

# Bank stability and market discipline: The effect of contingent capital on risk taking and default probability

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## Abstract

This paper proposes a tractable form of contingent capital (“coco”), a debt security that automatically converts into equity if asset value falls below a pre-determined threshold. We provide a closed form solution for the coco price and evaluate how the introduction of coco into a bank’s capital structure reduces the probability of default. We show that appropriate coco design (choice of conversion threshold and conversion ratio) can virtually eliminate stockholders’ incentives to risk-shift. This effect is robust to times of crisis and for regulators with different bank closure policies. Introducing coco into the capital structure may thus be able to cancel out adverse effects of equity-based compensation.

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# 1 Introduction

Bank capital, primarily in the form of common equity, provides banks with a buffer to absorb losses and protect creditors. In the recent financial crisis there has been a shortfall in the amount and quality of capital held by banks. Financial institutions were not able to raise significant new capital in the market and had to rely instead on governments to provide capital. The extensive amount of implicit guarantees and outright infusion of taxpayer money into large financial institutions has come under much scrutiny.

One of the most prominent suggested solutions for the shortfall of capital in bad times is “contingent capital” (contingent convertible bonds, often referred to as “coco”). Coco is subordinated debt that automatically converts into equity when a certain stress-related trigger is breached and thus can absorb losses on a going concern basis. Such an automatic debt-to-equity swap or “bail-in” is potentially valuable since it is executed in times of distress: Private investors provide a boost to loss-absorbing capital in a pre-arranged fashion at a time when the cost of raising equity in the market is prohibitively high.

In this paper we propose a tractable structure for contingent capital: The convertible bond has a fixed payoff unless at any time before maturity the value of assets falls below a pre-specified conversion threshold. In such an event the debt contract is converted automatically into common equity according to a pre-determined conversion ratio. Our general proposal for coco enables us to consider the effects of introducing coco into a bank’s capital structure. We present a closed form solution for the coco price, something that is a necessary condition for the development of a viable market. We then use our pricing model to show that issuing coco reduces the bank’s default probability and that coco design has important effects on the risk taking motivation of equity holders.

Contingent capital represents a growing area of interest. Flannery (2002) first suggests the idea of contingent capital in the context of financial institutions, assuming a conversion trigger based on the bank’s equity price. However, the issuance of such a security may lead to an undesired equilibrium in which conversion may not occur even if the corporation is in a bad standing and vice versa (see Duffie, 2009; Pennacchi, Vermaelen, and Wolff, 2010; Sundaresan and Wang, 2010). In contrast, we assume conversion resulting from a drop in asset value below

a threshold, which makes the analysis of coco tractable and removes the possibility of multiple equilibria. Other recent studies that analyze different types of contingent capital and their features include Albul et al. (2010) and McDonald (2010).<sup>1</sup>

The market for coco has already started developing. In November 2009 Lloyds Bank issued coco, where bonds will convert into ordinary shares if the consolidated core tier one ratio of Lloyds falls below 5%. In March 2010 Rabobank issued contingent capital where there is a debt write-down when its regulatory capital ratio falls below 7%.<sup>2</sup> Regulators and policymakers have advocated using contingent capital as a preferred tool for implementing prudential bank regulations. Swiss regulators have required additional capital for the country's systemically important institutions (Credit Suisse and UBS), stating that it can be issued in the form of contingent capital. In the U.S., recent legislation calls for the issuance of contingent capital to be an additional standard potentially faced by systemically important institutions. The Basel Committee and other regulators are also considering the role of contingent capital instruments in regulatory capital requirements.

We begin by presenting a closed form solution for the coco price. Our pricing model uses standard option pricing as developed by Black, Scholes, and Merton (1973, 1974), and Black and Cox (1976). A major concern with the widespread introduction of coco, raised by regulators, investors, and analysts, has been the ability of participants in the market to price the new capital instrument. A continuous time pricing model with closed form solutions, which is easy for implementation, is therefore important since it equips market participants with a benchmark pricing tool. As part of the analysis, we also provide closed form solutions for all other bank liabilities.

Using our analytic framework we consider the effect of coco on bank default probability and measure the stabilizing effect of coco. High bank default probability, in particular during times of crisis, is a concern that has been at the forefront of the debate regarding the introduction of contingent capital. To provide a level playing field we compare a bank issuing coco to one

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<sup>1</sup>We discuss the related literature in more detail in Section 1.1. With the exception of Flannery (2002) these studies were circulated after the first version of this paper.

<sup>2</sup>In both cases the design of the contingent capital securities is similar to the structure which we propose. In particular, both Lloyds and Rabobank use conversion triggers based on asset value.

instead issuing subordinated debt, which does not convert into equity in times of distress. We show that for relatively low asset risk the probabilities of default under the two alternative capital structures are both close to zero. As asset risk increases there is a significantly higher default probability for banks with subordinated debt instead of coco. The effectiveness of the coco as a stabilizing tool is thus higher for intermediate and high levels of a bank's asset risk. The difference can be very large: for reasonable parameter values the default probability for subordinated debt is two to three times larger than if the bank issues coco instead. Furthermore, we next show that a bank with subordinated debt may choose higher levels of risk, further increasing its default probability relative to a bank with coco.

We show that whether or not coco is included in the capital structure as well as coco design can have important effects on risk-taking motivation. Stockholders may be able to change the risk profile of the bank's assets (risk shift) in order to maximize the value of their own holding (Jensen and Meckling, 1976; Galai and Masulis, 1976).<sup>3</sup> Regulators and policymakers are trying to understand the relationship between coco, financial stability, and the risk taking motivation of financial institution equity holders (Acharya, Cooley, Richardson, and Walter, 2009). If stockholders are motivated to continually increase or reduce asset risk, it may become either too high, one of the main reasons for the financial crisis, or too low, which may result in a credit freeze and a lack of lending by financial institutions.<sup>4</sup> Furthermore, incentives that are sensitive to asset risk may reduce managements' attention to maximizing overall bank asset value. We identify coco terms that lead stockholders to be neutral to the level of asset risk and thus show that coco can reduce risk-shifting incentives present for a bank that instead issues subordinated debt and equity.

The two main terms of the coco are the conversion ratio and the conversion threshold. The conversion ratio is the percentage of the ownership in the post-conversion financial institution

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<sup>3</sup>Unlike firms in non-financial, unregulated industries, the primary creditors in a bank, the depositors, do not have sufficient incentives to monitor the bank, because of the implicit or explicit guarantees that are provided to commercial banks by national governments (see Saunders, Strock, and Travlos).

<sup>4</sup>Bebchuk, Cohen, and Spamann (2010) and Cheng, Hong, and Scheinkman (2010) provide examples of excessive risk taking and link it to the recent financial crisis. However, there is also evidence that during the financial crisis many financial institutions shied away from risk thus contributing to the low levels of new loans (Maddaloni et al., 2009).

that coco holders receive in case of conversion; previous shareholders receive the remainder. The conversion ratio can vary from 0 to 1. A level of zero (“stock-friendly”) means that in the event of conversion coco holders receive nothing and equity holders are faced with a lower level of debt. In contrast, a conversion ratio of 1 (“coco-friendly”) means that previous equity holders receive nothing and coco holders become the post-conversion equity holders.<sup>5</sup>

The conversion ratio has an important effect on risk taking incentives. For low conversion ratios stockholders have a motivation to increase asset risk, while they are motivated to decrease asset risk for high conversion ratios. For intermediate conversion ratios the incentives for stockholders to change asset risk are small making them close to indifferent to the chosen level of asset risk. The intuition for this effect is as follows: Increasing the level of asset risk makes conversion more likely. Conversion results in debt reduction but also in dilution of current stockholders. If dilution is small (low conversion ratio) stockholders want to increase risk while they want to decrease risk if dilution is large (high conversion ratio). What we show is that for intermediate levels of the conversion ratio costs and benefits of higher asset volatility offset and incentives to risk-shift can be virtually eliminated. Thus the presence of coco in the capital structure may reduce equity holders’ incentives to increase risk and provide an important additional channel through which stabilization can be achieved.

The conversion threshold, the level of asset value at which the coco is converted into equity, has only a relatively minor effect on incentives; the conversion ratio remains the dominant factor. For high and low conversion ratios different threshold levels all result in incentives to either increase or decrease risk, though the strength of incentives varies with the conversion threshold. Similarly, for intermediate levels of the conversion ratio incentives remain small for different conversion thresholds.<sup>6</sup>

The effect of the conversion ratio is also robust to different levels of leverage. For interme-

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<sup>5</sup>Interestingly, the conversion ratio varies across recently issued coco instruments. For example, Rabobank issued a security with a conversion ratio of 0. Lloyds Bank and Credit Suisse have issued cocos which are converted into the underlying stock at a relatively low or “stock-friendly” conversion ratio, where coco holders receive only a small share of the post-conversion equity.

<sup>6</sup>Although the conversion ratio is the dominant determinant of incentives, the exact level at which incentives are minimized may depend on the conversion threshold. This means that coco design that intends to minimize risk-taking motivation needs to take into account both parameters.

diated levels of the conversion ratio there is little effect of leverage on risk-taking. This is an important feature of the coco since it means that even when a bank moves closer to financial distress, as we would expect to happen during times of crisis, the temptation to risk shift does not increase substantially. Furthermore, incentives also do not depend on a regulator seizing banks as soon as there are early signs of distress (a high seizing threshold) or later on (a low seizing threshold).

Finally, we compare a bank issuing coco to alternative capital structures. Equity holders in a bank with deposits and equity only are always motivated to increase asset risk. Therefore, even though holding asset risk and the ratio of deposits to assets constant results in the same default probability of a bank with coco and one with equity only (Admati, DeMarzo, Hellwig, and Pfleiderer, 2010), including coco into the capital structure has the advantage of minimizing the motivation to increase risk. A bank with subordinated debt has both a higher default probability at the same level of asset risk as well as clear incentives to increase risk. Adding coco to a bank's capital structure may thus have important advantages over both alternative capital structures.

A common belief is that the financial crisis was in substantial part the consequence of flawed executive compensation formulas that gave senior managers at major financial institutions perverse incentives to pursue short-term profits by accepting risk and high leverage (see Binder, 2010; Bebchuk and Spamann, 2010). Since adding appropriately designed coco into the capital structure can eliminate the incentive of stockholders to risk shift, we show that using equity based compensation does not need to lead to undesirable incentives so long as the bank issues coco. This implies that there may not be any need for additional changes to compensation. Given recent interest we also consider the effect on incentives of coco like compensation and find that it represents another way to achieve low risk taking motivation.<sup>7</sup> However, as discussed, conventional equity based compensation can have low incentives to risk shift also, so long as well designed coco is in the capital structure.

The rest of the paper is organized as follows. We next briefly survey the literature on

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<sup>7</sup>On December 3, 2010 Reuters reported that the investment banking arm of Barclays is leaning towards issuing contingent capital securities to employees as part of their 2010 deferred compensation.

contingent capital. Section 2 presents the basic assumptions for the valuation of corporate liabilities of a bank with coco in its capital structure and for a bank with subordinated debt. We provide closed-form solutions for the valuation of a bank's claims using option replication. Section 3 compares quantitatively default probabilities for the cases of coco and subordinated debt. Section 4 analyzes the risk taking motivation of stockholders for different terms of the coco (conversion ratio and conversion threshold), leverage, regulatory seizing policy, and compares the effects on risk-taking of a bank with coco and one with subordinated debt or equity only. Section 5 concludes.

## 1.1 Recent literature on contingent capital

Before proceeding to the model section, we provide a more detailed overview of the literature on contingent capital. Flannery (2002, 2009), who introduces the idea of contingent capital for large financial institutions, considers a subordinated debt instrument that pays a fixed payment to its holders but converts automatically into equity when a certain stress-related trigger is breached. Importantly, conversion occurs at the market price of the bank's stock on the day of the conversion rather than at a predetermined price, a feature that may result in undesirable conversion scenarios and multiple equilibria.<sup>8</sup> In an effort to address this issue Sundaresan and Wang (2010) propose a contingent capital bond that pays a floating coupon. The number of shares issued at conversion multiplied by the trigger price equals the contingent capital bond's par value. Pennacchi, Vermaelen, and Wolff (2010) propose a coco with buyback options for equity holders and Duffie (2009) proposes a mandatory offering of new equity by banks facing financial distress.

Alternative proposals for the design of contingent capital have lead to work on valuation. Pennacchi (2010) compares several cases by simulating asset value in a jump diffusion model. Albul, Jaffee, and Tchisty (2010) obtain closed-form pricing expressions under the assump-

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<sup>8</sup>The market price of each corporate liability is a function of the ex-post possible payoffs that are contingent on the value of the underlying asset of the corporation. The fact that the payoff of a claim can be replaced by another payoff at some future time if its market value has touched some predetermined level may lead to an equilibrium in which conversion may not occur even if the corporation is in a bad standing and vice versa.

tion that all debt has infinite maturity. Glasserman and Nouri (2010) suggest a conversion mechanism process that converts just enough debt to meet the capital requirement each time a bank’s capital ratio reaches the minimum threshold.

Studies have also proposed other methods for capital provision in bad times. Kashyap, Rajan, and Stein (2008) suggest that an insurer could receive a premium for agreeing to provide an amount of capital to the bank in case of a systemic crisis. The policy would pay out upon the occurrence of a ‘banking systemic event,’ for which the trigger would be some measure of aggregate write-offs of major financial institutions over a year-long period. Caballero and Kurlat (2009) suggest that the central bank could issue tradable insurance credits, which would allow holders to attach a central bank guarantee to assets on their balance sheet. A threshold level or trigger for systemic panic would be determined by the central bank. Hart and Zingales (2010) suggest that financial institutions would issue capital when their credit default swap (CDS) spread rises above a certain level.

In the next section we present a tractable proposal of coco with finite maturity, a pre-determined conversion ratio, and a conversion trigger that is related to asset value. Our setup allows us to present in a straightforward way the effects of introducing coco into a bank’s capital structure, while leaving open the possibility of refinements, as discussed above. Furthermore, using the analytical solution allows us to examine the effect of coco design on default probability and risk-taking incentives of executive compensation.

## 2 A model for pricing bank capital

### 2.1 Bank capital structure

We consider a hypothetical bank with asset value denoted by  $V$ .<sup>9</sup> To finance its assets, the bank issues three types of claims as part of its capital structure: a single zero-coupon deposit, either contingent convertible (coco) or subordinated debt, and a residual equity claim. All the claims mature at time  $T$ , unless there is a security specific event previous to this time, details

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<sup>9</sup>To keep the notation as simple as possible, all variables without subscripts are present values.

of which we are about to discuss. We define the payoff of each of the claims in the capital structure and find their value by using an option pricing method with closed form solutions.

### 2.1.1 Contingent convertible bonds (coco)

We first consider the capital structure of a bank that issues contingent convertible bonds (coco) so that the capital structure includes deposits, coco, and equity.

The most senior claims in the capital structure are secured deposits with face value  $F^D$ . We assume that the government regulator will either seize the bank or force liquidation in the event of bank insolvency. Both of these events represent default which means that default occurs either at debt maturity,  $T$ , if the value of assets lies below the face value of the deposits,  $F^D$ , or if at any time  $t$  previous to maturity  $T$ , that is  $0 < t < T$ , the value of bank assets touches the default threshold  $K^D$ . We refer to such an event as early default  $\tau_D < T$  where the event is defined as:

$$\tau_D = \inf \{t > 0 | V_t \leq K^D\}. \quad (1)$$

We assume that the default threshold  $K^D$  is equal to  $F^D(1 - \gamma)$ , where  $0 \leq \gamma \leq 1$ , which means that the threshold is located below the face value of deposits. This assumption follows Black and Cox (1976) and captures the fact that the regulator has limited ability to seize the bank at the moment it becomes insolvent either because of imperfect information due to discrete audit frequency (Duffie and Lando, 2001), or simply choosing a policy where banks are not immediately seized (Cetin, Jarrow, Protter, and Yildirim, 2004). We can think of the size of  $\gamma$  as being related to the ability and willingness of the regulator to closely monitor and enforce bank solvency.<sup>10</sup> Close monitoring may be costly and may be difficult in the case of complicated bank balance sheets, for example those holding complex or illiquid assets such as tranches of collateralized debt obligations (CDO) of pools of mortgage backed securities (MBS). Figure 1 presents sample paths of asset value; default occurs either before maturity

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<sup>10</sup>The depletion of the FDIC's Deposit Insurance Fund (DIF) during the financial crisis is likely a consequence of the difficulty faced by the regulator in shutting down banks exactly at the moment when asset value reaches the value of liabilities.

(1.C) or at maturity (1.B).

The second debt security in the capital structure is the coco with face value  $F^C$  and market value  $C$ . The coco pays  $F^C$  at maturity,  $T$ , if there is no conversion event until debt maturity (Figure 1.A). In this case the stockholder is the residual claimant and receives  $(V_T - F^C - F^D)$ . In the event that at any time previous to maturity the value of the financial institution drops below the conversion threshold  $K^C$ , where  $V < K^C$ , the coco is converted into equity (Figure 1.B). In this event the coco holder receives a share  $\alpha$  ( $0 \leq \alpha \leq 1$ ) of the equity value and the previous shareholders receive the remaining  $(1 - \alpha)$  of equity. At maturity the payoff to the coco holder is therefore equal to  $\alpha(V_T - F^D)$  and the original stockholder receives  $(1 - \alpha)(V_T - F^D)$  unless default occurs, in which case both claimholders receive nothing (see Figure 1.C). The conversion threshold is defined as:

$$K^C = (1 + \beta)(F^C + F^D) \quad (2)$$

where  $\beta$  ( $\beta \geq 0$ ) measures the distance between the conversion threshold and the bank's book value of debt. Intuitively, a larger  $\beta$  implies conversion at times of lower leverage and lower probability of financial distress. The time of conversion is defined as

$$\tau_C = \inf \{t > 0 | V_t \leq K^C\}. \quad (3)$$

We note that  $\beta \geq 0$  implies that the event of conversion will always occur before the event of default, i.e.  $\tau_C \leq \tau_D$ . The payoff at maturity to the coco holders and to equity holders can be written as:

$$\begin{aligned} C_T &= F^C 1_{\{\tau_C > T\}} + \alpha \max \{(V_T - F^D), 0\} 1_{\{\tau_C < T < \tau_D\}} \\ E_T &= (V_T - F^C - F^D) 1_{\{\tau_C > T\}} + (1 - \alpha) \max \{(V_T - F^D), 0\} 1_{\{\tau_C < T < \tau_D\}} \end{aligned} \quad (4)$$

where  $1_\psi$  is an indicator function of the event  $\psi$ . Table 1 summarizes the payoffs to claimholders in different states of the world.

### 2.1.2 Subordinated debt

In this section we define a capital structure that includes subordinated debt instead of coco. It includes deposits, subordinated debt, and equity. Defining this capital structure allows us to compare the effects on default probability and risk taking motivation of the stockholders for cases where the capital structure contains coco instead of subordinated debt.

The subordinated debt has face value  $F^B$  and market value  $B$ . As in the previous case of coco in the capital structure, default occurs if the value of the bank's assets falls below the threshold  $K^D$  at any time before maturity  $T$ . If this event occurs, the subordinated debt holder receives nothing. Default can also occur if at maturity the asset value lies below the face value of total debt ( $V_T < F^D + F^B$ ). There is no mandatory conversion or increase in capital as a result of low asset value; instead, the subordinated debt holders have the ability to take legal action if at maturity the value of the financial institution is smaller than the sum of the face values of deposits and subordinated debt. This means that default can occur either if before maturity asset value falls below the threshold  $K^D$  or if at maturity asset value is below the total face value of debt instruments ( $F^D + F^B$ ). Therefore, as long as  $F^B$  is larger than zero, the default probability in the case of subordinated debt is, *ceteris paribus*, larger than in the case of coco.

The payoff to subordinated debt holders is equal to zero if the bank is taken over by the regulator before maturity ( $\tau_D < T$ ) or if at maturity the residual value of assets is below the value of deposits and there is no surplus to be paid to the subordinated debt holders ( $V_T < F^D$ ). If at maturity asset value lies between the total face value of debt and the face value of the deposits ( $F^D < V_T < F^D + F^B$ ), subordinated debt holders receive the residual assets of the financial institution after the deposit holders are paid in full. Otherwise, if there is no default, subordinated debt is paid in full ( $F^B$ ). The payoff can be summarized as:

$$B_T = (\max \{V_T - F^D, 0\} - \max \{V_T - F^D - F^B, 0\}) 1_{\{\tau_D > T\}}. \quad (5)$$

The equity holders' payoff at maturity, in the case that default has not occurred before that

time ( $\tau_D > T$ ), is equal to the residual value of assets:

$$E_T = \max \{V_T - F^D - F^B, 0\} 1_{\{\tau_D > T\}}. \quad (6)$$

If default occurs previous to maturity equity holders receive nothing. Table 2 reports the payoffs for the different states of the world.

## 2.2 Valuation of bank liabilities with coco or with subordinated debt

### 2.2.1 Valuation of claims when capital structure includes coco

We price each claim by replicating its payoff using a combination of different barrier options that all have closed form solutions. The appendix describes the payoffs and the pricing equations for all of the options that are used to replicate the different payoff functions. To the best of our knowledge we are the first to provide such analytic solutions for the pricing of the bank's claims where coco is part of the capital structure.

We first consider the case of a capital structure with coco. The value of the deposits derives from future cash flows that may be received in the events of early default and no default until debt maturity (two mutually exclusive events).<sup>11</sup> Intuitively, the payoff to deposit holders at maturity is equal to the payoff of a zero coupon bond and a European put option, as pointed out by the seminal paper of Merton (1974). The difference in our case is that since early default can occur both options are path dependent.

The value in the case of early default is equal to  $(1 - \gamma) F^D$  units of a *down-and-in* digital barrier option, defined as  $DB^{din}(K^D)$ . This option pays \$1 at the first time before maturity,  $T$ , that asset value,  $V$ , hits the default threshold,  $K^D$ , and nothing otherwise. The value in the case of no early default is replicated by using two *down-and-out* barrier options. If default does not occur at maturity, the payoff to the deposit holder is equal to the face value of the

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<sup>11</sup>In calculating the market value of the deposits we abstract here from the potential payments by the deposit insurer in case of default. We can calculate the expected cost of insurance to the regulator in our framework but we omit this for the sake of brevity.

deposits  $F^D$ , and therefore the payoff can be replicated by  $F^D$  units of a long position in a *down-and-out* digital barrier option, defined as  $DB^{dout}(K^D)$ . If default occurs at maturity, the deposit holder receives the residual assets of the financial institution and therefore the loss can be replicated by using a *down-and-out* put option  $PB^{dout}(K^D, F^D)$  with a strike price equal to the face value of the deposits  $F^D$  and a barrier level equal to the default threshold  $K^D$ . Both options have *down-and-out* features since early default can occur. Assuming a constant interest rate  $r$  the value of deposits can be expressed as:

$$\begin{aligned}
D &= E^Q \left[ \begin{aligned} &e^{-rT} \left( F^D \mathbf{1}_{\{V_T > F^D\}} + V_T \mathbf{1}_{\{F^D > V_T\}} \right) \mathbf{1}_{\{\tau^D > T\}} \\ &+ e^{-r\tau^D} F^D (1 - \gamma) \mathbf{1}_{\{\tau^D < T\}} \end{aligned} \right] \quad (7) \\
D &= F^D \left( DB^{dout}(K^D) + DB^{din}(K^D) \right) - PB^{dout}(K^D, F^D)
\end{aligned}$$

where  $E^Q[\cdot]$  denotes the expectation under the risk-neutral measure  $Q$ .

To price the coco we need to take into account three mutually exclusive events that affect its payoff. First, the value from the case of no conversion before maturity  $T$ , which is equal to  $F^C$  units of a *down-and-out* digital barrier option with a strike price of the total face value of the deposits and the coco ( $F^C + F^D$ ) and a barrier at the level of  $K^C$ . The value contributed to the second possible event (conversion and no default) and to the third possible event (default where the coco holder receives nothing) are replicated by a spread position in two *down-and-in* barrier call options with the same strike, but with different thresholds. The coco holder has a long position in  $\alpha$  units of a *down-and-in* call option on the value of assets  $CB^{din}(K^C, F^D)$ , with a strike price equal to the face value of deposits  $F^D$  and a threshold level of  $K^C$ . However, if asset value falls further and touches the default threshold the equity becomes worthless and the coco holder receives nothing. To capture this payoff structure we add a similar but short position in a *down-and-in* call option  $CB^{din}(K^D, F^D)$  with the same terms except having a barrier at the default threshold. This means that if asset value touches  $K^D$  the two option payoffs cancel each other. We can write the coco value as:

$$\begin{aligned}
C &= E^Q \left[ e^{-rT} \left( F^C 1_{\{\tau^C > T\}} + \alpha (V_T - F^D) 1_{\{\tau^C < T < \tau^D, V_T > F^D\}} \right) \right] \\
C &= F^C DB^{dout}(K^C) + \alpha \left( CB^{din}(K^C, F^D) - CB^{din}(K^D, F^D) \right).
\end{aligned} \tag{8}$$

The stock price is affected by the outcome of the same three mutually exclusive events as in the case of the coco: no early default and no conversion, no early default and conversion, and early default. The value of the stock can be expressed as follows:

$$\begin{aligned}
E &= E^Q \left[ e^{-rT} \left( \begin{aligned} &(V_T - F^C - F^D) 1_{\{\tau^C > T\}} \\ &+ (1 - \alpha) (V_T - F^D) 1_{\{\tau^C < T < \tau^D, V_T > F^D\}} \end{aligned} \right) \right] \\
E &= CB^{dout}(K^C, F^D + F^C) + (1 - \alpha) \left( CB^{din}(K^C, F^D) - CB^{din}(K^D, F^D) \right)
\end{aligned} \tag{9}$$

where  $CB^{dout}(K^C, F^D + F^C)$  is a *down-and-out* call option. This option is a European call option on the underlying bank asset value that pays off at maturity only if the asset value does not touch the conversion threshold  $K^C$  before maturity. In case of conversion, equity value is divided between the original stockholders, who receive  $(1 - \alpha)$ , and the coco holders, who receive a fraction  $\alpha$ . Therefore, as in the case of the coco we use a position in a *down-and-in* call option to take into account the stream of cash flows at the event of early conversion; the number of option units is equal to  $(1 - \alpha)$ .

### 2.2.2 Capital structure with subordinated debt

Figure 2 presents sample paths for the different scenarios in the case of a capital structure with subordinated debt. Since the event of conversion does not affect the payoff of the deposit holders there is no difference between the pricing of deposits in a bank with coco as part of its capital structure and an all else identical bank that does not have the conversion option.

The potential future cash flows to the subordinated debt holder depend on two mutually exclusive events: no default until maturity (Figure 2.A) and early default (Figure 2.B). As in the case of coco, at the event of early default, when the value of assets is equal to the default threshold  $K^D$ , the subordinated debt holder receives nothing. In the case that default does

not occur until debt maturity the payoff to the subordinated debt holders can be replicated by two options – a long position in a call option with a strike price equal to the face value of the deposits and a short position in a similar call option but with a higher strike which is equal to the total face value of debt instruments,  $(F^D + F^B)$ . If the value of assets touches the default threshold before maturity the subordinated debt holders receive nothing, which means that both options are *down-and-out* barrier options with the same barrier of  $K^D$ . The replicating options are identical to the Black and Cox (1976) framework for pricing subordinated debt and can be expressed as follows:

$$\begin{aligned}
B &= e^{-rT} E_t^Q \left[ \begin{array}{l} F^B 1_{\{\tau^D > T, V_T \geq F^B + F^D\}} \\ + \max(V_T - F^D, 0) \end{array} \right] 1_{\{\tau^D > T, (F^B + F^D) > V_T \geq F^D\}} \quad (10) \\
B &= CB^{dout}(K^D, F^D) - CB^{dout}(K^D, F^D + F^B).
\end{aligned}$$

The value of equity can be replicated by one *down-and-out* call option. While in the case of coco a similar option is used for replicating the payoff at the event of no early conversion, in the case of the subordinated debt the barrier is equal to the default threshold and not to the conversion threshold. The present value of the payoff can be written as:

$$\begin{aligned}
E &= e^{-rT} E_t^Q [\max(V_T - F^D - F^B, 0)] 1_{\{\tau^D > T\}} \quad (11) \\
E &= CB^{dout}(K^D, F^B + F^D).
\end{aligned}$$

### 2.3 Firm value process

The pricing of the different liabilities which compose the capital structure of a bank depends on the assumed stochastic process of the underlying asset. We assume that the dynamics of the bank's assets follow a simple Geometric Brownian Motion. This assumption is consistent with most existing structural models (Merton, 1974; Black and Cox, 1976; Brennan and Schwartz, 1978; Longstaff and Schwartz, 1995; Leland, 1994; Ericsson and Reneby, 1998) and with models for pricing a bank's claims (Albul, Jaffee, and Tchisty, 2010; Glasserman and Nouri, 2010;

and Sundaresan and Wang, 2010).<sup>12</sup> The assumption also has the advantage of having closed form solutions, which allows us to analyze in a straightforward way the sensitivity of the bank's stock price to changes in the value of asset risk. The detailed closed form solutions for the barrier options we use are developed by Merton (1973) and Rubinstein and Reiner (1991).

### 3 Default probability: Coco vs. subordinated debt

#### 3.1 The risk neutral probability of default

Policymakers and regulators are interested in monitoring the default probability of a financial institution due to the potentially harmful effect of default on the real economy. The default probability of a bank with subordinated debt as part of its capital structure is always equal or greater than the default probability of a bank with coco, since the subordinated debt holders can force default at maturity if their claims are not fully met. In contrast, the coco would always be converted into stock before the event of default and the regulator is the only entity that can force default at maturity.

The probability of default of a bank with subordinated debt is composed of two mutually exclusive events. Either before maturity asset value falls below the default threshold  $K^D$ , or at maturity the value of assets is below the total face value of debt ( $F^D + F^B$ ). Thus, the probability of default can be expressed as follows:

$$PD(B) = \Pr(\tau^D < T) + \Pr(V_T < (F^D + F^B) | \tau^D > T). \quad (12)$$

The probability of default for a bank with coco is also composed of two mutually exclusive events. However, if default has not occurred until debt maturity, only the regulator can enforce default when the value of the bank's assets lies below the face value of the deposits. The probability of default of a bank with a capital structure that includes coco can be written as:

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<sup>12</sup>Pennacchi (2010) instead considers a jump-diffusion formulation for the firm's underlying asset.

$$PD(C) = \Pr(\tau^D < T) + \Pr(V_T < F^D | \tau^D > T). \quad (13)$$

As long as there is subordinated debt in the capital structure ( $F^B$  is positive), the default probability in the case of subordinated debt is, ceteris paribus, larger than in the case of coco. The difference is equal to

$$PD(B) - PD(C) = \Pr((F^D + F^B) > V_T > F^D | \tau^D > T). \quad (14)$$

### 3.2 Calibrating the model

In order to understand the effect of coco in the capital structure we calibrate the model. We choose base case parameter values and then perform sensitivity analysis around the base case values. This way we can demonstrate the effects of each contract on the default probability of a bank. In the next section we then use the calibration to analyze the effect of capital structure and coco design on risk taking.

We assume that the bank has a capital structure composed of deposits with face value,  $F^D$ , which is normalized to 100, coco, with a face value,  $F^C$ , which is equal to 3% of the face value of the deposits, and time to maturity for both instruments of one year,  $T = 1$ . The value of the bank assets is equal to 108.02, which results from assuming that the quasi leverage ratio, which is defined as  $LR = \frac{(F^D + F^C)e^{-rT}}{V_t}$  is equal to  $LR = 0.93$ . The risk-free rate is equal to 2.5%, and the bank asset risk,  $\sigma$ , is equal to 5%. The coco conversion ratio  $\alpha$  is equal to 0.5 and the conversion threshold  $K^C$  is located 1% above the total face value of the two debt instruments so that conversion occurs before debt maturity at the first time that the value of the bank's assets is equal to  $(1 + \beta)$  times the total face value of the debt,  $(F^D + F^C)$ . The seizing policy of the regulator is to close the bank the first time that the value of assets is 3% below the face value of the deposits ( $\gamma = 3\%$ ). Table 3 summarizes and, where available, provides sources justifying the parameters used in the base case of our analysis. For the base case parameter values the 1-year risk neutral default probability for a bank with coco is 2.3% and for a bank with subordinated debt is 7.7%. The difference between the two default probabilities is 5.4%.

The lower default probability of a bank with coco is a reflection of the stabilizing effect that coco has on the capital structure. In the case of a deterioration in asset value instead of default there is an automatic provision of capital from the coco.<sup>13</sup>

### 3.3 Sensitivity to parameter values

While it is clear that, *ceteris paribus*, the probability of default of a bank with coco is equal to or lower than that of a bank with subordinated debt, the difference between the probabilities is affected by the leverage ratio and the level of asset risk. Table 4 performs sensitivity analysis with respect to asset volatility (1% to 9%) and leverage (0.91 to 0.95). For low levels of asset volatility the probabilities of default under the two alternative capital structures are almost identical. In both cases default probabilities are close to zero when asset risk is equal to 1% for any leverage ratios between 0.91 and 0.95. If asset risk is low, a large deterioration in asset value is unlikely which means that the effect of coco is small. In contrast, differences in default probabilities are much larger for higher levels of asset volatility. When asset risk is equal to 5%, the difference in default probability is equal to 2.4%, 5.4%, and 9.8% for leverage ratios of 0.91, 0.93, and 0.95 respectively. For intermediate levels of asset volatility including coco can thus significantly reduce the default probability. For high levels of volatility and leverage default probabilities are high in both cases, though they remain lower if coco is included. Introducing coco into the capital structure therefore may have important stabilizing effects on banks, which are pronounced in particular for intermediated values of asset risk.

## 4 The effect of coco on risk taking motivation

In addition to affecting the default probability introducing coco into a bank's capital structure can affect the risk-taking motivation of stockholders. While in the previous section we assumed

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<sup>13</sup>A couple of observations are useful when interpreting the base case default probabilities. First, we report risk neutral probabilities, which are larger than real default probabilities as long as there is a positive risk premium. Second, we are interested primarily in the relative magnitudes of the default probabilities which show that the default probability in the case of coco is significantly smaller than in the case of subordinated debt.

that asset risk is exogenous, in this section we assume that the bank’s management can determine the level of asset risk. We analyze the risk taking motivation under different terms of the coco contract, leverage ratios, and regulatory seizing policies, different capital structures (subordinated debt instead of coco, equity and deposits only), and assuming a pay package that consists of compensation in the form of coco. Our contribution in this section is related to Sundaresan and Wang (2010), Pennacchi (2010), and Albul et al. (2010) who suggest that a particular choice of conversion ratio may have implications for risk-shifting.

## 4.1 Risk taking with equity based compensation

### 4.1.1 Executive choice and limits on asset risk

We assume that the dominant component of the executive pay package that is sensitive to asset risk is equity-based compensation and that the executive chooses the level of asset risk that maximizes the value of the stock. In cases where the value of the stock is monotonically upward (or downward) sloping with respect to asset risk, the executive chooses the maximum (or minimum) possible level of risk to maximize the value of the compensation. We note that if all claimholders know what level of asset volatility bank management will choose there will be no scope for wealth transfers from debt to equity. Instead, we analyze here the incentives that management has to either increase or decrease risk given the existing contract.<sup>14</sup>

We restrict the level of asset risk of the financial institution with an upper and a lower boundary denoted by  $\sigma^L$  and  $\sigma^H$ . These levels are determined either by a regulator’s effort and ability to restrict the riskiness of a financial institution’s risk or by a limited set of technologies and projects in a specific time (similar to John, Saunders, and Senbet, 2000). The average asset risk of banks found in a large sample studied by Mehran and Rosenberg (2008) is equal to 5.3%, where the size of one standard deviation from this average is equal to 2.2%. Therefore, we set the upper bound for asset risk in our numerical analysis to 9% and the lower bound to

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<sup>14</sup>In theory it is possible to formulate a pricing model that takes into account that equity holders will maximize equity value by choosing asset volatility. We do not consider such a setup but instead analyze ex-post risk-shifting by management. This approach, which allows for transfers of wealth from debt to equity, is standard in the literature.

1%.

#### 4.1.2 Risk taking motivation and the terms of the coco

The conversion terms of the coco have important effects on the risk taking motivation of the stockholders, since the potential conversion event introduces additional complexity compared to capital structures with subordinated debt or with stock and deposits only. As the level of asset risk increases, the probability of touching the conversion threshold increases. This event has two opposing effects on equity value:

1. Equity value decreases, since in case of conversion stockholders lose some of their position to coco holders.
2. Equity value increases, since leverage decreases in case of conversion.

Figures 3 and 4 present the effect of the coco terms on the risk taking motivation of the stockholder. All else equal, the value of the stock decreases as the conversion ratio increases. However, the stockholder's choice of asset risk strongly depends on the conversion ratio. For a relatively low ("stock-friendly") conversion ratio the value of the stock increases with asset risk. Intuitively, an increase in asset risk results in a larger increase in equity value due to the potential decrease in leverage at conversion than the loss of equity value due to stock dilution at conversion. The reverse relationship may exist for a relatively high ("coco-friendly") conversion ratio. In Figure 3, when the conversion ratio is relatively low ( $\alpha = 0, 0.25$ ), the value of the stock increases with asset risk and when the ratio is relatively high ( $\alpha = 0.75, 1$ ), the value decreases with asset risk. Importantly the value of the stock may be almost insensitive to asset risk for some intermediate levels of the conversion ratio. For example, in Figure 3, the value of the stock is close to constant with respect to asset risk when the conversion ratio is equal to 50%.

These results have important effects on the optimal behavior of an equity holder who tries to maximize the value of stock by controlling asset risk. For a relatively low (high) conversion ratio the equity holder will choose the maximum (minimum) possible level of asset risk. For some intermediate level of the conversion ratio the stockholder may be indifferent to asset

risk, since the components that affect the value of the equity, as presented above, offset each other. From the point of view of a regulator, intermediate levels of the conversion ratio may be preferable. Strong risk-shifting incentives may lead to either very high or very low levels of asset risk, both of which are likely undesirable since they may lead either to excessive risk or sub-optimally low levels of risky lending. A lack of strong risk shifting incentives may therefore be optimal and, as we show, also achievable when introducing coco into the capital structure.

The second important term of the coco is the conversion threshold, i.e., the threshold at which conversion takes place. We find that the conversion threshold has little impact on the relationship between incentives and the conversion ratio, which is the dominant factor affecting risk taking motivation. Figure 4 shows the sensitivity of the stock price for three levels of  $\beta$  (0.5%, 1.5%, 2.5%), the percentage difference between asset value and the conversion threshold. We report equity value as a function of asset risk for three levels of the conversion ratio ( $\alpha = 0.1, 0.5, 0.9$ ), each time considering the effect of varying the conversion threshold. Figure 4.A shows that when the conversion ratio is relatively low (equal to 0.1, “stock friendly”) stockholders will try to choose the maximum level of asset risk for any level of the conversion threshold. Figure 4.C considers a conversion ratio that is relatively high (equal to 0.9, “coco-friendly”). In this case the relationship between the stock value and asset risk is downward sloping for all three conversion thresholds; thus stockholders prefer the minimum level of asset risk. For intermediate levels of the conversion ratio, risk-taking incentives remain small for the different conversion ratios. Figure 4.B presents the sensitivity of the stock price to asset risk for a conversion ratio of 50%, for which the effect of asset risk on equity value is small for all three levels of the conversion threshold. Specifically, a change of asset volatility from 1% to 9% causes a change in the stock price of less than 0.5% of asset value in all cases. Since there are likely costs associated with a change in the composition of a bank’s loan portfolio the stockholders may therefore have only a small incentive to shift asset risk.

Table 5 summarizes the asset risk choice of the stockholders for different terms. If stockholders have strong incentives to choose high or low levels of risk we report ‘Maximum’ or ‘Minimum’. We classify incentives as ‘Indifferent’ if the difference between stock value for high and low levels of risk is smaller than 0.5% of assets, since costs associated with risk-shifting

will likely make changes in asset risk not worthwhile. The results in the table emphasize the importance of the appropriate design of the coco terms. The stockholders are indifferent with respect to the level of asset risk when the conversion ratio is equal to 50% and the conversion threshold lies between  $\beta = 0\%$  and  $\beta = 2\%$ . For high levels of the threshold ( $\beta = 2\%$ ) the conversion ratio for which stockholders are indifferent is equal to 40% or 50%, while for low levels of the conversion threshold ( $\beta = 0\%$ ) incentives are low for levels of the conversion ratio between 50% and 80%. Thus, for all conversion thresholds, levels of indifference exist for an intermediate level of the conversion ratio, though the range of conversion ratios for which incentives are small depends on the conversion threshold.

### 4.1.3 Risk taking motivation and leverage

It is important to understand whether or not the effect of coco on incentives changes across different environments (e.g. good times: low leverage ratio, and bad times: high leverage ratio). We find that the effect of issuing coco on risk-taking is stable across different levels of leverage.

We consider three levels of leverage ( $LR = 0.91, 0.93, 0.95$ ) and find that the dominant factor that determines incentives is the conversion ratio, similar to the effect on risk taking motivation of varying the conversion threshold. Figures 5.A and 5.C show that for all levels of leverage conversion ratios that are relatively high ( $\alpha = 0.9$ ) or relatively low ( $\alpha = 0.1$ ) result in relationships between asset risk and stock value that are upward and downward sloping respectively. As leverage increases the value of the stock becomes more sensitive to asset risk but the direction of the incentives remains the same. In Figure 5.B the conversion ratio is set equal to 0.5%. For this conversion ratio the sensitivity of equity value to asset risk is small for all three levels of leverage. Thus for an intermediate level of the conversion ratio the value of the stock is insensitive to asset risk for different levels of leverage.

### 4.1.4 Risk taking motivation and regulatory policy

There may be differences in regulatory seizing policies and monitoring abilities. Our setup allows us to compare incentives for different levels of asset value at which the regulator seizes

the bank. In our model the level at which the bank is seized is measured by the percentage difference between the par value of the deposits and the value of the bank's assets (the parameter  $\gamma$ ). Figure 6 presents the value of stock versus asset risk for different regulatory seizing policies ( $\gamma = 1\%, 3\%, 5\%$ ). For all three levels the stock value is stable across different levels of asset risk. Specifically, moving from asset risk of 1% to 9% leads to only very small changes in equity value, which lie between 0.5% in asset value and which we therefore classify as not large enough to induce equity holders to change asset volatility. For high and low conversion ratios incentives are unaffected by different regulatory seizing policies, similar to the results when varying the conversion threshold and leverage. We omit graphical representations of the effects to save space. Regulators can be differentiated according to their ability and will to seize a bank in a financial distress. Our results show that the terms of the coco which lead to stable incentives that minimize stockholder risk shifting motivation do not need to be adjusted to different seizing policies.

To summarize, across different conversion thresholds, leverage ratios, and seizing policies, the primary determinant of incentives is the conversion ratio. The intuition is that these parameters do not affect the distribution of value in the event of conversion. Therefore, while they affect the overall likelihood of conversion taking place, they do not affect whether or not making conversion more likely is beneficial for stockholders.

## 4.2 Risk taking motivation under alternative capital structures

We next consider the risk taking motivation of stockholders in banks that are assumed identical except for having a capital structure that does not include coco. First, we consider a bank that issues subordinated debt instead of coco; second, a bank with deposits and equity only. Figure 7 graphs equity value against asset risk for both cases while varying the leverage ratio ( $LR = 0.91, 0.93, 0.95$ ).

In contrast to stockholders in a bank with coco, stockholders in a bank with subordinated debt are motivated to increase asset risk for any leverage ratio. Moreover, the sensitivity of the stock price to asset risk increases with leverage (see Figure 7.A), which means that as leverage increases the stockholders are more motivated to risk shift. We note that the sensitivity of the

stock price in a capital structure with coco is similar to that of a structure with subordinated debt when the coco's conversion ratio is relatively low (see Figure 5.A). However, as discussed, while the motivation of the stockholder in a structure with subordinated debt is always to increase asset risk, an appropriate design of the coco terms can eliminate the risk shifting motivation (see Figure 5.B).

We also analyze the risk taking motivation of stockholders in a bank with a capital structure which includes deposits and equity only (Figure 7.B). Such a capital structure is a special case of coco with conversion ratio and face value of zero. This is an important special case to consider. While the typical capital structure of large financial institutions is composed of deposits, stocks and subordinated debt, one criticism raised against coco is the claim that a capital structure which includes only deposits and stock has the same probability of default as a structure which includes coco, and that thus there is no advantage in introducing coco into a bank's capital structure (Admati et al., 2010). While for the same level of risk an equity only and coco bank have the same probability of default,<sup>15</sup> we show that risk-taking motivation also plays a role. In a capital structure with deposits and equity only, the value of the stock increases with asset risk for any level of leverage and thus the stockholders are motivated to increase asset risk to the highest possible level. As in the case of subordinated debt, risk shifting incentives are higher for higher levels of leverage (Figure 7.B). If changes in volatility are possible, the default probability of the bank with equity and deposits will therefore be higher compared to a bank with a well designed coco that has no strong motivation for risk shifting.

To summarize, incentives to risk shift are present both if the bank's capital structure includes subordinated debt or equity only. Both cases then may be inferior to a capital structure that includes a well designed coco.

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<sup>15</sup>In both cases default occurs only if the bank is seized before maturity or if at maturity asset value lies below the face value of deposits.

### 4.3 Risk taking with coco like compensation

The typical compensation of managers in financial institutions is composed of equity based compensation and therefore the analysis of the sensitivity of the stock price to asset risk is appropriate when trying to understand executive risk taking motivation (DeYoung et al., 2010).

Another possibility that has been proposed and recently introduced by Barclays is to replace conventional equity based compensation with coco like compensation. Given the recent interest we next briefly explore the risk taking motivation of a bank manager for whom coco is a major part of the compensation package, and who will be trying to maximize the value of compensation by changing the level of bank risk accordingly. Figure 8 graphs coco value against asset risk for different levels of the conversion ratio.

We find, not surprisingly, that the risk taking motivation created by coco compensation depends on the terms of the coco. A relatively high (low) conversion ratio motivates the manager to take the maximum (minimum) possible level of asset risk. This is intuitive: a high (“coco-friendly”) conversion ratio means that a higher level of risk makes conversion, which from the point of view of the coco holder is favorable, more likely. For example, when the conversion ratio is equal to 0.75 or 1, bank managers, who hold coco, are motivated to take the maximum possible level of asset risk, since the value of the coco increases with asset risk. A reverse motivation is present when the conversion ratio is low and equal to 0 or 0.25. In such cases coco holders would try to take the minimum level of asset risk due to the positive relationship between asset risk and coco price. For a conversion level of 50% the value of the coco is insensitive to asset risk and the coco holders are indifferent to the level of asset risk. Coco compensation can therefore minimize the risk-taking motivation of the manager. However, as discussed previously, coco compensation is not necessary to eliminate risk-taking motivation. All that is needed is for the capital structure to include appropriately designed coco. In that case equity based compensation results in virtually no incentive for managers to risk shift.

## 5 Conclusion

In this paper we propose a tractable model of contingent capital (coco): subordinated debt that converts into common equity if a pre-determined threshold is reached before maturity. We provide closed form solutions for coco and other bank liabilities, which is a necessary condition for a viable market. Using our pricing model we investigate the effect of coco design on default probability and risk-taking motivation.

Coco may be effective in stabilizing the banking sector. There are two important channels through which coco can achieve a reduction in risk. First, banks that issue coco instead of subordinated debt are significantly less likely to default. Second, coco may be an effective tool that can influence bank management with equity-based compensation to refrain from increasing risk. The coco conversion ratio (the fraction of post-conversion common equity that coco holders receive) has an important impact on risk-taking motivation. For relatively low (“stock-friendly”) conversion ratios stockholders have an incentive to increase asset risk, while a high (“coco-friendly”) conversion ratio leads to a desire to reduce risk. Importantly, we show that for intermediate levels of the conversion ratio, bank managements’ incentives to change risk can be virtually eliminated, encouraging banks to make the best possible investment decisions instead of trying to transfer wealth from depositors to equity holders by choosing high risk levels. Intuitively, coco in the capital structure means that an increase in risk may lead to conversion thus reducing equity holders’ stake in the company. Coco can therefore cancel out the adverse effects of equity-based compensation. Intuitively, well designed coco reduces the ability of shareholders to gain by transferring wealth from debt to equity.

We also show that risk-taking incentives for a bank with well-designed coco are smaller than for a bank with subordinated debt or one that issues equity only. Furthermore, coco incentives remain low even when a bank enters financial distress, while incentives to risk-shift increase with leverage for banks with subordinated debt or with equity only. Incentives also remain close to unaffected by different regulatory seizing policies (the level of bank losses at which the regulator takes over the institution).

During the recent financial crisis banks were severely undercapitalized and had to be bailed out. Introducing coco into banks’ capital structures represents a possibility to substantially

reduce incentives to increase bank risk, decrease the bank failure rate, and reduce the need for costly provision of capital.

## A Appendix: Option building blocks and pricing equations

We provide here the pricing formulas of the five barrier options that serve as building blocks for the valuation of the bank's liabilities. The formulas are derived by Merton (1973) and Rubinstein and Reiner (1991).

Following the Black and Scholes (1973) assumptions, the risk-free interest rate is constant over time and equal to  $r$ . The value of assets, denoted by  $S$ , is well described under the risk neutral probability by the following stochastic differential equation:

$$\frac{dS}{S} = rdt + \sigma dW$$

where  $W$  is a standard Brownian motion, and  $\sigma$  is the instantaneous constant standard deviation of the asset's rate of return. All options expire at time  $T$ , have a strike price of  $K$  and a barrier level of  $H$ .

1. *Down-and-in* call option: If  $S$  (the underlying asset) reaches the barrier  $H$ , the option becomes a vanilla European call option with strike price  $K$  and maturity  $T$ . If  $S$  does not reach the barrier  $H$  the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$CB^{din} = SN(d_1) - Ke^{-rT}N(d_2) + S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)}N(d_5) - Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)}N(d_6) - SN(d_3) + Ke^{-rT}N(d_4)$$

where  $N(\cdot)$  denotes the standard normal cumulative probability distribution function. The value of a *down-and-in* call option with a barrier which is smaller than the strike

price is:

$$CB^{din} = S \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} + 1 \right)} N(d_7) - K e^{-rT} \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} - 1 \right)} N(d_8).$$

2. *Down-and-out* call option: If  $S$  does not reach the barrier  $H$ , the option becomes a vanilla European call option with strike price  $K$  and maturity  $T$ . If  $S$  reaches the barrier  $H$  the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$CB^{dout} = SN(d_3) - K e^{-rT} N(d_4) - S \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} + 1 \right)} N(d_5) + K e^{-rT} \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} - 1 \right)} N(d_6).$$

The value of a *down-and-out* call option with a barrier which is strictly smaller than the strike price is:

$$CB^{dout} = SN(d_1) - K e^{-rT} N(d_2) - S \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} + 1 \right)} N(d_7) + K e^{-rT} \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} - 1 \right)} N(d_8).$$

3. *Down-and-out* put option: If  $S$  does not reach the barrier  $H$ , the option becomes a vanilla European put option with strike price  $K$  and maturity  $T$ . If  $S$  reaches the barrier  $H$  the option expires worthless. The value of the option with a barrier that is strictly smaller than the strike price is:

$$\begin{aligned} PB^{out} = & SN(d_1) - K e^{-rT} N(d_2) - S \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} + 1 \right)} N(d_5) + K e^{-rT} \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} - 1 \right)} N(d_6) \\ & - SN(d_3) + K e^{-rT} N(d_4) + S \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} + 1 \right)} N(d_7) - K e^{-rT} \left( \frac{H}{S} \right)^{\left( \frac{2r}{\sigma^2} - 1 \right)} N(d_8) \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{K}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_2 &= \frac{\ln\left(\frac{S}{K}\right) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \\ d_3 &= \frac{\ln\left(\frac{S}{H}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_4 &= \frac{\ln\left(\frac{S}{H}\right) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \\ d_5 &= \frac{\ln\left(\frac{H}{S}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_6 &= \frac{\ln\left(\frac{H}{S}\right) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \end{aligned}$$

$$d_7 = \frac{\ln\left(\frac{H^2}{SK}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \quad d_8 = \frac{\ln\left(\frac{H^2}{SK}\right) + T\left(r - \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} .$$

4. *Down-and-out* digital call option: If  $S$  does not reach the barrier  $H$ , the option pays one unit of currency at maturity  $T$  if the value of assets at maturity is greater than the strike price  $K$ . If  $S$  reaches the barrier  $H$  the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$DB^{dout} = Ke^{-rT} \left[ N(d_4) - \left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2} - 1\right)} N(d_6) \right] .$$

The value of a *down-and-out* digital call with a barrier which is strictly smaller than the strike price is:

$$DB^{dout} = Ke^{-rT} \left[ N(d_2) - \left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2} - 1\right)} N(d_8) \right] .$$

5. *Down-and-in* digital call option (with payoff at touch): If  $S$  reaches the barrier  $H$  the option pays one unit of currency at touch. If  $S$  does not reach  $H$  the option expires worthless. The value of the option is:

$$DB^{din} = \left(\frac{H}{S}\right)^{\frac{2r}{\sigma^2}} N(d_5) + \frac{S}{H} N(d_9)$$

where

$$d_9 = \frac{\ln\left(\frac{H}{S}\right) - T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} .$$

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**Table 1: Payoffs for capital structure with coco**

Liability Type	Scenario			
	$\tau_c > T$	$\tau_D \geq T \geq \tau_c$		$T > \tau_D$
	(1) No early conversion	(2) Early conversion		(3) Early default
		$V_T \geq F^D$	$V_T < F^D$	
		(2.A) No default	(2.B) Default	
Secured debt	$F^D$	$F^D$	$V_T$	$F^D(1-\gamma)$
Coco	$F^C$	$\alpha(V_T - F^D)$	0	0
Equity	$V_T - (F^C + F^D)$	$(1-\alpha)(V_T - F^D)$	0	0

**Table 2: Payoffs for capital structure with subordinated debt**

	<b>Scenario</b>			
<b>Liability type</b>	$T > \tau_D$ (1) No early default			$T \geq \tau_D$ (2) Early default
	$V_T \geq (F^D + F^S)$ (1.A) No default	$(F^D + F^S) > V_T \geq F^D$ (1.B) Default only on subordinated debt	$F^D > V_T$ (1.C) Default	
Secured debt	$F^D$	$F^D$	$V_T$	$F^D(1-\gamma)$
Subordinated debt	$F^S$	$V_T - F^D$	0	0
Equity	$V_T - (F^D + F^S)$	0	0	0

**Table 3: Parameter definitions and calibration base values**

Maturity (T): following Marcus and Shaked (1984) and Ronn and Verma (1986) a one-year maturity is reasonable with the annual frequency of regulatory audits. Leverage ratio (LR): We let the leverage ratio  $LR = Fe^{-rT} / V$  range between 0.91 and 0.95. This range is typical for commercial and investment banks before and during the crisis (Hoening, 2009). Therefore, we set the face value of the financial institution's debt (F) to 100 and we calculate for each level of the leverage ratio the appropriate level for a firm's asset value, V. Asset risk ( $\sigma_v$ ): Asset risk is set equal to 5.0%, similar to the average asset risk of banks found in a large sample studied by Mehran and Rosenberg (2008); maximum and minimum asset risk levels are equal to mean asset risk plus or minus one standard deviation of the cross-sectional standard deviation of risk. Interest Rate (r): We choose a continuous constant rate of 2.5%. This is on par with average annual returns on 3-month treasury bills between 2000-2007. The coco principal amount is equal to 3% of the deposit face value, similar to the ratio in the case of Llyods' coco. The sensitivity of the other parameter in the model is analyzed in the paper by using a base case assumptions where the conversion ratio is set initially to  $\alpha=50\%$ , the conversion threshold is located 1% above the total face value of debt ( $\beta=1\%$ ) and the regulator policy is to seize the bank at the first time that the value of assets is 3% below the face value of the deposits ( $\gamma=3\%$ ).

Description	Notation	Value
<b>Market Parameters</b>		
Bank's assets	V	108.02
Leverage ratio	$LR=(F^D + F^S) e^{-rT}/V$	0.93
Assets' volatility	$\sigma$	5.3%
Risk free rate	r	2.5%
<b>Claims Parameters</b>		
Deposit's principal amount	$F^D$	100
Deposit and debt maturities	T	1
Coco principal amount	$F^C, F^S$	3
Conversion ratio	$\alpha$	50%
Conversion threshold	$K^C$	$(F^S + F^D)(1 + \beta)$
Distance between the total debt face value and the conversion threshold	$\beta$	1%
<b>Regulatory Parameters</b>		
Default threshold	$K^D$	$F^D(1 - \gamma)$
Minimum possible level of asset risk	$\sigma^L$	1%
Maximum possible level of asset risk	$\sigma^H$	9%
The distance between the secured debt face value and the default threshold	$\gamma$	3%

**Table 4: Default probabilities for a bank with coco and for a bank with subordinated debt**

The table presents the risk neutral default probabilities in percentage for a bank with capital which is composed of coco, deposits and stock and for a bank with subordinated debt, deposits and stocks. Except for the conversion option of coco the two banks are all else identical. The levels of the parameters chosen are equal to those reported in Table 3.

	Asset risk	Leverage ratio		
		91%	93%	95%
<b>coco</b>	1%	0.0	0.0	0.0
	3%	0.0	0.0	0.4
	5%	0.7	2.3	6.2
	7%	4.9	9.6	17.2
	9%	12.3	19.3	28.9
<b>Subordinated debt</b>	1%	0.0	0.0	0.0
	3%	0.1	0.8	4.5
	5%	3.1	7.7	16.0
	7%	9.7	16.5	26.0
	9%	17.4	25.3	35.2
<b>The different between the probabilities</b>	1%	0.0	0.0	0.0
	3%	0.1	0.8	4.1
	5%	2.4	5.4	9.8
	7%	4.8	6.9	8.8
	9%	5.1	6.0	6.3

**Table 5: Optimal level of asset risk for different features of the coco ( $\alpha, \beta$ )**

The table shows the level of asset risk that a stockholder trying to maximize wealth would choose for different levels of conversion ratio ( $\alpha$ ) and distance between the total debt face value and the conversion threshold ( $\beta$ ). We assume that the stockholder can choose asset risk that range from 1% to 9%. At each case, there are three possible results: (1) Max -The executive chooses the maximum possible level of asset risk (2) Min -The executive chooses the minimum possible level of asset risk (3) Indifference- since the difference between the maximum value of the stock and the minimum value of the stock within the range is below 0.5% of asset value, the executive is indifferent to the level of asset risk. All other data are the same as in Table 1.

Conversion Ratio ( $\alpha$ )	distance between the total debt face value and the conversion threshold ( $\beta$ )				
	0.0%	0.50%	1.00%	1.50%	2.00%
0%	Max	Max	Max	Max	Max
10%	Max	Max	Max	Max	Max
20%	Max	Max	Max	Max	Max
30%	Max	Max	Max	Max	<b>Indifferent</b>
40%	Max	Max	Max	<b>Indifferent</b>	<b>Indifferent</b>
50%	<b>Indifferent</b>	<b>Indifferent</b>	<b>Indifferent</b>	<b>Indifferent</b>	Min
60%	<b>Indifferent</b>	<b>Indifferent</b>	<b>Indifferent</b>	<b>Indifferent</b>	Min
70%	<b>Indifferent</b>	Min	Min	Min	Min
80%	Min	Min	Min	Min	Min
90%	Min	Min	Min	Min	Min
100%	Min	Min	Min	Min	Min

## Figure 1: Time line of the model for the coco

There are three possible states in the model:

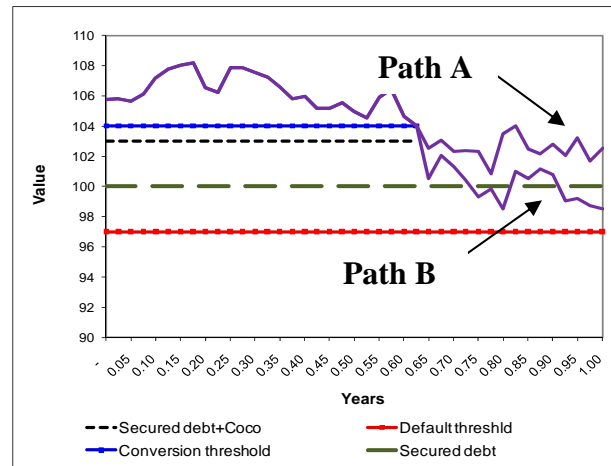
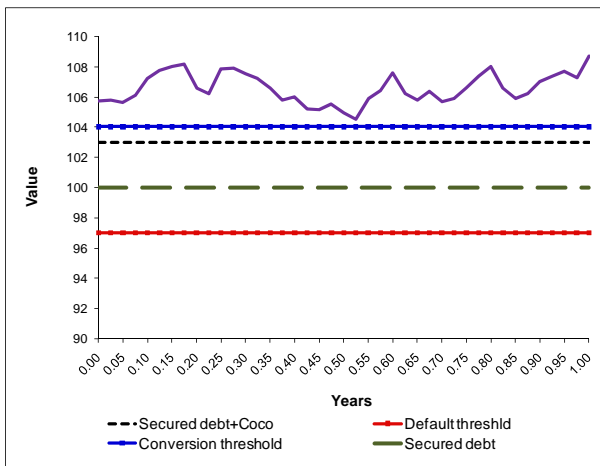
(1.A) No conversion event: The value of the bank's assets does not touch the conversion threshold until debt maturity ( $\tau_c > T$ ) and the coco holder is fully paid, while the stockholder receives the residual assets.

(1.B) Conversion event with no default: The value of the bank's assets has reached the lower conversion threshold and as a result the coco holder receives a predetermined ratio of the bank's stocks in exchange for unwinding its debt obligation ( $\tau_d > T \geq \tau_c$ ). At maturity, if the value of the bank's assets is above the face value of debt the Coco holders and the original shareholders share the residual assets after payments are made to the secure debt holders (Path A). Otherwise, default occurs and the coco holder and the original stockholder receive nothing (Path B).

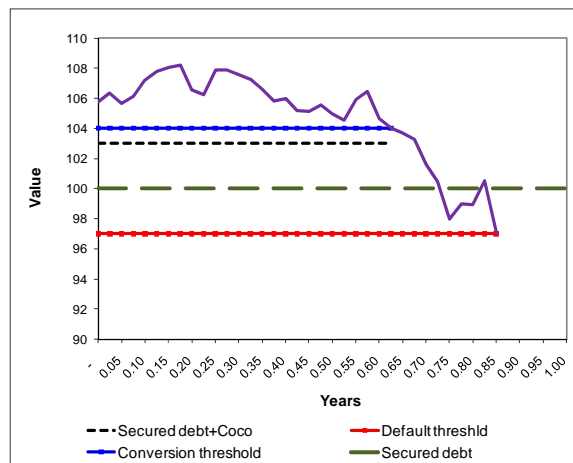
(1.C) Early default: The value of the bank's assets has touched the default threshold before maturity. ( $\tau_d \leq T$ ). The coco holder and the initial stockholders receive nothing.

### (1.A) No conversion

### (1.B) Early conversion and no early default



### (1.C) The case of early default



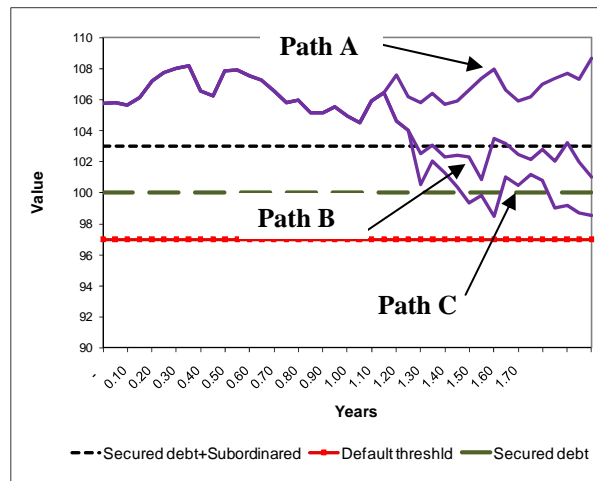
## Figure 2: Time line of the model for the subordinated debt

There are two possible states in the model:

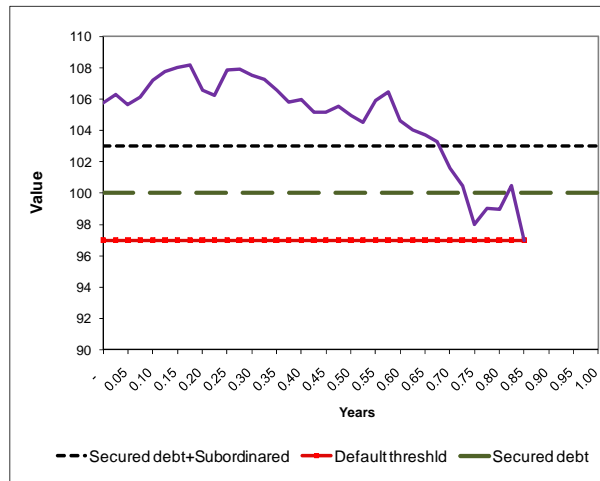
(2.A) No early default: The value of the bank's assets does not touch the default threshold until debt maturity ( $\tau_d > T$ ). The payoff to the claimholders depends on the value of the bank's assets at maturity. If the value of assets is above the total face value of debt the subordinated debt holder is fully paid and the stockholder receives the residual assets (Path A). If the value of asset is between the deposit face value and the total value of debt obligation, the subordinated debt holder receives the residual assets after the deposit holder is paid in full and the stockholder receives nothing (Path B). If the value of assets is below the face value of debt and both the subordinated debt holder and the stockholder receive nothing (Path C).

(2.B) Early default: The value of the bank's assets touches the default threshold before maturity ( $\tau_d \leq T$ ). The subordinated debt holder and the stockholder receive nothing.

### (2.A) The case of no early default

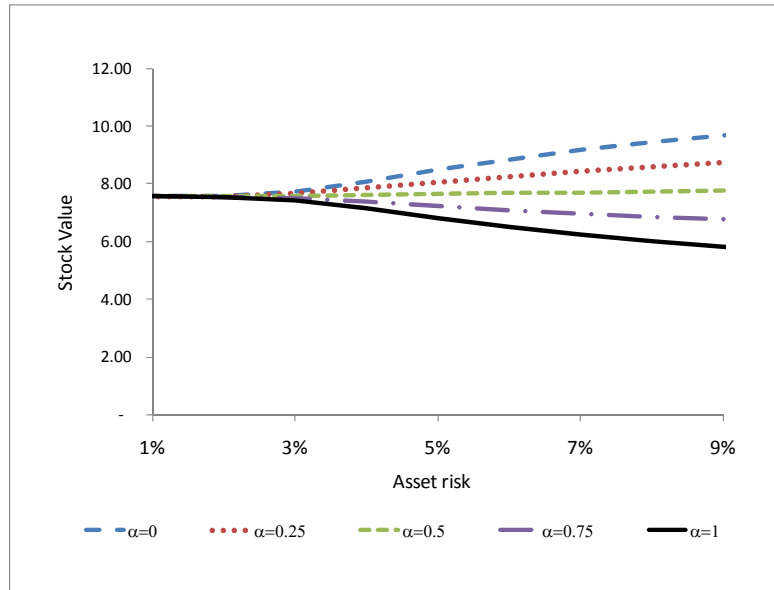


### (2.B) The case of early default



### Figure 3: Stock value versus asset risk for different conversion ratios

The figure presents the value of stock versus asset risk for different conversion ratios ( $\alpha$ ) where the bank capital structure is composed of coco, deposits and stocks. All other parameters are identical to the base case parameters (see Table 3).

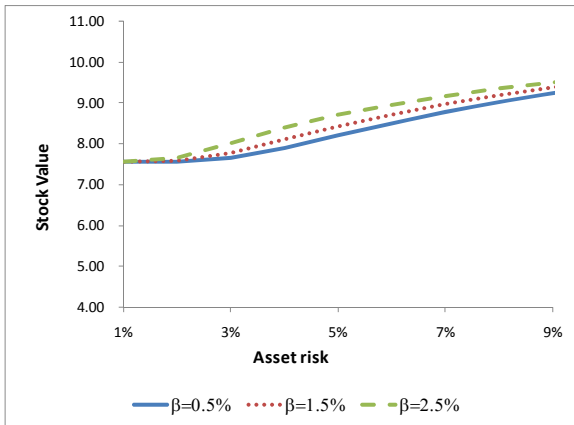


## Figure 4: Stock value versus asset risk for different conversion thresholds

The figure presents the value of a stock of a financial institution with Coco as part of its capital structure versus assets risk for different distance between the conversion threshold and the firm's book value of debt ( $\beta$ ). The conversion ratio ( $\alpha$ ) is equal at panel (4.A) to 0.9, and to 0.5 and 0.1 at panels (4.B) and (4.C) respectively. All other parameters are identical to the base case parameters (see Table 3).

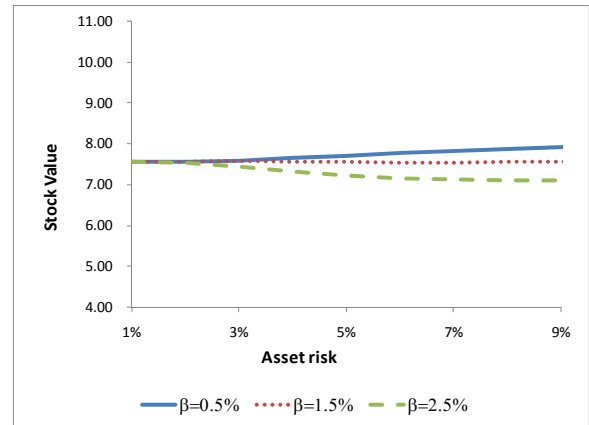
### (4.A) "Stock friendly"

Low conversion ratio ( $\alpha=0.1$ )



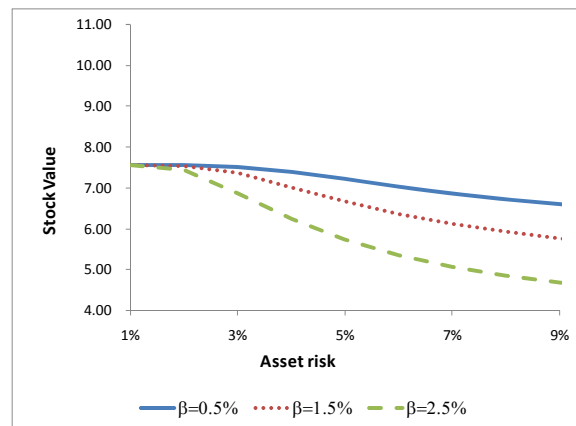
### (4.B) Moderate conversion ratio

The conversion ratio ( $\alpha$ ) is equal to 0.5



### (4.C) "Coco friendly"

High conversion ratio ( $\alpha=0.9$ )

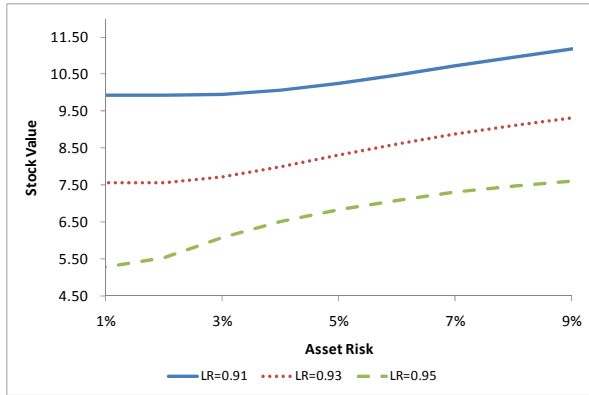


## Figure 5: Stock value for versus asset risk for different leverage ratios

The figure presents the value of a stock versus asset risk in a bank with a capital structure which includes coco, deposits and stocks. The conversion ratio ( $\alpha$ ) is equal in panel (5.A) to 10% (“Stock Friendly”) and to 90% (“Coco friendly”) in panel (5.B). In each case the leverage ratio receives values of 0.91, 0.93 and 0.95. All other parameters are identical to the base case parameters (See Table 3).

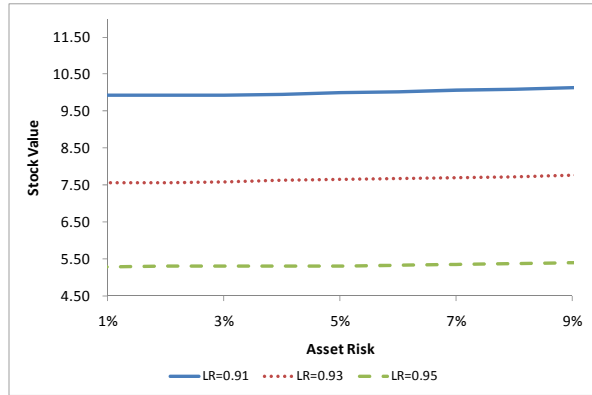
### (5.A) “Stock friendly”

Low conversion ratio ( $\alpha=0.1$ )



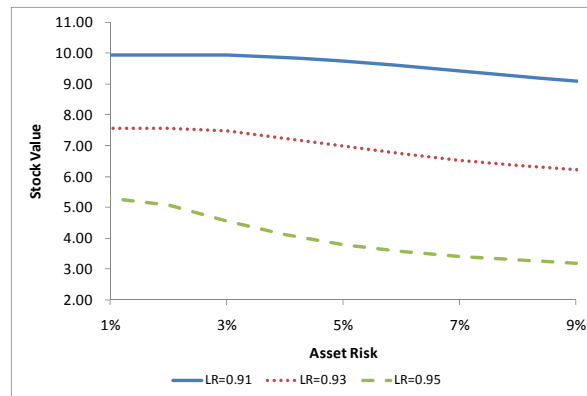
### (5.B) “Moderate conversion ratio”

Conversion ratio is equal  $\alpha=0.5$



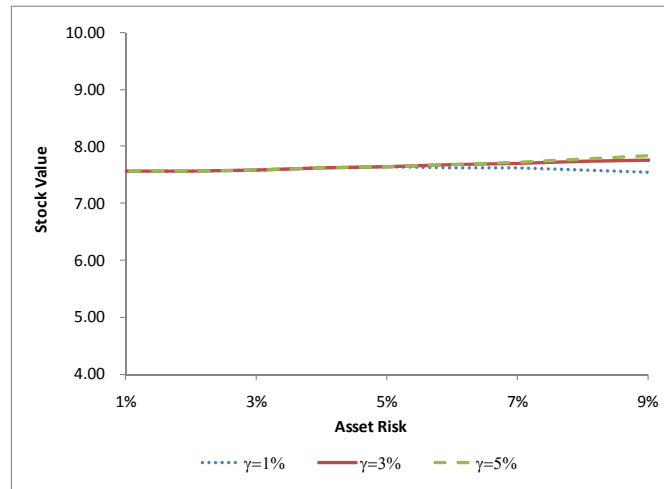
### (5.C) “Coco friendly”

High conversion ratio ( $\alpha=0.9$ )



## Figure 6: Stock value versus asset risk for different regulatory policies

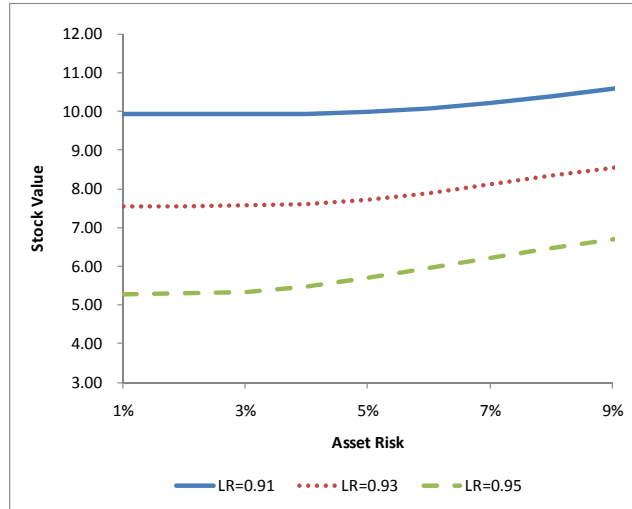
The figure presents the value of a stock versus asset risk in a bank with a capital structure which includes deposits, coco and stocks for different regulatory seizing policies. The ability and will of the regulator determine the level of the threshold in which the bank is seized by the regulator. This level is expressed by the parameter  $\gamma$ , which is the percentage negative difference between the face value of the deposits and the seizing threshold. All other parameters (conversion ratio, conversion threshold etc.) are identical to the base case parameters (see Table 3).



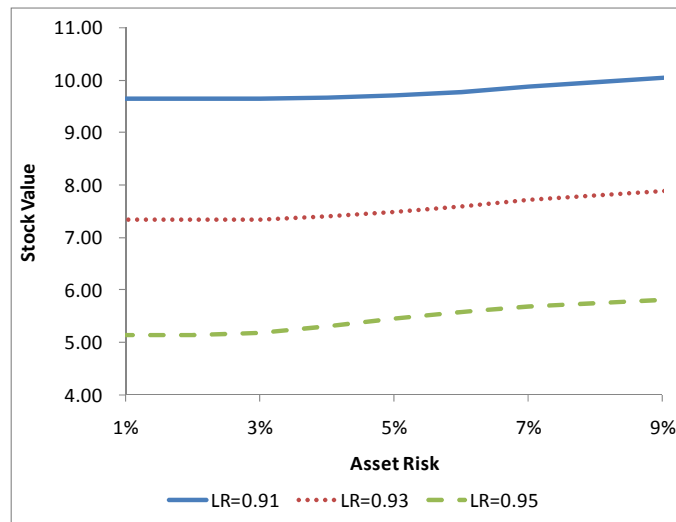
## Figure 7: Stock value versus asset risk for different capital structures

Figure (7.A) presents the value of a stock versus asset risk for a bank with a capital structure which includes coco, deposits and stocks. Figure (7.B) presents the value of a stock versus asset risk for a bank with only deposits and stocks. In each case the leverage ratio receives values of 0.91, 0.93 and 0.95. All other parameters are identical to the base case parameters (See Table 3).

### (7.A) Capital structure with subordinated debt



### (7.B) Capital structure with stock only



## Figure 8: Coco value versus asset risk for different conversion ratios

The figure presents the value of coco versus asset risk for a financial institution with a capital structure that is composed of coco, deposits, and stock. We vary the level of the conversion ratio ( $\alpha$ ). All other parameters are identical to the base case parameters (see Table 3).

