Liquidity Derivatives

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ABSTRACT

It is well established that investors price market liquidity risk. Yet, there exists no financial claim contingent on liquidity. We propose a contract to hedge uncertainty over future transaction costs, detailing potential buyers and sellers. Introducing liquidity derivatives in Brunnermeier and Pedersen (2009) improves financial stability by mitigating liquidity spirals. We simulate liquidity option prices for a panel of NYSE stocks spanning 2000 to 2020 by fitting a stochastic process to their bid-ask spreads. These contracts reduce the exposure to liquidity factors. Their prices provide a novel illiquidity measure reflecting cross-sectional commonalities. Finally, stock returns significantly spread along simulated prices.

JEL classification: G12, G13, G17

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1 Introduction

Derivatives contracts allow market participants to exchange money and risk. Prominent examples include interest rate and volatility derivatives. But investors are still exposed to the risk of changes in market liquidity. The market liquidity of a security refers to the degree to which an order can be executed at a price close to the consensus value of the asset (Foucault et al., 2013). Liquidity risk, defined as the uncertainty about future market liquidity, has a strong impact on asset returns (Amihud and Mendelson, 1986). For instance, the aggregate liquidity factor of Pástor and Stambaugh (2003) earns a sizeable annualized risk premium of 4.81% over the period 1968-2020 and of 5.68% between 2000 and 2020, despite the general improvement in liquidity conditions in the last years. From an aggregate perspective, liquidity is known to exacerbate marketwide swings. Market liquidity deteriorates the most precisely at times of economic downturns, as trading frenzies widen the gap between sell and buy volumes.[1]

In this paper, we propose a new class of financial instruments designed to hedge against fluctuations in market liquidity. Liquidity derivatives are contracts which condition their payoff on metrics of liquidity of the underlying asset, such as the bid-ask spread. Informed by a structural model, we show empirically that heterogeneous agents have the incentive to trade liquidity derivatives and that these instruments have the potential to mitigate systemic risk by allowing leveraged investors to derive counterbalancing profits when prices deviate from fundamentals. Ultimately, liquidity derivatives are a natural remedy to the amplification dynamics that are typical of financial crises, which often require the intervention of Central Banks in their role of lenders of last resort.

Liquidity derivatives have already sparked the interest of the financial industry. In 2010, Citigroup pondered the rollout of derivatives based on a liquidity index reflecting five indicators from the swap and options market to allow investors to trade on liquidity, the CLX. Terry Benzschawel, managing director at Citi, commented on the product as follows.

> "We want to get natural buyers and sellers of liquidity together. We do have an explicit hedging programme, based on the underlying assets in the index. There is a basis risk, but the beauty is that as this widens, the strategy involves buying up assets whose prices are falling, thereby providing liquidity to the market."

Risk

[1] *Illiquidity – the difficulty of selling assets at a reasonable price – is at the heart of all financial crises.* (The Economist).

To date, liquidity derivatives have not yet been marketed, perhaps because the challenges of trading on liquidity are manyfold. For one, liquidity is multifaceted, requiring the selection of a suitable proxy. Several measures of liquidity are available, but none can be replicated by a portfolio. Further, academics and regulators might worry about the tail risk of the contract which requires the seller to have deep pockets, limited exposure, and a hedging strategy. This paper addresses these challenges and shows that liquidity derivatives are a market-based solution to downward price-liquidity spirals. The lack of a cash-and-carry arbitrage is common to several financial products. For instance, in the large market for commodity derivatives the volatility of the underlying price is unspanned (Trolle and Schwartz, 2009), and in the case of weather derivatives traded risks originate even outside of the real economy. It is imporant to recognize that the risk of extreme liquidity dry-ups is already borne by designated market makers (DMMs), who routinely agree with firms to quote a maximum bid-ask spread on their stocks and a minimum depth in exchange for an annual fee (Venkataraman and Waisburd, 2007). The principle of efficient allocation of resources in financial markets suggests that liquidity risk can be optimally distributed among investors by means of contingent claims.

We begin with a simple model to show theoretically that the lack of a market for liquidity is responsible for the spiral between market and funding liquidity, highlighting the benefits of the proposed class of derivatives for financial stability. In Brunnermeier and Pedersen (2009), an adverse shock to arbitrageurs' wealth reduces their liquidity provision to customers, in turn impacting prices and leading financiers to increase margin requirements, which again tighten the capital constraint of arbitrageurs. As prices deviate from fundamentals, arbitrageurs might also come across losses on existing positions. Liquidity derivatives appreciate precisely when such deviations arise, stabilizing markets and correcting both amplifying dynamics. Arbitrageurs disproportionately value payoffs at times of low market liquidity, and have the reciprocated incentive to provide customers with a payment in exchange for insurance against illiquidity. Deviations of prices from fundamentals increase the value of arbitrageurs' liquidity derivatives holdings, whose collateral value also rises exactly at times of margin calls, thus softening the blow of both loss and margin spirals.

Next, we propose a contract designed to effectively strip liquidity risk from financial instruments in exchange for an upfront payment. We view liquidity derivatives as option contracts based on the evolution of the relative bid-ask spread of a reference asset as recorded by an independent reporting entity, whose payoff cumulates the transaction costs per unit of notional in excess of their value at the initiation of the contract. Thus, liquidity derivatives appreciate when illiquidity is high or sustained. While other classes of derivatives could also reference market liquidity, we focus on options which resonate with insurance purposes since money changes hands when the parties close the deal, reflect the volatility of the underlying process, and build on a vastly explored pricing theory. We detail potential buy and sell sides of the market, which we derive from a comparative argument advantage. Clearly, transaction costs are relatively more important for traders with high turnover such as hedge funds than they are for buy-and-hold investors like pension funds, who reap the benefit of higher returns mandated by unmarketable instruments without their actual liquidation. These two categories of investors have opposite downsides with respect to liquidity risk. Empirical support to this idea comes from the juxtaposition between the exposure to the Pástor and Stambaugh (2003) (PS) liquidity factor of two publicly available aggregate indices, the Barclay Hedge Fund Index and the S&P Insurance Select Industry Index, after controlling for the Fama and French (2015) factors plus momentum. While the loadings on PS are similar in absolute value, insurance companies are positively exposed to liquidity risk and hedge funds are negatively exposed to it, suggesting that arbitrageurs are naturally eager to trade contracts based on liquidity with long-term investors. At the daily frequency, relative bid-ask spreads are positive, mean reverting, and exhibit volatility clusters. Guided by Cox et al. (1985), we price liquidity derivatives in an equilibrium framework where the risk compensation reflects the estimated magnitude and volatility of future transaction costs and their comovement with the market returns, along the lines of the liquidity-adjusted CAPM by Acharya and Pedersen (2005).

The third contribution of the paper is to assess empirically the properties of the proposed pricing method. We use Monte Carlo techniques to simulate a representative panel of model-consistent liquidity option prices for CRSP stocks traded on the NYSE in the period spanning 2000 to 2020, and analyse their empirical properties. The resulting median premium for a three-month horizon liquidity derivative is 67 basis points per unit of notional, and the distribution of prices is positively skewed and follows the recent decline in illiquidity documented in the literature. Liquidity derivatives significantly reduce the exposure of stock returns to the PS liquidity factor. Simulated prices provide a novel measure of illiquidity that reflect commonalities across stocks and peak during the NBER recessions, in line with other traditional measures such as Amihud (2002) and Pástor and Stambaugh (2003). Differently from benchmark liquidity proxies, our proposed measure embeds a compensation for the risk-adjusted forecasted comovement between illiquidity and the market. Equipped with the evidence that simulated prices capture well the component of returns induced by illiquidity, we show that portfolios of stocks sorted on their instrument-level liquidity risk generate significant abnormal returns with respect to classical asset pricing models including Fama and French (2015) factors plus momentum, both at daily and at monthly frequencies, suggesting that liquidity option prices represent a risk dimension yet to be explored. This finding is robust after controlling for the confounding effect of size, volatility, volume, turnover, and relative spread itself.

Literature Review Examples of academic research leading to innovation in the financial industry include Brenner and Galai (1989), who propose options referencing a volatility index which have been a useful guide to structure the contracts nowadays traded on the VIX. The seminal paper by Amihud and Mendelson (1986) documents that expected returns increase in assets' trading costs as measured by their bid-ask spread. Brennan and Subrahmanyam (1996) and Brennan et al. (1998) corroborate these findings by using alternative measures of illiquidity. Mahanti et al. (2008) measure liquidity as the degree to which assets are held by investors who are expected to trade more frequently. Other notable contributions in this area include Amihud (2002), who measures illiquidity as the price response to trading volume and shows that such measure commands higher expected returns, and Pástor and Stambaugh (2003), who develop a measure of liquidity based on return reversals. The insight of reversals as measures of liquidity is further explored in Nagel (2012). Amihud et al. (2015) find evidence that less liquid assets earn higher returns and shows that a portfolio of illiquid-minus-liquid stocks (IML) produces significant risk-adjusted returns in an international sample. Relatedly, Amihud and Noh (2021) find a time-varying IML premium which rises at times of financial distress. Unlike their work, we characterize a financial contract designed to separate the compensation commanded by the instrument-level market liquidity risk from asset returns. Derivatives based on market liquidity have not yet been explored in the literature. The only exception is Golts and Kritzman (2010), who propose a cliquet option on the S&P 500 as a reference process for market-wide illiquidity.[2] Our work explicitly focuses on contracts based on a measure of market liquidity, the relative bid-ask spread. We leave to future research the extension to other proxies such as the market depth. Beyond proposing a simple pricing model for liquidity derivatives, we simulate their prices for a representative panel of NYSE firms to evaluate their hedging properties. In doing so, we introduce a

[2] Bhaduri et al. (2007) discuss how to hedge the funding liquidity risk which originates from the lock-up of capital in managed funds, not to be confused with market liquidity risk.

novel price-based measure of liquidity risk. Further, we detail potential buyers and sellers of liquidity derivatives and analyse the systemic implications of such instruments for price stability.

We thus naturally relate to the theoretical asset pricing literature. Among others, Longstaff (1995) derives an upper bound to the discount resulting from the lack of marketability of a security. More recently, Acharya and Pedersen (2005) present a liquidity-adjusted CAPM where a stock's compensation depends on the interplay between its illiquidity and returns with market illiquidity and market returns. Our work is similar in spirit to Bongaerts et al. (2011), who develop an equilibrium framework with heterogeneous agents where illiquidity premia depend on the wealth, risk aversion, and trading horizon of short-sellers. We complement this approach by providing a pricing model for financial claims which condition their payoff on market liquidity in exchange for a prespecified cash amount. Indeed, early resolution of uncertainty over future transactions commands a premium in the cross-section (Schlag et al., 2021). Market liquidity is intimately connected to funding liquidity (Gromb and Vayanos, 2002; Garleanu and Pedersen, 2007; Pelizzon et al., 2016) and limits to arbitrage (surveyed in Gromb and Vayanos, 2010). We contribute to this literature by showing that a market for liquidity risk can improve efficiency and financial stability by preventing self-fulfilling impairments to orderly markets.

The remainder of the paper is organized as follows. Section 2 introduces an equilibrium model with heterogeneous agents and liquidity derivatives, and Section 3 proposes a contract contingent on liquidity suggesting a simple pricing algorithm. Section 4 develops testable hypotheses and provides empirical results. Section 5 concludes.

2 Theoretical Properties

We use a well-established, stylized environment to investigate the potential implications of a market for liquidity risk. Such framework should have heterogeneous agents, to investigate the scope for a market, and should allow to evaluate the systemic effects of liquidity on market stability. The natural reference is Brunnermeier and Pedersen (2009) (henceforth, B&P), that we enrich with a derivative contingent on the future liquidity of traded assets.[3] Section 2.1 briefly describes our framework. For ease of comparison with

[3] The model in B&P is the cornerstone of the intermediary asset pricing theory. Its empirical performance is strongly validated in Adrian et al. (2014), who estimate that the leverage of intermediaries is able to explain 77% of the cross-

the previous literature, we maintain intact the anatomy of the B&P model, preserving its assumptions and notation. Indeed, our framework includes B&P as a special case with only shares traded. Section 2.2 next shows that a derivative on liquidity is welfare-improving and alleviates liquidity spirals in the otherwise identical B&P model. In doing so, we make a clean case that the appetite for liquidity derivatives is a natural result of agents heterogeneity, and that the lack of a market for liquidity is responsible for the feedback loop between firesales and margin calls causing financial instability. We further show that this characteristic is not shared by other derivatives.

2.1 Model setup

Consider a B&P economy that features J risky assets traded at times $t = 0, 1, 2, 3$. At time $t = 3$, each security pays the fundamental value v^j that is a random variable defined in dollars amount on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The conditional expectation of the value of each stock j is $v_t^j = E_t[v^j]$ and follows ARCH dynamics, so that shocks to economic fundamentals increase future volatility. Formally,

$$\begin{aligned} v_{t+1}^j &= v_t^j + \Delta v_{t+1}^j = v_t^j + \sigma_{t+1}^j \varepsilon_{t+1}^j, \qquad && \varepsilon_t^j \overset{i.i.d.}{\sim} \mathcal{N}(0,1) \\ \sigma_{t+1}^j &= \underline{\sigma}^j + \theta^j |\Delta v_t^j| && \end{aligned} \tag{1}$$

with $\underline{\sigma}^j, \theta^j \geq 0$. However, the price of each stock p_t^j might in general differ from the associated expected fundamental value v_t^j because of temporary imbalances in the order flow (Grossman and Miller, 1988). Our measure of illiquidity is

$$\Lambda_t^j = p_t^j - v_t^j \tag{2}$$

This model features two contracts. Stocks with price p_t are ownership rights of a business that has fundamental value v^j. Liquidity derivatives are contingent claims on the illiquidity of the stock that are characterized by a price λ_t^j to be determined in equilibrium and a payoff $|\Lambda_{t+1}^j|$ that increases when subsequent price deviations from fundamentals occur either from above or from below. The buyer of the derivative pays a fixed premium at time t in exchange for a payoff contingent on illiquidity in date $t+1$, and the counterparty takes the opposite side of the trade. There are three groups of market participants, namely "customers" and "speculators," who trade assets and are informed about funda-

sectional variation in stock returns. The introduction of a derivative dovetails with the received Grossman and Miller (1988), on which B&P builds upon, who use a similar structure to address both the futures market and the underlying stock market.

mentals, and "financiers" who finance speculators but observe only prices as noisy signals of the value of businesses.

2.1.1 Customers

Three risk-averse customers are indexed by the time they start trading, $k = 0, 1, 2$. At time 0, costumer k has wealth W_0^k and zero shares, but discovers that she will experience an endowment shock of $\mathbf{z}^k = \{z^{1,k}, \ldots, z^{J,k}\}$ shares at time $t = 3$. The endowment shocks are random variables that for each stock sum to zero across customers. Denote the vector of total demand shock of customers that arrived to the market up to time t by $Z_t \equiv \sum_{k=0}^{t} \mathbf{z}^k$. Customers arrive sequentially to the exchange and it is their demand pressure that causes prices to temporarily deviate from fundamentals until the date $t = 2$ when all customers are present and $Z_2 = 0$. Before a customer arrives to the marketplace, her demand vectors for stocks and liquidity derivatives are $\mathbf{y}_t^k = 0$ and $\mathbf{c}_t^k = 0$, respectively. After arrival, customers choose their positions in each period to maximize exponential utility over terminal wealth $U(W_3^k) = -\exp\{-\gamma W_3^k\}$. Customers' wealth evolves according to

$$W_{t+1}^k = W_t^k + (\mathbf{p}_{t+1} - \mathbf{p}_t)'(\mathbf{y}_t^k + \mathbf{z}_k) + (\boldsymbol{\lambda}_t - |\boldsymbol{\Lambda}_{t+1}|)'\mathbf{c}_t^k \tag{3}$$

2.1.2 Speculators

Speculators, such as hedge funds, are risk-neutral maximizers of terminal wealth W_3 and derive profits from the order flow imbalance by providing immediacy to customers. On each date, they select their positions in stocks and derivatives $(\mathbf{x}_t, \mathbf{c}_t)$. Speculators' wealth is affected by an independent standard normal shock η_t, and evolves according to

$$W_t = W_{t-1} + (\mathbf{p}_t - \mathbf{p}_{t-1})'\mathbf{x}_{t-1} + (|\boldsymbol{\Lambda}_t| - \boldsymbol{\lambda}_{t-1})'\mathbf{c}_{t-1} + \eta_t \tag{4}$$

While speculators are leveraged, the total margin on their positions cannot exceed their capital W_t. Collateral requirements are set according to the following portfolio margining rule.[4]

$$\sum_j (x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}) + \sum_j (l_t^{j+} n_t^{j+} + l_t^{j-} n_t^{j-}) \leq W_t \tag{5}$$

[4]Portfolio margining, a margin-setting methodology based on the greatest projected net loss of the positions in a portfolio, has become a standard business practice in the financial industry. See, e.g., https://www.cboe.com/us/options/margin/portfolio_margining_rules/ and https://www.eurex.com/ec-en/services/margining/eurex-clearing-prisma.

where x_t^{j+} and x_t^{j-} are long and short positions in stocks, and l_t^{j+} and l_t^{j-} long and short open positions in liquidity derivatives at time t, respectively. Finally, m_t^{j+} (m_t^{j-}) indicates the amount of capital borrowed per unit of long (short) stock positions, and similarly n_t^{j+} (n_t^{j-}) denotes how much financing speculators can borrow against each unit of long (short) derivative positions.

2.1.3 Financiers

Financiers provide capital to speculators, but observe only the stock price sequence. Conditioning on this information, they set margins to limit counterparty credit risk targeting a π value-at-risk (VaR), that is

$$\pi = Pr(-\Delta p_{t+1}^j > m_t^{j+}) = Pr(\Delta p_{t+1}^j > m_t^{j-}) \tag{6}$$

Financiers accept derivative positions as collateral with the same VaR rule.

$$\pi = Pr(|\Lambda_{t+1}^j| < n_t^{j+}) = Pr(-|\Lambda_{t+1}^j| > n_t^{j-}) \tag{7}$$

Since financiers are uninformed about fundamental values, the above probabilities are conditional on the filtration generated by market prices $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \ldots, \mathbf{p}_t\}$. For example, the VaR specification requires that price drops that exceed margins on long stock positions only happen with probability π. Similarly, the collateral value of derivatives cannot exceed the minimum expected payoff resulting from the position with confidence level $1 - \pi$. Margins m on stocks increase in price volatility and can increase in market illiquidity (Brunnermeier and Pedersen, 2009, Proposition 3). Differently, margins n on liquidity derivatives *decrease* as prices deviate from fundamentals and derivatives appreciate.

2.1.4 Equilibrium

An equilibrium is a pair of price processes $(\boldsymbol{p}_t, \boldsymbol{\lambda}_t)$ such that (i) the vectors $(\boldsymbol{x}_t, \boldsymbol{c}_t)$ maximize speculators' expected terminal profits subject to the margin constraint; (ii) For each customers k, the choices $(\boldsymbol{y}_t^k, \boldsymbol{c}_t^k)$ maximize the expected utility after the arrival to the marketplace and is zero beforehand; (iii) margins are set according to the VaR rule; and (iv) the markets clear, namely $\boldsymbol{x}_t^k + \sum_{k=0}^2 \boldsymbol{y}_t^k = \mathbf{0}$, and customers and speculators agree on the amount $\boldsymbol{c}_t = \sum_{k=0}^2 \boldsymbol{c}_t^k$ of derivatives transactions. A B&P economy obtains when $\boldsymbol{c}_t = \mathbf{0}$.

2.2 Model solution

Date $t = 3$ is a terminal condition for valuing the securities as of time 2 (see also Grossman and Miller, 1988). By backward induction, $p_2 = v_2$ and $\Lambda_2 = 0$, whence $\lambda_1 = 0$ follows, because the equilibrium price of a contingent claim whose payoff degenerates to zero must equal zero. Price deviations from fundamentals arise at time $t = 1$ and $t = 0$, that implies that liquidity derivatives are traded at time $t = 0$ as customer $k = 0$ and speculators populate the marketplace, and settled at time $t = 1$. That is, any equilibrium is such that $\lambda_2 = \lambda_1 = 0$.[5] The solution rests on a recursive optimization argument. Throughout, let Γ denote a customer's value function and J a speculator's value function. Liquidity increases in the wealth of speculators. In fact, when speculators finances are unconstrained, illiquidity is zero as all arbitrage opportunities are executable. The basic illiquidity problem arises because speculators have funding constraints in the form of the VaR rules in Equations (6) and (7). Thus, speculators cannot exploit all arbitrage opportunity, and have to cherry-pick the most profitable investments. Speculators' shadow cost of capital in $t = 1$, denoted as ϕ_1, is one plus the maximum profit per dollar invested.

$$\phi_1 = 1 + \max_j \left\{ \max \left(\frac{v_1^j - p_1^j}{m_1^{j+}}, \frac{p_1^j - v_1^j}{m_1^{j-}} \right) \right\} \tag{8}$$

At time $t = 0$, speculators maximize

$$\mathbb{E}_0[J_1(W_1, p_1, v_1, p_0, v_0)] = \mathbb{E}_0[W_1 \phi_1] \tag{9}$$

subject to the margin constraint. For ease of comparison with Brunnermeier and Pedersen (2009), we consider the case in which speculators are unconstrained at time $t = 0$. The first order condition for speculators' j-th stock holding is $\mathbb{E}_0[\phi_1(p_1^j - p_0^j)] = 0$, and for a position in derivative written on j is $\mathbb{E}_0[\phi_1(\lambda_0^j - |\Lambda_1^j|)] = 0$. Therefore,

$$\begin{aligned} p_0^j &= \frac{\mathbb{E}_0[\phi_1 p_1^j]}{\mathbb{E}_0[\phi_1]} = \mathbb{E}_0[p_1^j] + \frac{\text{Cov}_0[\phi_1, p_1^j]}{\mathbb{E}_0[\phi_1]} \\ \lambda_0^j &= \frac{\mathbb{E}_0[\phi_1 |\Lambda_1^j|]}{\mathbb{E}_0[\phi_1]} = \mathbb{E}_0[|\Lambda_1^j|] + \frac{\text{Cov}_0[\phi_1, |\Lambda_1^j|]}{\mathbb{E}_0[\phi_1]} \end{aligned} \tag{10}$$

During funding liquidity crises, the stochastic discount factor ϕ_1 is higher than its expected value and prices deviate from fundamental values because speculators are con-

[5] Intuitively, illiquidity requires two periods to materialize, but the discussion that follows would result from any finite time model with $T \geq 3$.

strained. Therefore, the term $\text{Cov}_0[\phi_1, |\Lambda_1^j|]$ is positive. From Equation (10), we recognize that arbitrageurs are willing to pay a premium for liquidity derivatives that is higher than the expected payoff in order to obtain insurance against fluctuations in illiquidity that occur in states of the world where the shadow cost of capital is high. Moreover, in equilibrium arbitrageurs are eager to trade both assets.[6]

Lemma 1. *The solution to the customer's mean-variance problem is*

$$\begin{aligned} y_0^{j,k} &= \frac{v_0^j - p_0^j}{\gamma(\sigma_1^j)^2} - z^{j,k} \\ c_0^{j,k} &= \frac{\lambda_0^j - \mathbb{E}_0|\Lambda_1^j|}{\gamma\, Var_0(|\Lambda_1^j|)} \end{aligned}$$

Proof. See Appendix A ■

Corollary 1. *In equilibrium, customers sell liquidity derivatives to arbitrageurs.*

Proof. Replace Equation (10) in the expression for $c_0^{j,k}$ that is given by Lemma 1. ■

Importantly, the two markets load on different risk factors. The stock exchange is a market for the fundamental value of the business, and the market for liquidity allows for the transfer of liquidity risk. The attractiveness of the stock for customers results from their endowment shock. On the other hand, customers are willing to provide arbitrageurs with insurance against illiquidity as long as they are offered a larger premium than the expected future payoff. As is standard with mean-variance preferences, the amount of insurance the customers are willing to provide decreases with their risk aversion γ and the variance of illiquidity $\text{Var}_0(|\Lambda_1^j|)$. The next result is a welfare characterization.

Proposition 1. *Liquidity derivatives are welfare-improving.*

Proof. Notice that the introduction of liquidity derivatives does not change the value function of speculators. On the other hand, the value function of the customer Γ_0 increases because of the expanded possibility frontier. Formally,

$$\Gamma_0(W_0^k, p_0, v_0) = -\exp\left[-\gamma W_0^k - \frac{1}{2}\left(\sum_{j\in J} \frac{(v_0^j - \mathbb{E}_0 p_1^j)^2}{(\sigma_1^j)^2} + \underbrace{\sum_{j\in J} \frac{(\lambda_0^j - \mathbb{E}_0|\Lambda_1^j|)^2}{(\text{Var}_0(|\Lambda_1^j|))^2}}_{\text{Welfare improvement}} + \sum_j \frac{(v_1^j - p_1^j)^2}{(\sigma_2^j)^2} \right) \right]$$

[6] Assume by contradiction that $\mathbb{E}_0[\phi_1(p_1^j - p_0^j)] > \mathbb{E}_0[\phi_1(\lambda_0^j - |\Lambda_1^j|)]$. Arbitrageurs' first order condition would proscribe them to buy stocks exerting upward pressure on p_0 until the equivalence between the profitability of their investment opportunities is restored.

that corresponds exactly to the customer value function in the B&P model aside from a welfare-improving term that results from a market for liquidity derivatives. ■

Intuitively, liquidity derivatives are contracts that give traders the option to hedge their exposure and help move toward market completeness. Interestingly, their introduction leaves indifferent the speculators and improves the welfare of value investors such as customers. A comparative advantage argument explains well the scope for a market on liquidity. In the model, agents differ in their risk attitudes, access to funding, and preferred turnover. Limits to arbitrage that arise from imbalance in the order flow and funding liquidity constraints cause the pricing kernel of speculators to reflect the availability of capital. Speculators are willing to pay a premium over and above the expected value of the payoff for an asset whose reward is high in states where funding is constrained, markets are illiquid, and the dollar remuneration per unit of capital is large. Conversely, customers do not have access to funding and are eager to take the other side of a trade earning positive expected return. The next result characterizes the systemic implications of these instruments.

Proposition 2. *Liquidity derivatives attenuate both margin and loss spirals, mitigating the effect of adverse shocks to speculators' wealth on market prices.*

Proof. First, note that in equilibrium the speculator is always long in the derivative, $c_0 > 0$. To simplify notation, we prove the statement for the case of $J = 1$ assets. Consider the case $Z_1 > 0$, that implies $p_1 \leq v_1$, $\Lambda_1 < 0$, and $x_1 \geq 0$. In equilibrium, the funding constraint binds and

$$m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) + n_1^+ l_1 = b_0 + p_1 x_0 + c_0 |\Lambda_1| + \eta_1$$

Combining the implicit function theorem with the market clearing conditions,

$$\frac{\partial m_1^+}{\partial p_1}\frac{\partial p_1}{\partial \eta_1} x_1 + m_1^+ \frac{2}{\gamma(\sigma_2)^2}\frac{\partial p_1}{\partial \eta_1} + \frac{\partial n_1^+}{\partial p_1}\frac{\partial p_1}{\partial \eta_1} c_0 = \frac{\partial p_1}{\partial \eta_1} x_0 + \frac{\partial |\Lambda_1|}{\partial p_1}\frac{\partial p_1}{\partial \eta_1} c_0 + 1$$

It is useful to recall that $\frac{\partial |\Lambda_1|}{\partial p_1} = \frac{|\Lambda_1|}{\Lambda_1} = \text{sign}(\Lambda_1)$. Importantly, the sensitivity of the liquidity derivative's payoff to the price of the underlying has the opposite sign of the direction of the order flow imbalance Z_1. The latter is an unique key property to counter loss spirals, which arise if the order flow imbalance pushes prices away from fundamentals in the opposite direction to previous exposures of arbitrageurs. After rearranging terms,

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2} m_1^+ + \frac{\partial m_1^+}{\partial p_1} x_1 + \frac{\partial n_1^+}{\partial p_1} c_0 - x_0 + c_0}$$

Liquidity derivatives mitigate both margin and loss spirals. Indeed, when financiers are uninformed about the fundamental value of the security the impact of the term $\frac{\partial m_1^+}{\partial p_1} x_1 < 0$ that gives rise to a margin spiral is mitigated by the effect of $\frac{\partial n_1^+}{\partial p_1} c_0 > 0$. Intuitively, while on the one hand financiers require more skin in the game when observing prices moving further below fundamentals, the collateral value of the derivative position that profits from illiquidity increases. Speculators further encounter a loss spiral if x_0 is of the same sign as x_1, because price drops are accompanied by losses on the previous positions. The latter effect is offset by the long position in the liquidity derivative that appreciates precisely when the price is lower than fundamental value of the business, i.e., when the demand pressure moves against the arbitrage position. Appendix B proves the converse case with $Z_1 \leq 0$. ■

Importantly, liquidity derivatives stabilize market prices against loss spirals by conditioning the revenues of speculators to future realizations of order flow imbalance. As share prices move adversely and speculators are forced to post more collateral to finance their stock positions, portfolio margining enables to borrow against the increased value of derivatives position.[7] Summarizing, liquidity derivatives have three features worth emphasizing. Arbitrageurs always demand insurance against market illiquidity, because it positively correlated with their shadow cost of capital. Margins on liquidity derivatives are countercyclical. Liquidity derivatives condition the wealth of arbitrageurs to imbalance in the order flow, mitigating loss spirals on existing positions. Since empirically illiquidity and volatility are correlated (Stoll, 1978), one might wonder whether these characteristics are spanned by volatility derivatives.[8] Without making additional assumptions, it is not possible to establish whether arbitrageurs enter the market for volatility as buyers or as sellers, so as to pin down the effect of these derivatives on margin spirals as generally as in the case of liquidity derivatives.[9] However, the next result

[7]The results achieved by maintaining for ease of comparison the specification of preferences as in B&P are conservative. Real-world arbitrageurs are arguably risk averse over margin calls and losses on previous positions and would have a larger insurance motive, thus strengthening the stabilizing effects of liquidity derivatives on financial markets.

[8]We thank Tarun Chordia and Davide Tomio for this observation.

[9]Specifically, to establish the direction of the exposure of arbitrageurs to volatility derivatives we would require an assumption about the sign of the covariance of their payoff with the arbitrageurs' stochastic discount factor, $\text{Cov}_0[\phi_1, \sigma_1^j] = \theta^j \text{Cov}_0[\phi_1, \varepsilon_1^j]$. In fact, shocks to the fundamentals ε_1^j impact positively both the illiquidity $|\Lambda_1^j|$, at the numerator of the shadow cost of capital ϕ_1, and the margins on stocks m_1, at the denominator of ϕ_1. Which of the two effects prevails is

shows that volatility derivatives do not prevent loss spirals.

Lemma 2. *Volatility derivatives do not have the stabilizing effects on market prices achieved by liquidity derivatives.*

Proof. For clarity of notation, we prove the statement in the one-asset case $J = 1$ without loss of generality. From Equation (1), at $t = 0$ the value of σ_1 is known. Volatility derivatives are contingent claims with market price s_t and payoff σ_2. It is worth emphasizing that, since $p_2 = v_2$, σ_2 represents the time $t = 2$ volatility of both the fundamental value and the stock price, as well as the expected value of σ_3. The sensitivity of the volatility derivative payoff to the price of the underlying obtains from the ARCH specification which models the future volatility as a function of the current fundamental value of the business.

$$\frac{\partial \sigma_2}{\partial p_1} = \frac{\partial \sigma_2}{\partial v_1}\frac{\partial v_1}{\partial p_1} = \frac{\partial \sigma_2}{\partial v_1} \tag{11}$$

$$= \frac{\partial \underline{\sigma} + \theta |v_1 - v_0|}{\partial v_1} \tag{12}$$

$$= \theta \text{ sign } (\Delta v_1) = \theta \text{ sign } (\varepsilon_1) \tag{13}$$

Recall that arbitrageurs' positions in the stock x_1 depends on the order flow imbalance Z_1. Innovations to the fundamental value ε_1 might or might not be of the same sign as the imbalance in the order flow Z_1, a distinct source of randomness which results from the distribution of endowment shocks across customers. Therefore, volatility derivatives do not hedge speculators against loss spirals, which hit the speculators if the imbalance in the order flow has the opposite sign as their previous exposures x_0. ■

In general, liquidity spirals are induced by external stakeholders uninformed about fundamental values, who require collateral when the two legs of an arbitrage position widen following price swings caused by demand and supply forces. We have discussed the case of financiers, but investors in hedge funds are similarly uninformed and might redeem their quotas when the legs of an arbitrage position diverge (Shleifer and Vishny, 1997). Payoffs contingent on market liquidity effectively hedge speculators in those states of the world by establishing a countercyclical connection between market and funding liquidity which prevents self-fulfilling impairments to orderly markets.

ultimately an empirical question.

3 A Market for Liquidity Risk

We have formally argued that liquidity derivatives improve welfare and financial stability. In the B&P model, both customers and speculators observe the fundamental value of stocks. Real world trading venues are however more complex. In Section 3.1 we characterize the potential buy and sell sides of the market for liquidity derivatives, and in Section 3.2 we describe a suitable proxy for price deviations from fundamental values. Section 3.3 illustrates desirable properties for the payoff structure of derivatives based on liquidity. Section 3.4 develops a simple pricing technique for liquidity derivatives.

3.1 Liquidity Derivatives: Demand and Supply

The scope of liquidity derivatives is to facilitate the transfer of market liquidity risk between investors. As the model in Section 2 illustrates, the appetite for derivatives on market liquidity is a natural result of agents' heterogeneity. Traders with high turnover, such as hedge funds (arbitrageurs) typically hold assets for short investment horizons, which exposes their business model to the risk of large transaction costs and of their comovement with the market. At the opposite end of the spectrum, buy-and-hold investors (customers) such as insurance companies and pension funds do not face high execution costs since they trade infrequently, and view favorably the return compensation resulting from periods of prolonged illiquidity.[10]

As an illustration of the magnitude of the differential exposures of investors to liquidity risk, we carry the following empirical exercise. We measure the average performance of hedge funds and of insurance companies by collecting publicly available data from the Barclay Hedge Fund Index and the S&P Insurance Select Industry Index.[11] These series consist of monthly returns from June 2003, the first value of the S&P Insurance Index, to December 2020, and quantify the aggregate performance of businesses in these two sectors. Table 1 presents the results of time-series regressions of the industry returns indexes on the benchmark Fama and French (2015) model plus momentum augmented with the Pástor and Stambaugh (2003) liquidity factor, which is a long-short portfolio of stocks sorted according to their sensitivity to aggregate liquidity shocks.[12] The coefficients

[10] The insight that clientele group holding periods lead to differential exposures to market liquidity dates back to the asset pricing model developed by Amihud and Mendelson (1986). More recently, Bongaerts et al. (2011) build a model where lock-up investors differ from high-turnover traders in their exposure to transaction costs.

[11] The data sources are https://portal.barclayhedge.com/cgi-bin/indices/displayIndices.cgi?indexID=hf and https://www.spglobal.com/spdji/en/indices/equity/sp-insurance-select-industry-index, respectively.

[12] Data are collected from Robert Stambaugh's website (http://finance.wharton.upenn.edu/~stambaug/), which provides further details about the construction of this variable.

on the control variables display values and magnitudes in line with the economic theory. Importantly, insurance companies and hedge funds have opposite exposures to liquidity risk. The return to the average firm in the hedge fund industry rises with PS, with an economically large coefficient estimate of 0.11. Conversely, the return to the average firm in the insurance industry moves in the other direction, with a statistically significant coefficient of -0.15. The similarity in size but the difference in sign of these estimates suggest that the scope for risk transfers between market participants who are differently exposed to liquidity risk is substantial.

Investors differ in their exposures to liquidity risk along many other dimensions. Clearly, some traders have a comparative advantage in processing information about fundamentals, and others are more knowledgeable about the dynamics of the order flow (Pasquariello and Vega, 2007). Within the banking sector, the most liquid assets are held by shadow banks, and the least liquid by commercial banks (Hanson et al., 2015). The group of investors exposed to liquidity risk is numerous, and includes DMMs, who are committed to provide traders with immediacy when liquidity dries up. These intermediaries, often referred to as liquidity providers of last resort, are themselves heavily exposed to liquidity risk (Menkveld and Wang, 2013). The potential sell-side of the market is rich as well. For instance, Berkshire Hataway, who exemplifies well the customers in our model interested in the fundamental value of the stocks, uses extensively the sale of derivatives to generate revenues and cash (Frazzini et al., 2018). These reasons lead us to investigate further how to price liquidity derivatives.

3.2 The Reference Process

Liquidity has many faces. However, the liquidity risk of a position refers to uncertainty about its future transaction costs, and is traditionally identified as a nontraded risk factor (Pástor and Stambaugh, 2003). To fix ideas, suppose an investor has a long (short) stock position and wants to hedge against fluctuations in the bid (ask) price, while willing to retain other risks associated with the asset on the portfolio. The time t immediacy cost is captured by the distance between the market order execution and the midquote m_t,

$$p_t = \frac{1}{2}\Big(a_t + b_t + d_t(a_t - b_t)\Big) = m_t + \frac{1}{2}d_t\Big(a_t - b_t\Big) \tag{14}$$

where a_t and b_t are respectively the best ask and bid quotes, and $d_t = 1$ for buyer initiated trades and -1 otherwise. The parallel with Eq. (2) from B&P is apparent. Eq. (14) decomposes the time t value of a position in its midprice, proxying for fundamental value, and trading costs, capturing the cost of immediate liquidation.[13] Asset returns result from the evolution of both terms (Amihud and Mendelson, 1986). Of course, high prices are bad news for buyers and low prices penalize sellers, so that for each value of d_t the execution costs map to the half bid-ask spread.

The above example clarifies that the bid-ask spread is a reasonable proxy for the instrument-level market liquidity risk of standard-sized positions, i.e., those with negligible price impact, when fundamental values are not observable. To achieve comparability across firms, we consider the time t execution costs c_t, defined as one half times the relative spread.

$$c_t = \frac{1}{2} \frac{a_t - b_t}{m_t} \tag{15}$$

Empirically, relative spreads are a feasible reference process, and do not suffer from estimation issues as other liquidity measures such as price impact. Contemporary data on the relative spread in Eq. (15) are readily available from third-party reporting entities. Market prices are carefully monitored both at the SEC and at the exchange level to prevent and sanction manipulative conducts, and the quotes posted by market makers are tied to the tightest composite bid-ask spread resulting from competition between trading venues.[14]

3.3 Hedging Liquidity Risk

We term liquidity derivatives financial claims contingent on the market illiquidity of the underlying asset. Many are however the payoff structures which respond to that criterion. We have argued that the relative bid-ask spread is a suitable reference process. In order to impose further discipline, we engineer these instruments to separate liquidity risk from fundamental risk over a specified horizon. It is worth noting that returns embed a periodic compensation for both fundamental and liquidity risk, and that modern accounting practices often involve the mark-to-market appraisal of financial assets. Because of these reasons, it is appropriate to condition the derivative's payoff on the dynamics of illiquid-

[13] We leave to future research a generalization of round-trip transaction costs to the price impact of large positions, noting that Eq. (14) naturally extends to an arbitrary q-sized position. Indeed, denoting through upper bars weighted averages of best prices at the quantities quoted on the limit order book, $p_t(q) = \frac{1}{2}\left(\bar{a}_t(q) + \bar{b}_t(q) + d_t\left(\bar{a}_t(q) - \bar{b}_t(q)\right)\right)$ measures the depth.

[14] Among other NYSE provisions available at https://nyseguide.srorules.com/rules, rule 104 (a) prescribes that prices entered by DMMs shall be not more than the Designated Percentage away from the then current National Best Bid Offer (NBBO) available across US exchanges. Rule 6140 (d) explicitly forbids exchange members or organizations to participate or have any interest, directly or indirectly, in the profits of a manipulative operation or knowingly manage or finance a manipulative operation.

ity of the underlying asset over the holding period.[15] Consider a payoff function of the following form.

$$\mathcal{H}_T = \frac{1}{2} \max \left\{ H_T - K, 0 \right\} \tag{16}$$

H_T accumulates through the investment horizon T the relative spreads c_i of the underlying in excess of its value at inception of the contract, and K is a strike price.

$$H_T = \sum_{i=t+1}^{T} \max\{c_i - c_t, 0\} \tag{17}$$

This instrument compensates the holder of an arbitrary asset for large deviations of transaction costs from their level at the beginning of the contract, earning more the higher and longer-lasting is illiquidity. As an example, Figure 1 displays the 2000Q1 time-series behavior of the relative spread of Walmart Inc., and the corresponding step-wise option payoff plotted against time.

Together, Eq. (16) and (17) mirror the configuration of weather options actively traded over the counter, where market participants agree on a designed institution to measure the reference process. Importantly, the structure we posit is consistent with the insurance purpose of avoiding large losses resulting from high or sustained illiquidity, and maps the difficult issue of measurement of liquidity risk to the well-understood field of option pricing theory. Undoubtedly, there are other types of derivatives that in principle could reference the bid-ask spread as underlying, such as futures or swaps. We focus on options mainly for two reasons. First, their limited downside risk, that differs from the obligations to trade at maturity that a future imposes, makes them a natural candidate for insurance-like purposes. Second, differently from swaps, which involve continuous exchanges of cash flows without any initial outflow, options require the payment of their premium *una tantum* that fits well with the idea of a "hedge-and-forget" strategy.

In the literature, there have been some suggestions on how to trade on liquidity at the aggregate level, essentially by buying and selling long-short portfolios of stocks sorted on firm-level liquidity measures (e.g. Amihud et al. (2015)) or on their sensitivity to aggregate liquidity shocks, as in the case of the Pástor and Stambaugh (2003) liquidity factor. Special interest for instrument-level liquidity derivatives arises as illiquidity is not reduced through a standard diversification argument as other risks (Amihud, 2018).

[15]The approach resonates well with the Basel III liquidity regulation, which requires high-quality liquid assets to be continuously marketable.

Illiquidity is additive: buying and selling a portfolio of illiquid assets means bearing the sum of the illiquidity costs of its components, hence using an aggregated instrument like IML to hedge a specific portfolio of assets is ill-suited. If, on one side, investors may be concerned about high trading costs when holding exactly their optimal portfolio of securities, on the other side they have the disadvantage of deviating from optimality if they need to hold a portfolio imperfectly correlated with their personal optimum.

We are aware of negative effects that could take place due to the introduction of liquidity derivatives. A short position in liquidity derivatives might result in a substantial downside tail risk. Fortunately, such risk can be partially mitigated by exploiting the phenomenon of "flight-to-liquidity," whereby illiquid markets induce investors to rebalance their portfolios toward liquid assets. Another well-known property of liquidity is its strong degree of comovement in the cross section of assets often referred to as "commonality in liquidity." As an example, Chordia et al. (2000) report a coefficient of 0.79 when regressing the firm-level percentage changes in relative quoted bid-ask spreads on their cross-sectional average. Thus, liquidity option sellers can diversify their exposures to several underlying assets and retain on their portfolios the risk of liquidity at the market level. It is advisable that issuers of liquidity derivatives invest into money market instruments such as T-Bills to derive offsetting gains when market liquidity deteriorates. This strategy represents a partial hedge for the option seller. To further mitigate credit risk, the contract could include a provision ruling out the option exercise if the seller experiences a loss larger than a pre-specified threshold. Reducing the risk the seller has to bear, such clauses would lower the cost premium of the contract. For simplicity, this feature is not considered here but could be discussed in future research.

3.4 Pricing Liquidity Derivatives

At the core of liquidity risk is the impossibility of a replication in the Black and Scholes (1973) tradition. The lack of stable correlation patterns for liquidity is intriguing, and hinders statistical arbitrage. In these regards, liquidity risk resembles the unspanned risk that weather derivatives channel to active financial markets since two decades (Alaton et al., 2002). More generally, the pricing of derivatives often assumes a stochastic process for the underlying source of randomness. To choose a suitable specification, we first look at the empirical properties of relative bid-ask spreads at the daily level, following the approach used in the seminal work of Grünbichler and Longstaff (1996) in the framework of the valuation of volatility options. In our sample, relative spreads are strictly positive by construction and relatively persistent, with a cross-sectional median autocorrelation that decays slowly (from 0.57 for the first order until 0.41 with 20 lags).[16] This behavior is similar to what documented in Groß-KlußMann and Hautsch (2013) at the intraday level.[17] Relative spreads seem also to be mean reverting. The cross-sectional median first-order autocorrelation in the first difference of the spread is -0.46 while higher-order autocorrelations are close to zero. These figures are close to what reported in Harvey and Whaley (1992) for changes in the volatility of S&P500 options, and indicate that the conjecture of mean reversion is reasonable. To further test this property, we regress the squared changes in spreads on their level and find a slope that is significant at the 1% level in 95% of the stocks included in our sample. To sum up, at the daily frequency, relative bid-ask spreads are positive, mean reverting, and exhibit volatility clusters. These empirical facts suggest that a stochastic process in the tradition of Cox et al. (1985) (henceforth CIR) is a reasonable specification to describe the dynamics of the relative bid-ask spreads in Eq.(15). Grünbichler and Longstaff (1996) reach the same conclusion by using the same tests on stock volatility when proposing a pricing model for volatility options.[18] Tractability of the process under both the physical and the risk-neutral measure is also appealing. Thus, we specify a CIR process for the relative spread.[19]

$$dc_t = \alpha(\mu - c_t)dt + \sqrt{c_t}\sigma dB_t \tag{18}$$

In Eq. (18), the parameters (α, μ, σ) have the usual interpretation as mean-reversion speed, long-run mean, and standard deviation of the square root process, respectively, and are required to conform to the Feller condition to ensure non-negativity of the spread.

[16]Our sample of NYSE-traded stocks for the period 2000-2020 is described in Section 4 below.

[17]A crucial difference between daily and intraday spreads is that the former do not exhibit the "seasonality" patterns that can be observed at higher frequencies. Chan et al. (1995) and Chung et al. (1999) document that spreads are higher in the beginning of a trading day and decrease in the course of the trading session, a pattern explained by a higher adverse-selection component due to the processing of overnight information in the morning. Focusing on the daily frequency (end-of-day), we side-step this component thereby working with a variable reflecting more closely transaction costs investors incur.

[18]Earlier, we showed theoretically that liquidity and volatility represent two distinct, albeit correlated, sources of risk. And indeed, empirically the median correlation between firm-level return volatility and relative spread across the stocks in the sample is only 0.36, i.e. around two thirds of liquidity variation are due to different factors other than volatility. Furthermore, liquidity risk is priced in the cross-section after controlling for volatility. The IML portfolio (Amihud et al., 2015; Amihud and Noh, 2021) is built grouping stocks into illiquidity portfolios *within* volatility terciles and yet delivers a Fama-French-Carhart-adjusted premium of 0.40% (t-statistic 3.12) for the period 1998-2017 (Amihud, 2018). Thus, liquidity risk is unspanned by volatility derivatives.

[19]It is worth stressing that using the CIR to represent the behavior of the relative spread does not mean we consider it an exogenous variable. The spread reflects in fact a variety of elements among which trading behavior driven by informational asymmetries and adverse selection, although mitigated using the daily frequency, and the impact of order flow imbalances, as illustrated in the model in Section 2. We use a stochastic process just for pricing purposes instead of modelling explicitly what drives the spread, which is not what we are looking for, exactly as it happens for stocks in the Black and Scholes (1973) model, that does not conflict with stock prices being equilibrium outcomes and, as such, endogenous to the choices of market participants.

These parameters can be easily estimated by Maximum Likelihood of the transition density of the CIR process, which is proportional to a noncentral χ^2 distribution, using OLS regressions as initial values (Kladívko, 2007). Under the risk-neutral measure,

$$dc_t = \tilde{\alpha}(\tilde{\mu} - c_t)dt + \sqrt{c_t}\sigma dB_t^{\mathbb{Q}}, \qquad \tilde{\alpha} = \alpha + \varrho, \;\; \tilde{\mu} = \frac{\alpha\mu}{\alpha + \varrho} \tag{19}$$

In incomplete markets, non arbitrage is silent about the market price of risk, which requires an equilibrium argument. Guided by Cox et al. (1985), we let ϱ denote the stock-specific ratio of the covariance between changes in relative bid-ask spreads and percentage changes in optimally invested wealth (approximated through market returns) to the relative spread. Eq. (19) shows that a more negative ϱ implies a slower speed of reversion and a higher long-run mean. Both features increase the standard deviation of the risk-adjusted CIR process (Hördahl and Vestin, 2005). The market price of risk takes the form $\theta = \frac{\varrho\sqrt{c}}{\sigma}$, thus retaining the property of a higher risk compensation for negative comovements of liquidity with the market (remindful of the liquidity-adjusted CAPM in Acharya and Pedersen, 2005). Rephrasing, a liquidity option on a stock whose transactions cost are particularly high at times of negative marketwide returns demands higher premia.

We compute the model-implied prices of liquidity derivatives C_t through Monte Carlo techniques, by simulating multiple times the underlying process under the $\mathbb{Q}$-measure and averaging the resulting discounted payoffs, as is standard with path-dependent instruments.

$$C_{t,T} = e^{-rT}\mathbb{E}_t^{\mathbb{Q}}[\mathcal{H}_T] \tag{20}$$

which can be evaluated numerically by using a large number N of simulated paths.

4 Empirical Estimation

We have shown that liquidity derivatives are beneficial to financial markets and provided a simple pricing method. We now turn to the data and study the empirical properties of simulated liquidity option prices, henceforth LOPs. For simplicity, we focus on stocks as most bonds trade over-the-counter. Section 4.1 illustrates descriptive statistics about option prices for a panel of stocks traded on the NYSE. Section 4.2 develops testable hypotheses illustrating the economic motivation behind them, with empirical findings presented in the subsequent sections. In more detail, Section 4.3 provides evidence that liquidity options effectively strip liquidity risk out of financial assets. Section 4.4 introduces a novel measure of aggregate illiquidity based on option prices. Finally, Section 4.5 documents strong links between LOPs and stock returns which survive risk adjustments and the confounding effect of correlated variables.

4.1 Data Description and Summary Statistics

We apply our pricing formula to CRSP stocks listed on the NYSE during the period January 2000-December 2020. The sample is confined to stocks listed on the NYSE to avoid the effect of differences in microstructure (Amihud, 2002; Reinganum, 1990) and in trading algorithms between exchanges (Korajczyk and Sadka, 2008).[20] The NYSE is the world's largest stock exchange by market capitalization. It combines an auction market system with the obligations for DMMs to maintain continuous, two-sided quotes to their assigned securities to guarantee that all auction orders are fully executed. Thus, the exchange ensures the continuity of the bid-ask spread reference process required by liquidity derivatives. At the same time, DMMs contribute with human judgment as well as capital to determine closing prices that accurately reflect the mix of buy and sell interest at the end of the day. For instance, DMMs incorporate news releases into the close prices, which are crucial references for equity-linked products such as derivatives.[21]. The data selection closely follows Amihud (2002) with details reported in Appendix C. This procedure leaves us with a sample that include only a limited number of relatively small stock.[22] Nevertheless, it is important to address the potential concern that some stocks are so small and so illiquid that liquidity derivatives would simply not be traded on them. However, this is unlikely because investors often have the interest to actively trade firms with small market capitalization as a large extent of profitable investment strategies exploiting anomalies make use of such firms. Novy-Marx and Velikov (2016) show that microcaps (stocks below the NYSE 20^{th} size percentile) earn high gross Sharpe ratios in most anomalies relative to other size groups, but the difference considerably shrinks after accounting for transaction costs. Hou et al. (2020) find that 65% of anomalies are statistically insignificant after excluding microcaps. This considerations motivate the interest for financial claims hedging uncertainty about transaction costs for smaller stocks. Besides, the results below hold also with more restrictive selection criteria. We

[20] For example, while trading on Nasdaq takes place mostly through market makers, the majority of trades on the NYSE occurs between buying and selling investors directly.

[21] For additional information, see `https://www.nyse.com/article/nyse-closing-auction-insiders-guide`.

[22] Table 2 shows that the 5^{th} percentile for market capitalization is 160 millions of dollars.

use Bloomberg data because of better reporting quality of bid and ask quotes.

As discussed in Section 3.4, we price liquidity derivatives at the end of each month by fitting a CIR process to the relative bid-ask spreads at the market close using one-year estimation windows. We then obtain the price of liquidity options by simulating $N = 100,000$ bid-ask spread paths per stock over a maturity of $T = 3$ months and averaging across the resulting payoffs. The investment horizon is standard, such as for instance the time interval between successive delivery dates of benchmark futures contracts. Without loss of generality, we normalize the strike price to zero since computational costs prevent us from calculating large panels of option prices for an arbitrary set of strike values. The daily risk-free rate from Kenneth French's website is used for discounting.[23] French's market factor returns proxy for changes in optimally invested wealth when computing their covariance with changes in relative spreads.

Our sample includes 1,755 listed firms for a total of 192,746 firm-month LOPs. Table 2 shows descriptive statistics for firm characteristics and simulated option prices. The median price required to remove uncertainty about transaction costs over the next three months amounts to 67 bps of the stock price at the contract initiation. Option prices exhibit a substantial degree of variation (the standard deviation is 612 bps) and a skewness of 4.45, with the average price roughly four-fold as big as the median. These and other interesting facts are summarized in Figure 2, which represents cross-sectional option prices deciles every year.[24] Looking at the time-series dimension, liquidity options are particularly expensive during the *dot-com bubble* in 2001 and in the course of the financial crisis peaking in 2009. Unsurprisingly, the premium required to hedge against illiquidity includes also a component related to market-wide conditions. Overall, prices are higher for the first part of the sample and remarkably drop after 2009, in line with the general decline in illiquidity observed in recent years (Amihud and Mendelson, 2015). From a cross-sectional perspective, the distribution is strongly right-skewed. In other words, liquidity options are cheap for most stocks and thus appealing for investors willing to pay upfront a small amount to engage in a hedge-and-forget strategy.

Aware of the fact that the CIR process used in Eq. (18) is an approximation aimed at delivering tractability, we explore how well it captures the behavior of the relative bid-ask spread for pricing purposes with a simple exercise. We compute the realized payoff over a maturity of 3 months that one would have obtained by investing in liquidity options if

[23] For details, see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

[24] We exclude the year 2000 from the picture as it would represent data about only December, 2000, which is when liquidity option prices are first available using a one-year estimation window.

these were available, discounting it back to the pricing time. Then, we sort options into 10 portfolios based on their price at the end of each month t and regress equal-weighted portfolio payoffs at month $t+3$ on the corresponding time-t equal-weighted portfolio prices using OLS. Simulated option prices predict 52% of the variation in realized payoffs, with a regression slope coefficient of 0.28 and a t-statistic above 51. Figure 3 helps visualizing this performance by showing the time average of portfolios payoffs against the time average of portfolio prices. With the exception of portfolio 10, the two quantities line up nicely with a corresponding R^2 of 0.79. In addition to the pricing algorithm, simulated option prices and realized payoff might diverge because of differences between the physical and the risk-neutral measure. Since prices are smaller than payoffs, these risk adjustments are negative, as it should indeed occur for instruments hedging risks. Intuitively, deviations from the 45-degree line in the plot increase along the x-axis because more expensive portfolios require higher adjustments. These findings confirm that the pricing procedure is reliable as it satisfies fundamental asset pricing restrictions.

4.2 Hypotheses Development

First, we empirically investigate whether the proposed liquidity options effectively strip liquidity risk out of financial assets. Let us assume excess returns follow well-established multifactor models like the Fama and French (1993) three-factors model, augmented with a traded risk factor like the one suggested in Pástor and Stambaugh (2003) to account for liquidity risk. If liquidity options provide a valid hedge against liquidity risk, we expect that a portfolio containing one stock and the corresponding option exhibits returns which are less exposed to liquidity factors than non-hedged stock returns.

Hypothesis 1. *The return of a portfolio composed of one stock and one corresponding liquidity option is significantly less exposed to a proxy for liquidity risk than the stock return alone.*

Second, we have reason to believe that LOPs reflect commonalities in the cross-section. Other than being affected by illiquidity at the instrument level, stocks are exposed also to aggregate liquidity risk (Acharya and Pedersen, 2005; Pástor and Stambaugh, 2003) and as such they covary with it. Hence, we expect the price of liquidity derivatives to pick up some of the exposure to common liquidity shocks, in such a way that an aggregated measure of simulated option prices captures market-wide conditions in similar fashion to other well-known measures of illiquidity (Amihud, 2002; Pástor and Stambaugh, 2003). In particular, we expect it to spike during crisis periods.

Hypothesis 2. *A cross-sectionally aggregated measure of liquidity option prices reflects market-wide liquidity conditions, similarly to the aggregate measures in (Amihud, 2002) and Pástor and Stambaugh (2003).*

Third, we explore the link between LOPs and stock returns. If Hypothesis 1 and 2 hold, liquidity options are clearly relevant for financial markets, and as such they are prone to impact the cross-section of returns. Since the payoff of liquidity options accumulates in the realized bid-ask spread, which is known to positively affect stock compensation (Amihud and Mendelson, 1986), we expect stock returns to increase in LOPs. Furthermore, since LOPs reflect asset-specific liquidity risk, they capture a component of returns that is not spanned by traditional factor models lacking an explicit liquidity factor. Thus, we expect that portfolios of stocks sorted according to simulated option prices generate abnormal returns in such a framework.

Hypothesis 3. *Stock returns increase in liquidity option prices. Portfolios sorted on option prices violate the mean-variance efficiency of multifactor models which do not include a liquidity risk factor.*

4.3 Hypothesis 1: Liquidity-hedged Portfolios

Returns of single stocks ("raw" positions) are compared with "hedged" positions obtained by financing the purchase of the corresponding liquidity options by selling a part of the shares with the same value, in such a way that the initial investment is equal to the initial stock price in both cases. The return of a liquidity-hedged position between time t and the maturity of the option at $t+3$ consists of two components, namely the ordinary stock appreciation and the liquidity option payoff, relative to the initial outflow. Dividends and eventual differences between the mid-price and the adjusted price used to calculate returns are accounted for. To reflect the compensation of positions financed by borrowing at the risk-free rate, we focus on excess returns. A thorough explanation of the procedure is provided in the Appendix D.

Next, we test the hypothesis that hedged positions are less exposed to liquidity risk. As a proxy for it, we use the traded liquidity factor provided by Pástor and Stambaugh (2003). In order to reduce the noise in the estimation of factor loadings, we first sort stocks into 10 portfolios based on their LOPs, and we then regress raw portfolio returns and hedged portfolio returns on Fama and French (1993) model plus momentum plus PS on a rolling basis using 60-months windows. Rolling estimation delivers time series of liquidity factor loadings that allows to formally test Hypothesis 1. Table 3 shows that the absolute mean exposure to PS across portfolios almost halves thanks to liquidity options, passing from 0.0235 to 0.0119.[25] Single-portfolio loadings substantially shrink, exhibiting sometimes more than a 10-fold reduction. In three cases the exposure actually increases, but this happens for portfolios starting with low betas (portfolios 3, 4, and 7). We stress that, in addition to sample uncertainty, we cannot expect to observe exactly zero loadings because PS is meant to capture price impact, a dimensions of liquidity only partially overlapping with bid-ask spreads. Importantly, portfolios heavily exposed to PS remarkably benefit from liquidity options. A prominent example is portfolio 10, whose liquidity beta is more than 20 times smaller when hedged. This result is of primary importance as portfolio 10 contains the stocks with the highest LOPs and thus those for which liquidity concerns are most urgent.

As an explicit statistical test for Hypothesis 2, we use a t-test for difference in means allowing for different variances by using the loadings obtained with a rolling estimation, following the approach of Savor and Wilson (2013, 2014). The last column of the table reports p-values for the null hypothesis that the absolute mean liquidity beta of a raw position is less than or equal to the hedged position one. The null is rejected in 7 cases, suggesting that liquidity options significantly alter the liquidity risk profiles of the portfolios considered, in line with the objective of their design. To sum up, these contracts strongly reduce the exposure of portfolios to aggregate liquidity risk, thereby revealing strong potential for the financial industry.

4.4 Hypothesis 2: Commonality in Liquidity

In the presence of strongly skewed prices, as emerged from Figure 2, a natural cross-sectional measure for LOPs is the median of the distribution in each month, as in Cakici and Zaremba (2021). This constitutes an option-based market-wide proxy for illiquidity capturing transaction costs, that we name OPT. We compare it to two well-known measures. The first is $ILLIQ$ (Amihud, 2002) at the monthly frequency. The second one is the aggregate liquidity from Pástor and Stambaugh (2003).[26] To transform it into an *illiquidity* measure, we flip its sign, and denote it with $PSLIQ$. OPT is also contrasted with a simpler aggregate built using the cross-sectional median of the relative bid-ask spread, the underlying of liquidity options, that we call $SPREAD$. Standardized time-series of these four measures are plotted in Figure 4, where shaded regions correspond to

[25]It is important to focus on absolute mean values to avoid the comparison to be biased due to potentially negative betas.

[26]Notice that this corresponds to what is plotted in their Figure 1, which differs from the innovations u_t (Eq. (8) in their paper).

NBER recession periods.

OPT is reported in red. As other measures, it spikes up in 2001 to reflect investors' concerns about liquidity during the *dot-com bubble*. After a general decrease in the subsequent years, all the four aggregates reach high levels during the sub-prime crisis, but with different timing. $PSLIQ$ and $SPREAD$ come to their respective peak of the period first, closely followed by $ILLIQ$. OPT keeps rising during the entire recession, hitting its local highest point some months later. In the remaining years, all measures stay below their mean, except for $ILLIQ$ and $PSLIQ$. Both these two clearly jump upwards towards the end of the sample to reflect the shock induced by the Covid-19 pandemic, but the former remains high also afterwards. OPT and $SPREAD$ are less affected by this phenomenon. The measures are clearly persistent, with a first-order autocorrelation above 0.9, apart from $PSLIQ$, whose distinctive trait is a strong mean reversion (autocorrelation of 0.05).

To a large extent, OPT captures important changes in liquidity at the aggregate level, but exhibits some differences with respect to other measures. This happens because of the complex nature of illiquidity. While $ILLIQ$ and $PSLIQ$ reflect price responses associated with every dollar of trading activity and temporary price changes accompanying order flows, respectively, the bid-ask spread is influenced by different aspects of market liquidity. LOPs, in particular, reflect the no-arbitrage compensation required to insure against uncertainty over future transaction costs relative to the spread at the time of pricing, thereby capturing a dimension of liquidity which differ not only from the two we have just mentioned, but also from $SPREAD$, which instead reflects only point-wise deviations from the fundamental price that do not accumulate over the holding period to reflect concerns about cumulative illiquidity. Moreover, OPT includes a risk-adjustment component consistent with an equilibrium model, a feature absent in the other metrics. Finally, it must be stressed that since liquidity options are not actually observed, an option-based illiquidity measure is the result of a simulation exercise. With traded options, OPT would gain a genuine forward-looking behavior, something unprecedented in the liquidity literature. Given these considerations, it is not surprising that $ILLIQ$, $PSLIQ$ and $SPREAD$ span together only 77% of the variation of OPT in a time-series regression, leaving unexplained almost one fourth of the movements in the market-wide illiquidity captured by LOPs. OPT stands therefore as a complementary illiquidity measure to traditional ones that can be employed in future research.

4.5 Hypothesis 3: Abnormal Returns

As a first step to investigate the relation between stock returns and instrument-level liquidity risk, we perform univariate portfolio sorting based on LOPs, which are firm-specific, at the end of each month, grouping stocks into 10 equal-weighted portfolios. Figure 5 shows portfolio daily average excess returns in bps. As expected, there is a clear increasing relation between option prices and returns that is notably strong for stocks whose liquidity options are more expensive. The difference between portfolio 10 and portfolio 1 is 4 bps with a t-statistic of 5.11, which means returns significantly spread along the dimension of liquidity option prices. The results hold also at the monthly frequency, where the top-minus-bottom-decile average excess return is 94 bps (t-statistic=5.20).

Second, we test well-known empirical asset pricing models on the 10 option-prices-sorted portfolios. Table 4 reports the intercept and the R^2 for time-series regressions of the type

$$r_{t,t+1} = \alpha + \beta F_t + \varepsilon_t \tag{21}$$

where α is the intercept, $r_{t,t+1}$ is an $N \times 1$ vector of excess returns from t to $t+1$, β is the loading matrix on the K traded factors in F_t which in turn includes the market excess return (CAPM), Fama and French (1993) factors (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) factors (FF5) and FF5 plus Momentum (FF6), and ε_t is the error term. t-statistics are computed using Newey and West (1987) standard errors with 5 lags. Overall, traditional factor models struggle to explain portfolios sorted according to LOPs. Despite achieving a high R^2, they consistently fail to clear the asset pricing restriction of zero alpha. In particular, 9 out of 10 alphas are significant for CAPM, FF3 and FF4, and 5 for FF5 and FF6. The GRS statistic (Gibbons et al., 1989) rejects the null hypothesis of zero alphas well below the 1% significance level for all models. Importantly, significant alphas concentrate in portfolios with more expensive liquidity options and increase along this dimension, in line with economic intuition: risk adjustments from well-known factors do not alter the findings of Figure 5. A portfolio going long in the top decile and short in the bottom decile produces a daily alpha of at least 3.7 bps in all models with a t-statistic never below 4.79. In other words, the majority of its mean excess return over the period (4 bps) cannot be explained through the exposure to traditional factors. As a benchmark, the mean excess returns of the market factor, SMB, HML, Momentum, RMA and CMA are 3.36, 1.35, -0.21, 1.41, 0.57 and 0.79 bps, respectively. The abnormal remuneration of the long-short portfolio is therefore not only statistically but also economically significant. Results with monthly data (bottom panel) are very similar. The GRS test hypothesis is rejected in all models and the LOP-long-short portfolio earns a significant alpha of at least 84 basis points.

The reader may be concerned that the positive relation between returns and LOPs could be partially driven by correlation with confounding variables, as often argued in the literature (Amihud and Mendelson, 2015). Small firms are tipycally more illiquid than larger ones. Moreover, size is well-known in the anomaly literature since it proxies for the exposure to SMB, an important traded risk factor. Volatility and illiquidity are positively correlated (Stoll, 1978), and volatility heavily affects the impact that illiquidity has on stock returns (Spiegel and Wang, 2005). Volume (in dollars), turnover (Datar et al., 1998) and relative bid-ask spread are three alternative measures of illiquidity at the stock level which may partially overlap with LOPs. The correlations between LOPs and these variables have the sign one would expect: -0.21 with size (log market equity); 0.07 with volatility (measured by monthly standard deviation of stock returns following Amihud and Noh (2021)); -0.07 with volume; 0.04 with turnover and 0.30 with relative spread. Table 5 reports the time average of these firm characteristics for the 10 equal-weighted portfolios sorted on LOPs. Mean values are normalized into the $[0, 1]$ interval so that the portfolio with lowest value will display a zero and the one with the highest value will have a 1. Portfolios with high option prices are composed of relatively small and volatile stocks, which are illiquid according to both relative spread and dollar volume. The relation with turnover is instead non-monotonic, increasing at first and then dropping for the last two portfolios.

To account for the influence of these variables on excess returns of option-price-sorted portfolios, we use bivariate conditional sorts as nonparametric tool and carry out a similar analysis to the univariate case. At the end of each month, stocks are first sorted into terciles based on one of the variables just mentioned and *then* grouped with respect to LOPs into 5 portfolios within each tercile. We thus end up with 15 control-and-LOP-sorted portfolios in the spirit of Amihud et al. (2015). The patterns of average excess returns of the resulting double-sorted portfolios are summarized in Figure 6, which is the two-dimensional counterpart of Figure 5. It shows that stock returns increase in liquidity option prices within all terciles of the five controls we consider. A more detailed representation is provided in Table 6. This includes also t-statistics for the average return of LOP-long-short portfolios within each conditioning tercile in the second-to-last column, a high hurdle to gauge the robustness of the findings described in the univariate case. As a benchmark, the last column shows that returns follow the patterns documented in the literature for the control variables. The increasing relation between option prices and returns persists after controlling for every variable, i.e. returns are generally larger for higher option prices within all the control-terciles. With the only exception of stocks very frequently traded, the difference between the 5^{th} portfolio ("Expensive") and the 1^{st} one ("Cheap") is always significant. Importantly, this happens even for the first size-tercile and the last volatility-tercile, alleviating concerns that results are driven by micro-caps or by volatility. Returns significantly spread along the LOP dimension also after conditioning on turnover or the relative bid-ask spread underlying the options. This confirms that the relation between liquidity derivative prices and excess returns goes beyond what traditional liquidity measures capture, thanks to their intrinsic risk-adjustment and their ability to compensate for cumulative illiquidity, as discussed in Section 4.4. Results are largely confirmed at the monthly frequency, as shown in Table 7.

The previous findings document that liquidity option prices capture an "illiquidity dividend" embedded in stock returns which persists at every level of a set of covariates correlated with illiquidity. We now examine whether this conclusion survives the risk-adjustment by testing traditional factor models against the newly built double-sorted portfolios. Results are presented in Table 8, which reports the value of the GRS statistic together with the R^2 for the models listed on the rows. As for the univariate sorting, Fama-French models generate a good fit for the data, yet they do not pass asset pricing tests: the null hypothesis of mean-variance efficiency is rejected for all models and for all the sets of portfolios considered (the 1% critical value for the GRS test is 2.04). To investigate whether the remuneration of the long-short portfolios based LOPs after controlling for size, volatility, volume, turnover and relative spread is due to the exposure to Fama-French factors, the last three columns show their alphas and the relative t-statistic for each characteristic-tercile. For example, the column $1_5 - 1_1$ of the first panel contains the intercept for a long-short portfolio obtained as the difference between the highest-LOP portfolio and the lowest-LOP portfolio within the smallest size-tercile. As the economic intuition would suggest, pricing errors on long-short portfolios are somewhat smaller than in the univariate case because double-sorting nets out the effect of the conditioning variables, which are known to impact stock returns. The largest alphas occur for small, volatile stocks which are rarely traded with low turnover and high bid-ask spreads. Notably, alphas are strikingly similar to the average excess returns displayed in Table 6, even when 6 factors are considered. Put differently, the spread in stock returns along the LOP dimension cannot be explained as a compensation for traditional risk factor exposures even after controlling for the effect of confounding correlated variables. These findings persist also with monthly data, as reported in Table 9. The GRS statistics in fact always largely exceed the 1% critical value (2.12) and the within-tercile long-short portfolio alphas are almost always significant and very close to the average excess returns of the same portfolios.

5 Conclusion

The large degree of time-series variation in market liquidity underlines the business need of investors to hedge their exposures to its fluctuations. We show theoretically that the lack of a market for liquidity is responsible for the spirals between market and funding liquidity. Guided by the principle of efficient allocation of resources in financial markets, we propose a novel liquidity derivative to fill this gap and improve financial stability, and suggest a simple pricing algorithm. We view liquidity derivatives as offering a payoff that accumulates every time the relative bid-ask spread of the underlying financial asset exceeds the transaction costs per unit of notional at the beginning of the contract which appreciates when illiquidity is high or sustained. Liquidity derivatives are a clear and interpretable measure reflecting market participants' expectations about future transaction costs. As such, they have the potential of becoming informative indicators for market watchers as well as for policymakers who want to assess the most appropriate course of action needed to avoid sudden adverse market outcomes. The hedging property of these instruments is supported empirically. Stock returns significantly spread along instrument-level liquidity risk and portfolios of stocks sorted on their simulated liquidity derivative prices generate anomalies that persist after controlling for variables correlated with illiquidity.

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Appendix A: Proof of Lemma I

Proof. We have shown that liquidity derivatives only trade at time $t = 0$, therefore the customers' value function at time $t = 1$ is the same as in B&P, to which we direct the reader for a derivation.

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp\left\{ -\gamma\left[W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2}\right]\right\} \tag{A.1}$$

At time $t = 0$, customer $k = 0$ arrives to the market and maximizes $\mathbb{E}_0[\Gamma_1(W_1^k, p_1, v_1)]$.

$$\begin{aligned} \max_{(y_0^k, c_0^k)_{j\in J}} \quad & \Gamma_1(W_1^k, p_1, v_1) = \\ \max_{(y_0^k, c_0^k)_{j\in J}} \quad & -\mathbb{E}_0[\exp\{-\gamma W_1^k\}] \\ & = -\exp\{-\gamma\mathbb{E}_0[W_1^k] - \frac{\gamma}{2}\text{Var}_0[W_1^k]\} \end{aligned} \tag{A.2}$$

Where the first equality holds because customers are price takers. Replacing Equation (1) into Equation (3), we get

$$\mathbb{E}_0[W_1^k] = W_0^k + (\mathbb{E}_0\mathbf{v}_1 - \mathbf{p}_0)'(\mathbf{y}_0^k + \mathbf{z}_k) + (\boldsymbol{\lambda}_0 - \mathbb{E}_0|\boldsymbol{\Lambda}_1|)\mathbf{c}_0^k \tag{A.3}$$

$$\text{Var}_0[W_1^k] = (\mathbf{y}_0^k + \mathbf{z}_k)^2\boldsymbol{\sigma}_1^2 + (\mathbf{c}_0^k)^2\text{Var}_0(|\Lambda_1^j|) \tag{A.4}$$

where $\boldsymbol{\sigma}_1 = \text{diag}(\sigma_1^1, \ldots, \sigma_1^J)$. We obtain the solution to the customer's problem by taking the first order conditions for an interior optimum.

$$\begin{aligned} y_0^{j,k} &= \frac{v_0^j - p_0^j}{\gamma(\sigma_1^j)^2} - z^{j,k} \\ c_0^{j,k} &= \frac{\lambda_0^j - \mathbb{E}_0|\Lambda_1^j|}{\gamma\text{Var}_0(|\Lambda_1^j|)} \end{aligned} \tag{A.5}$$

■

Appendix B: Proof of Proposition II

Proof. The case of $Z_1 \leq 0$, that implies $p_1 \geq v_1$, $\Lambda_1 > 0$, and $x_1 \leq 0$ is analogous. We have

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2}m_1^- + \frac{\partial m_1^-}{\partial p_1}x_1 + \frac{\partial n_1^+}{\partial p_1}c_0 + x_0 + c_0} \tag{B.1}$$

When financiers are uninformed, the term $\frac{\partial m_1^-}{\partial p_1}x_1 < 0$, which gives rise to a margin spiral, is attenuated by $\frac{\partial n_1^+}{\partial p_1}c_0 > 0$. The speculator who was previously short-selling the stock faces a loss spiral when upward price movements drive prices further away from fundamentals. Note that losses on initial positions are balanced by larger benefits from liquidity derivatives. ■

Appendix C: Sample Selection

The sample selection closely follows Amihud (2002). We select stocks from CRSP restricting to ordinary common shares (first digit of CRSP code is 1) of non-financial firms (SIC-code excluding the interval $[6000, 6999]$). In every year, only stocks with available closing price for more than 200 days in that year and with price greater than \$5 at the end of the year are included, to avoid that "penny stocks" drive the results. The existence of a lower bound imposed by the SEC to the bid-ask spread would make estimation more noisy for such firms (Amihud, 2002). Finally, all firms must have data on market capitalization at the end of the previous year. This excludes derivatives like American Depositary Receipts of foreign stocks and scores and primes.

Because of better reporting quality regarding bid and ask quotes, we then use data from Bloomberg, including returns and accounting measures. Daily observations where the absolute spread is non-positive or greater than 5\$ are deleted (Chung and Zhang, 2014; Korajczyk and Sadka, 2008). We drop stale data points about bid and ask quotes: if the same ask and bid appear for more than 5 days in a row, only the first 5 observations are kept. If the daily closing price is missing, the midquote is used instead. Volume is winsorized at the 1^{st} and 99^{th} percentile of its cross-sectional distribution in each year.

Liquidity option prices are simulated for firms with less than 100 missing data for the relative spread in each year t. This is a lower bound that ensures reliability of parameter estimates. LOPs equal to zero and prices with negative MLE estimates of σ in Eq. (19) are deleted. Although we impose a high level of precision in our numerical procedure (100,000 Monte Carlo simulations), economically non-meaningful results due to purely computational limits can still occur. Moreover, we drop prices for which the Akaike Information Criterion (AIC) relative to the CIR parameter estimation exceeds the 99^{th} percentile of each year to ensure data are of sound quality. Finally, prices are trimmed at the 99th percentile of the distribution of the corresponding year, and multiplied by 10000 to obtain values in basis points (bps) to improve readability. Whenever a price is lower than 1 bps (8029 observations), it is set to 1 bps to make it a meaningful value.

Appendix D: Liquidity-hedged Positions

We describe here in detail the computation of returns used in Section 4.3. We focus on 3-month returns to match the time to maturity of the 3-months European liquidity options we simulate. For each stock, the raw (non-hedged) return between time t and $t+3$, where t represents months, is simply the percentage change between the dividend-adjusted prices, i.e. $r_{t+3} = p_{t+3}/p_t - 1$. We build a hedged position such that the initial investment is still p_t to make the two strategies comparable. This is achieved by selling a part of the stock with equal value to the price of one liquidity option. Since option prices are expressed in bps of the mid-price at t, m_t, one needs to account for potential differences between p_t and m_t. Let C_t be the price of a 3-month option, now expressed in decimals of m_t. A hedged position ζ_t has initial value

$$\zeta_t = p_t \left(1 - \frac{m_t}{p_t} C_t\right) + p_t \left(\frac{m_t}{p_t} C_t\right) = p_t \tag{D.1}$$

Multiplying C_t by the ratio m_t/p_t converts it into dollars and ensures $\zeta_t = p_t$ even with $p_t \neq m_t$. At time $t+3$, i.e. at the maturity of the option, the investor obtains the corresponding payoff X_{t+3}, which is also expressed in bps per unit of notional. Hence, the value of the position at $t+3$ will be

$$\zeta_{t+3} = p_{t+3} \left(1 - \frac{m_t}{p_t} C_t\right) + m_t X_{t+3} \tag{D.2}$$

The ratio of ζ_{t+3} to p_t minus 1 represents the return of a hedged position, which can be compared with r_{t+3}. Hypothesis 2 is tested after subtracting the risk-free rate such that we compare investments financed by borrowing p_t.

Figures

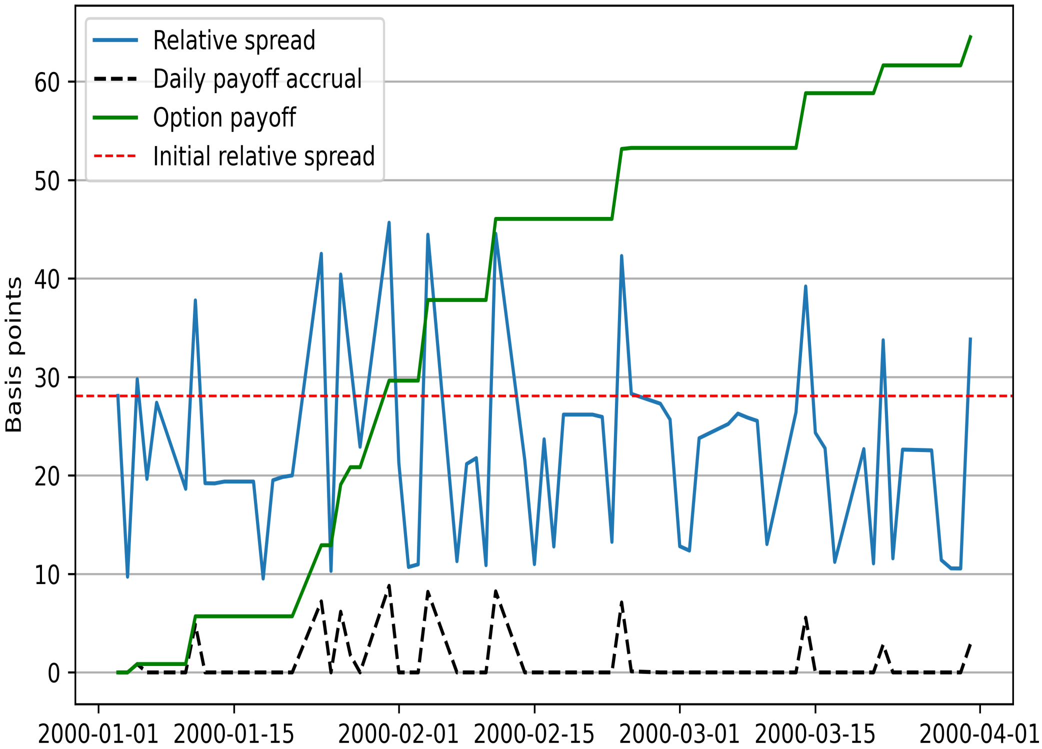

FIGURE 1: **Example of liquidity option payoff**
This Figure shows the payoff of a liquidity option on the relative spread of Walmart Inc. for the first quarter of 2000. The daily payoff accrual is one half the difference between the relative spread and its value at the initiation of the contract, when such difference is positive, and zero otherwise. The option payoff is the sum of the daily payoff accruals at the maturity.

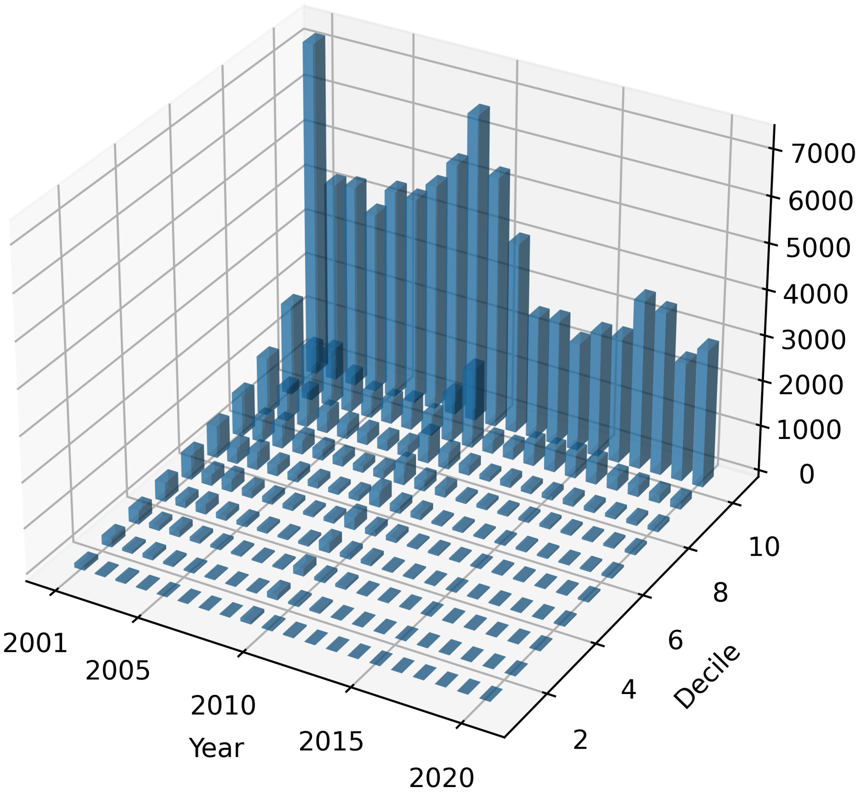

FIGURE 2: **Liquidity option prices distribution**
This Figure shows yearly cross-sectional liquidity option prices deciles (bps) from 2001 to 2020.

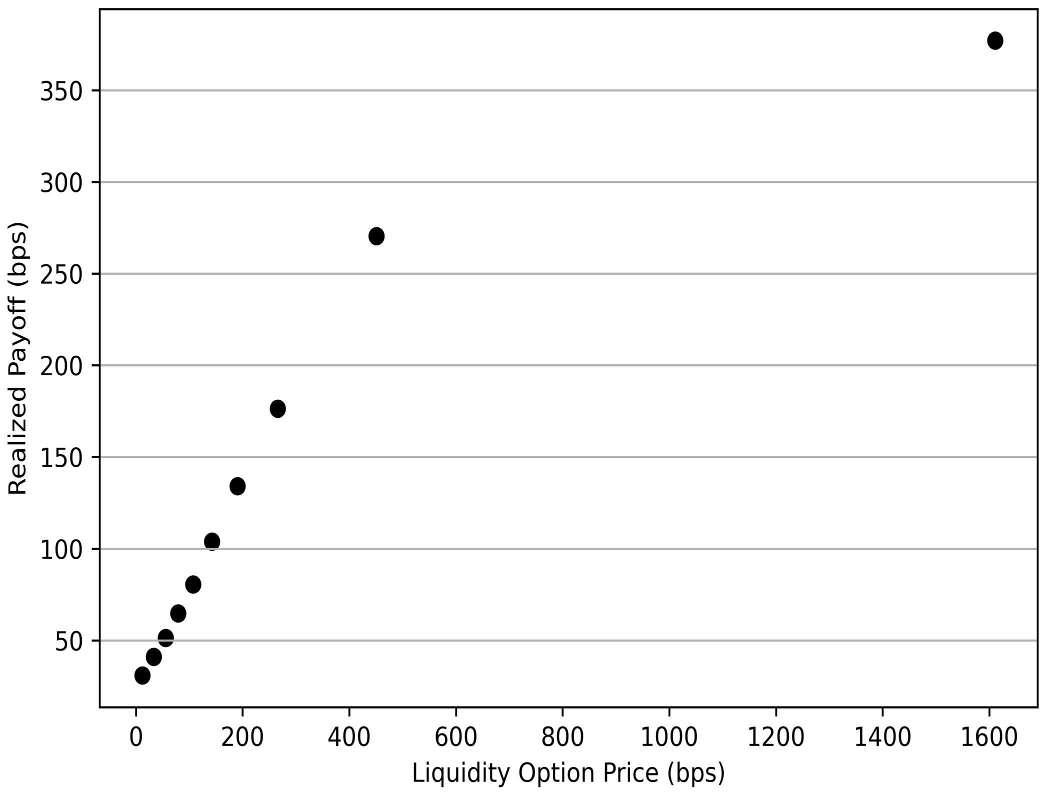

FIGURE 3: **Liquidity option prices vs Realized payoffs**
This Figure shows average liquidity option prices (bps, x-axis) against average realized payoffs (bps, y-axis) for 10 equal-weighted portfolios sorted on option prices. Data from January 2001 to December 2020.

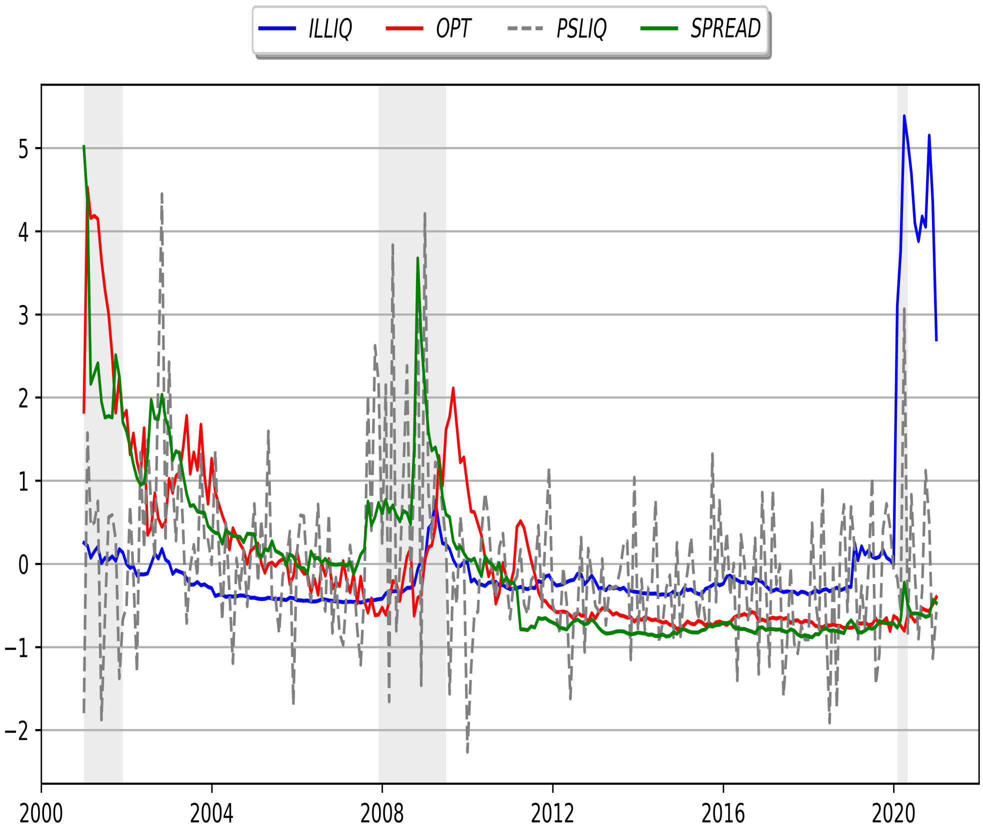

Figure 4: **Comparison of liquidity measures**
This Figure shows market-wide illiquidity measures from December 2000 to December 2020. $ILLIQ$ refers to Amihud (2002). OPT is the monthly cross-sectional median option price. $PSLIQ$ is the aggregate liquidity measure from Pástor and Stambaugh (2003), with flipped sign. $SPREAD$ is the monthly cross-sectional median relative bid-ask spread. Series are standardized. Grey shaded areas represent NBER recessions.

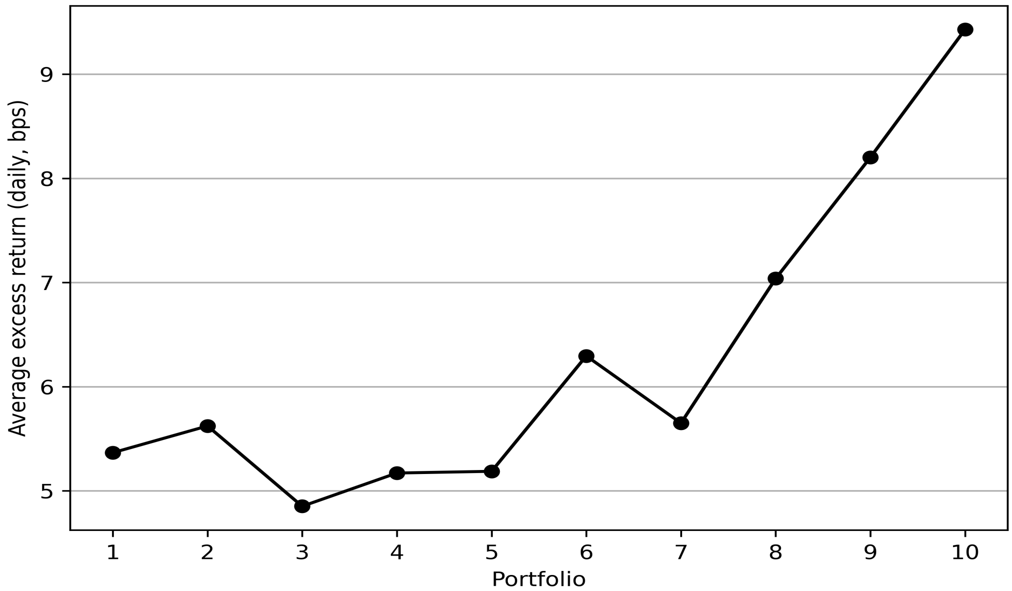

Figure 5: **Average excess returns: 10 equal-weighted portfolios**
This Figure shows the average excess returns of 10-equal weighted portfolios of stocks sorted on liquidity option prices on a monthly basis. Daily data from January 2001 to December 2020.

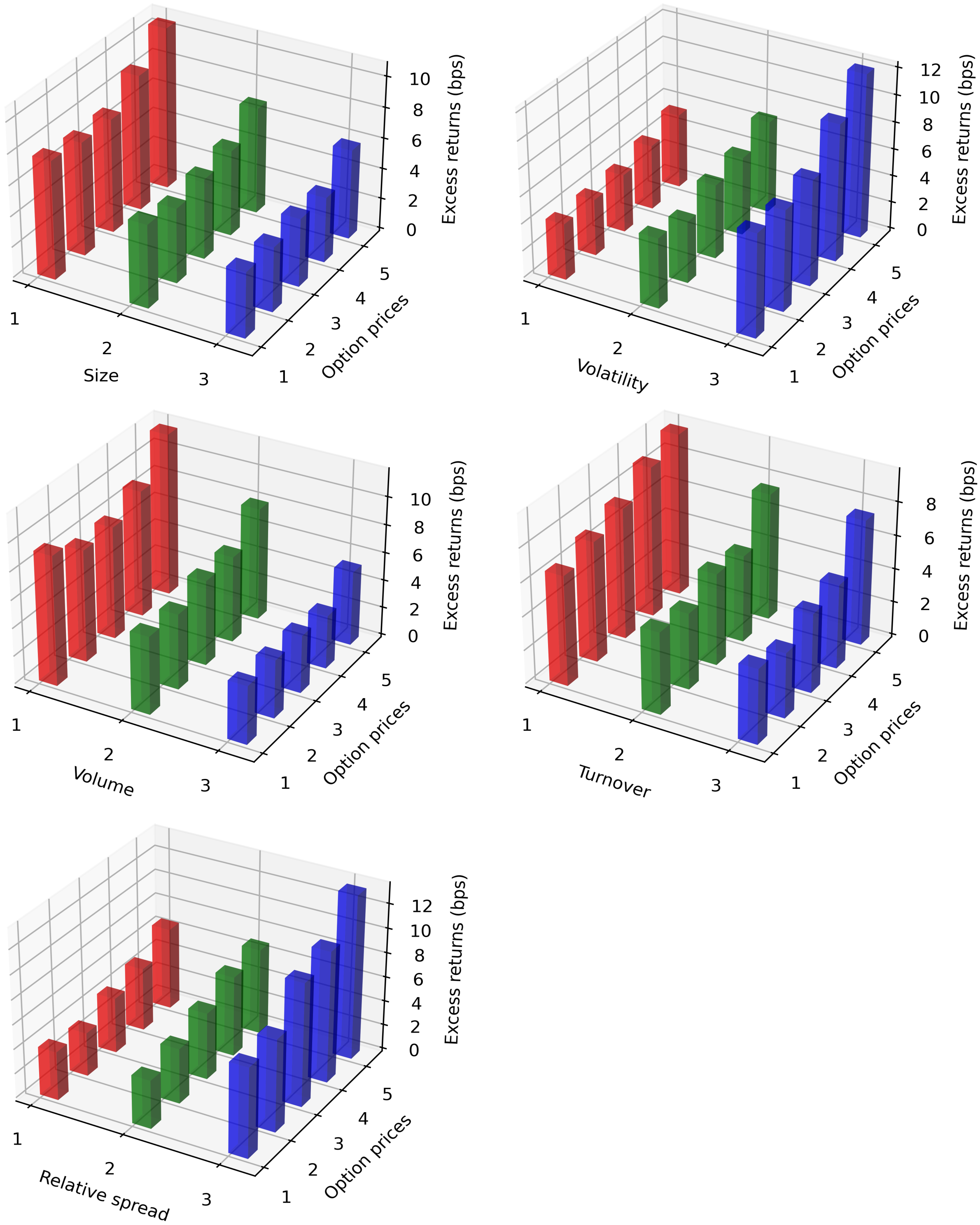

FIGURE 6: **Average excess returns: conditional double-sorted portfolios**
This Figure shows average excess returns of conditional double-sorted portfolios. Portfolios are formed on a monthly basis by sorting stocks first into terciles based on size (top-left panel), volatility (top-right), dollar volume (middle-left), turnover (middle-right) or relative bid-ask spread (bottom-left) and then into quintiles based on liquidity option prices for each tercile. Daily data from January 2001 to December 2020.

Tables

Table 1: **Heterogeneous Liquidity Exposures**

This Table reports estimates from the monthly time-series regressions of the Barclay Hedge Fund Index and the Standars and Poor's Insurance Index on the Fama and French factors plus momentum augmented with the Pastor and Stambaugh aggregate liquidity factor. The data range from June 2003, the first value of the Standars and Poor's Insurance Index, to December 2020. Heteroskedasticity-robust standard errors in parentheses.

	Barclay Hedge Fund Index	S & P Insurance Index
$R_m - R_f$	0.35***	0.99***
	(0.02)	(0.06)
SMB	0.02	-0.04
	(0.03)	(0.09)
HML	0.01	0.58***
	(0.05)	(0.10)
RMW	-0.06	-0.18
	(0.04)	(0.13)
CMA	-0.07	-0.28*
	(0.06)	(0.16)
MOM	-0.03	-0.05
	(0.02)	(0.04)
PS	0.11***	-0.15***
	(0.02)	(0.06)
Observations	210	210
R^2	0.82	0.83

Table 2: **Summary statistics**

This Table reports summary statistics of the sample consisting of CRSP stocks traded on the NYSE with Bloomberg data from January, 2000 to December, 2020 with corresponding distribution percentiles. Market Cap. is market capitalization expressed in million of dollars. Volume represents monthly average traded volume in millions of dollars. Rel. spread is the monthly average ratio of the difference between the best bid and ask quotes to the midquote, measured in basis points. $ILLIQ$ refers to the Amihud (2002) illiquidity measure at the stock and at the monthly level. C denotes liquidity option prices expressed in basis points per units of notional for a contract maturity of three months. m is the monthly average midquote. Volatility is the monthly standard deviation of stock returns. Turnover is the monthly average ratio of dollar volume to market capitalization.

							Percentiles						
	N	Mean	SD	Skewness	Min	Max	1%	5%	25%	50%	75%	95%	99%
Market Cap.	241401	10734	29439	6.96	1.07	580934	53.03	160.17	821.91	2444.45	7998.35	43749	160282
Volume	291545	20.73	44.14	6.16	0.00	2419	0.01	0.12	1.50	6.13	20.38	88.14	221.13
Rel. Spread	291733	38.28	200.36	23.50	0.28	16471	1.27	1.86	4.71	10.98	23.52	113.15	479.64
$ILLIQ$	290608	0.1465	1.6788	20.3975	0.00	45.6963	0.00	0.0001	0.0007	0.0025	0.0122	0.2081	1.5712
C	192746	279.72	611.51	4.45	1.00	7974.33	1.00	1.98	20.64	67.38	233.59	1325.58	3318.96
m	291733	42.01	93.50	22.47	0.09	6421.76	1.98	4.89	14.17	25.65	45.67	114.75	275.91
Volatility	289183	0.02	0.26	482.57	0.01	134.29	0.01	0.01	0.01	0.02	0.03	0.05	0.09
Turnover	241261	0.37	0.58	19.41	0.00	55.06	0.01	0.03	0.14	0.24	0.43	1.07	2.25

Table 3: **Liquidity factor loadings: raw and hedged portfolios**

This Table presents absolute mean loadings on Pástor and Stambaugh (2003) liquidity factor in a multifactor model including Fama and French (1993) plus momentum for 10 equal-weighted portfolios sorted on liquidity option prices. Loadings refer to raw returns and to returns hedged with liquidity options for each stock in each portfolio. The last column reports the *p*-value for the null hypothesis that the mean absolute exposure of raw position is less than or equal to that of a hedged position, from a *t*-test for difference in means that allows for unequal variances based on rolling estimates following the approach in Savor and Wilson (2013, 2014). Data from January 2001 to December 2020.

Portfolio number	Absolute mean exposure		P-value difference in means
	Raw	Hedged	
1	0.0223	0.0053	0.004
2	0.0351	0.0198	0.000
3	0.0025	0.0214	0.962
4	0.0004	0.0151	0.991
5	0.0207	0.0040	0.022
6	0.0241	0.0010	0.000
7	0.0013	0.0104	0.799
8	0.0514	0.0359	0.003
9	0.0274	0.0037	0.002
10	0.0494	0.0024	0.001
Mean	0.0235	0.0119	

Table 4: **Alphas and R^2 of equal-weighted portfolios sorted on liquidity option prices**

This Table shows alphas (bps) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on 10 equal-weighted portfolios sorted on liquidity option prices at the end of each month. The top and the bottom panel present daily and monthly data, respectively, from January 2001 to December 2020. $t-$statistics in brackets are obtained by using Newey and West (1987) standard errors with 5 lags.

	Decile portfolio											
	1	2	3	4	5	6	7	8	9	10	10-1	R^2
	Daily data											
CAPM alpha	1.79	2.13	1.36	1.63	1.67	2.77	2.02	3.36	4.49	5.91	4.12	84.53
	(2.32)	(2.89)	(1.84)	(2.23)	(2.28)	(3.78)	(2.61)	(4.04)	(4.74)	(6.04)	(5.18)	
FF3 alpha	1.42	1.81	1.00	1.28	1.26	2.29	1.41	2.66	3.71	5.31	3.89	92.23
	(2.39)	(3.18)	(1.79)	(2.28)	(2.39)	(4.36)	(2.79)	(5.24)	(6.39)	(7.76)	(5.00)	
FF4 alpha	1.58	1.96	1.08	1.36	1.29	2.29	1.35	2.60	3.58	5.32	3.74	92.40
	(2.75)	(3.49)	(1.90)	(2.41)	(2.41)	(4.34)	(2.67)	(5.14)	(6.27)	(7.71)	(4.79)	
FF5 alpha	0.91	1.22	0.28	0.61	0.64	1.74	0.89	2.23	3.26	4.87	3.96	92.83
	(1.57)	(2.24)	(0.53)	(1.17)	(1.29)	(3.49)	(1.85)	(4.53)	(5.76)	(7.17)	(5.10)	
FF6 alpha	0.95	1.25	0.30	0.64	0.65	1.75	0.89	2.22	3.24	4.88	3.93	93.09
	(1.75)	(2.42)	(0.57)	(1.24)	(1.32)	(3.51)	(1.84)	(4.52)	(5.78)	(7.23)	(5.09)	
	Monthly data											
CAPM alpha	25.57	33.84	19.24	25.29	26.87	50.04	33.65	64.16	85.96	116.89	91.32	80.37
	(1.43)	(2.07)	(1.12)	(1.46)	(1.54)	(2.60)	(1.61)	(3.08)	(3.87)	(3.91)	(5.02)	
FF3 alpha	20.51	29.84	15.14	21.86	21.29	43.30	24.85	52.65	75.52	109.31	88.81	89.68
	(1.58)	(2.83)	(1.37)	(1.67)	(2.07)	(3.38)	(2.05)	(4.75)	(6.18)	(4.96)	(4.14)	
FF4 alpha	27.34	32.75	16.36	23.56	22.87	45.19	28.02	53.60	74.60	111.60	84.26	89.90
	(2.12)	(3.21)	(1.52)	(1.67)	(2.23)	(3.39)	(2.22)	(4.25)	(5.12)	(4.64)	(3.77)	
FF5 alpha	2.30	13.96	-1.56	-2.36	4.06	22.75	11.92	37.06	56.99	86.59	84.28	90.93
	(0.20)	(1.35)	(-0.15)	(-0.19)	(0.41)	(2.01)	(1.00)	(3.65)	(4.83)	(4.52)	(4.16)	
FF6 alpha	2.81	14.23	-1.38	-2.11	4.26	22.99	12.18	37.21	57.06	86.86	84.06	91.58
	(0.27)	(1.49)	(-0.13)	(-0.17)	(0.45)	(2.1)	(1.05)	(3.63)	(4.81)	(4.72)	(4.07)	

TABLE 5: **Normalized average portfolio characteristics**

This Table presents average firm characteristics for 10 equal-weighted portfolios sorted on liquidity option prices (C). Average characteristics are normalized into the $[0, 1]$ interval. Size is the log market equity. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in million of dollars. Turnover is the ratio of dollar volume to market capitalization. The relative bid-ask spread is the bid-ask spread divided by the midquote. Data from January 2001 to December 2020.

Portfolio number	C	Size	Volatility	Volume	Turnover	Relative spread
1	0.000	0.943	0.153	0.921	0.241	0.082
2	0.013	1.000	0.047	1.000	0.268	0.031
3	0.027	0.989	0.000	0.939	0.501	0.006
4	0.042	0.917	0.044	0.805	0.637	0.000
5	0.060	0.781	0.108	0.590	0.749	0.035
6	0.082	0.628	0.220	0.429	0.974	0.091
7	0.112	0.453	0.381	0.250	1.000	0.170
8	0.159	0.225	0.672	0.096	0.920	0.310
9	0.275	0.000	1.000	0.000	0.243	0.624
10	1.000	0.257	0.812	0.267	0.000	1.000

TABLE 6: **Average excess returns of conditional double-sorted portfolios (daily)**

This Table presents average excess returns for conditional double-sorted portfolios formed at the end of each month by sorting stocks first on the left-hand variable and then on liquidity option prices. Size refers to market capitalization. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in dollars. Turnover is the ratio of dollar volume to market capitalization. The relative bid-ask spread is the bid-ask spread divided by the midquote. The second-to-last column reports in brackets the t-statistic for the average returns of LOP-long-short portfolio within each tercile of the conditioning variable. Daily data from January 2001 to December 2020.

	Liquidity Option Price						
	Cheap	2	3	4	Expensive	5-1	All
Size							
Small	7.71	7.41	7.47	8.93	10.69	2.98 (2.69)	8.08
Medium	5.41	4.88	5.26	5.62	6.98	1.56 (1.99)	5.77
Large	4.26	4.20	4.44	4.26	5.86	1.60 (2.33)	4.83
Volatility							
Stable	4.14	4.17	4.29	4.81	5.52	1.38 (2.72)	4.77
Medium	5.16	4.55	5.48	5.81	6.82	1.66 (2.35)	5.79
Volatile	7.59	7.41	7.80	10.14	12.12	4.53 (3.58)	8.77
Volume							
Infrequent	9.26	8.03	8.09	9.14	11.76	2.50 (2.31)	8.22
Medium	5.55	5.40	6.11	6.18	8.07	2.52 (2.86)	6.33
Frequent	4.15	4.14	4.12	3.82	5.31	1.16 (1.49)	4.48
Turnover							
Low	6.52	7.16	7.81	9.00	9.75	3.23 (3.46)	7.94
Medium	4.85	4.53	5.46	5.19	7.58	2.72 (3.50)	5.74
High	4.42	3.86	4.71	5.01	7.47	3.05 (2.95)	5.01
Relative bid-ask spread							
Low	3.95	3.55	4.52	5.16	6.69	2.74 (3.21)	4.95
Medium	3.94	4.49	5.45	6.55	6.99	3.04 (3.19)	5.55
High	7.32	7.53	10.06	10.71	13.42	6.11 (5.08)	8.53
All	5.49	5.01	5.74	6.35	8.82	3.32 (4.88)	

Table 7: **Average excess returns of conditional double-sorted portfolios (monthly)**

This Table presents average excess returns for conditional double-sorted portfolios formed at the end of each month by sorting stocks first on the left-hand variable and then on liquidity option prices. Size refers to market capitalization. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in dollars. Turnover is the ratio of dollar volume to market capitalization. The relative bid-ask spread is the bid-ask spread divided by the midquote. The second-to-last column reports in brackets the *t*-statistic for the average returns of LOP-long-short portfolio within each tercile of the conditioning variable. Monthly data from January 2001 to December 2020.

	Liquidity Option Price						
	Cheap	2	3	4	Expensive	5-1	All
Size							
Small	148.29	142.76	154.19	177.12	220.14	71.85 (3.07)	160.91
Medium	104.33	93.64	101.93	112.13	135.68	31.35 (1.98)	112.73
Large	82.78	78.47	85.14	80.48	115.92	33.14 (2.17)	93.26
Volatility							
Stable	80.70	83.14	83.82	96.39	111.76	31.07 (3.09)	95.20
Medium	98.79	87.52	105.46	114.11	133.22	34.44 (2.74)	114.02
Volatile	141.04	141.00	151.23	201.61	253.45	112.41 (3.83)	169.16
Volume							
Infrequent	187.73	162.76	171.92	185.27	252.85	65.12 (2.43)	163.41
Medium	105.21	108.98	121.23	115.98	156.99	51.78 (2.92)	123.15
Frequent	80.06	74.83	75.00	74.16	99.68	19.61 (1.24)	85.36
Turnover							
Low	128.48	141.60	154.62	187.79	198.70	70.22 (3.78)	158.21
Medium	92.27	87.37	104.37	102.36	152.95	60.68 (3.51)	112.77
High	81.99	68.67	87.41	95.83	146.27	64.29 (3.04)	95.86
Relative bid-ask spread							
Low	71.40	79.07	82.53	67.96	97.53	26.14 (2.45)	84.97
Medium	102.30	94.01	107.02	117.87	113.08	10.78 (0.65)	111.79
High	197.44	207.65	201.69	217.38	288.92	91.48 (3.06)	175.13
All	104.90	96.78	111.30	125.47	179.87	74.96 (5.05)	

Table 8: **GRS statistic and R^2 of conditional double-sorted portfolios (daily)**

This Table reports the GRS F-statistic (Gibbons et al., 1989) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on cross-sections of 3x5 equal-weighted (conditionally) double-sorted portfolios. Alphas are reported for long-short portfolios for each tercile of the conditioning variable. $t-$ statistics in brackets are obtained using Newey and West (1987) standard errors with 5 lags. Daily data from January 2001 to December 2020.

	Bivariate portfolios		Long-short portfolio alpha		
	GRS statistic	R^2 on the PFs (%)	$1_5 - 1_1$	$2_5 - 2_1$	$3_5 - 3_1$
		Size x Option price			
CAPM	3.16	79.55	3.27	1.58	1.47
			(3.13)	(2.05)	(2.16)
FF3	6.10	90.05	3.27	1.54	1.41
			(3.13)	(1.99)	(2.09)
FF4	6.09	90.23	2.97	1.35	1.29
			(2.91)	(1.75)	(1.9)
FF5	5.30	90.76	3.43	1.72	1.54
			(3.31)	(2.24)	(2.29)
FF6	5.33	91.05	3.37	1.68	1.52
			(3.37)	(2.23)	(2.27)
		Volatility x Option price			
CAPM	4.47	79.98	1.37	1.85	4.86
			(2.70)	(2.81)	(3.82)
FF3	6.29	88.65	1.14	1.60	4.59
			(2.47)	(2.51)	(3.63)
FF4	6.21	89.55	1.12	1.45	4.08
			(2.41)	(2.30)	(3.39)
FF5	5.59	89.27	1.35	1.80	4.33
			(2.93)	(2.80)	(3.45)
FF6	5.59	90.16	1.34	1.77	4.25
			(2.91)	(2.81)	(3.57)
		Volume x Option price			
CAPM	4.12	79.03	2.73	2.41	0.89
			(2.61)	(2.81)	(1.19)
FF3	8.52	89.64	2.73	2.22	0.73
			(2.61)	(2.62)	(0.99)
FF4	8.74	89.94	2.41	1.94	0.51
			(2.39)	(2.35)	(0.71)
FF5	7.83	90.22	2.81	2.33	1.07
			(2.72)	(2.75)	(1.47)
FF6	8.00	90.62	2.75	2.28	1.03
			(2.76)	(2.8)	(1.48)
		Turnover x Option price			
CAPM	5.23	81.50	3.26	2.52	2.90
			(3.62)	(3.24)	(2.92)
FF3	7.50	88.90	2.92	2.16	2.58
			(3.49)	(2.98)	(2.65)
FF4	7.54	89.09	2.75	2.02	2.28
			(3.28)	(2.83)	(2.41)
FF5	6.75	89.61	2.88	2.23	2.69
			(3.45)	(3.07)	(2.77)
FF6	6.81	89.91	2.85	2.21	2.63
			(3.40)	(3.11)	(2.81)
		Relative spread x Option price			
CAPM	5.58	79.40	2.14	2.59	6.39
			(2.84)	(2.99)	(5.28)
FF3	8.00	89.15	1.62	2.06	6.19
			(2.49)	(2.74)	(5.14)
FF4	8.25	89.54	1.52	1.93	5.96
			(2.32)	(2.56)	(4.91)
FF5	7.40	89.70	1.87	2.26	6.41
			(2.87)	(3.01)	(5.33)
FF6	7.60	90.20	1.85	2.24	6.37
			(2.89)	(3.00)	(5.29)

Table 9: GRS statistic and R^2 of conditional double-sorted portfolios (monthly)

This Table reports GRS F-statistic (Gibbons et al., 1989) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on cross-sections of 3x5 equal-weighted (conditionally) double-sorted portfolios. Alphas are reported for long-short portfolios for each tercile of the conditioning variable. $t-$ statistics in brackets are obtained using Newey and West (1987) standard errors with 5 lags. Monthly data from January 2001 to December 2020.

	Bivariate portfolios		Long-short portfolio alpha		
	GRS statistic	R^2 on the PFs (%)	$1_5 - 1_1$	$2_5 - 2_1$	$3_5 - 3_1$
		Size x Option price			
CAPM	2.95	75.01	83.70	35.05	28.33
			(3.88)	(2.36)	(1.92)
FF3	4.48	87.45	85.33	35.85	25.47
			(4.00)	(2.35)	(1.75)
FF4	4.59	87.79	73.94	34.21	27.12
			(3.48)	(1.99)	(1.53)
FF5	3.27	88.60	91.12	42.69	33.42
			(3.77)	(2.78)	(2.22)
FF6	3.34	89.43	90.45	42.55	33.46
			(3.69)	(2.75)	(2.20)
		Volatility x Option price			
CAPM	3.12	75.37	26.58	39.27	122.43
			(2.58)	(3.83)	(3.65)
FF3	4.41	85.38	21.68	36.39	121.42
			(2.66)	(3.79)	(3.60)
FF4	4.28	86.96	20.13	32.60	105.20
			(2.58)	(3.49)	(3.16)
FF5	3.10	86.46	22.45	35.14	108.95
			(2.69)	(3.77)	(3.37)
FF6	3.10	88.52	22.36	34.93	108.12
			(2.79)	(3.85)	(3.27)
		Volume x Option price			
CAPM	2.93	74.54	73.88	53.83	15.03
			(3.15)	(3.06)	(1.00)
FF3	4.32	85.81	75.36	51.67	11.73
			(3.07)	(2.94)	(0.80)
FF4	4.74	86.49	61.67	46.52	8.92
			(2.58)	(2.38)	(0.55)
FF5	3.18	87.20	72.71	57.76	18.62
			(2.91)	(3.36)	(1.26)
FF6	3.43	88.52	71.96	57.43	18.42
			(2.73)	(3.31)	(1.29)
		Turnover x Option price			
CAPM	4.25	75.83	66.81	55.25	66.09
			(3.28)	(2.98)	(3.39)
FF3	4.97	84.89	61.72	49.96	62.99
			(3.54)	(3.33)	(3.43)
FF4	5.54	85.73	58.51	46.70	56.22
			(3.06)	(2.8)	(2.87)
FF5	4.22	85.96	59.82	50.06	61.41
			(3.49)	(3.30)	(3.02)
FF6	4.46	87.32	59.65	49.88	61.04
			(3.45)	(3.28)	(3.03)
		Relative spread x Option price			
CAPM	3.93	72.44	21.40	19.78	106.45
			(2.09)	(1.37)	(3.62)
FF3	4.60	83.77	18.05	18.85	106.14
			(1.77)	(1.41)	(3.59)
FF4	4.77	85.17	14.89	8.35	91.53
			(1.46)	(0.69)	(3.28)
FF5	3.40	85.11	20.17	16.73	102.24
			(2.00)	(1.03)	(3.56)
FF6	3.53	87.30	19.98	16.15	101.44
			(2.08)	(1.2)	(3.53)