A Simple Non-Parametric Approach to the Term Structure and Time Decomposition of Credit Default Swap Spreads

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Abstract

This study introduces a simple non-parametric approach to pricing credit default swaps (CDS) and other single-name credit-risky securities. The method relies exclusively on closed-form solutions, allows any term structure of CDS spreads to be reproduced, and implies lower pricing errors than conventional models. The study extends the theme by providing an equally simple and intuitive approach to the time decomposition of CDS spreads, which is similar to, but also remarkably different from the traditional decomposition of spot risk-free rates into forward rates. The overall research conclusions are supported by a case study of the Eurozone sovereign debt crisis.

JEL classification: G12, G13, G14.

Keywords: No-arbitrage credit risk pricing, term structure of CDS spreads, spot and forward CDS contracts.

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1. Introduction

The term structure of credit default swap (CDS) spreads represents a valuable piece of information for pricing credit-risky securities; mainly (and unsurprisingly), actual positions in CDS contracts. Pricing models, based on the term structure of CDS spreads, can be classified as either parametric or semi-parametric. Parametric models have been used, among others, by Pan and Singleton (2008), Chen et al. (2013), and Jarrow et al. (2019), and generally work in the following way. First, a stochastic (parametric) process for the risk-neutral default intensity and a distribution function for the CDS spreads' pricing errors are assumed. Second, based on these assumptions, the model parameters are estimated using the maximum-likelihood method or a similar optimization rule. Finally, the estimated model can be used to price existing CDS contracts and other credit-risky securities (e.g., risky bonds). An appealing characteristic of parametric models is the fact that all prices rely on a few parameter values, which dependence on more fundamental variables can also be analyzed. For the same reason, the main limitation of these models is that pricing errors can be minimized but never completely eliminated. In other words, the use of these models for pricing purposes necessarily assumes some degree of market-mispricing in observed CDS spreads, model-mispricing, or a combination of the two.[1]

When the primary interest is marking-to-market CDS contracts, the conventional approach consists of a semi-parametric model. While different variations exist (Hull and White, 2003; O'Kane and Turnbull, 2003), the core assumptions can be summarized as follows: the risk-free interest process and the default time are risk-neutrally independent,

[1] The previous discussion refers to so-called reduced-form models (Jarrow and Turnbull, 1995). Structural credit risk models (Merton, 1974) constitute a different family of parametric models. Du et al. (2019) offer a good example of the technical challenges associated with replicating an observed term structure of CDS spreads, based on a structural credit risk model. These later models are not addressed in the present study.

and (forward risk-neutral) default probabilities have a piecewise constant profile. Based on these assumptions, the term structure of default probabilities can be estimated sequentially, from the lowest to the highest maturity of available CDS spreads, such that these observed quotes are perfectly refitted by the model. As a constant default probability model represents the clearest example of a parametric model, a piecewise constant default probability model can be effectively described as semi-parametric.

It is sensible to require a CDS pricing model to replicate observed CDS spreads. However, we may reasonably wonder whether this is sufficient—and if we can go further. Figure 1 provides an illustrative example. Panel 1A reflects the hypothetical *complete* term structure of CDS spreads (CTSCDS henceforth; black solid line, left axis). Generated using a particular parameterization of the Nelson-Siegel model, this comprises all possible maturities over a 10-year horizon: from one to 3,650 calendar dates.[2] However, the CTSCDS is not observed in practice. As the figure indicates, the *observed* term structure (OTSCDS; red points, left axis) is typically reduced to 6m, 1y, 2y, 3y, 4y, 5y, 7y, and 10y maturities. Panel 1A incorporates the predicted CTSCDS, in accordance with the piecewise constant default probability model discussed below (PWCDP; blue dashed line, left axis) and the corresponding absolute pricing errors (APE; black dotted line, right axis). As the panel shows, the PWCDP model provides a perfect fit for the observed CDS spreads, although there may be non-negligible pricing errors for the rest of the curve.

Now consider Panel 1B. It contains the same *true* CTSCDS as Panel 1A, but the OTSCDS is used in this case to perform a straight interpolation between the observed quotes. The precise method applied is Shape-Preserving Piecewise Cubic Hermite

[2] The precise parameter values in the Nelson-Siegel model are $\beta_0 = 50$; $\beta_1 = 0$; $\beta_2 = 1250$; $\alpha_1 = 10$.

Interpolation (PCHIP).[3] As the figure makes clear, the PCHIP method offers a more accurate representation of the true CTSCDS than the PWCDP model. In numerical terms, the mean absolute pricing errors (MAPE) are 2.61 bp for the PWCDP model and 0.11 bp for the PCHIP method. As this example shows, even a relatively simple interpolation scheme may provide a better fit for the CTSCDS than a PWCDP model, particularly for maturities up to 6m, where these kinds of models impose a flat term structure. By extension, the example also suggests that, instead of assuming a piecewise constant default probability *ex-ante* and estimating the CTSCDS *ex-post*, the problem of model-mispricing can be minimized by first fitting the most plausible CTSCDS, based on the observed quotes, and having a pricing model capable of reproducing next the entire curve.

<Figure 1 about here>

This study makes two main contributions to the literature on credit risk pricing and on the pricing of CDS contracts, in particular. As a first contribution, it derives an extremely simple, non-parametric pricing model that allows to refit any pre-specified CTSCDS. The model draws on the following three core elements. First, the price of a CDS contract can always be expressed as a simple function of a reduced number of well-established building blocks in credit risk pricing, or credit risk discount factors (CRDF), initially defined by Lando (1998). Second, in a discrete-time economy, where all future asset maturities and possible defaulting times are the same (i.e., all future calendar dates, consistent with the proposed interpolation), a set of no-arbitrage conditions can be derived between the values of those CRDFs for any two consecutive maturities. Finally, based on these results, and provided that a CTSCDS is determined ex-ante, an equation system exits that allows the immediate bootstrap of such CRDFs for all possible maturities.

[3] PCHIP refers to the code used in the commercial software Matlab® for this interpolation method.

Regarding the implementation of the model, a reasonable concern is that refitting a CTSCDS (a total of 3,650 quotes in our example) may imply much higher complexity and/or computational costs than reproducing a limited number of observed quotes (only eight values in the same example), as the PWCDP model does. However, the opposite turns out to be true. As previously discussed, the price of a CDS contract is a simple function of the CRDFs, and as the present study shows, the same simple structure applies to the no-arbitrage conditions between these CRDFs. This implies that both the bootstrapping process and the posterior mark-to-market of any position in a CDS contract are very easy to implement. Specifically, the bootstrapping procedure is based exclusively on closed-form solutions. Thus, unlike conventional pricing models, it does not involve a sequence of root-search algorithms or any other optimization procedure.

In terms of pricing errors, this study uses the Eurozone sovereign debt crisis as a research field to compare the performance of four different pricing approaches. On the one hand, the PWCDP model. On the other hand, the non-parametric model introduced in this study, where the CTSCDS has been estimated ex-ante using either a linear (Linear), PCHIP, or Cubic Spline (Spline) interpolation between the observed quotes. Based on this empirical analysis, we can conclude that the non-parametric model with a PCHIP interpolation provides the lowest MAPE, while the PWCDP model generates the highest MAPE.

The second main contribution of the present study relates to a less obvious, but important application of the proposed pricing model: the time decomposition of CDS spreads. This refers to the problem of determining the percentage of a CDS spread that can be reasonably attributed to the protection of specific time intervals within the contracts' maturity. Despite its intrinsic interest, this question has not received explicit attention in the academic literature.[4] This study shows that the time decomposition of CDS spreads is similar to, but also remarkably different from the usual decomposition of spot (risk-free interest) rates into forward rates. Analogous to a spot rate decomposition, a CDS spread decomposition follows from the possibility of representing a spot CDS contract as a portfolio of forward CDS contracts. However, in contrast to a forward rate agreement, the enforcement of a forward CDS contract is conditional on the survival of the underlying bond before the initiation date. The principal implication is that, unlike a spot rate, a CDS spread is not a simple mean of forward CDS spreads, and this is because forward CDS spreads may never be paid. Nevertheless, once the pricing of spot and forward CDS contracts is settled directly in relation to the CRDFs, this challenging characteristic of a CDS spread decomposition and its implications for the shape of the term structure of CDS spreads are both very easy to address.

The remainder of this paper is organized as follows. Section 2 defines the basic setting and introduces the no-arbitrage conditions between the CRDFs. Section 3 reviews the pricing of CDS contracts, based on these CRDFs. Section 4 incorporates additional assumptions and describes the bootstrapping process. A conventional PWCDP is presented in Section 5 as a restricted case. Section 6 discusses the possible applications of the term structure of CRDFs, including the time decomposition of CDS spreads. Section 7 presents the case study that serves to compare the performance of different pricing approaches and to illustrate some of their possible applications. Section 8 provides a summary of the main conclusions.

[4] In the context of reduced-form models, it is common to express a CDS spread as a weighted average of risk-neutral hazard rates (Schönbucher, 2003; Lando and Mortensen, 2005), which may be seen as a form of time decomposition. However, the definition presented here is not the same and can be considered more general.

2. Basic Setting and No-Arbitrage Conditions between Credit Risk Discount Factors

2.1. Setting

This study focuses on the pricing of CDS contracts and other single-name credit-risky securities at the current (non-defaulting) time 0. With this goal in mind, a simple discrete-time economy with a daily time interval is assumed. Traded assets include (but are not restricted to) default-free and risky zero-coupon bonds of all possible maturities.[5] These maturities are denoted T, and correspond to all future calendar dates up to time τ—that is, $T \in \{\Delta, 2\Delta, \dots, \tau\}$, with $\Delta = 1/365$. The price of a default-free zero-coupon bond with nominal \$1 and maturity T is denoted $Z(T)$.[6] For risky bonds, default may occur at any future calendar date and represents an absorbing state. The default time is denoted τ^d, while the minimum between τ^d and T is denoted L_d^T. In the event of default, bond holders receive (irrespective of the possible coupon) a fraction θ of its face value and the asset is liquidated. Markets are complete and arbitrage-free.

2.2. Credit Risk Discount Factors and No-Arbitrage Conditions

In our particular setting, the three basic CRDFs are defined as follows:

- $A(T)$: The present value of an asset class A paying a constant annuity of \$$\Delta$ every Δ years until L_d^T (included).
- $B(T)$: The present value of an asset class B paying \$1 at τ^d, provided $\tau^d \leq T$.
- $C(T)$: The present value of an asset class C paying \$1 at T, provided $\tau^d > T$.

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[5] This assumption is made for convenience and can be easily relaxed. In particular, as in Jarrow and Turnbull (1995), the only real requirement is that enough traded assets exist to allow the prices of default-free and risky zero-coupon bonds to recover for all possible maturities.

[6] Because all prices are determined at current time 0, a simple notation is used to avoid emphasizing the present time 0. Please also note that $Z(T) \equiv e^{-r(T)T}$, where $r(T)$ is the spot rate with maturity T.

It is important to stress that, in the case of asset class A with maturity T, a default time $\tau^d \leq T$ implies the cancelation of the periodic stream of payments from $\tau^d + \Delta$ onwards. This includes $\tau^d + \Delta$, but not τ^d itself. While such clarification would be meaningless in a continuous-time model (Lando, 1998), it is a key element in the present case. In addition, the discrete-time setting considered in this study makes it possible to introduce a fourth convenient CRDF:

- $E(T)$: The present value of an asset class E paying \$1 at T, provided $\tau^d > T - \Delta$

Hence, the difference between assets C and E with the same maturity T is that the payment of \$1 at T is conditional on survival at time T in the case of C, and on survival at the previous date $T - \Delta$ in the case of E.

Figure 2 depicts the payment structure associated with the four contingent claims. Along with the assumptions made in Section 2.1, this payment structure implies two no-arbitrage conditions that must hold for any two consecutive maturities, $T - \Delta$ and T.

<Figure 2 about here>

The first no-arbitrage condition (NAC1) relates $A(T)$, $A(T - \Delta)$ and $E(T)$:

$$A(T) = A(T - \Delta) + \Delta E(T), \tag{1}$$

with $A(0) = 0$.

Equation (1) reflects the fact that the present value of a daily annuity of \$$\Delta$ paid until time T or default must be equal to the sum of: (a) the present value of a daily annuity of \$$\Delta$ paid until time $T - \Delta$ or default, and; (b) the present value of \$$\Delta$ paid with certainty at time T, conditional on no default at time $T - \Delta$ or before. This second component follows from the previous comment about the effect of a default event on asset class A payments.

The second no-arbitrage condition (NAC2), which must hold for any two consecutive maturities $T - \Delta$ and $T,$ is as follows:

$$C(T) + B(T) - B(T - \Delta) = E(T), \tag{2}$$

with $B(0) = 0$.

On the left-hand side of Equation (2), $C(T)$ is the present value of \$1 paid at time T, conditional on no default at that time or before. In addition, $B(T) - B(T - \Delta)$ equals the present value of \$1 paid at T in the case of default at that precise moment and not before. Taken as a whole, the left side of Equation (2) equals the present value \$1 paid with certainty at time $T,$ conditional on no default at time $T - \Delta$ or before, and this is exactly what $E(T)$ on the right side of said equation represents. Combining Equations (1) and (2) leads to the following related condition:

$$A(T) = A(T - \Delta) + \Delta[C(T) + B(T) - B(T - \Delta)]. \tag{3}$$

Equation (3) provides a necessary relationship between the three core CRDFs for any two consecutive maturities $T - \Delta$ and T. One important observation is that this equilibrium condition relies exclusively on the payment structure associated with assets A, B, and C and the assumptions made in Section 2.1. In other words, it does not depend on any risk-neutral pricing model.[7]

[7] A further intuitive implication of Equation (3) is that $A(T) = \Delta\left[\sum_{h=1}^{T/\Delta} C(h\Delta) + B(T)\right]$.

3. Credit Default Swap Spreads as a Function of Credit Risk Discount Factors

The value of a position in a CDS contract with maturity T equals the difference between its premium leg and protection leg. The daily structure of the premium leg is shown in Figure 3. This figure reflects a key feature of a CDS contract. Namely, while the annual premium per dollar of protected debt, cds, is generally paid in quarterly installments, the liquidation of the contract in the case of default implies the payment of the premium accrued since the last quarterly payment. For this reason, a non-defaulting state on a given day implies a consolidated right to accrue Δcds the following day, regardless of whether or not a default occurs on that posterior day. If we further assume no counterparty risk from the protection buyer's side, such a consolidated right to accrue Δcds can be considered as a risk-free income on a given day, conditional on no default on the previous day. Because this payment structure mimics that of asset A, scaled by cds, the present value of the premium leg is simply:

$$X(T) = cdsA(T), \tag{4}$$

where the nominal value of the protected bond is normalized to 1.

<Figure 3 about here>

The daily structure of the protection leg is shown in Figure 4. On any given day, the protection payment is 0 in the case of no default, and a fraction $(1-\theta)$ of the protected bond's face value in the case of default. Thus, the payment structure of the protection leg reproduces that of asset B scaled by $(1-\theta)$, and the same applies for its present value for a nominal of 1:

$$Y(T) = (1-\theta)B(T). \tag{5}$$

<Figure 4 about here>

The break-even CDS spread, $cds(T)$, is finally obtained by equating the premium and protection legs of the contract (see also Duffie and Singleton, 2003):

$$cds(T) = \frac{(1-\theta)B(T)}{A(T)}. \quad (6)$$

4. Additional Assumptions and the Bootstrapping of Credit Risk Discount Factors

All previous results are based on no-arbitrage arguments alone, implying that they do not rely on any particular risk-neutral pricing model. However, a convenient additional assumption is that the risk-free interest rate process and default time are risk-neutrally independent (Jarrow and Turnbull, 1995; Jarrow, et al., 1997; Duffie and Singleton, 2003; Hull and White, 2003; O'Kane and Turnbull, 2003). If we denote $S(T)$ the risk-neutral survival probability at time T (as seen at current time 0), this new assumption allows us to decompose $C(T-\Delta)$ and $E(T)$ as follows: $C(T-\Delta) = Z(T-\Delta)S(T-\Delta)$; and $E(T) = Z(T)S(T-\Delta)$. If we further denote $f(T-\Delta,T) \equiv -(1/\Delta)log[Z(T)/Z(T-\Delta)]$ the forward rate between $T-\Delta$ and T, we then obtain:

$$E(T) = e^{-f(T-\Delta,T)\Delta}C(T-\Delta), \quad (7)$$

with $C(0) = 1$. The interpretation of the previous equation is straightforward. Under the assumption of risk-neutral independence between the risk-free interest rate process and the default time, $E(T)$ can be obtained by discounting first from T to $T-\Delta$ at the forward rate, and then from $T-\Delta$ to current time 0, using the discount factor $C(T-\Delta)$.

Let us now assume that $A(T-\Delta)$, $B(T-\Delta)$, and $C(T-\Delta)$ values are available for a given maturity $T-\Delta$. In such a case, and assuming that the forward rate $f(T-\Delta,T)$

is also available, Equations (1), (2), (6) and (7) lead to a system of three equations and three unknowns—$A(T)$, $B(T)$, and $C(T)$—with a simple closed-form solution:

$$A(T) = A(T-\Delta) + \Delta e^{-f(T-\Delta,T)\Delta} C(T-\Delta); \tag{8a}$$

$$B(T) = \frac{cds(T)A(T)}{(1-\theta)}; \tag{8b}$$

$$C(T) = e^{-f(T-\Delta,T)\Delta} C(T-\Delta) - B(T) + B(T-\Delta). \tag{8c}$$

Several aspects of this result deserve special attention. First, because Equation System (8) links $\{A(T), B(T), C(T)\}$ to $\{A(T-\Delta), B(T-\Delta), C(T-\Delta)\}$, it allows us to bootstrap the full term structure of CRDFs, based on a previously settled CTSCDS and the initial values $\{A(0), B(0), C(0)\} = \{0,0,1\}$. Second, because the solution is also in closed-form (trivial and unique), such an estimation does not require the implementation of a series of root-search algorithms or any other optimization rule. In other words, the full term structure of CRDFs can be obtained instantaneously, simply with a spreadsheet. Third, these term structures converge naturally toward their risk-free counterparts as the CTSCDS tends to zero: $B(T)$ tends to zero, $C(T)$ tends to $Z(T)$, and $A(T)$ tends to $\Delta \sum_{h=1}^{T/\Delta} Z(h\Delta)$.[8] Fourth, the solution is also free from any specific assumption about the risk-free interest rate process or default time. The unique imposed assumption is that they are independent in a risk-neutral way. Last, and related to the above, the solution does not even involve the estimation of risk-neutral survival (or forward default) probabilities. As the next section demonstrates, these can be obtained easily as a *sub-product* of the

[8] This last expression indicates that $A(T)$ tends to the present value of a risk-free daily annuity of \$$\Delta$ paid until time T. This result is a direct implication of the previous remarks on $B(T)$ and $C(T)$ and the observation made in footnote 7.

bootstrapping process; however, such additional results are not needed for any of the applications considered in this study.

Table 1 provides a numerical example, in which the CTSCDS is estimated from the same OTSCDS as in Figure 1, but using a simple linear interpolation.[9] The table reflects the CDS spreads for the observed maturities (6m, 1y, 2y, 3y, 4y, 5y, 7y, and 10y) and some of the interpolated values. For the interval (0,6m] it could be presumed either a flat term structure or the same slope as that in the interval [6m,1y]. For this and other cases of linear interpolation, the latter option is adopted. The example also assumes a constant risk-free rate of 2% and recovery rate of 40%. The final estimates of the CRDFs for the selected maturities are presented in Table 1, while Figure 5 plots the results for all possible maturities. It is worth stressing that, although different interpolation schemes (i.e., Linear, PCHIP, Spline) can be considered in the first step, the posterior bootstrapping process will be always the same.

<Table 1 about here>

<Figure 5 about here>

5. A Restricted Case: The Piecewise Constant Default Probability Model

This section describes a PWCDP model as a restricted version of the non-parametric model introduced above. First, we can show that Equation System (8) produces the same results as those generated by a model that, based on a CTSCDS, estimates the risk-neutral default probability at any time T, conditional on no previous default. If we denote $q(T)$ as the elements of this term structure of forward risk-neutral default probabilities, the risk-neutral survival probability at time T is

[9] The Excel file containing this example is available at www.santiagoforte.com.

$$S(T) = \prod_{u=0}^{T/\Delta} [1 - q(u\Delta)], \tag{9}$$

while the risk-neutral probability of default at time T (and not before) is

$$H(T) = q(T) \prod_{u=0}^{(T-\Delta)/\Delta} [1 - q(u\Delta)]. \tag{10}$$

It should be noted that $q(0) = 0$. From Equations (9) and (10), and assuming again that the risk-free interest process and default time are risk-neutrally independent, we obtain the following expressions for $A(T)$, $B(T)$, $C(T)$, and $E(T)$:

$$A(T) = \Delta \sum_{h=1}^{T/\Delta} \{Z(h\Delta) S[(h-1)\Delta]\} = \Delta \sum_{h=1}^{T/\Delta} \left\{ Z(h\Delta) \prod_{u=0}^{h-1} [1 - q(u\Delta)] \right\}; \tag{11}$$

$$B(T) = \sum_{h=1}^{T/\Delta} \{Z(h\Delta) H(h\Delta)\} = \sum_{h=1}^{T/\Delta} \left\{ Z(h\Delta) q(h\Delta) \prod_{u=0}^{h-1} [1 - q(u\Delta)] \right\}; \tag{12}$$

$$C(T) = Z(T) S(T) = Z(T) \prod_{u=0}^{T/\Delta} [1 - q(u\Delta)]; \tag{13}$$

$$E(T) = Z(T) S(T - \Delta) = Z(T) \prod_{u=0}^{(T-\Delta)/\Delta} [1 - q(u\Delta)]. \tag{14}$$

It is a relatively simple task (addressed in the Appendix) to show that Equations (11)–(14) satisfy both NAC1 and NAC2. Based on Equations (6), (11), and (12), it also holds that:

$$cds(T) = \frac{(1-\theta)\sum_{h=1}^{T/\Delta}\{Z(h\Delta)q(h\Delta)\prod_{u=0}^{h-1}[1-q(u\Delta)]\}}{\Delta\sum_{h=1}^{T/\Delta}\{Z(h\Delta)\prod_{u=0}^{h-1}[1-q(u\Delta)]\}}. \quad (15)$$

Equation (15) provides the break-even CDS spread for a contract with maturity T, as a function of all forward risk-neutral default probabilities from 0 to T. From this equation, it is actually possible to isolate $q(T)$ as a function of all previous probabilities:

$$q(T) = \frac{cds(T)\Delta\sum_{h=1}^{T/\Delta}\{Z(h\Delta)\prod_{u=0}^{h-1}[1-q(u\Delta)]\} - (1-\theta)\sum_{h=1}^{(T-\Delta)/\Delta}\{Z(h\Delta)q(h\Delta)\prod_{u=0}^{h-1}[1-q(u\Delta)]\}}{(1-\theta)Z(T)\prod_{u=0}^{(T-\Delta)/\Delta}[1-q(u\Delta)]}. \quad (16)$$

Hence, it is indeed possible to bootstrap a complete term structure of $q(T)$ values from a CTSCDS by means of Equation (16). This term structure can be used to determine the core CRDFs from Equations (11)–(13) and, finally, to price different single-name credit-risky securities, as described in Section 6. However, this task is arduous and unnecessary. This is because Equation System (8) leads to exactly the same result in a much easier way. In addition, even if the intention is to estimate the term structure of the $q(T)$ and/or $S(T)$ values, this aim can be achieved more easily (and with identical results) by incorporating two additional equations into Equation System (8):

$$S(T) = \frac{C(T)}{Z(T)}; \quad (17)$$

$$q(T) = 1 - \frac{S(T)}{S(T-\Delta)}. \quad (18)$$

Please note that $S(0) = 1$.

Consider now the implementation of a conventional PWCDP model. In our particular setting, this would entail the following. First, it is assumed that $q(T)$ is in effect piecewise constant, where changes coincide with the maturity of observable CDS spreads.

Second, the term structure of $q(T)$ values is estimated sequentially so that Equation (15) fits the observed CDS spreads perfectly. It is worth mentioning that a bootstrapping process of this sort implies the implementation of a sequence of root-search algorithms (one for each observed quote). Finally, based on the term structure of $q(T)$ values obtained, we can determine the term structure of the core CRDFs and the prices of different single-name credit-risky securities, as previously described.[10]

Comparing the two pricing approaches, the PWCDP model imposes ex-ante a piecewise constant profile on the term structure of $q(T)$, leading ex-post to an effective interpolation of the observed CDS spreads (see again Figure 1, Panel 1A). By contrast, the non-parametric model imposes ex-ante a particular interpolation scheme between the observed quotes, implying ex-post a complete term structure of $q(T)$ values. Figure 6 provides a numerical example. In this figure, the term structure of $q(T)$ and $S(T)$ is estimated by assuming the same OTSCDS as in Figure 1, and four different *pricing models*: the PWCDP model on the one hand, and the non-parametric model with a linear (NP/Linear), PCHIP (NP/PCHIP), or Spline (NP/Spline) interpolation on the other. As the figure makes clear, the PWCDP model entails the most discontinuous term structure of $q(T)$ values, while the NP/Spline model generates the smoothest term structure. Nevertheless, because of the small marginal effect of $q(T)$ on $S(T)$, the term structure of $S(T)$ exhibits no evident jumps in any case.

The ability to generate a smooth term structure of forward risk-neutral default probabilities is, of course, a nice property for a credit risk pricing model, although it is

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[10] We can also analyze the limit case where $q(T)$ is a constant parameter equal to q. In such a case, and based again on Equation (15), the CDS spread is also constant and given by $cds = (1 - \theta)q/\Delta$. This result is nothing but the discrete-time version of the so-called *credit risk triangle* between the CDS spread, the recovery rate, and the constant hazard rate in a continuous-time model (see e.g., O'Kane, 2008). As is evident, this naïve version of a fully parametric model will only be consistent with a flat OTSCDS.

actually of second-order importance. What is important to stress here is that the non-parametric model is less restrictive and much easier to implement than the PWCDP model. Moreover, as the empirical evidence in Section 7 demonstrates, it leads to lower MAPE. Before addressing this empirical analysis, Section 6 reviews some possible applications of the term structure of CRDFs, with a special focus on a novel one: the time decomposition of CDS spreads.

<Figure 6 about here>

6. Applications

6.1. Pricing of CDS Contracts

The clearest application of the term structure of CRDFs obtained from the term structure of CDS spreads (TSCDS) is the marking-to-market of any position in a CDS contract.[11] In the case of a long position with a previously settled spread, cds, this value is simply:

$$V(T) = (1 - \theta)B(T) - cdsA(T). \tag{19}$$

By extension, this also implies a simple approach to estimating CDS returns (Berndt and Obreja, 2010; Augustin et al., 2020).

[11] The distinction between OTSCDS and CTSCDS is relevant for presenting different estimation approaches for the full term structure of CRDFs, but less relevant for describing the potential applications of those CRDFs. In addition, the interpretation of the possible TSCDS shapes provided in this section is equally valid for the OTSCDS and CTSCDS. Thus, unless an alternative is necessary, the generic expression TSCDS will be used in the remainder of this paper.

6.2. Pricing of Risky Bonds

Consider a risky bond with coupon b, nominal p, and maturity T. Let us also denote T_m as the maturity of the m^{th} coupon payment, where $m = 1, \dots, M$, and $T_M = T$. The present value of this bond will be:

$$d(T) = b \sum_{m=1}^{M} C(T_m) + pC(T) + \theta p B(T). \tag{20}$$

The first term on the right side of the equation reflects the present value of the stream of coupon payments. The second term accounts for the payment of the nominal amount at maturity, in the case of no default. Finally, the last term incorporates the present value of the fractional recovery of the nominal value in the case of default.

6.3. Pricing of Forward CDS Contracts

Now, consider a forward CDS contract signed at current time 0 for credit protection between T_j and T_k, with $0 \leq T_j < T_k$. More precisely, the initiation date is T_j, conditional on $\tau^d > T_j$, so the first effective date with the accrual of premium payments and delivery of the bond in exchange for the bond's face value in the case of default is $T_j + \Delta$. The daily structure of this contract is, in fact, the structure described in Figures 3 and 4 for a spot contract. The sole difference is that the starting date is now T_j rather than 0, and the ending date is T_k. To derive the present value of the premium leg of the forward contract based on the CRDFs, let us define (for any T^* and T, with $0 \leq T^* < T$):

- $A(T^*, T)$: The present value of the same asset class A paying a constant annuity of \$$\Delta$ every Δ years, but this time between T^* and T with the following conditions: (i) the first payment is at $T^* + \Delta$, conditional on $\tau^d > T^*$ (otherwise, the asset is liquidated at τ^d), and; (ii) provided that $\tau^d > T^*$, the last payment is at L_d^T (included).

Based on the definition of $A(T)$ and $A(T^*, T)$, it holds that

$$A(T^*, T) = A(T) - A(T^*). \tag{21}$$

If we use $fcds$ to denote the spread of the forward CDS contract described above, the present value of the premium leg is:

$$X(T_j, T_k) = fcdsA(T_j, T_k). \tag{22}$$

We can also derive the present value of the protection leg based on the CRDFs. Let us define:

- $B(T^*, T)$: The present value of the same asset class B paying \$1 at τ^d, provided this time that $T^* < \tau^d \leq T$.

From the definition of $B(T)$ and $B(T^*, T)$, it must hold that

$$B(T^*, T) = B(T) - B(T^*), \tag{23}$$

and the present value of the protection leg is:

$$Y(T_j, T_k) = (1 - \theta)B(T_j, T_k). \tag{24}$$

The value of a long position in the forward CDS contract is thus:

$$FV(T_j, T_k) = (1 - \theta)B(T_j, T_k) - fcdsA(T_j, T_k). \tag{25}$$

By imposing $FV(T_j, T_k) = 0$, we finally obtain the break-even forward CDS spread:

$$fcds(T_j, T_k) = \frac{(1 - \theta)B(T_j, T_k)}{A(T_j, T_k)}. \tag{26}$$

It is worth noting that $fcds(0, T) = cds(T)$.

6.4. A Note on Portfolio Management

Previous results on the pricing of single-name credit-risky securities apply, regardless of the exact pricing model used to determine the CRDFs. However, the non-parametric model introduced in this study offers clear advantages for portfolio management. As has been shown, the combination of a CTSCDS (obtained directly from the OTSCDS) and the term structure of risk-free interest rates (TSIR) provides a direct estimate of the term structure of CRDFs. Moreover, the prices of those securities are simple functions of these CRDFs. Consequently, the model allows for a straight mapping between observable market risk factors (OTSCDS and TSIR) and the prices of the most common single-name credit-risky securities (spot and forward CDS contracts and risky bonds). The final implication is the possibility of translating the predicted distribution function for such market risk factors into a distribution function for the values of different credit-risky portfolios, using Monte Carlo simulations. By extension, this represents an easy path for integrating market and credit risk.[12]

6.5. Time Decomposition of CDS Spreads

It is not entirely clear why, to date, despite the large amount of work done on the pricing of CDS contracts, the academic literature has never provided an explicit formulation of the time decomposition of CDS spreads. The explanation may be that any representation of a CDS spread, based on either a parametric or semi-parametric model, depends strongly on the assumptions and parameters of the specific model. Equation (15) provides a good example; in such a representation, the time decomposition of CDS spreads does not emerge intuitively. As will be shown below, the situation changes

[12] Clearly, this extension to portfolio management should incorporate the probability of a default event at the future pricing date. Accordingly, the empirical evidence of the connection between historical/current CDS levels and the probability of a future default event should be accounted for.

considerably when spot and forward CDS spreads are expressed as simple functions of the CRDFs.

In line with the case of a spot rate decomposition, the time decomposition of CDS spreads follows from the possibility of representing a long (short) position in a CDS contract as a portfolio of long (short) positions in N consecutive forward CDS contracts. If we define $T_0 = 0$ and $T_N = T$, then

$$X(T) = \sum_{i=1}^{N} X(T_{i-1}, T_i), \tag{27}$$

that is, the present value of the cost of credit protection up to time T must be equal to the present value of the cost of credit protection for an arbitrary number of consecutive (but not necessarily identical) time intervals between time 0 and time T. Consequently,

$$cds(T) = \sum_{i=1}^{N} w(T_{i-1}, T_i; T) fcds(T_{i-1}, T_i), \tag{28}$$

where

$$w(T_{i-1}, T_i; T) = \frac{A(T_{i-1}, T_i)}{A(T)} \in [0,1]; \tag{29}$$

with

$$\sum_{i=1}^{N} w(T_{i-1}, T_i; T) = 1. \tag{30}$$

Based on Equations (28)–(30), we can divide the maturity T of a CDS contract into an arbitrary number of intermediate time intervals and express the associated CDS spread as a weighted average of the forward CDS spreads corresponding to those time slots. The weight of a particular forward spread $fcds(T_{i-1}, T_i)$ on the spot spread $cds(T)$ is given by the weight of $A(T_{i-1}, T_i)$ in $A(T)$. Among the factors that influence this ratio (relative time length; time value of money), it is worth highlighting the risk of default up to initiation date T_{i-1}. All things being equal, the higher this risk, the lower the present value of any stream of payments in the time interval $(T_{i-1}, T_i]$, conditional on no previous default—and therefore, the lower the influence of the corresponding forward spread on the spot spread. This reflects one main difference with the time decomposition of spot rates: unlike the forward rates embedded in a spot rate, the forward CDS spreads contained in a CDS spread may never be paid, and this is properly reflected in their weights. In summary, a distinguishing aspect of a CDS spread decomposition is the fact that the risk of default enters both the forward spreads and their weights.

As a corollary to the previous results, the level and steepness of the TSCDS are to a certain extent related. To understand this more clearly, let us consider the following simple decomposition:

$$cds(T) = \left[1 - \frac{A(T^*, T)}{A(T)}\right] cds(T^*) + \frac{A(T^*, T)}{A(T)} fcds(T^*, T); \tag{31}$$

and rearranging terms,

$$[cds(T) - cds(T^*)] = \frac{A(T^*, T)}{A(T)} [fcds(T^*, T) - cds(T^*)]. \tag{32}$$

Equation (32) provides an intuitive interpretation of the possible forms of the TSCDS (increasing, decreasing, or hump-shaped) and its steepness. Concerning the possible forms, a forward spread $fcds(T^*, T)$ higher (lower) than the spot spread $cds(T^*)$

implies, as expected, a positive (negative) slope in the interval $[T^*, T]$. However, the steepness depends not only on the absolute difference between $fcds(T^*, T)$ and $cds(T^*)$, but also on the ratio $A(T^*, T)/A(T)$. Following previous arguments, the higher the risk of default up to T^*, the lower this ratio and, assuming that all other things are equal, the flatter the TSCDS in the interval $[T^*, T]$. Thus, in effect, when it comes to analyzing a TSCDS, the level and steepness cannot be completely dissociated.[13]

A final implication of the previous results is the possibility of interpreting $w(T_{i-1}, T_i; T)fcds(T_{i-1}, T_i)$ as the total contribution of the time interval $(T_{i-1}, T_i]$ to the CDS spread with maturity T. Accordingly, the relative contribution will be

$$Q(T_{i-1}, T_i; T) = \frac{w(T_{i-1}, T_i; T)fcds(T_{i-1}, T_i)}{cds(T)} = \frac{B(T_{i-1}, T_i)}{B(T)} \in [0,1], \qquad (33)$$

with

$$\sum_{i=1}^{N} Q(T_{i-1}, T_i; T) = 1. \qquad (34)$$

Equation (33) indicates that the relative contribution of one particular time interval $(T_{i-1}, T_i]$ to the spread of a CDS contract with maturity T is given by the ratio between the following: the present value of $1 paid at default if this happens during that particular

[13] Please note that the discussion above refers to the expected influence of level on the steepness, not the sign (positive or negative) of the slope. Also, it leaves aside the underlying economic reasons for the actual shape of the TSCDS. For example, Bhat et al., (2016) provide empirical evidence which supports the predictions of Duffie and Lando (2001) as regards the effect of higher accounting transparency on the level, slope, and concavity of the TSCDS for corporate issuers, while Augustin (2018) explores the influence of global and country-specific risk on the shape of sovereign TSCDS. It is also worth noting that practitioners usually think of forward CDS spreads as a function of spot spreads. Likewise, the term structure of forward CDS spreads is typically analyzed based on the TSCDS. From a practitioner's point of view, this makes complete sense. As trading normally concentrates on liquid spot contracts, a forward contract can be constructed synthetically from such spot contracts if needed. When applying a strict economic interpretation, however, it makes more sense to think of spot spreads as a product of forward spreads.

time interval, and the present value of $1 paid at default if this happens at any time during the life of the contract. The proximity of these two values implies that the risk of default is concentrated in that specific time interval, thus making a significant contribution to the spread of the CDS contract. The opposite result is achieved if there is a significant difference between the two aforementioned values.

7. Case Study: The Eurozone Sovereign Debt Crisis

The aim of this section is twofold. First, to compare the performance of a conventional PWCDP model with that of the non-parametric model presented in this study. In the latter case, different interpolation schemes for the ex-ante estimation of the CTSCDS will be considered. Second, to illustrate some possible applications of the term structure of the CRDFs obtained—in particular, the estimation of forward CDS spreads and time decomposition of (spot) CDS spreads.

The Eurozone sovereign debt crisis provides an interesting framework for addressing these two questions. Within a short period of time, it combines issuers with relatively low and extremely high CDS spread levels. In addition, we can expect the liquidity of a CDS contract to be higher for France and Ireland than for an average corporation with a similar default risk. For the following analyses, weekly data on CDS spreads with maturities ranging from 6m to 10y and a CR/CR14 restructuring clause are collected from Markit. The period considered is 2010–2019, and the selected countries are France, Spain, Italy, Ireland, Portugal, and Greece. In the particular case of Greece, the sample period closed earlier, on October 18, 2011. While this choice may seem somewhat arbitrary, it corresponds to the first observation of a 6m-CDS spread above 10,000 bp, with no further drop below that level until the effective default of Greek sovereign debt.[14] The risk-free interest rates are approximated using the German TSIR.[15] Finally, following market conventions, a recovery rate of 40% is assumed. Table 2 presents the main descriptive statistics for each country's CDS spread.

<Table 2 about here>

7.1. Semi-Parametric vs. Non-Parametric Estimation: Relative Pricing Errors

As an initial step, the performance of the four different pricing models—PWCDP, NP/Linear, NP/PCHIP, and NP/Spline—is evaluated based on their relative pricing errors. To address this comparison, we need to solve the problem that, unlike the illustrative example in Section 1, the true CTSCDS is not known. As only a limited number of observed quotes are actually available, the comparison proceeds as follows. For each issuer and date, the four models are estimated using all but the 6m-CDS spread. Next, the results are used to determine the 6m-CDS spread predicted by each model, with absolute pricing errors gauged through comparison with the actual quote. The process is repeated for other available maturities to obtain a final sample of absolute pricing errors. Three important clarifications must be made regarding this empirical test. First, strictly speaking, the comparison will be made between a pricing model (PWCDP) and three direct interpolation schemes between the observed quotes (Linear, PCHIP, and Spline). However, because the non-parametric model can reproduce any of these CTSCDS, the analysis represents, in effect, a comparison between a conventional PWCDP model and

[14] The initial proposal for a bond exchange with a nominal discount of 50% on notional Greek debt was made during the Euro Summit held on October 26, 2011, and formally announced on February 21, 2012 (see Zettelmeyer et al., 2013 for details). On February 28, 2012, the International Swaps and Derivatives Association (ISDA) accepted a question related to a potential Hellenic Republic credit event. The occurrence of a credit event was initially denied by the ISDA on March 1, 2012, but was finally accepted on March 9, 2012, after a second question was formulated.

[15] Zero and negative spot rates are frequently observed along the sample period. To avoid potential problems associated with non-positive risk-free interest rates, a minimum value of 0.01% is imposed.

the non-parametric model introduced in this study. Second, although the overall results effectively allow for the comparison of different pricing models, the numbers obtained will tend to underestimate their real accuracy. This is because some information that could be used to estimate them will always be ignored. Third, while pricing errors for the 6m maturity (the first OTSCDS element) are considered for comparison purposes, those for the 10y (the final element) are excluded. This is because, in practice, the lowest possible CDS contract maturity that might need to be priced is one day. Hence, whatever the lowest available maturity may be within the OTSCDS, some form of *extrapolation* will always be required to complete the left side of the CTSCDS. By contrast, the maturity of an existing CDS contract will never be higher than that of the last available quote in the OTSCDS. For this reason, it is unnecessary to investigate potential pricing errors beyond the longest available maturity and may actually distort conclusions.[16]

Table 3 provides the main descriptive statistics for the absolute pricing errors. Among the four competing approaches, the NP/PCHIP model implies the lowest MAPE, while the PWCDP model generates the highest MAPE (the same conclusion applies to the median). A rough calculation suggests that, on average, pricing errors from NP/PCHIP are half of those generated by PWCDP, and even the simple NP/Linear approach generally provides more accurate results.

<Table 3 about here>

Figure 7 presents a more in-depth analysis of the pricing errors generated by each model. The first two panels represent the MAPE by country and year and clearly suggest that the overall credit risk level is an important determinant of the individual and relative

[16] Please note that, for liquidity reasons, CDS spreads with maturities beyond 10y (i.e., 15y, 20y, and 30y) are not covered in this case study. However, the same reasoning would apply, regardless of the length of the longest available maturity in practice.

performance of various pricing models. To arrive at this result, we can compare the MAPE of France with that of Greece and Portugal (Panel 7A), and also compare the results during and after the sovereign debt crisis (7B).[17] Because the introductory example in Section 1 suggests that the maturity of the CDS contract may be also an important factor in the relative performance of the different pricing models, the last two panels of Figure 7 explicitly account for these two factors. Regarding maturity (7C), and consistent with the introductory example, PWCDP underperforms all other models for the shortest maturities. However, these differences tend to decline and even revert, in the case of the longest values. That said, NP/PCHIP outperforms PWCDP for all maturities in the range of 6m–5y, and offers a similar result for the 7y. It is worth noting that the accuracy of any model in the (0,5y] interval is particularly relevant. As the 5y is by far the most traded maturity, most existing CDS contracts have a remaining maturity in that specific range. Regarding the credit risk level (7D), this is approximated by the 5y-CDS spread. The results confirm that the higher this level, the higher both the MAPE of all models and the benefits of considering the non-parametric approach with an appropriate interpolation scheme.

<Figure 7 about here>

Tables 4 and 5 provide additional details on the maturity and credit risk level effects. Specifically, Table 4 presents the MAPE by model, crossing different maturities and 5y-CDS spreads, while Table 5 highlights the difference between the MAPE of the model at hand, in each case, and that of the most accurate model. Thus, a value of 0.00 bp indicates that the corresponding model is, on average, the most accurate approach for

[17] One possible concern is that the yearly results are merely the reflection of a "Greece Effect." However, the 2012 results are comparable to those for 2011. This is observed even though the sample period for Greece closed on October 18, 2011.

that specific maturity and 5y-CDS spread level. In contrast to Figure 7, Table 5 offers a consistent but richer picture of the relative performance of the four pricing models. For example, as Figure 7 indicates, PWCDP tends to generate more accurate results than other models for the 7y maturity, although this is not always the case. For the (300,1000] interval of the 5y-CDS spread level, it is actually the simplest NP/Linear model that generates the lowest MAPE. Another example is provided by NP/Spline. According to Figure 7, this is the most accurate model for a 5y-CDS spread level above 1,000 bp, but the least accurate approach for the 7y maturity. Table 5 clarifies that the NP/Spline model is the preferred method for some, but not all maturities when the 5y-CDS spread is in the highest range. In particular, for the 7y maturity, NP/Spline is the least accurate model.

<Table 4 about here>

<Table 5 about here>

The overall results in Figure 7 and Tables 4 and 5 suggest that no single model is superior under all circumstances. Nonetheless, they also support the conclusion that the NP/PCHIP model represents the most sensible option if a particular approach must be chosen. This conclusion is particularly true when the NP/PCHIP model is compared with the PWCDP model, and sustained via the following observations: first, NP/PCHIP provides the lowest overall MAPE (Table 3); second, when sorting by maturity and credit risk level (Table 5), this is the only model that never deviates more than 1.5 bp from the most accurate alternative. Thus, in what follows, results will be based on the NP/PCHIP approach.

7.2. Examples of the Term Structure and Time Decomposition of CDS Spreads

We can now proceed to analyze different examples of the term structure and time decomposition of CDS spreads. To simplify the exposition and interpretation of the results, only CDS spreads with a time interval of one year, and up to the most liquid 5y maturity (i.e., 1, 2, 3, 4, and 5y-CDS spreads), will be inspected. Similarly, the estimated forward CDS spreads will always refer to contracts with a total length of one year, initiated at time 0 (identical to a 1y spot CDS spread), 1 2, 3, or 4. Finally, the decomposition of CDS spreads will also concentrate on those specific time slots.

Figure 8 analyzes the particular case of France (July 27, 2010) and contains six panels. Panel 8A plots the TSCDS and reflects both the effectively observed and the interpolated spreads. The resulting term structures of $A(T)$, $B(T)$, and $C(T)$ values are presented in Panels 8B, 8C, and 8E, respectively. Taken together, the information provided in these panels is representative of an investment-grade issuer. First, the TSCDS has a low overall level and a positive slope, reflecting the higher level of uncertainty associated with future time periods. In addition, the smooth decline in $C(T)$ as maturity increases seems to reflect the time value of money more than it reflects a significant increase in the risk of default. Consistent with this perception, $B(T)$ remains low, while $A(T)$ grows steadily. Panel 8E again depicts the TSCDS, although this time in combination with the estimated term structure of forward CDS spreads (TSFCDS) and (for comparison purposes) the term structure of the simple mean of forward CDS spreads (TSMFCDS). The panel also contains the actual weight of each forward CDS spread for different spot spreads. In this particular example, such weights are always close to $1/T$; in other words, all relevant forward CDS spreads have approximately the same influence on a given CDS spread. Consequently, the TSCDS is very similar to the TSMFCDS. Panel 8F shows the final contribution of each year of protection on each CDS spread. If we focus on the time decomposition of the 5-year CDS spread, the exact contributions of years 1, 2, 3, 4, and 5 are 11%, 17%, 21%, 24%, and 26%, respectively. As shown in Panel 8E, such differences can be explained by the TSFCDS alone. The actual weight of each forward spread on the 5-year spot spread is roughly the same; in fact, it is slightly decreasing.

<Figure 8 about here>

Figure 9 reproduces Figure 8 in the case of Greece (September 13, 2011), and it reveals a completely different situation. In Panel 9A, the risk of imminent default is already reflected in the somewhat extreme short-run CDS spreads. Consistent with this situation, the $C(T)$ value (Panel 9D) declines rapidly, reaching just €0.50 for 6-month maturity and €0.10 for 5-year maturity. The $B(T)$ value (Panel 9C) moves in the opposite direction: around the same €0.50 for 6-month maturity, and €0.90 for 5-year maturity. Moreover, for the same maturities, the $A(T)$ value (Panel 9B) only reaches €0.35 and €1.22, respectively. The results in Panel 9E reflect the predicted connection between level and steepness in the TSCDS (Section 6.5). Due to the low present value of future payments, conditional on no previous default, the corresponding forward spreads have a small weight on the spot spreads. This, in turn, translates into a TSCDS that is significantly flatter than the TSMFCDS. For instance, while the weight of the first forward CDS spread in the 5-year spot CDS spread is 21% in the example of France, this same weight jumps to 45% in the case of Greece. This effectively explains why the significant drop in successive forward CDS spreads does not translate into a proportional reduction in spot spreads. Finally, as Panel 9F reflects, the combination of a high first forward CDS spread (the highest, in fact) and a high weight for this spread causes the year-one protection to account for 75% of the 5-year CDS spread. This is in sharp contrast to the corresponding value of 11% in the case of France.

<Figure 9 about here>

Figure 10 repeats the analysis used in the case of Ireland (October 25, 2011). It provides an example of an intermediate, hump-shaped TSCDS, which, for the rest, is consistent with previous results.

<Figure 10 about here>

7.3. The European Central Bank Intervention

"The ECB is ready to do whatever it takes to preserve the Euro, and believe me; it will be enough."

Mario Draghi, President of the European Central Bank. July 26, 2012.

The risk of a Eurozone collapse forced the European Central Bank (ECB) to change its policy. It is well known that Draghi's statement on July 26, 2012, and the decisions that followed had a significant impact on credit spreads within the Eurozone. We can now analyze the impact of this episode in more detail. For the sake of concreteness and conciseness, the analysis will focus on the cases of Spain and Italy. The results for other countries exhibit a similar pattern and they are available on request.

Figure 11, which presents the results of the analysis of Spain, consists of four panels. Panels 11A and 11B depict the evolution of spot and forward CDS spreads; both reflect the significant impact of Draghi's statement on the cost of credit protection. Panels 11C and 11D focus on the composition of the 5-year CDS spreads. Regarding the weight of each forward spread (Panel 11C), the weight of the first forward CDS spread reached its peak immediately before the announcement, while the weight of the last forward spread hit bottom. After Draghi's remark, the weight of all forward spreads began to converge until they finally reached a situation typical of an investment-grade issuer (see the example of France in Figure 8). The evolution of each year's contribution to the 5-year CDS spread (Panel 11D) confirms that Draghi's statement and the ECB's posterior policy change had a significant impact not only on the level, but also on the composition of the CDS spreads. The analysis of Italy in Figure 12 reveals a similar picture, with one main difference: the significant impact of Giuseppe Conte’s resignation announcement on May 27, 2018.

<Figure 11 about here>

<Figure 12 about here>

8. Conclusions

This study introduces a simple non-parametric approach to pricing CDS contracts and other single-name credit-risky securities. Like the traditional estimation of implied discount factors in risk-free bond prices, this method provides direct estimates of credit risk discount factors from the term structure of CDS spreads. Its implementation is based exclusively on closed-form solutions, implying that no root-search algorithm or any other form of optimization are required. Empirical evidence from the Eurozone likewise confirms that this model leads to fewer pricing errors than a conventional semi-parametric (piecewise constant default probability) model, which can, in fact, be seen as a restricted and computationally demanding version of the non-parametric model presented in this study. On the whole, the proposed model is shown to be an effective alternative to semi-parametric models, when the intention is marking-to-market CDS positions. It is equally fair to say that the proposed model should not be seen as a substitute for parametric models, which have their own path.

As a second main contribution, the present study formalizes the concept of time decomposition of CDS spreads. This is defined as the problem of determining the percentage of a CDS spread, which can be reasonably attributed to the protection of specific time intervals within the contract’s maturity. Parallel to the traditional decomposition of spot rates into forwards rates, the decomposition of CDS spreads follows from the fact that a spot CDS contract can be reproduced by a portfolio of forward CDS contracts. However, unlike a spot rate, a spot CDS spread is not a simple mean of forward spreads. The reason for this is that forward spreads may never be paid, and this is properly reflected in the weight they impose on spot spreads. Despite this peculiarity, the time decomposition of CDS spreads is extremely easy to implement and interpret, once spot and forward spreads are defined in terms of credit risk discount factors.

As regards future research, an important element of the pricing model presented in this study is the ex-ante interpolation of observed CDS spreads. Thus, the relative performance of alternative interpolation schemes is a question that deserves further investigation.

Appendix

This appendix shows that Equations (11)–(14) satisfy both NAC1 and NAC2. As regards NAC1:

$$A(T) = \Delta \sum_{h=1}^{T/\Delta} \{Z(h\Delta)S[(h-1)\Delta]\}$$

$$= \Delta \sum_{h=1}^{(T-\Delta)/\Delta} \{Z(h\Delta)S[(h-1)\Delta]\} + \Delta Z(T)S(T-\Delta)$$

$$= A(T-\Delta) + \Delta E(T).$$

Concerning NAC2:

$$C(T) + B(T) - B(T-\Delta) = Z(T)S(T) + \sum_{h=1}^{T/\Delta} \{Z(h\Delta)H(h\Delta)\} - \sum_{h=1}^{(T-\Delta)/\Delta} \{Z(h\Delta)H(h\Delta)\}$$

$$= Z(T)S(T) + Z(T)H(T)$$

$$= Z(T)\left\{\prod_{u=0}^{T/\Delta}[1-q(u\Delta)] + q(T)\prod_{u=0}^{(T-\Delta)/\Delta}[1-q(u\Delta)]\right\}$$

$$= Z(T)\prod_{u=0}^{(T-\Delta)/\Delta}[1-q(u\Delta)]$$

$$= Z(T)S(T-\Delta)$$

$$= E(T).$$

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Tables and Figures

Table 1. Numerical example of the bootstrapping of credit risk discount factors.

Obs. Mat.	T	$cds(T)$	$A(T)$	$B(T)$	$C(T)$
	0	-	**0.00000**	**0.00000**	**1.00000**
	1/365	52.13	0.00274	0.00002	0.99992
	2/365	52.28	0.00548	0.00005	0.99984
	…	…	…	…	…
	182/365	80.15	0.49469	0.00661	0.98350
6m	**183/365**	**80.31**	0.49739	0.00666	0.98339
	184/365	80.46	0.50008	0.00671	0.98329
	…	…	…	…	…
	364/365	108.33	0.98009	0.01770	0.96270
1y	**1**	**108.49**	0.98272	0.01777	0.96258
	…	…	…	…	…
2y	**2**	**159.52**	1.92044	0.05106	0.91053
	…	…	…	…	…
3y	**3**	**203.90**	2.80097	0.09519	0.84879
	…	…	…	…	…
4y	**4**	**242.35**	3.61675	0.14609	0.78158
	…	…	…	…	…
5y	**5**	**275.51**	4.36408	0.20039	0.71232
	…	…	…	…	…
7y	**7**	**328.22**	5.65471	0.30933	0.57757
	…	…	…	…	…
10y	**10**	**380.30**	7.12610	0.45168	0.40580

This table presents a subsample of the numerical example results, where the term structure of $A(T)$, $B(T)$, and $C(T)$ is estimated based on a CTSCDS and Equation System (8). In this case, the CTSCDS is obtained through a linear interpolation of CDS spreads with observed maturities (Obs. Mat.). These observed spreads and the initial CRDF values are indicated in bold format. The example assumes a constant risk-free rate of 2% and a recovery rate of 40%.

Table 2. Main descriptive statistics for the CDS spreads.

France	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	14.60	16.37	21.94	28.01	35.20	42.09	52.27	62.35
Median	5.86	7.09	11.15	16.83	23.29	30.55	45.35	57.72
Min	1.18	1.56	3.59	5.63	8.18	10.88	17.06	23.12
Max	115.24	120.95	135.82	146.12	160.58	170.36	177.06	184.06
SD	21.32	22.40	25.73	28.04	30.64	32.45	32.10	31.51

Spain	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	70.88	80.51	98.93	112.73	122.11	130.51	142.04	151.00
Median	23.30	31.13	47.49	60.21	70.79	80.76	99.17	116.40
Min	3.64	5.25	9.58	13.72	17.79	24.02	34.18	45.25
Max	375.53	409.00	486.14	503.25	504.31	504.15	485.57	460.42
SD	88.78	93.07	101.95	105.10	104.26	103.08	96.13	87.02

Italy	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	76.47	91.10	117.82	138.68	152.89	164.74	180.32	191.74
Median	41.76	54.58	82.16	104.46	119.48	131.94	154.24	172.94
Min	10.03	17.11	31.06	42.42	52.25	62.13	80.29	97.35
Max	551.72	555.99	544.32	530.58	514.43	501.52	486.33	467.85
SD	89.01	95.11	96.93	95.62	92.40	89.42	81.61	73.44

Ireland	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	144.86	155.00	168.30	172.97	170.93	171.10	174.79	174.42
Median	10.90	14.78	23.69	33.34	43.16	52.12	70.73	88.19
Min	2.07	2.98	5.39	8.47	11.87	15.36	21.53	27.83
Max	1,356.41	1,359.22	1,341.46	1,302.16	1,211.33	1,149.98	1,089.27	1,016.41
SD	263.51	272.69	277.81	266.16	242.10	224.19	202.96	178.28

Portugal	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	221.62	259.25	293.77	302.30	303.47	307.92	312.54	310.02
Median	71.88	94.46	134.17	161.19	192.80	213.54	239.03	255.92
Min	2.68	4.35	9.96	15.71	22.61	27.79	42.01	57.82
Max	1,669.43	2,122.97	2,133.04	1,857.78	1,670.09	1,554.03	1,391.93	1,232.70
SD	341.17	396.15	408.94	369.77	327.59	299.34	260.92	224.02

Greece	cds(0.5)	cds(1)	cds(2)	cds(3)	cds(4)	cds(5)	cds(7)	cds(10)
Mean	1,829.69	1,768.99	1,651.31	1,545.61	1,430.87	1,349.55	1,252.76	1,164.89
Median	952.66	956.15	973.97	956.46	917.90	870.20	826.59	771.31
Min	224.09	223.90	235.93	246.24	252.39	256.23	255.73	255.30
Max	12,822.95	10,934.82	9,531.71	8,686.78	8,096.37	7,669.84	7,113.72	6,918.56
SD	2,659.22	2,314.64	1,931.05	1,704.00	1,552.66	1,446.31	1,310.84	1,214.67

This table presents the main descriptive statistics for the CDS spreads of France, Spain, Italy, Ireland, Portugal, and Greece. Data are collected weekly from January 2010 to December 2019, inclusive. The exception is Greece, where the last observation corresponds to October 18, 2011.

Table 3. Main descriptive statistics for the absolute pricing error by pricing model.

	PWCDP	NP/Linear	NP/PCHIP	NP/Spline
Mean	8.51	6.26	4.64	5.07
Median	3.49	1.78	1.46	1.61
Min	0.00	0.00	0.00	0.00
Max	1,888.13	942.77	781.65	658.98
SD	34.45	25.17	18.06	16.00

This table presents the main descriptive statistics for the absolute pricing error by pricing model: PWCDP NP/Linear, NP/PCHIP, and NP/Spline. The absolute pricing error for each observed maturity is estimated by ignoring the specific quote in the estimation process; next, the actual and predicted CDS spread for that maturity are compared. The reported statistics correspond to pricing errors for 6m, 1y, 2y, 3y, 4y, 5y, and 7y maturities.

Table 4. Mean absolute pricing error by maturity and 5y-CDS spread level.

PWCDP	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	1.35	2.77	4.11	5.56	6.57	16.99
1y	1.49	2.92	3.98	4.53	4.38	10.10
2y	1.68	2.67	3.37	3.19	5.00	10.24
3y	1.52	1.93	2.19	2.81	5.44	7.17
4y	1.49	1.55	1.69	1.61	2.08	2.60
5y	1.71	1.86	1.45	1.98	2.49	3.35
7y	1.63	1.46	1.52	2.27	3.42	4.30

NP/Linear	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	1.08	1.66	2.59	4.31	5.48	14.63
1y	0.90	1.36	2.12	3.52	4.48	11.95
2y	1.03	1.19	1.87	3.32	5.24	9.69
3y	1.12	1.23	1.88	2.92	4.36	6.05
4y	1.25	0.92	1.37	1.34	2.87	5.31
5y	1.45	1.36	2.23	2.37	3.53	5.80
7y	2.08	2.21	2.45	2.00	2.61	5.88

NP/PCHIP	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	1.07	1.82	2.75	4.16	5.04	12.68
1y	0.88	1.34	2.03	3.07	3.69	9.18
2y	1.06	1.20	1.63	2.34	4.52	8.12
3y	1.15	1.01	1.36	2.39	3.49	4.96
4y	1.28	0.83	1.13	1.57	3.01	3.69
5y	1.45	1.12	1.52	1.84	2.68	3.72
7y	1.92	1.57	1.51	2.05	3.43	4.42

NP/Spline	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	1.22	2.01	2.89	4.08	5.84	11.62
1y	0.93	1.34	1.92	2.71	3.88	7.72
2y	1.08	1.26	1.82	2.56	3.67	7.30
3y	1.16	1.00	1.33	2.08	3.47	5.08
4y	1.29	0.89	1.23	1.90	3.12	3.87
5y	1.44	1.02	1.37	1.92	2.49	3.18
7y	2.03	2.08	2.79	3.91	5.08	6.49

This table presents MAPE by maturity (vertical axis) and 5y-CDS spread level (horizontal axis). The pricing models considered are PWCDP, NP/Linear, NP/PCHIP, and NP/Spline.

Table 5. Difference between the mean absolute pricing error of each model and that of the most accurate model by maturity and 5y-CDS spread level.

PWCDP	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	0.28	1.11	1.52	1.48	1.53	5.37
1y	0.61	1.58	2.06	1.82	0.69	2.38
2y	0.65	1.49	1.74	0.85	1.33	2.94
3y	0.40	0.93	0.86	0.73	1.97	2.20
4y	0.24	0.73	0.57	0.27	0.00	0.00
5y	0.27	0.84	0.08	0.14	0.00	0.17
7y	0.00	0.00	0.02	0.28	0.81	0.00

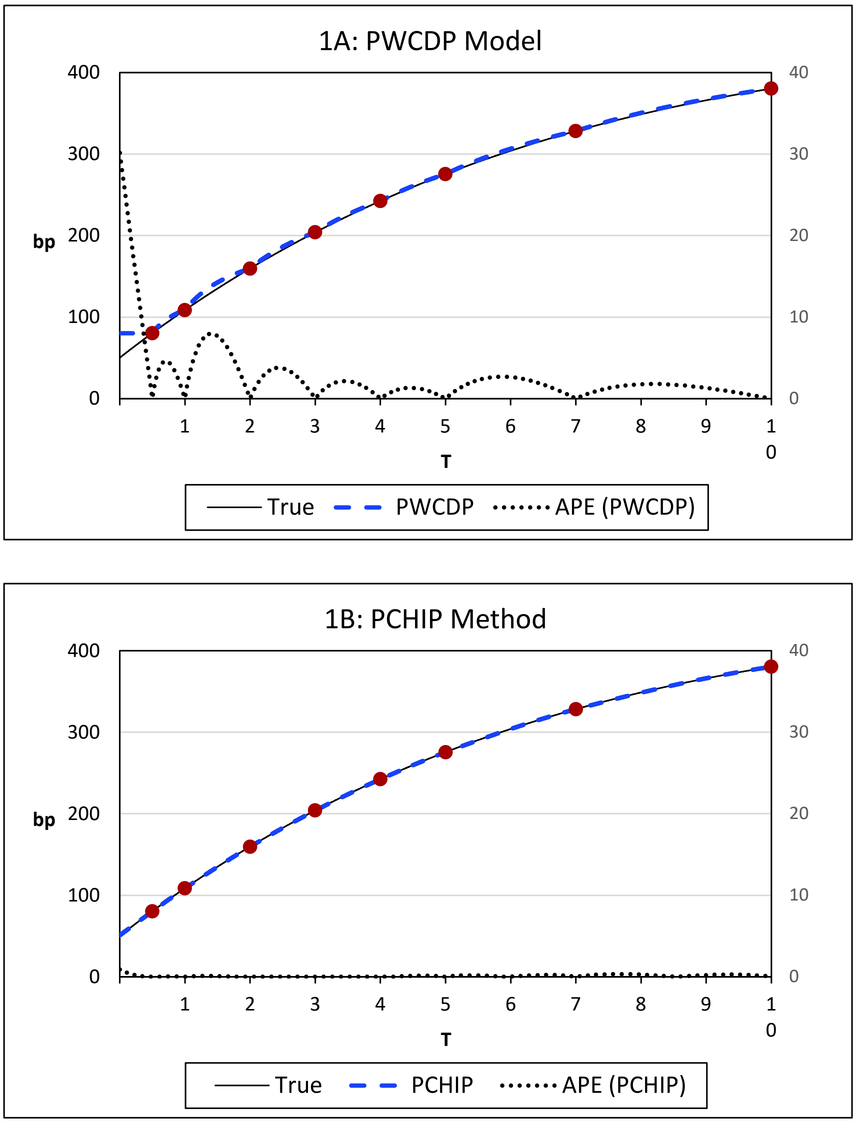
NP/Linear	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	0.00	0.00	0.00	0.23	0.45	3.01
1y	0.02	0.02	0.20	0.81	0.79	4.23
2y	0.00	0.00	0.24	0.99	1.57	2.39
3y	0.00	0.23	0.55	0.84	0.89	1.09
4y	0.00	0.09	0.24	0.00	0.79	2.71
5y	0.00	0.34	0.86	0.53	1.04	2.62
7y	0.45	0.75	0.94	0.00	0.00	1.58

NP/PCHIP	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	0.00	0.16	0.16	0.08	0.00	1.06
1y	0.00	0.00	0.11	0.36	0.00	1.46
2y	0.02	0.01	0.00	0.00	0.85	0.82
3y	0.03	0.01	0.03	0.31	0.02	0.00
4y	0.03	0.00	0.00	0.24	0.93	1.09
5y	0.01	0.10	0.15	0.00	0.19	0.54
7y	0.29	0.10	0.00	0.06	0.82	0.12

NP/Spline	(0,50]	(50,150]	(150,300]	(300,500]	(500,1000]	>1000
6m	0.15	0.35	0.30	0.00	0.80	0.00
1y	0.05	0.00	0.00	0.00	0.19	0.00
2y	0.04	0.08	0.19	0.23	0.00	0.00
3y	0.04	0.00	0.00	0.00	0.00	0.12
4y	0.04	0.06	0.10	0.56	1.04	1.27
5y	0.00	0.00	0.00	0.08	0.00	0.00
7y	0.40	0.62	1.28	1.91	2.47	2.18

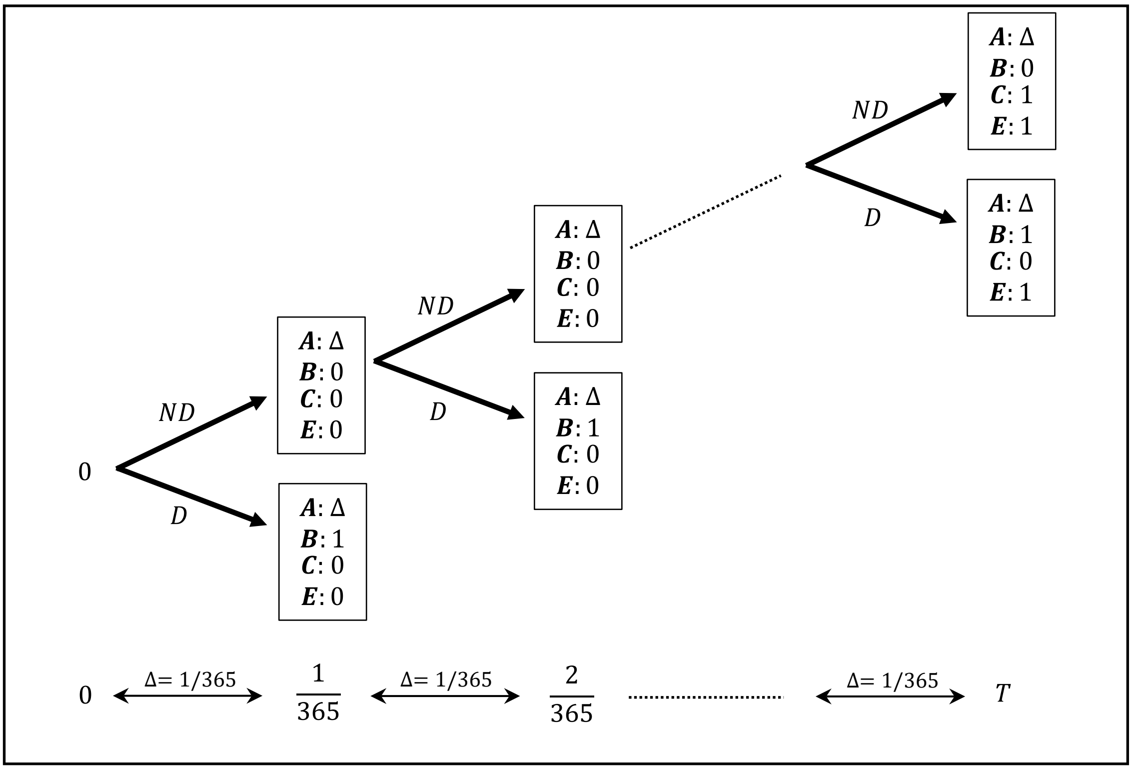
This table reports the difference between the MAPE of each pricing model and that of the most accurate model. The differences are sorted by maturity (vertical axis) and 5y-CDS spread level (horizontal axis). The pricing models considered are PWCDP, NP/Linear, NP/PCHIP, and NP/Spline.

Figure 1. Example of estimation approaches for the complete term structure of CDS spreads.



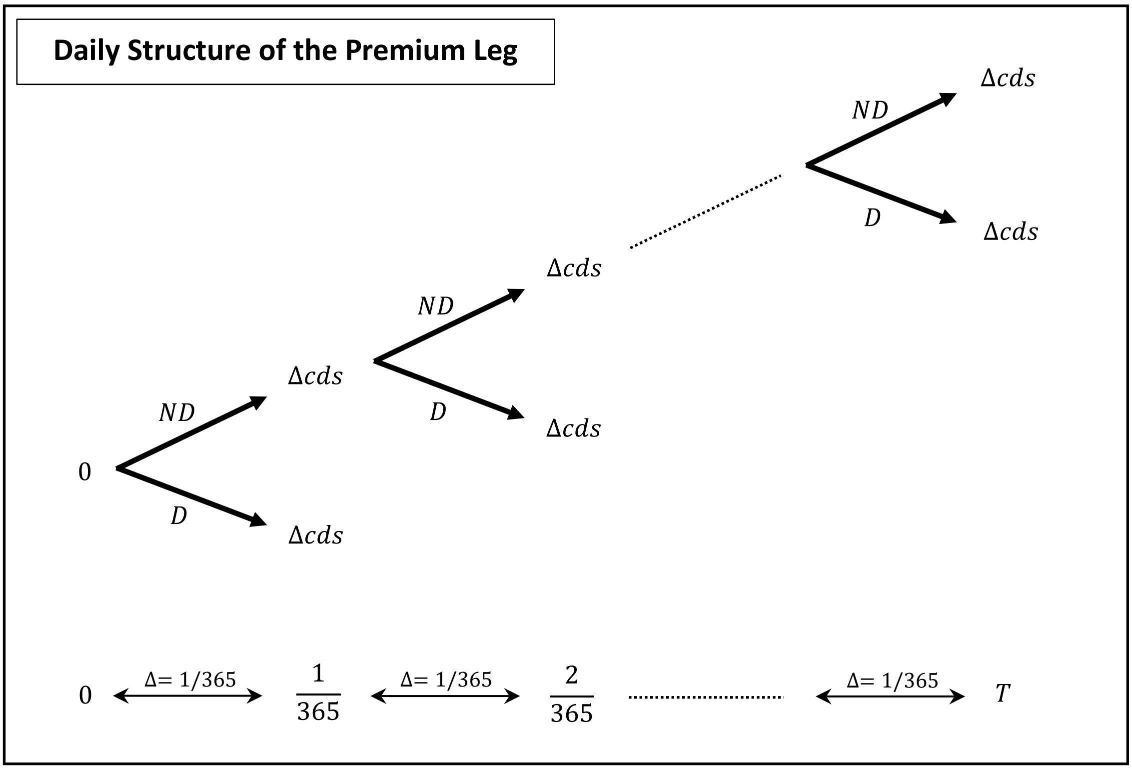
This figure provides an example of two possible estimation approaches for the CTSCDS (True; black solid line, left axis) based on the OTSCDS (red points, left axis). These estimation approaches are the PWCDP model (Panel 1A; blue dashed line, left axis), and the PCHIP method (Panel 1B; blue dashed line, left axis). It is assumed that the actual CTSCDS corresponds to a particular parametrization of the Nelson-Siegel model. The figure also incorporates the absolute pricing errors from each estimation method (APE; black dotted line, right axis).

Figure 2. Structure of payments for assets A, B, C, and E with maturity T.



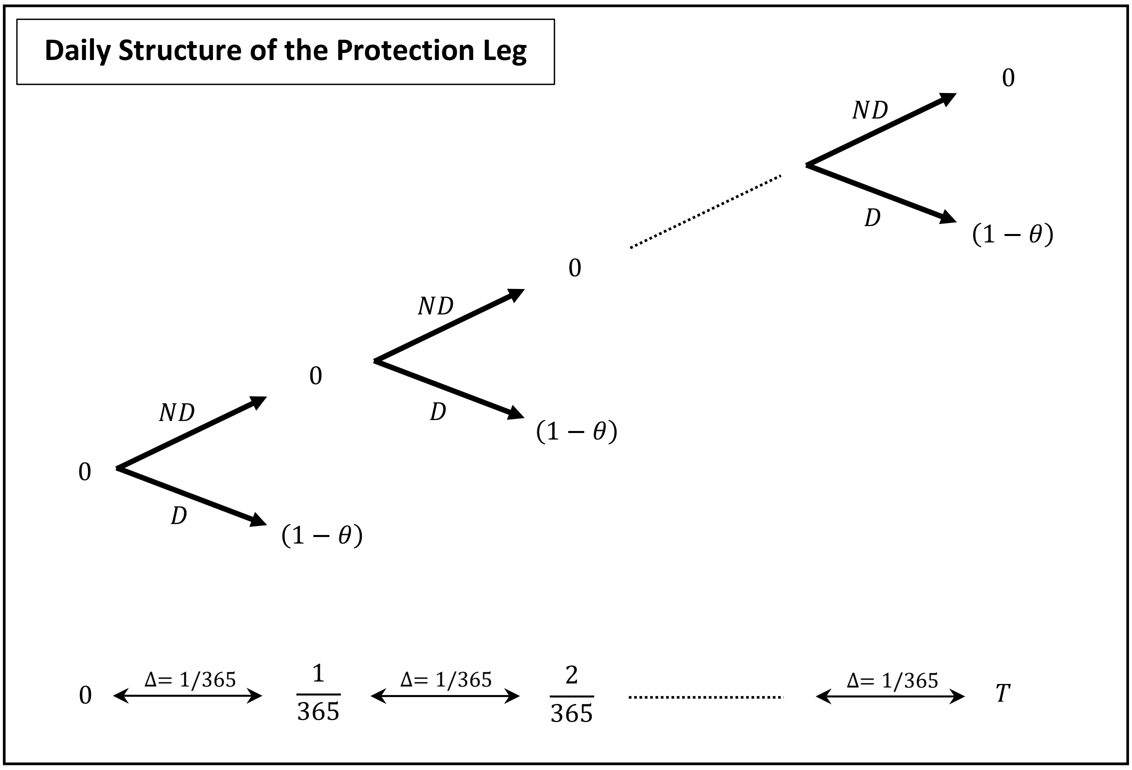
This figure presents the structure of payments for assets A, B, C, and E with maturity $T > 0$. The possible outcomes for each day are no default (ND) or default (D).

Figure 3. Daily structure of the premium leg in a CDS contract with maturity T.



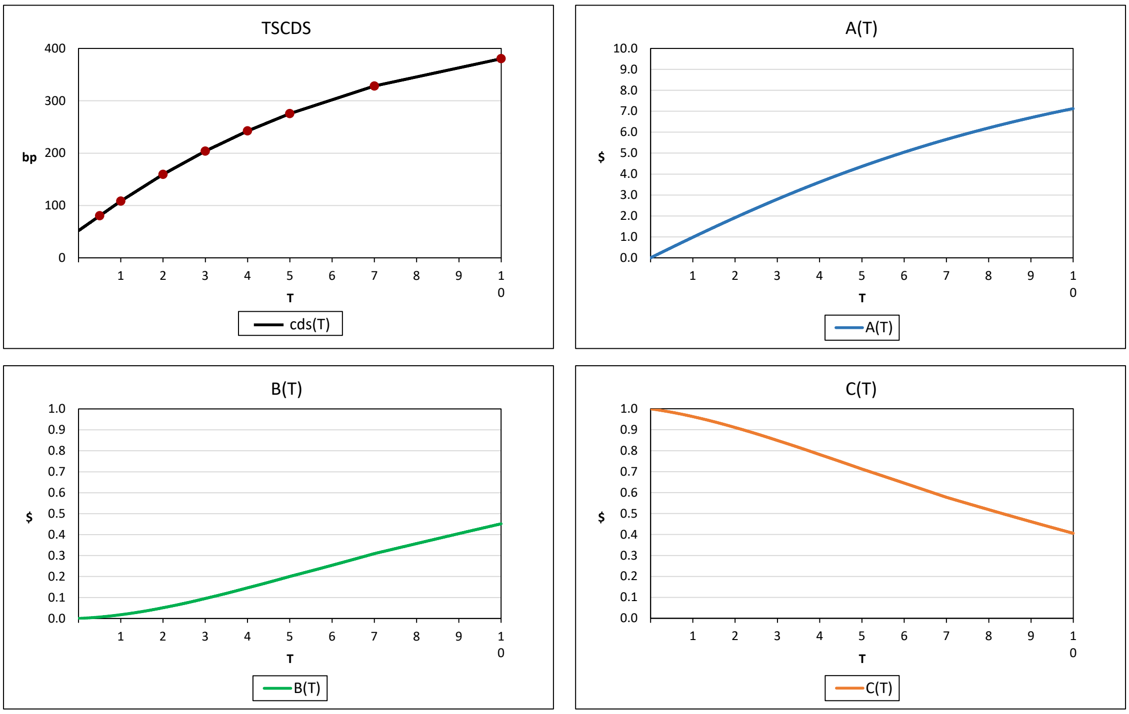
This figure presents the daily structure of the premium leg in a CDS contract with maturity $T > 0$. The possible outcomes for each day are no default (ND) or default (D).

Figure 4. Daily structure of the protection leg in a CDS contract with maturity T.



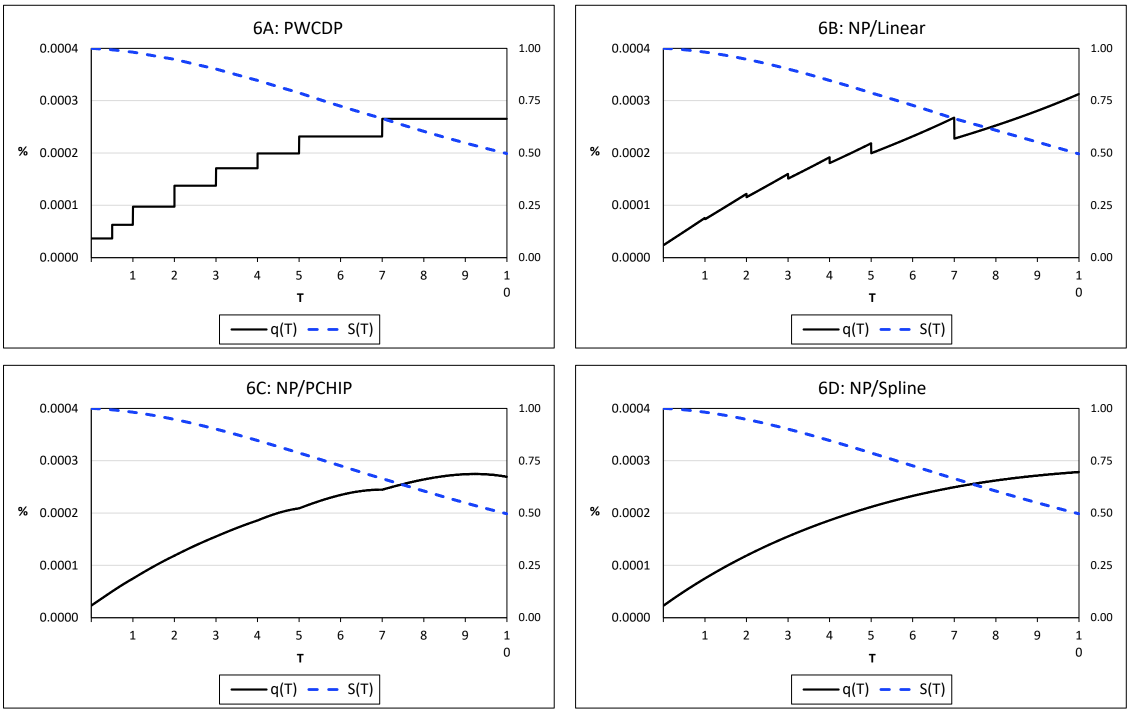
This figure presents the daily structure of the protection leg in a CDS contract with maturity $T > 0$. The possible outcomes for each day are no default (ND) or default (D).

Figure 5. Numerical example of the bootstrapping of credit risk discount factors.



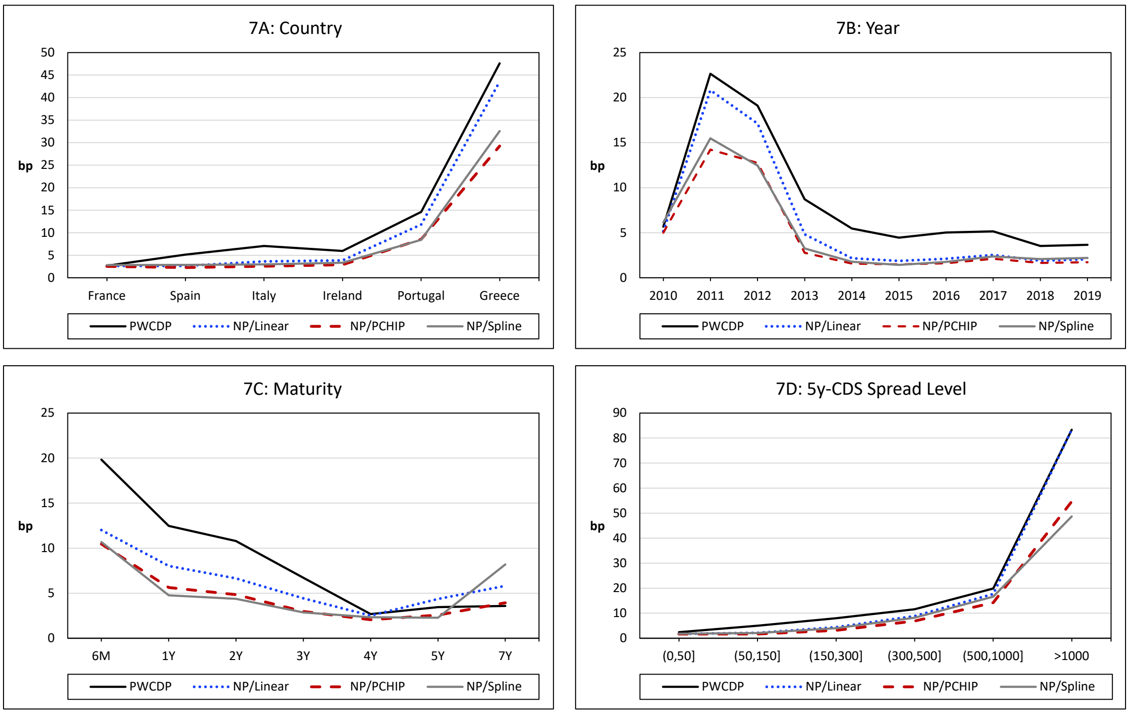
This figure plots the numerical example results, where the term structures of $A(T)$, $B(T)$, and $C(T)$ are estimated based on the CTSCDS and Equation System (8). The red point indicates that the CDS spread corresponds to an observed maturity: 6m, 1y, 2y, 3y, 4y, 5y, 7y, or 10y. In this case, the CTSCDS is obtained via a linear interpolation of the observed quotes. The example assumes a constant risk-free rate of 2% and a recovery rate of 40%.

Figure 6. Example of the term structures of forward risk-neutral default and survival probabilities by pricing model.



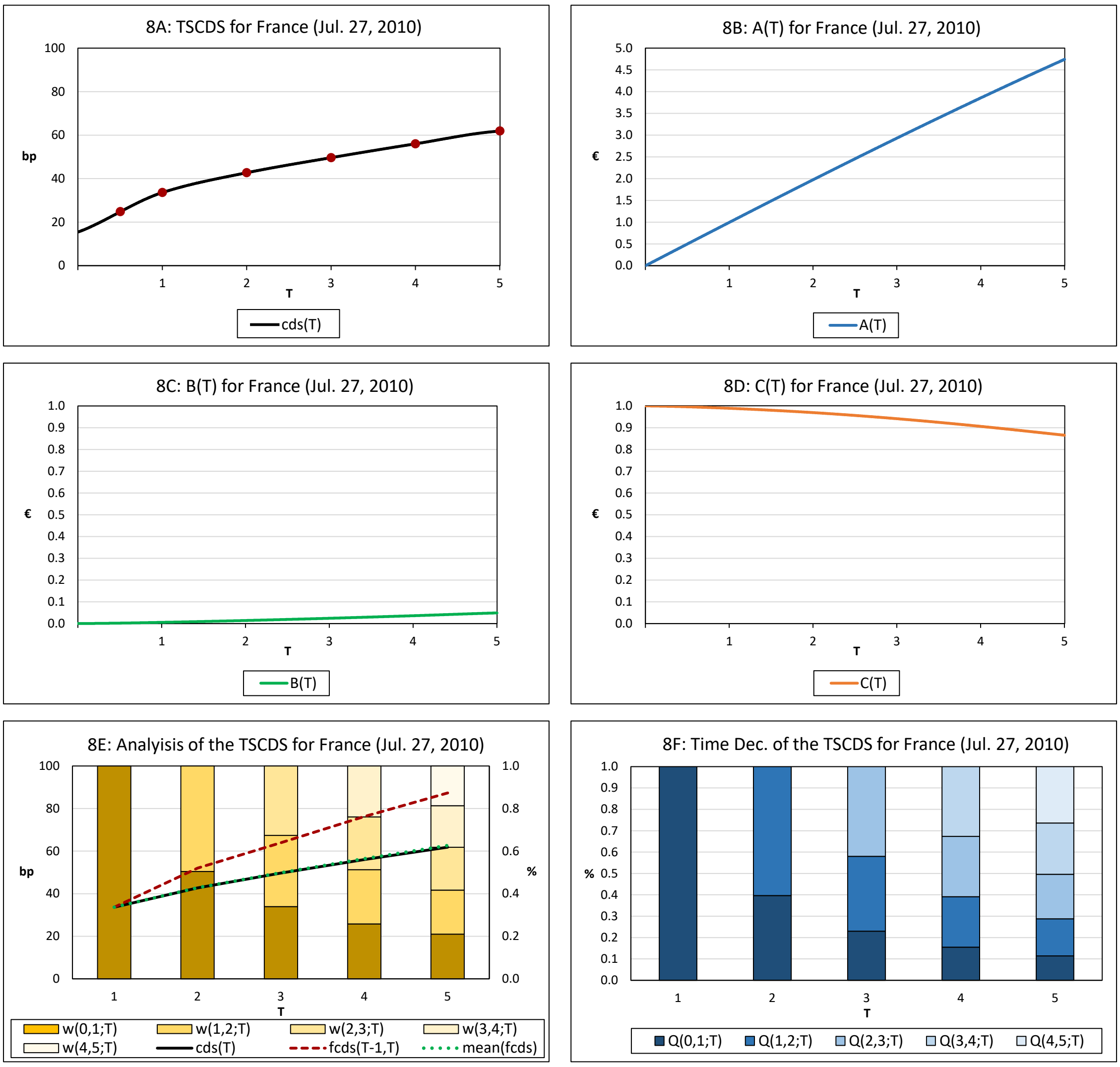
This figure plots the term structures of $q(T)$ (black solid line, left axis) and $S(T)$ (blue dashed line, right axis) for four different pricing models: PWCDP (Panel 6A), NP/Linear (6B), NP/PCHIP (6C), and NP/Spline (6D). All cases assume the same OTSCDS with the following maturities: 6m, 1y, 2y, 3y, 4y, 5y, 7y, and 10y.

Figure 7. Mean absolute pricing error by country, year, maturity, and 5y-CDS spread level.



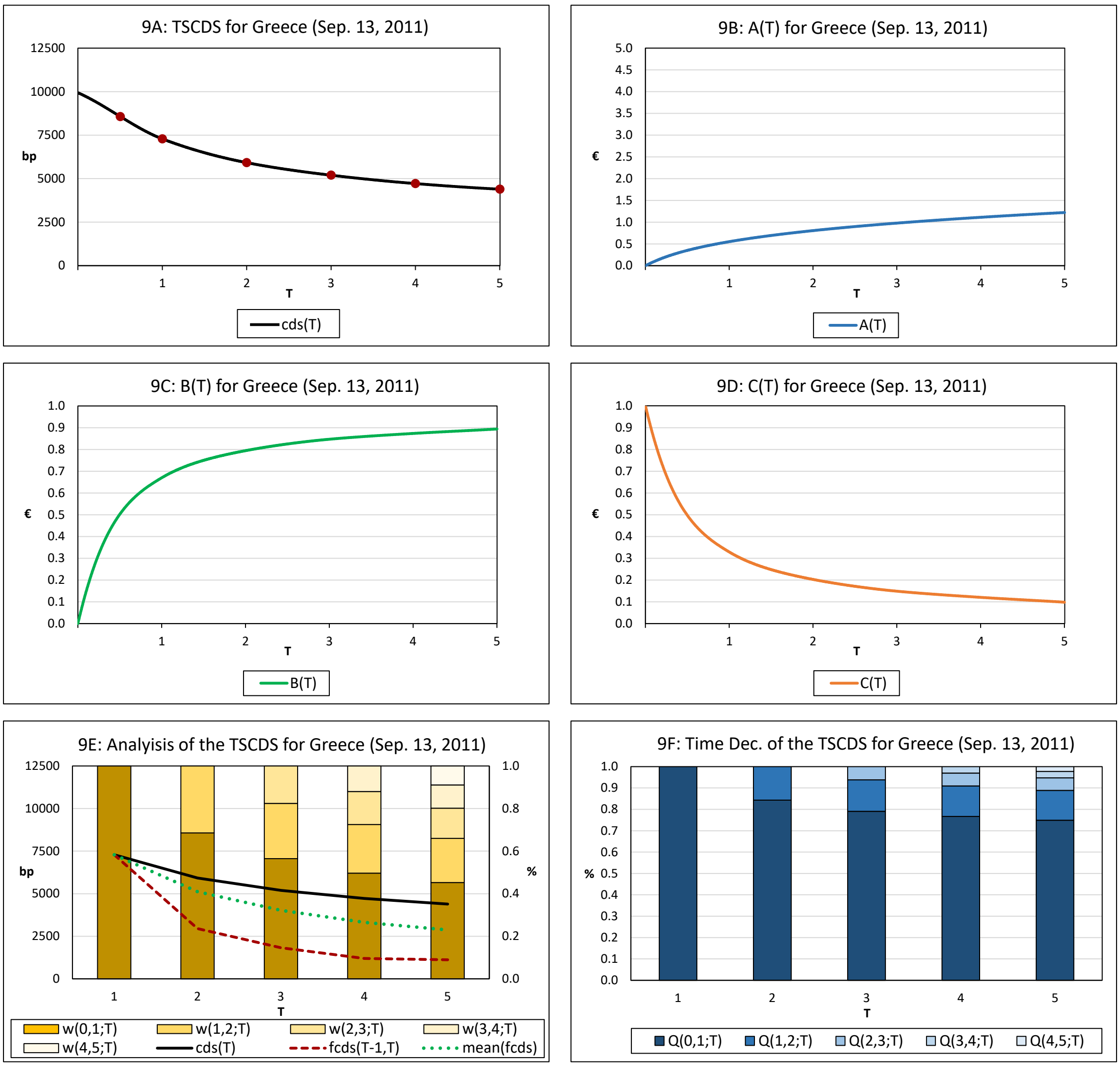
This figure plots the MAPE by country (Panel 7A), year (7B), maturity (7C), and 5y-CDS spread level (7D). The pricing models considered are PWCDP (black solid line), NP/Linear (blue dotted line), NP/PCHIP (red dashed line), and NP/Spline (grey solid line).

Figure 8. Term structure and time decomposition of CDS spreads for France—July 27, 2010.



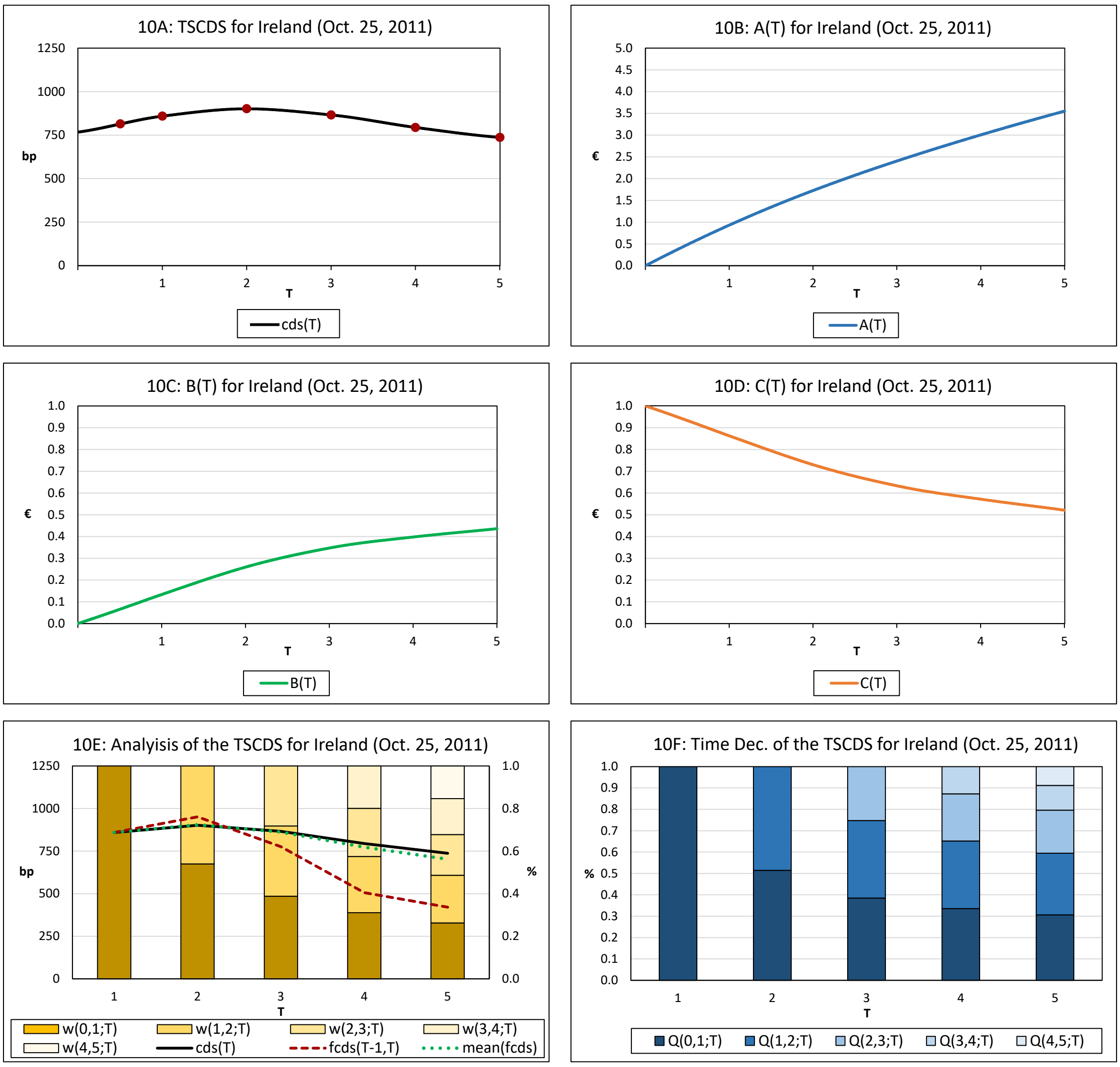
This figure comprises six panels, which present the term structure and time decomposition of CDS spreads for France on July 27, 2010. Panel 8A plots the CTSCDS, estimated based on the observed quotes (red points) and the PCHIP method. Panels 8B, 8C, and 8D present the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 8E plots the OTSCDS (1 to 5y; black solid line), TSFCDS (red dashed line), and TSMFCDS (green dotted line). It also shows the actual weight of each forward CDS spread on each spot CDS spread. Finally, Panel 8F shows the corresponding decomposition of each CDS spread.

Figure 9. Term structure and time decomposition of CDS spreads for Greece—September 13, 2011.



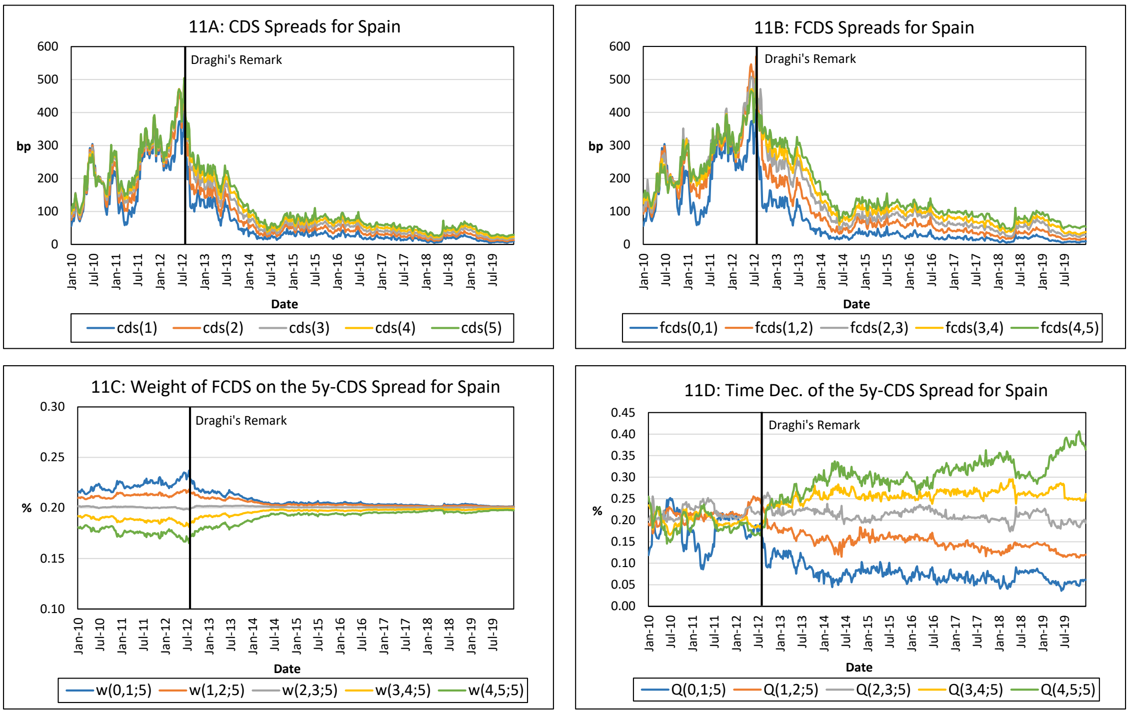
This figure comprises six panels, which present the term structure and time decomposition of CDS spreads for Greece on September 13, 2011. Panel 9A plots the CTSCDS, estimated based on the observed quotes (red points) and the PCHIP method. Panels 9B, 9C, and 9D present the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 9E plots the OTSCDS (1 to 5y; black solid line), TSFCDS (red dashed line), and TSMFCDS (green dotted line). It also shows the actual weight of each forward CDS spread on each spot CDS spread. Finally, Panel 9F shows the corresponding decomposition of each CDS spread.

Figure 10. Term structure and time decomposition of CDS spreads for Ireland—October 25, 2011.



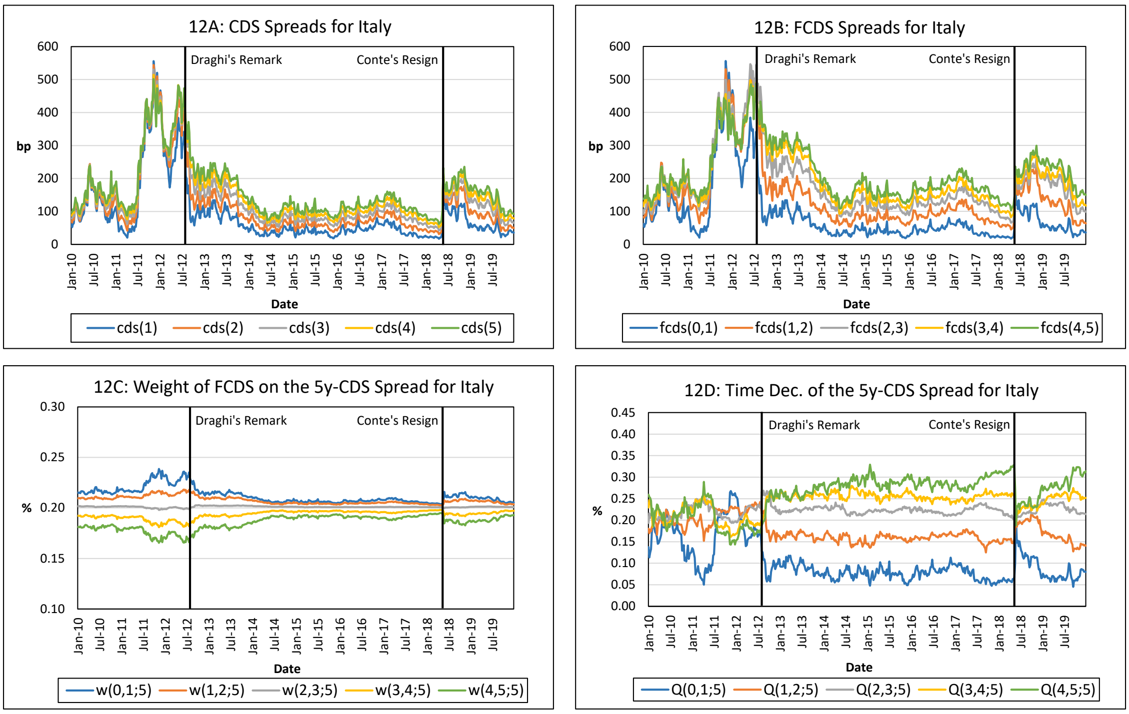
This figure comprises six panels, which present the term structure and time decomposition of CDS spreads for Ireland on October 25, 2011. Panel 10A plots the CTSCDS, estimated based on the observed quotes (red points) and the PCHIP method. Panels 10B, 10C, and 10D present the term structures of $A(T)$, $B(T)$, and $C(T)$, respectively. Panel 10E plots the OTSCDS (1 to 5y; black solid line), TSFCDS (red dashed line), and TSMFCDS (green dotted line). It also shows the actual weight of each forward CDS spread on each spot CDS spread. Finally, Panel 10F shows the corresponding decomposition of each CDS spread.

Figure 11. Time decomposition of the 5-year CDS spread for Spain, January 2010–December 2019.



This figure comprises four panels, which show the time decomposition of the 5-year CDS spread for Spain from January 2010 to December 2019, inclusive. Panel 11A plots the time series of 1- to 5-year CDS spreads. Panel 11B presents the time series of forward CDS spreads. Panel 11C shows the evolution of the weight of each forward CDS spread on the 5-year CDS spread. Panel 11D plots the time series of the final decomposition of the 5-year CDS spread. The results in the last three panels are based on weekly estimates of the CTSCDS, generated based on the observed quotes and the PCHIP method.

Figure 12. Time decomposition of the 5-year CDS spread for Italy, January 2010–December 2019.



This figure comprises four panels, which show the time decomposition of the 5-year CDS spread for Italy from January 2010 to December 2019, inclusive. Panel 12A plots the time series of 1- to 5-year CDS spreads. Panel 12B presents the time series of forward CDS spreads. Panel 12C shows the evolution of the weight of each forward CDS spread on the 5-year CDS spread. Panel 12D plots the time series of the final decomposition of the 5-year CDS spread. The results in the last three panels are based on weekly estimates of the CTSCDS, generated based on the observed quotes and the PCHIP method.