	0.012	22.617	-0.413	4.096
FINANCE HOLDINGS	-0.022	23.263	-0.532	2.490
FINANCE MISC.	0.004	40.803	-2.175	18.698
REAL ESTATE	-0.004	18.273	-0.346	8.988
FINANCE SERVICES	0.013	25.923	0.411	6.975

Table 2: Summary statistics – Daily data – Annualised standard deviations.

	GENERAL	**INDUSTRIALS**	**SERVICES**	**FINANCE**
GENERAL	1	0.924	0.922	0.953
INDUSTRIALS	0.924	1	0.799	0.849
SERVICES	0.922	0.799	1	0.804
FINANCE	0.953	0.849	0.804	1

Table 3: Empirical correlations.

FOOD	CARS	PAPER	CHEMICALS	CONSTRUCTION	ELECRONICS	PLANTS & MACHINE	INDUSTRIALS MISC	MIN	TEX	DISTRIBUTION	MEDIA	PUB. UTIL. SERV.	TRANS & TOURISM	INSURANCE	BANKS	FINANCE HOLDINGS	FINANCE MISC.	REAL ESTATE	FINANCE SERVICES
1	0.571	0.422	0.592	0.587	0.480	0.496	0.275	0.453	0.542	0.396	0.311	0.562	0.491	0.621	0.611	0.645	0.193	0.407	0.449
---	1	0.479	0.653	0.626	0.568	0.529	0.296	0.479	0.596	0.448	0.374	0.618	0.496	0.698	0.676	0.719	0.225	0.444	0.530
---	---	1	0.495	0.517	0.452	0.430	0.252	0.384	0.473	0.325	0.303	0.476	0.414	0.512	0.516	0.531	0.182	0.338	0.401
---	---	---	1	0.664	0.636	0.549	0.307	0.507	0.630	0.469	0.419	0.657	0.520	0.695	0.705	0.726	0.259	0.481	0.524
---	---	---	---	1	0.604	0.563	0.346	0.486	0.621	0.489	0.378	0.637	0.562	0.686	0.701	0.728	0.261	0.520	0.539
---	---	---	---	---	1	0.475	0.276	0.399	0.584	0.491	0.559	0.716	0.513	0.650	0.677	0.729	0.362	0.492	0.556
---	---	---	---	---	---	1	0.254	0.534	0.532	0.392	0.318	0.513	0.468	0.598	0.602	0.579	0.224	0.409	0.442
---	---	---	---	---	---	---	1	0.242	0.303	0.245	0.177	0.288	0.272	0.297	0.293	0.346	0.134	0.249	0.234
---	---	---	---	---	---	---	---	1	0.461	0.334	0.290	0.513	0.440	0.557	0.552	0.514	0.165	0.364	0.386
---	---	---	---	---	---	---	---	---	1	0.463	0.431	0.628	0.494	0.671	0.680	0.657	0.255	0.436	0.494
---	---	---	---	---	---	---	---	---	---	1	0.406	0.509	0.404	0.499	0.518	0.544	0.225	0.357	0.398
---	---	---	---	---	---	---	---	---	---	---	1	0.494	0.351	0.466	0.496	0.539	0.317	0.359	0.391
---	---	---	---	---	---	---	---	---	---	---	---	1	0.526	0.731	0.725	0.733	0.277	0.454	0.538
---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.569	0.586	0.590	0.219	0.441	0.462
---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.832	0.750	0.275	0.493	0.599
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.760	0.299	0.515	0.627
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.339	0.549	0.607
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.266	0.271
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.433
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1

Table 4: Empirical correlations.

FOOD	CARS	PAPER	CHEMICALS	CONSTRUCTION	ELECRONICS	PLANTS & MACHINE	INDUSTRIALS MISC	MIN	TEX	DISTRIBUTION	MEDIA	PUB. UTIL. SERV.	TRANS & TOURISM	INSURANCE	BANKS	FINANCE HOLDINGS	FINANCE MISC.	REAL ESTATE	FINANCE SERVICES
1	0.547	0.400	0.577	0.552	0.483	0.482	0.277	0.436	0.510	0.378	0.319	0.537	0.474	0.597	0.580	0.628	0.189	0.401	0.415
---	1	0.457	0.635	0.592	0.569	0.501	0.291	0.454	0.559	0.443	0.387	0.603	0.479	0.672	0.645	0.713	0.227	0.438	0.501
---	---	1	0.471	0.491	0.447	0.404	0.231	0.359	0.454	0.340	0.304	0.460	0.398	0.499	0.494	0.526	0.190	0.332	0.372
---	---	---	1	0.640	0.625	0.533	0.309	0.483	0.597	0.473	0.415	0.641	0.505	0.684	0.690	0.715	0.249	0.469	0.485
---	---	---	---	1	0.597	0.546	0.327	0.462	0.588	0.482	0.397	0.609	0.551	0.660	0.680	0.705	0.255	0.494	0.493
---	---	---	---	---	1	0.478	0.278	0.406	0.567	0.483	0.483	0.721	0.517	0.637	0.675	0.728	0.320	0.477	0.515
---	---	---	---	---	---	1	0.266	0.516	0.509	0.381	0.332	0.507	0.456	0.571	0.581	0.570	0.220	0.388	0.407
---	---	---	---	---	---	---	1	0.255	0.280	0.243	0.169	0.282	0.268	0.303	0.308	0.340	0.129	0.239	0.240
---	---	---	---	---	---	---	---	1	0.426	0.337	0.322	0.491	0.439	0.525	0.534	0.501	0.171	0.369	0.371
---	---	---	---	---	---	---	---	---	1	0.444	0.410	0.606	0.463	0.628	0.652	0.637	0.242	0.420	0.452
---	---	---	---	---	---	---	---	---	---	1	0.364	0.503	0.402	0.495	0.519	0.532	0.218	0.357	0.382
---	---	---	---	---	---	---	---	---	---	---	1	0.477	0.339	0.460	0.496	0.501	0.230	0.320	0.330
---	---	---	---	---	---	---	---	---	---	---	---	1	0.529	0.705	0.713	0.728	0.262	0.447	0.512
---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.563	0.575	0.578	0.223	0.427	0.430
---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.814	0.736	0.271	0.486	0.543
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.755	0.303	0.504	0.576
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.308	0.527	0.576
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.244	0.219
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0.407
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1

Table 6: CCC correlation estimates.

	ω	α	γ	β
FOOD	0.007667 *0.044489*	0.17429 *0.041231*	-0.042721 *0.016308*	0.945534 *0.019298*
CARS	-0.018458 *0.033559*	0.181820 *0.025871*	-0.049772 *0.014939*	0.960047 *0.012635*
PAPER	0.133547 *0.080686*	0.190099 *0.027615*	-0.115743 *0.021219*	0.903522 *0.031548*
CHEMICALS	0.06875 *0.052705*	0.210777 *0.048487*	-0.042927 *0.01917*	0.905615 *0.028169*
CONSTRUCTION	-0.017683 *0.040712*	0.206656 *0.035078*	-0.038557 *0.015415*	0.945024 *0.013988*
ELECRONICS	-0.105743 *0.047497*	0.273387 *0.043656*	-0.037565 *0.018623*	0.960144 *0.016466*
PLANTS & MACHINE	-0.001862 *0.052135*	0.257848 *0.038020*	-0.135494 *0.018234*	0.9174 *0.022963*
INDUSTRIALS MISC	0.043335 *0.049291*	0.200922 *0.031524*	-0.033979 *0.015484*	0.931596 *0.021698*
MINERALS METALS	0.136145 *0.112448*	0.254659 *0.051434*	-0.06675 *0.025988*	0.867494 *0.048532*
TEXILE CLOTHING	-0.006179 *0.067197*	0.304888 *0.055493*	-0.103177 *0.021758*	0.896652 *0.023522*
DISTRIBUTION	0.126355 *0.080918*	0.356644 *0.056348*	-0.104257 *0.021247*	0.840823 *0.041691*
MEDIA	-0.091437 *0.039332*	0.371298 *0.039381*	-0.004515 *0.018767*	0.924889 *0.017853*
PUB. UTIL. SERV.	0.056750 *0.088477*	0.164620 *0.045733*	-0.022391 *0.018623*	0.929796 *0.038658*
TRANS & TOURISM	0.034109 *0.086364*	0.341412 *0.041971*	-0.018215 *0.021881*	0.871284 *0.037845*
INSURANCE	0.145923 *0.110749*	0.267765 *0.041751*	-0.062281 *0.021792*	0.857678 *0.049524*
BANKS	-0.004814 *0.080188*	0.339686 *0.046219*	-0.046271 *0.026547*	0.892956 *0.036466*
FINANCE HOLDINGS	0.003824 *0.054225*	0.247872 *0.045694*	-0.025745 *0.020524*	0.923353 *0.025147*
FINANCE MISC.	-0.009788 *0.034614*	0.443871 *0.034797*	-0.086374 *0.023577*	0.906579 *0.014537*
REAL ESTATE	-0.067388 *0.035504*	0.312774 *0.026917*	0.014675 *0.016420*	0.923310 *0.016421*
FINANCE SERVICES	0.125354 *0.075030*	0.334182 *0.043659*	-0.055903 *0.023690*	0.859555 *0.036751*

Table 5: GARCH specifications and parameter estimates (standard deviations in italic).

Parameters	Estimates	Standard deviations	z-statistics
α	0.02077	8e-5	263.543
β	0.48970	0.00339	144.275
	Log Likelihood: -9810.1293		

Table 7: DCC estimates – full sample – all sectors

INDUSTRIALS			
Parameters	Estimates	Standard deviations	z-statistics
α	0.01224	0.00085	14.344
β	0.80220	0.02581	31.081
	Log Likelihood: -4975.616		
SERVICE			
Parameters	Estimates	Standard deviations	z-statistics
α	0.02726	0.00037	74.414
β	0.51168	0.01071	47.790
	Log Likelihood: -3973.119		
FINANCE			
Parameters	Estimates	Standard deviations	z-statistics
α	0.02651	0.00039	68.191
β	0.93092	0.00114	813.988
	Log Likelihood: -1351.378		

Table 8: DCC estimates – full sample – macro sectors

Parameters	Estimates	Standard deviations	z-statistics
c_1	0.706864	0.00510	138.60
c_2	0.999311	0.01574	63.488
c_3	0.039117	0.00026	150.45
a_1	0.09456	0.00186	50.760
a_2	0.06454	0.00280	22.979
a_3	0.01931	0.00048	40.172
b_1	0.63063	0.00472	133.53
b_2	0.98445	0.00018	5292.6
b_3	0.92461	0.00096	956.65
	Log-Likelihood -9321.6949		

Table 9: FDCC estimates – full sample – all sectors

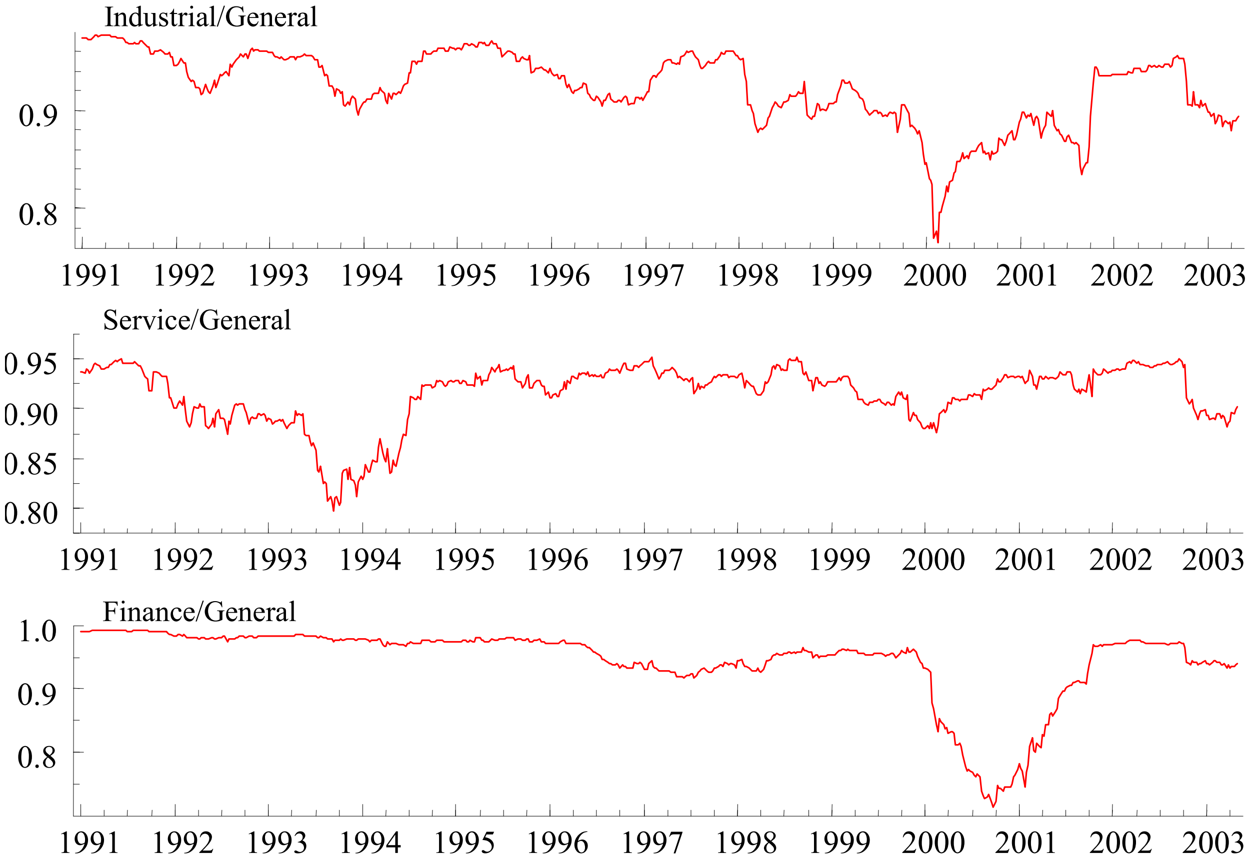

Figure 1: Correlation between Mibtel general index and major sectors indexes.

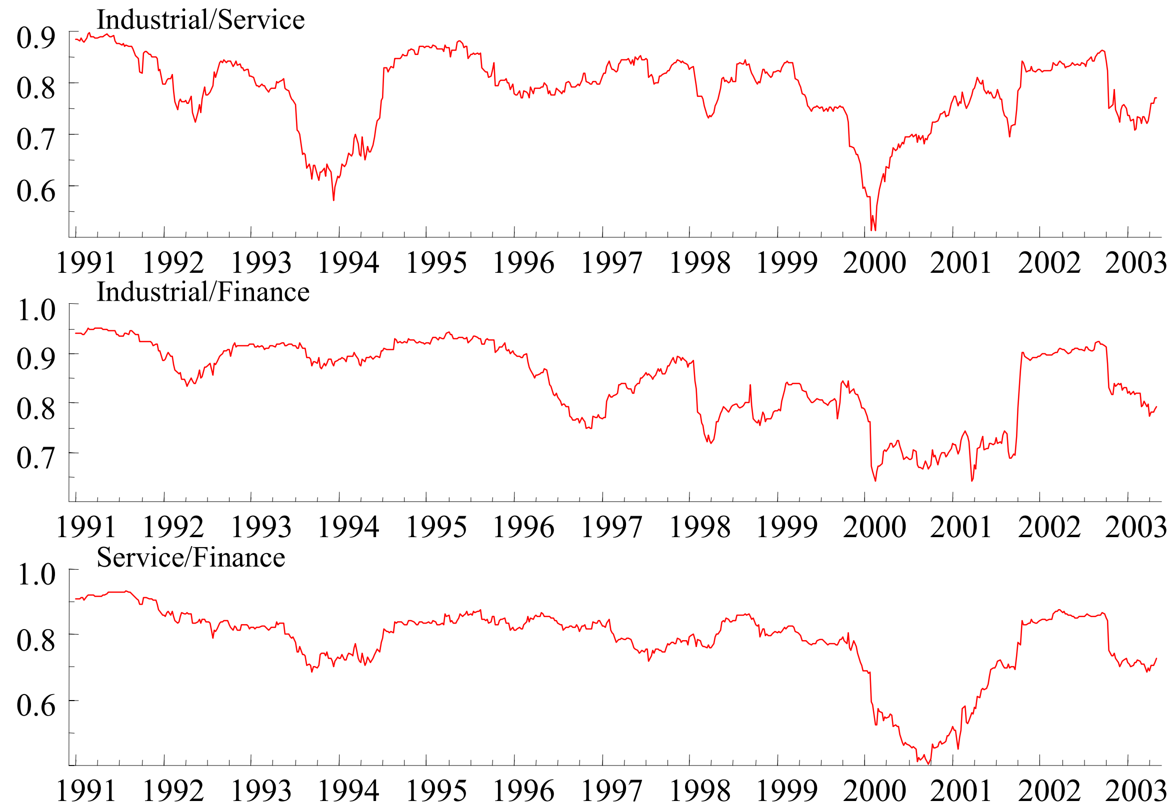

Figure 2: Correlation between major sector indexes.

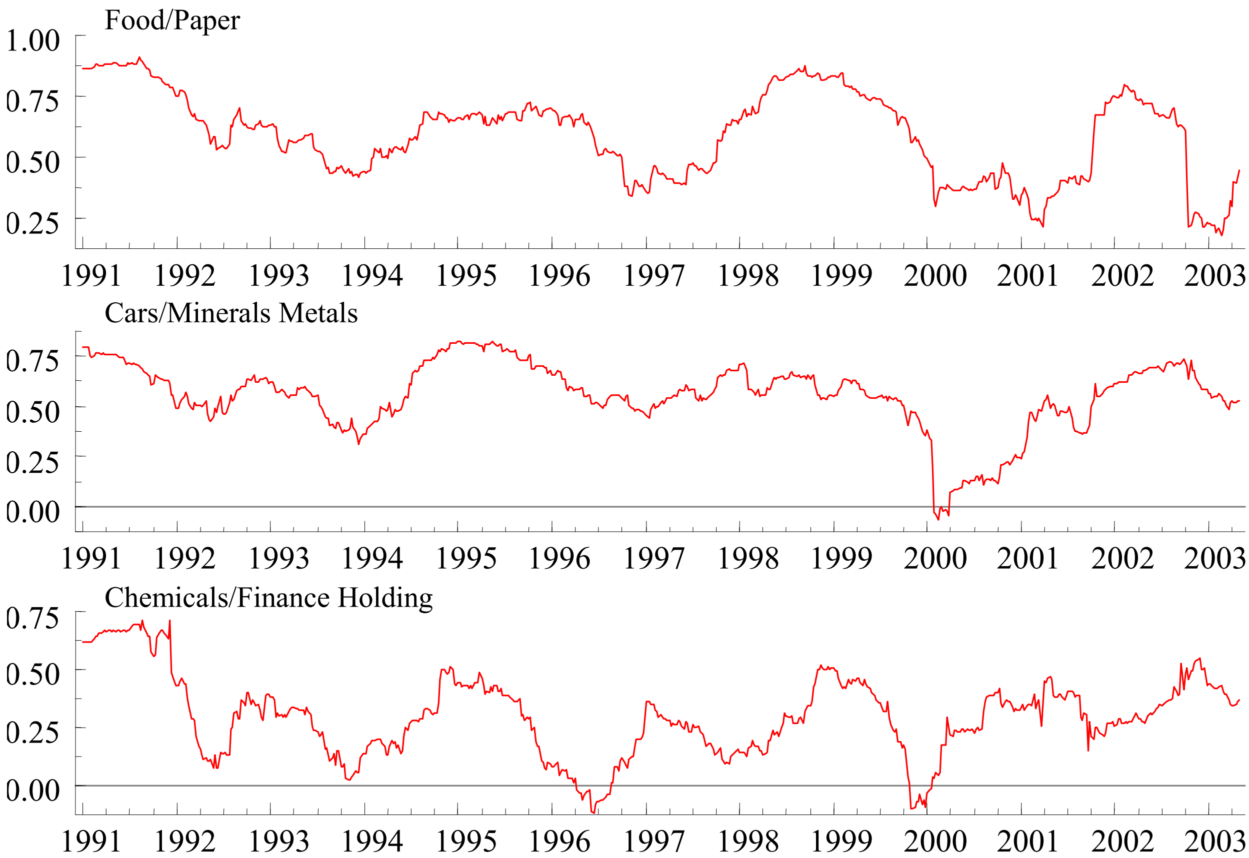

Figure 3: Correlation dynamics.

CCC	DCC	FDCC		CCC	DCC	FDCC
0.050	0.048	0.050	*FOOD*	0.064	0.063	0.063
---	---	---	*CARS*	-0.030	-0.031	-0.031
0.013	0.014	0.026	*PAPER*	0.030	0.031	0.026
---	---	---	*CHEMICALS*	-0.076	-0.076	-0.061
0.038	0.038	0.066	*CONSTRUCTION*	0.096	0.096	0.101
---	---	---	*ELECRONICS*	0.027	0.026	0.015
0.010	0.011	0.007	*PLANTS MACHINE*	0.051	0.051	0.040
0.268	0.267	0.230	*INDUSTRIALS MISC*	0.249	0.249	0.220
---	---	---	*MINERALS METALS*	-0.037	-0.036	-0.011
---	---	---	*TEXTILE CLOTHING*	-0.018	-0.018	-0.032
0.071	0.069	0.065	*DISTRIBUTION*	0.068	0.067	0.060
---	---	---	*MEDIA*	0.032	0.036	0.006
---	---	0.017	*PUBLIC UTILITY SERVICES*	0.113	0.113	0.056
0.192	0.193	0.203	*TRANSPORT TOURISM*	0.173	0.174	0.174
---	---	---	*INSURANCE*	-0.059	-0.066	-0.012
---	---	---	*BANKS*	-0.046	-0.043	-0.026
---	---	0.022	*FINANCE HOLDINGS*	0.004	0.006	0.082
---	---	---	*FINANCE MISC*	-0.025	-0.025	-0.018
0.358	0.359	0.306	*REAL ESTATE*	0.364	0.365	0.328
---	---	0.007	*FINANCE SERVICES*	0.019	0.019	0.022

Table 11: Portfolio allocation in the Markovitz approach in a constrained and a non constrained problem (without risk free asset) – global optimal portfolios

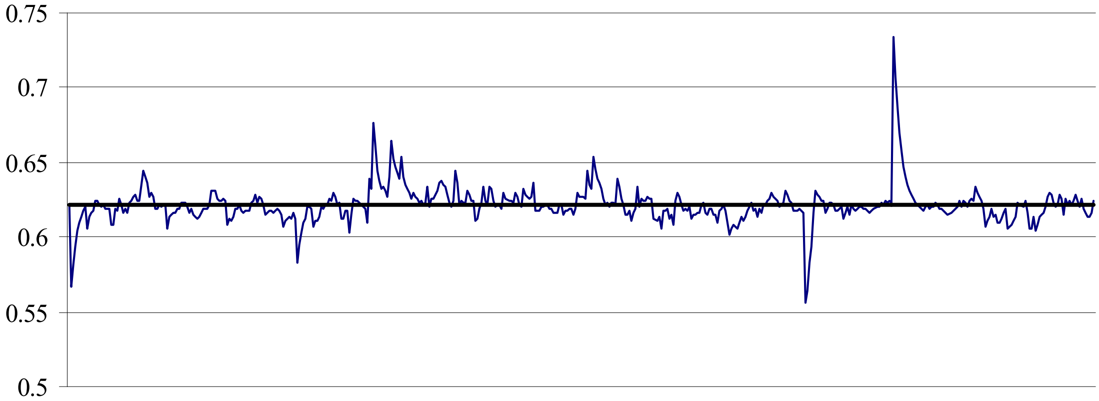

Figure 4: Static (CCC) and Dynamic correlations (bold and blue lines, respectively) between Real Estate and Finance Service indices (last two years)

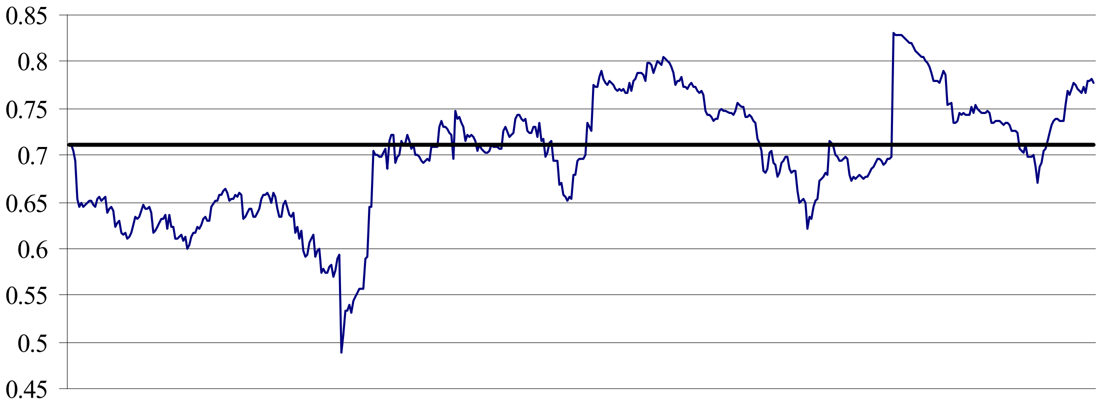

Figure 5: Static (CCC) and Dynamic correlations (bold and blue lines, respectively) between Food and Cars indices (last two years)

Type	Portfolio Return	Correlation Model		
		CCC	DCC	FDCC
Unconstrained	Equally Weighted	6.642	6.637	6.304
	Mibtel	6.291	6.285	6.051
	Global Optimal	6.276	6.270	6.051
	20%	6.732	6.728	6.387
Constrained	Equally Weighted	7.433	7.419	6.870
	Mibtel	6.781	6.765	6.308
	Global Optimal	6.759	6.743	6.314
	Max index return	17.231	17.229	16.741

Table 10: annualised optimal portfolio variances based on last two months of the sample; last two months annualised Mibtel standard deviation is 8.214; optimal portfolio variances depend on the required portfolio return which is in turn set equal to: last two month return of an equally weighted portfolio; last two month return of the Mibtel index; global optimal portfolio; 20% annual return (unconstrained only); among the 20 sectorial indices, the maximum last two moths return.

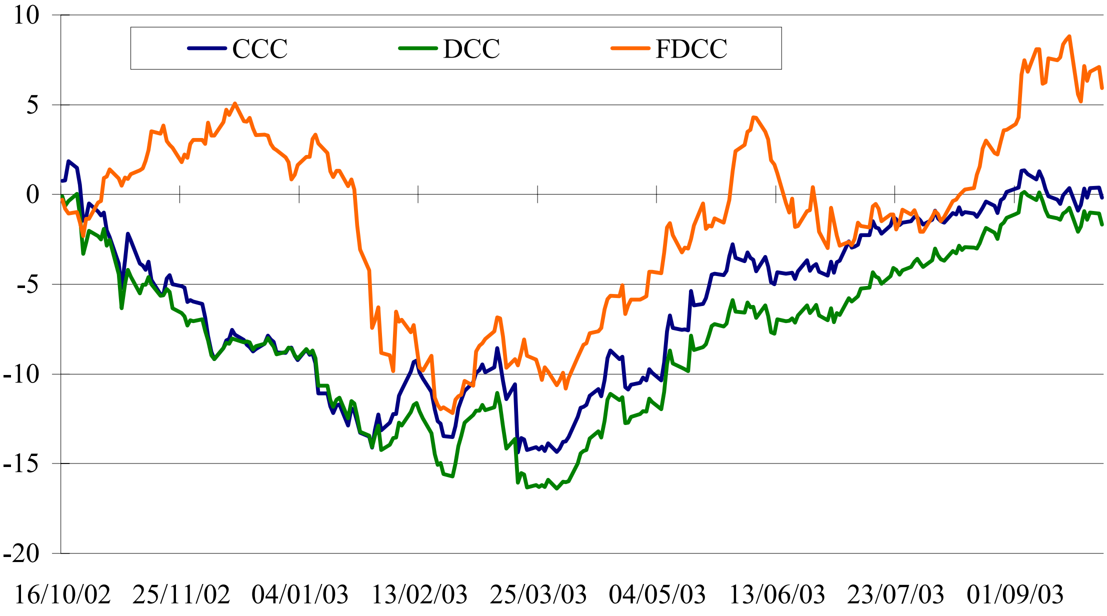

Figure 6: portfolio returns - unconstrained Markovitz approach with objective return set to 20% (annual return)