Structural Models: A Microscope on the Credit Crisis

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Outline of Talk

OBJECTIVE:

- ***To use a structural model to examine the consistency of CDS and equity option prices during the crisis of 2007-2010.***
  - In a structural model, both prices are driven by the *underlying stochastic process of firm asset value* *--do both prices imply the same asset volatility??*

- First see how model with constant parameters performs, and then examine how ***parameters must change*** with time to make CDS and option prices consistent. In particular, we consider
  - Changes in jump intensity
  - Changes in CDS liquidity (future work)
  - Changes in recovery rates (future work, some comments)

THEORY:

1) **Need a Structural Model that Includes *Jumps, Illiquidity***
[my Princeton Lectures 2006; see Appendix here]

- Illiquidity premium on bonds (additional discount)
- Simple Poisson jump to default with large loss
(“catastrophe” as in Barro; example Lehman Bros.)
- Both are necessary to explain *short term credit spreads and* default rates (sufficient for aggregate bond stats)
- Also considered important elements in crisis

2) **Model Values *Equity*, *Bonds*, *Options* and *CDSs* of firms with**

- Endogenous Default (except in catastrophe)
- Arbitrary Maturity of Debt (exponentially declining)
- Jumps in underlying firm asset value (mixed jump-diff.)

APPLICATIONS:

Examine ***consistency of CDS and equity option markets,*** end 2006-2010

- Are prices of equity, equity options, and CDS rates consistent through time, when asset values and asset volatility change but
  - Jump intensity, default costs, and liquidity of bond markets are constant?
  - If not, can we explain differences by changes in perceived jump intensity, default costs, and/or liquidity?
    - To be consistent, no further parameter changes expected
- Focus at this point is on two financial firms:
  - Goldman Sachs and JPM (mostly former)
  - Hope to extend subsequently

THE APPROACH: Initial Calibration (focus on Goldman)

Parameters (initially fixed through full period 12/31/06 – 8/03/10)

- Net tax advantage to debt is 25%
- Default costs if "diffusion default" = 2% (repos secured)
- Default costs if jump to catastrophe = 91.4% (Lehman loss)
- Payout (dividends, after-tax interest) on total assets = 5.5%
  - Changes through time

At initial date 3/31/07, choose asset value, volatility, and coupon rate so that

- Leverage = 90.1% (from balance sheet) determines debt
- Average maturity of debt = 1.2 years
  - 85% short term initially (now 78% with 1.5 yr. avg. mat.)
- Coupon set so bond sells at par initially
- Equity Price and 6-mo. ATM option prices matched
- Jump intensity parameter consistent with CDS rates
  - Calibrated jump intensity = 15 bps for 1 yr. CDS, 25 bps for 5 yr. (upward sloping jump intensity)

At subsequent weekly intervals,

- Update debt principal, average maturity by interpolating quarterly balance sheet data from GS 10Qs
- Update interest (coupon) rate on GS debt, based on swap rates
- Update Equity value
- Update option values (BBG ATM implied vols, price using B/S)

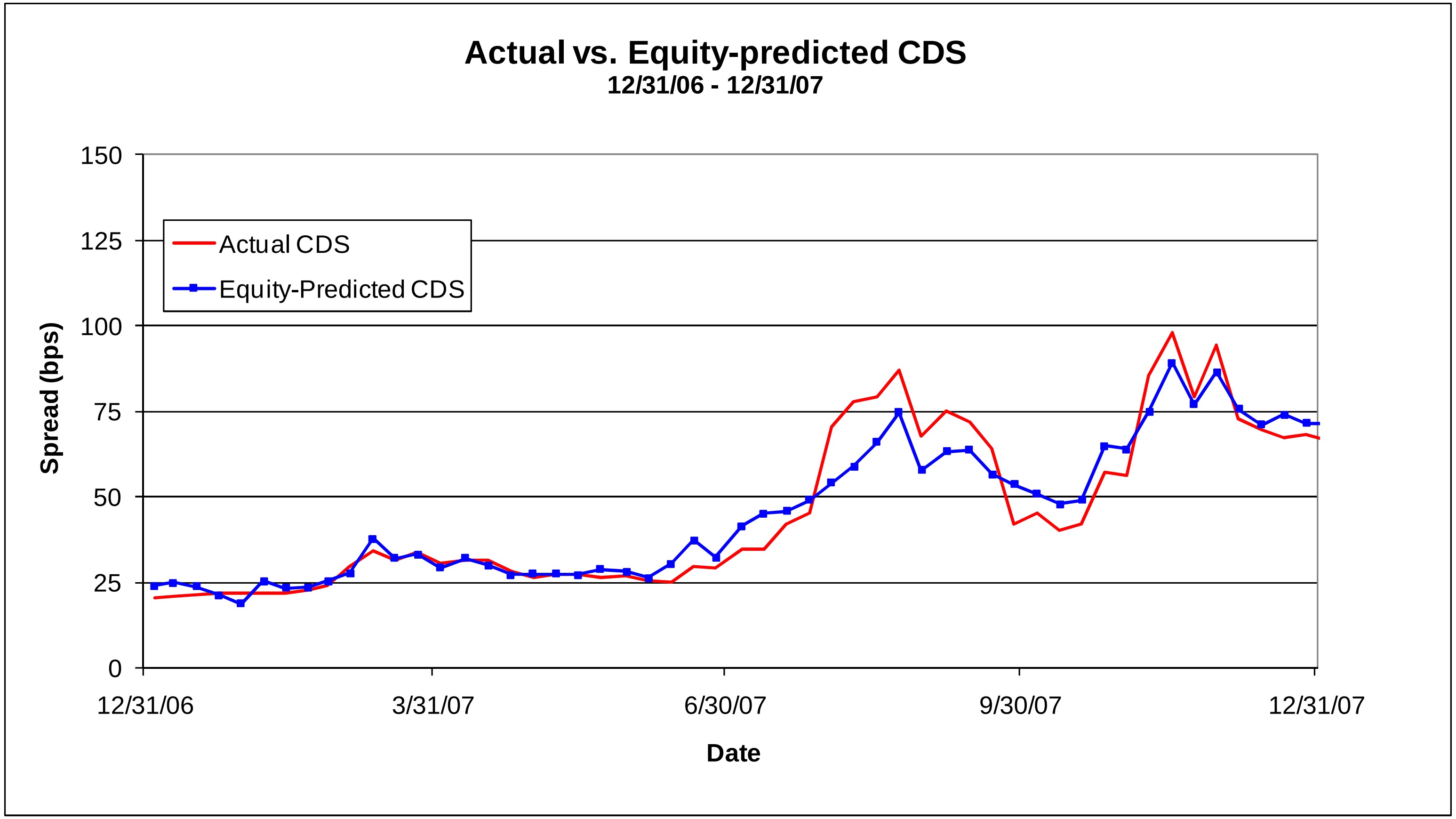
Assuming jump intensity remains fixed at 25 bps, can compute consistent

Asset value and asset diffusion volatility, use these to predict CDS rates

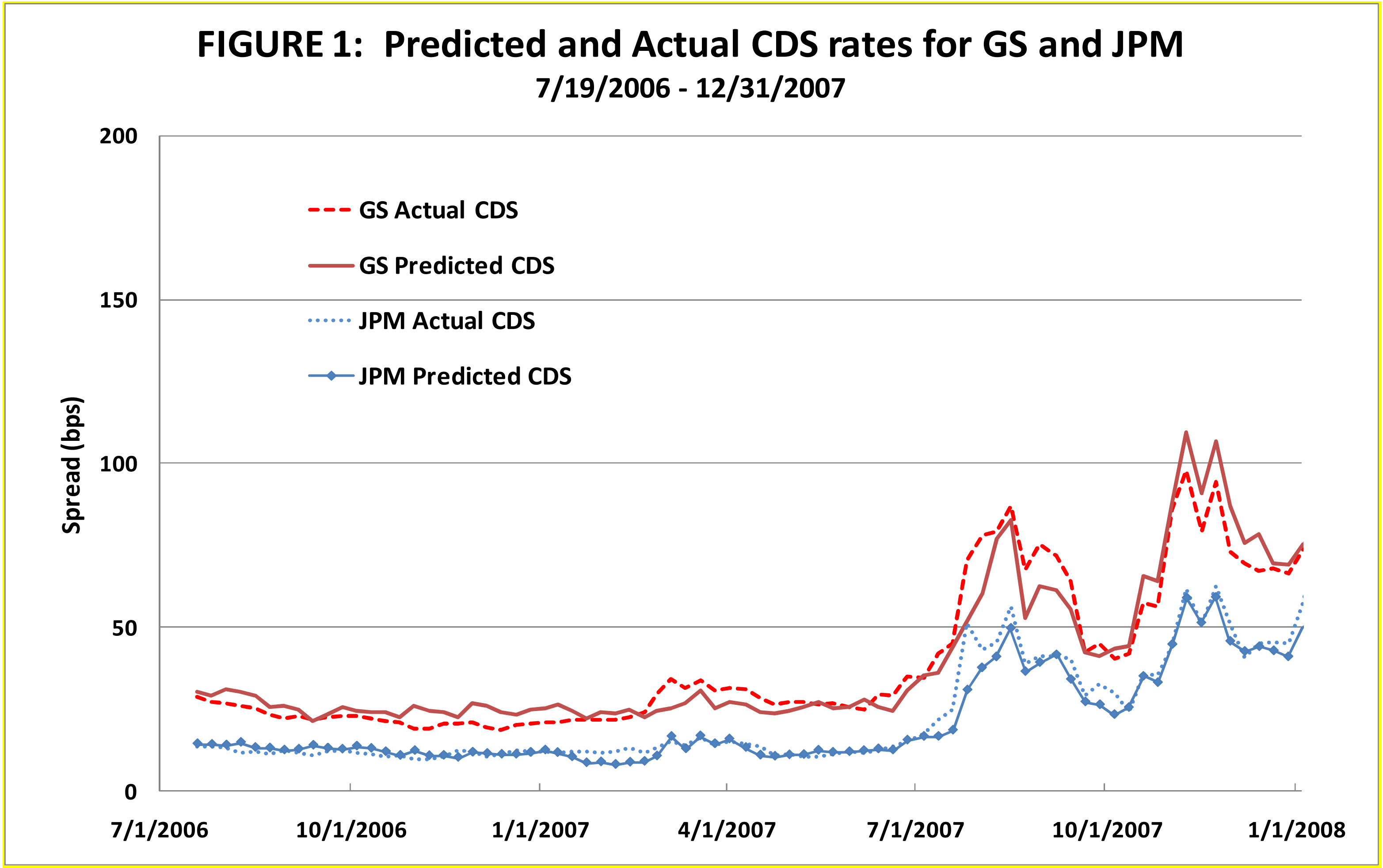
*Alternatively, we could match CDS rates instead of option prices to back out asset value and asset volatility, then predict **option** prices*

—basically, we compare implied volatilities of CDSs vs. options

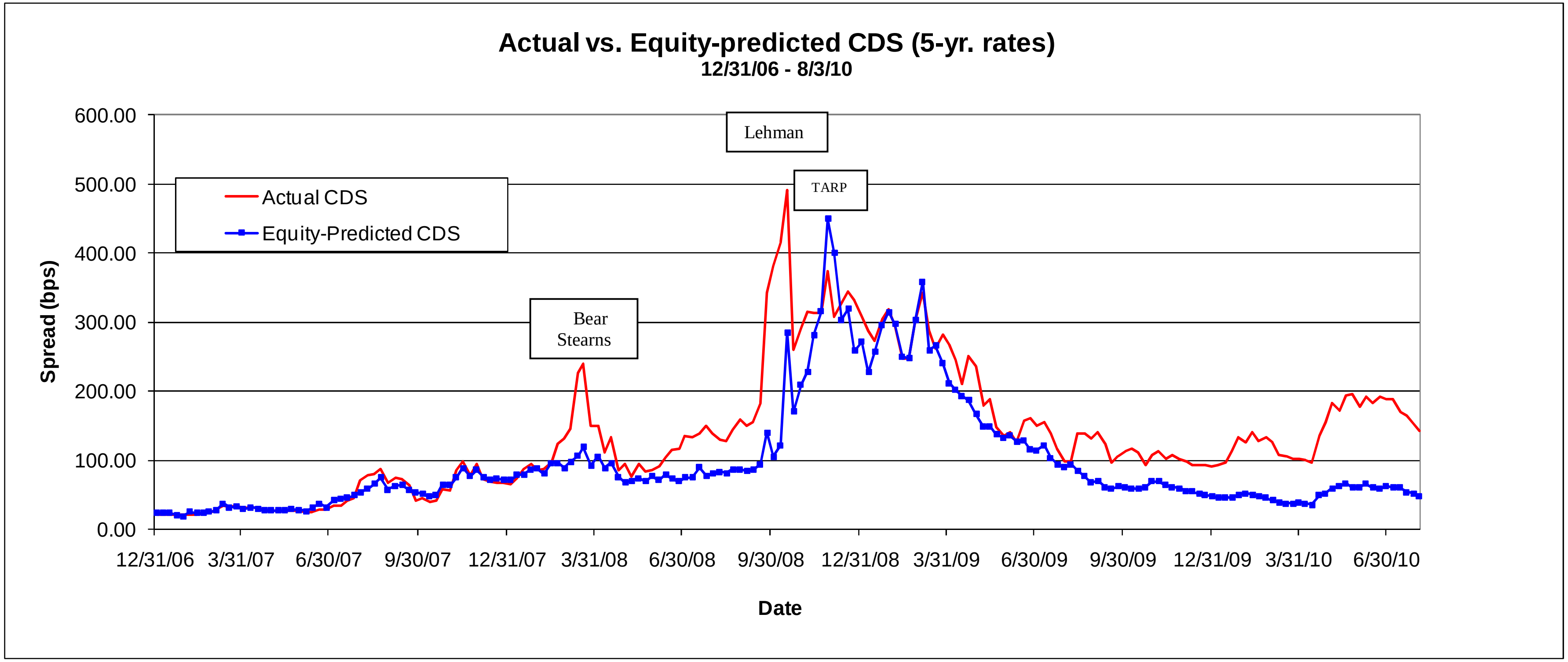
How does it work? Initially, quite well:



Here's an earlier calibration that includes JPM:



But it gets considerably worse as crisis worsens and continues:

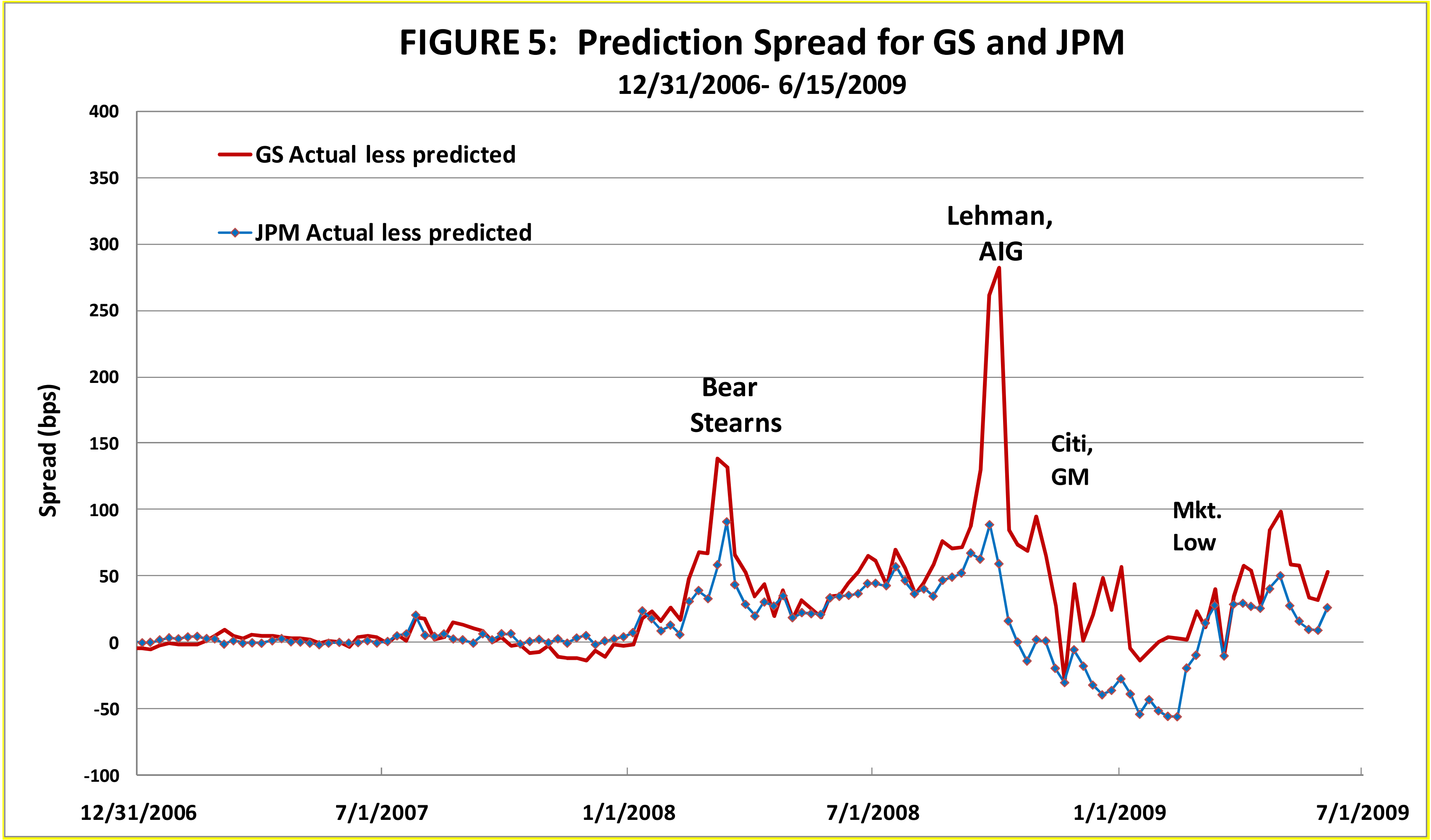


Observe somewhat strange reversal in mid-late November 2008.

GS hits low, implied volatilities soar briefly 11/14 – 11/28.

Currently, far off.

Reversal in November 2008 not limited to GS: [from earlier calibration]



How to explain?

- The Model is wrong (let's reject this!)
- Markets may not have been arbitraged (consider at end of talk)
- **The Model assumes constant parameters, but they may vary with time**

Candidates for changing parameters that could explain high CDS rates

- **Default costs**
- **(Il)-liquidity costs for bonds, repos**
- **Jump Intensity**

Default costs.

- As diffusion default costs α rise, with constant option prices, ***asset volatility must fall.***
  - This in turn implies CDS rates must fall (even though recovery less)
  - So rise in diffusion default costs can't explain high CDS rates
- Jump-to-default costs can rise from 91.35% to 100%
  - But this results in a very small (insufficient) rise in CDS rates

Increase in (il)liquidity of bonds

- This will raise cost of debt relative to before (e.g., h from 60 to 200 bps)
  - Coupon rises substantially

- To be consistent with constant option prices, asset volatility scarcely changes
  - This in turn implies CDS rates barely change (or fall), so
  - Illiquidity of bonds cannot explain high CDS rates, ***ceteris paribus***

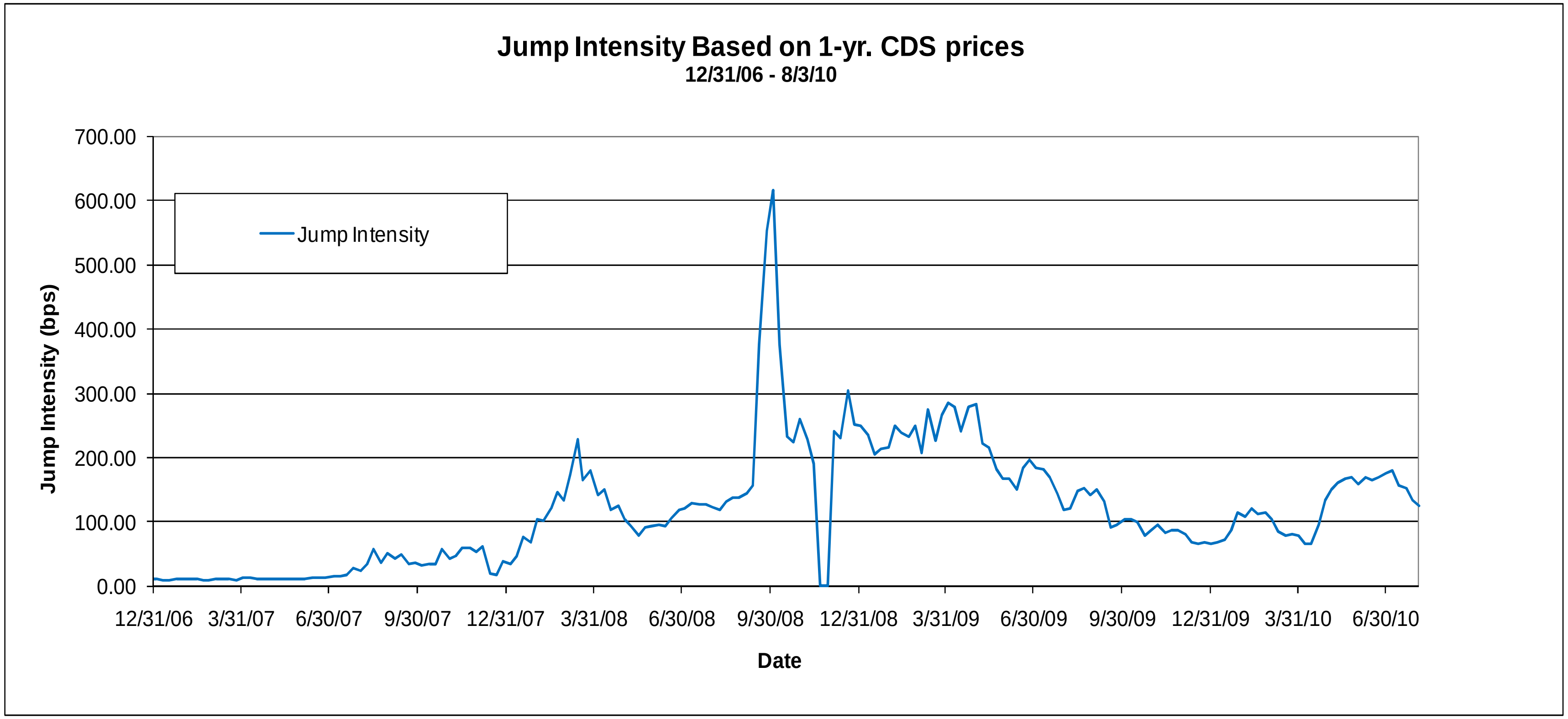
Increase in Jump Intensity

- Idea: Given that option prices remain constant, a higher jump intensity (and therefore lower diffusion volatility)

  may affect "out of money" CDS rates more,
  and thus raise them relative to options.

- Application: now allow ***jump intensity to vary*** as well as asset value and asset volatility.

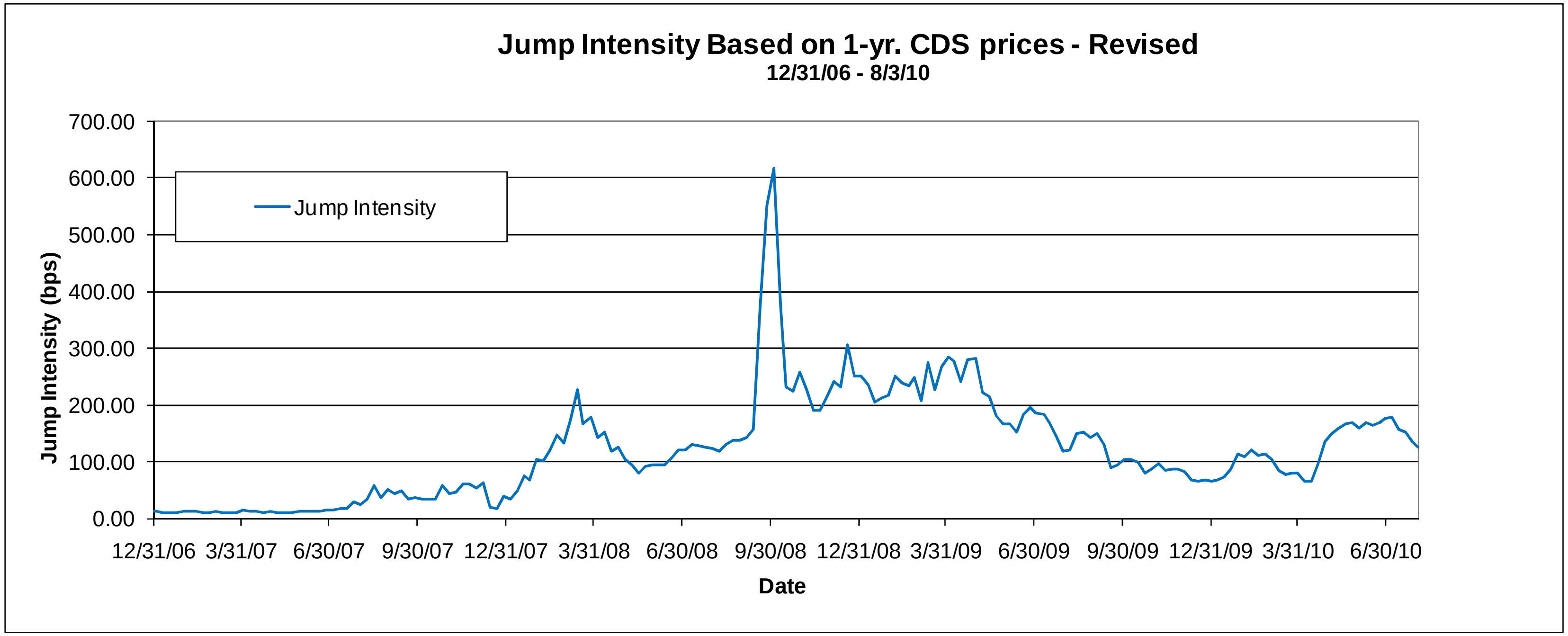
o ***Can now match equity value, option value, <u>and CDS value.</u>***



A bit scary that the current implied jump intensity is so high!

But history of option IVs vs. CDS rates seems bizarre 11/7 – 12/5/08

Date:	11/07	11/14	11/21	11/28	12/5
CDS (bps)	386	387	436	376	407
Reported IV(%)	90	97	131	127	91
Pseudo IV(%)	90	97	100	92	91



If time, discuss

- ➢ Possible market imbalances (Duffie work)
- ➢ 5-yr. CDS and term structure of jump intensity
- ➢ Find better data sources for CDS, option-implied vols.

Future developments (for someone else!)

- ***Look at more firms*** (very tedious, alas…)
- ***Develop model that explicitly recognizes parameter uncertainty***
  (also tedious, and question about fixed parameters at some stage of the parameter dynamics)

Conclusions:

- Structural models with ***fixed*** volatility and jump intensity cannot possibly explain both option prices and CDS rates in the structural model presented
- Even allowing freely varying diffusion volatility cannot explain both option prices and CDS rates
- Allowing jump intensity ***and*** diffusion volatility to change seems to provide a decent fit
- Market imperfections (positive arbitrage opportunities) may provide an alternative explanation.

Stochastic Process: **Assume $CF(t)$ is current (after tax) cash flow, paid out to security holders, with risk-neutral diffusion and jump components:**

$$dCF(t) = \mu CF(t)dt + \sigma CF(t)\, dZ(t) \quad \text{if no jump at or prior to } t$$

$$= -kCF(t_-) \quad \text{if jump at } t$$

where k is the fractional loss of cash flow if a jump occurs at t.

The jump is a Poisson process with constant risk-neutral intensity λ; thus the probability of no jump before time t is $e^{-\lambda t}$.

The expected growth rate of cash flow is: $E[dCF(t)/ CF(t)] = (\mu - \lambda k)dt$

Recall: default occurs at t if the diffusion value $V(t)$ hits barrier V_B, or if a jump occurs. If so, debt is in default and receives value $(1- \alpha)$ if the barrier V_B is hit, or $(1 - k)V(t)$ if there is a jump.

WITHOUT LOSS OF GENERALITY, let current time t = 0, V = V(0)

Riskfree rate: r

V, the value of unlevered firm at t= 0 $V = CF/(r - \mu + \lambda k)$,

V(t), *excluding a jump*, has a risk-neutral process $dV/V = gdt + \sigma dZ$

where to give a risk-neutral return r, $g = r - \delta + \lambda k$

Dividend rate (fraction of pre-jump value): $\delta = CF/V = r - \mu + \lambda k$

Combining results above, we note that $g = \mu$

Bankruptcy costs: α

Cumulative default frequency at t: $F[t; V, V_B]$ (or F)

First passage density $f[t; V, V_B]$ (or f)

Clearly these latter functions depend on growth rate g and σ. I have suppressed these arguments here.

Let h denote the liquidity premium, implying debt holders discount expected cash flows at rate $r + h$. Given h, the

VALUE OF DEBT

$$D(h) = \int_0^\infty e^{-(r+h)t}(C + mP)e^{-mt}(1-F)e^{-\lambda t}dt + (1-\alpha)V_B\int_0^\infty e^{-(r+h)t}e^{-\lambda t}e^{-mt} f\, dt$$
$$+ (1-k)\int_0^\infty e^{-(r+h)t}e^{-mt}(e^{gt}V)\lambda e^{-\lambda t}(1-F)dt$$

The first term is the discounted coupon plus principal payments, which decline exponentially at the rate m as debt is retired. Note that coupons are paid only if (i) the default barrier has not been reached, with probability $1 - F$, and that no jump has occurred, which is with probability $e^{-\lambda t}$. The second term is discounted payoffs if the barrier is reached at time t, times the probability that a jump has not occurred. Note e^{-mt} appears in this term and the next because current debt only has claim to fraction e^{-mt} of value. The final term is the value if the jump occurs at time t, which occurs with probability $\lambda e^{-\lambda t}$, reduced by $(1 - F)$, the probability the boundary V_B is reached before the jump. Note that default by jump gives expected value $(1 - k)V(t)$, where the expected value of $V(t) = V e^{gt}$ and V is the current firm value. Conditional on no prior jumps, $V(t)$ grows at rate g, whereas inclusive of expected jump loss, $V(t)$ grows at rate r.

Integrating the first term and last terms by parts gives:

$$D(h) = \frac{C+mP}{r+m+\lambda+h}(1-\int_0^\infty e^{-(r+m+\lambda+h)t} f\,dt) + (1-\alpha)V_B \int_0^\infty e^{-(r+m+\lambda+h)t} f\,dt$$
$$+ \frac{\lambda(1-k)V}{r+m+\lambda+h-g}(1-\int_0^\infty e^{-(r+m+\lambda+h-g)t} f\,dt)$$

We now make use of a key result on first passage times $f(t;\, V_0,\, V_B)$ where dV/V follows a log Brownian motion with drift rate g:

$$dV/V = gdt + \sigma dZ$$

$$q(z,V,V_B) \equiv \int_0^\infty e^{-zt} f(t;V,V_B)dt = \left(\frac{V}{V_B}\right)^{-y(z)},$$

where

$$y(z) = \frac{(g-.5\sigma^2)+((g-.5\sigma^2)^2+2z\sigma^2)^{0.5}}{\sigma^2}$$

Note that we have suppressed the arguments (g, σ) of the stochastic process in the definitions of h and y, which we continue to do hereafter. Recalling $g = \mu$, and we can rewrite the debt value function as

$$D(h) = \frac{C+mP}{r+m+\lambda+h}(1-\left(\frac{V}{V_B}\right)^{-y1}) + (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-y1} + \frac{\lambda(1-k)V}{r+m+\lambda+h-g}(1-\left(\frac{V}{V_B}\right)^{-y2})$$

where

$$y_1(h) = y(r+m+\lambda+h) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2+2((r+m+\lambda+h)\sigma^2]^{0.5}}{\sigma^2}$$
$$y_2(h) = y(r+m+\lambda+h-\mu) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2+2((r+m+\lambda+h-g)\sigma^2]^{0.5}}{\sigma^2}$$

VALUE OF CASH FLOWS TO EQUITY HOLDERS OF A LEVERED FIRM:

Equity holders discount cash flows without an additional risk premium.[1] The value of equity in a levered firm will reflect the value of the unlevered firm V_0, plus the value of tax savings provided by deductibility of coupon payment, less the value of default costs, less the value (to shareholders) of the cash flows to debt. These cash flows are discounted at rate r rather than $r + h$, and will have value $D(0)$. Thus equity has value

$$E = V + TS - DC - D(0),$$

where tax savings provide a constant cash flow τC when the firm is solvent, and zero otherwise.

The value of ***tax savings*** is

$$TS = \int_0^{\infty} e^{-rt} \tau C (1 - F) e^{-\lambda t} dt$$

$$= \frac{\tau C}{r + \lambda} (1 - \left(\frac{V}{V_B} \right)^{-y3})$$

where

$$y_3 = y(r + \lambda) = \frac{(g - .5\sigma^2) + [(g - .5\sigma^2)^2 + 2((r + \lambda)\sigma^2]^{0.5}}{\sigma^2}$$

Default costs (incurred by default from diffusion) are given by

[1] Alternatively, equity holders could also discount at a rate including a risk premium. Our rate r could be viewed as including such an equity premium (although r would exceed Treasury rates, assuming equity is less liquid than Treasuries). In this case, h would be the incremental liquidity premium for debt relative to equity, which could in fact be negative.

$$DC = \alpha V_B \int_0^\infty e^{-(r+\lambda)t} f\, dt$$
$$= \alpha V_B \left(\frac{V}{V_B}\right)^{-y3}$$

OPTIMAL DEFAULT LEVEL V_B:

Equity value for arbitrary V is given by

$$E = V + TS - DC - D(0)$$
$$= \ V \ + \frac{\tau C}{r+\lambda}(1-\left(\frac{V}{V_B}\right)^{-y3}) - \alpha V_B \left(\frac{V}{V_B}\right)^{-y3}$$
$$-\frac{C+mP}{r+m+\lambda}(1-\left(\frac{V}{V_B}\right)^{-y4}) - \ (1-\alpha)V_B\left(\frac{V}{V_B}\right)^{-y4} - \frac{\lambda(1-k)V}{r+m+\lambda-\mu}(1-\left(\frac{V}{V_B}\right)^{-y5})$$

NOTE we need to assume h = 0 here, since equity value does not discount bond payments as bondholders do.

Default occurs at the optimal (smooth pasting) level of V where $dE(V)/dV\,|_{\,V=VB}\ = \ 0$, implying

$$V_B = \frac{\dfrac{(C+mP)y_4}{(r+m+\lambda)} - \dfrac{TCy_3}{(r+\lambda)}}{1+(1-\alpha)y_4 + \alpha\, y_3 - \dfrac{\lambda(1-k)}{(r+\lambda+m-\mu)}y_5}$$

where

$$y_4 = y(r+m+\lambda) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2 + 2((r+m+\lambda)\sigma^2]^{0.5}}{\sigma^2}$$
$$y_5 = y(r+m+\lambda-\mu) = \frac{(g-.5\sigma^2)+[(g-.5\sigma^2)^2 + 2((r+m+\lambda-g)\sigma^2]^{0.5}}{\sigma^2}$$