The Credit Ratings Game - revisited

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Abstract

I analyze credit rating agencies and competition. A shortcoming of existing models is that they only consider competition in duopoly, although the U.S. market consists of three major players, and even ten organizations that are designated as Nationally Recognized Statistical Rating Organizations. I develop a framework using Evolutionary Game Theory to analyze the interaction of credit rating agencies in a competitive market with more than two agencies. I show that significant changes in market structures and outcomes can happen for any arbitrary current market size, for example when one new agency enters a market currently consisting of two, three, or ten agencies. Furthermore, honest rating behavior can indeed be achieved as a result of high competition. Alternatively, it can be implemented by regulatory measures like abolishing the "issuer pays" model or by a centralized monitoring of ratings quality.

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1 Introduction

Is more competition between credit rating agencies (CRAs) good or bad for the quality and informativeness of credit ratings? CRAs are widely considered to have been a major factor within the development of the recent financial crisis. Therefore, the question whether regulators should encourage more competition or rather monopolize the market for credit ratings is fundamental and potentially crucial for avoiding the next crisis.

Conventional wisdom and the regulators' view are speaking in favor of competition. However, recent research finds the opposite. This holds both for theoretical and empirical studies, see for example Bolton et al. (2011) and Becker and Milbourn (2011), respectively. As a shortcoming of existing theoretical models, I identify that they only consider competition in duopoly. Therefore, I develop a framework using Evolutionary Game Theory to analyze the interaction of CRAs in a competitive market with an arbitrary number of agencies. I aim to highlight effects of competition that cannot be captured in a duopoly model.

The U.S. market is characterized by a limited number of approved CRAs, so-called Nationally Recognized Statistical Rating Organizations. First there were only Moody's and S&P, and since approximately 1997, Fitch has been there as the third agency. In the meantime, seven more agencies have been approved, so there are now ten CRAs that are designated as Nationally Recognized Statistical Rating Organizations.[1] Therefore it is questionable whether the market can anymore adequately be described as a duopoly.

The current market for credit ratings is characterized by the "issuer pays" business model. It was switched from an earlier "investor pays" model due to difficulties in collecting sufficient fees and information drain. However, the "issuer pays" model has an inherent conflict of interest for the CRA: It may be profitable to inflate ratings, especially for issuers with a lot of possible future business. The question whether competition can increase ratings quality is thus related to whether it helps to prevent ratings inflation.

There are a couple of empirical studies on the topic. The most prominent and related is by Becker and Milbourn (2011). They take the market entry of Fitch as a natural experiment to analyze the effect of increasing competition. Overall, they document a

[1] See Becker and Milbourn (2011) and http://www.sec.gov/answers/nrsro.htm, retrieved May 29, 2011.

decrease in ratings quality. He et al. (2010) examine whether rating agencies reward large issuers of mortgage-backed securities. After controlling for deal characteristics, they can analyze a situation in which small and large issuers differ only in the amount of possible future business. They find evidence for a positive bias of CRAs towards large issuers and thus for ratings inflation. The question remains whether more competition will be beneficial for ratings quality and helps to avoid ratings inflation.

The existing theoretical literature mostly supports the view that competition is bad for ratings quality and makes ratings inflation worse. Mathis et al. (2009) analyze reputation cycles and ratings inflation for a monopolistic CRA. Camanho et al. (2010) extend Mathis et al. (2009) by competition effects. They state that "competition results in greater ratings inflation." Skreta and Veldkamp (2009) find that competition makes ratings shopping worse. The article by Bolton et al. (2011) models a setting similar to mine, but is still limited to the comparison of monopoly vs. duopoly. The authors state that "competition among CRAs may reduce market efficiency since it facilitates ratings shopping by issuers".

Still, there is also some theoretical literature that gives hope for a possible cure of ratings inflation. Stolper (2009) suggests that the problem might be solved by a proper regulatory approval scheme for CRAs. Doherty et al. (2009) show both theoretically and empirically that the market entry of a new CRA can improve ratings quality and precision. Their story is that the entrant CRA can attract business from good issuers that have been pooled with worse quality issuers. By using a more precise rating scale, the entrant CRA allows the good issuers to receive higher prices for the investments they sell. More general economic research by Hörner (2002) develops a reputational theory where competition may increase quality if existence of competitive choice is required to make loss of reputation a real threat. One of the first papers analyzing the trade-off between building up a long-term reputation and making higher short-term profits by misbehaving is by Klein and Leffler (1981). So in principle, if more competition should be a cure of ratings inflation, it would need to affect this trade-off towards the benefit of building up a long-term reputation.

My hypothesis is that reputation costs might be too low in a market with a small number of CRAs, as the investors and issuers do not have a sufficient number of alternatives to effectively punish CRAs that deliver bad quality. Therefore the transition from monopoly to duopoly, or even to a market with three participants, might still not provide sufficient alternatives. This might explain that Becker and Milbourn (2011) do not document an increase in ratings quality following the market entry of Fitch. However, the change might come for an even larger number of CRAs, some of whom possibly offering true alternatives to the existing ones.

Theoretical researchers usually argue that the big three CRAs have such a high market share that it can be justified to neglect the remaining market participants. Then, additional arguments might justify why a model with only two CRAs (apart from the advantage of being more tractable) can adequately describe a market with three of them. One counterargument is, however, that it might be particularly interesting to determine the conditions under which a new rating agency that possibly has different ethical standards and business practices can successfully invade the market, even if it starts off with a tiny market share.

In the present paper, I model the CRAs' incentives to inform the investors honestly about the quality of investments, rather than to inflate ratings, as an interplay with investors' sophistication level. I apply the methodology of Evolutionary Game Theory, which allows an arbitrary number of market participants, as well as the analysis of new behavioral traits to possibly successfully enter an established market. First I show that dependent on the parametrization, the market can develop into different equilibria, where either honest or inflating CRAs dominate in the end. In a second step, I explain the model's parameters as functions of the number of CRAs in the market. Thus I can show directly how different market structures and outcomes result from changing the number of CRAs. Existing theoretical models can only distinguish between a monopolistic CRA and competition in duopoly. In contrast, I show that significant changes in market structures and outcomes can happen at any transition, for example when one new CRA enters a market currently consisting of two, three, or ten CRAs.

As a conclusion, I aim to give policy recommendations, e.g. under which conditions it might be good to encourage competition, or which other parameters of the market for credit ratings should possibly be influenced to reach a socially desirable outcome.

The paper is structured as follows: In Section 2, I introduce the modeling framework. Following is an analysis of the model in Section 3. In Section 4, I visualize and discuss the results for an arbitrary number of CRAs. Next, I discuss the effect of competition and explicitly focus on the number of CRAs in Section 5. Section 6 concludes the paper. In the Appendix, Section A, I illustrate the effect of an alternative specification of payoffs.

2 Model

2.1 Discussion of Modeling Alternatives and Limitations

I develop a framework for competition between more than two CRAs. The aim of my model is to examine whether (and under which conditions) competition is favorable. More precisely, I expect that there are incentives for inflation in a monopolistic market (one CRA). These incentives might be amplified for the case of two CRAs, as previous theoretical research like Bolton et al. (2011) and Camanho/Deb/Liu (WP 2010) shows. However, I aim to find out whether honest behavior might be established for further increasing competition, i.e., more than two CRAs, and possibly examine the limit case of a market with infinitely many CRAs.

To achieve this goal, I see two possible approaches. The first is via simulation and numerical solution of a multiple-agent model. Such an approach is expected to become computationally more and more expensive in the number of players (here, CRAs). The nature of the approach precludes parameter-free, analytical results.

The second approach, which I will follow in the remainder of the paper, is Evolutionary Game Theory, see e.g. Weibull (1997). Here, the idea is to show that dependent on market characteristics and the CRAs' opponents, honest behavior might be more successful. In that case, the population of (arbitrarily many) CRAs consists of more and more honest CRAs. But that might have effects on the composition of opponents' populations and vice versa. Advantages of the second approach are the rather simple analysis and analytical solutions. Also, the approach does not specify the number of players, and it implicitly covers the limit case of infinitely many CRAs. A disadvantage of the approach is that the model flexibility is limited, compared to a simulation approach. For example, the model assumes a random pairing between investors and CRAs, and it is not possible to remember and update the individual CRA's reputation over time. The only state variables are the population shares. At least, as the market share of each type of CRA is related to its reputation, this share can also be seen as a proxy for the CRA's current reputation in the model. Also, the model provides useful implications that are similar to those expected for a model with a more complex treatment of reputation effects.

A possible critique of my approach is that the model's parameters, particularly the fees charged by CRAs, the reputation cost faced by a CRA caught inflating, and the monitoring costs borne by the sophisticated investors, are not derived from an equilibrium model. My model is flexible enough, though, to define these parameters as functions of some explanatory variables in a preceding model. One example for such an explanatory variable is the number of CRAs in the market, an approach that will be pursued in Section 5. Any such parametrization will result in one of the cases described in Section 4, as long as the parameters stay constant over time.

Also, it is possible to define the parameters as functions of the two state variables, i.e., the population shares. This can possibly change also the dynamic properties of the model into situations that are not described in Section 4. One example is that the fees that issuers are willing to pay to the CRAs for a good rating might reasonably be argued to depend on the fraction of trusting investors on the market. I analyze this case in the Appendix, Section A. One could also justify that the reputation cost faced by a CRA caught inflating depends on the fraction of honest CRAs on the market, as the latter increase the choice of alternatives for the investors and issuers.

The implementation of the many alternative specifications is quite straightforward, but will not be discussed in further detail, apart from the two mentioned extension sections. Also, the calibration of the model parameters to practical requirements and an empirical test of the predictions are left to further research.

2.2 Model Setup

The model is based on the methodology of Evolutionary Game Theory, see e.g. Weibull (1997). More precisely, I build on an article by Schuster et al. (1981), which derives results for evolutionary games between two populations. The economic setting of the model is given by Bolton et al. (2011), from which I also use the notation as far as possible, to ensure comparability.

I consider a market with two types of investments: First, a good investment, which is present on the market with a share $\lambda \in [0,1]$. It provides a payoff $1 + R > 1$ upon investment of 1, i.e., a net payoff of $R > 0$. Second, a bad investment, which is present on the market with a share $(1-\lambda)$. It provides a payoff of zero, which can be interpreted as default, upon investment of 1. So the net payoff of the bad investment is (-1).

There are two populations of players that interact with each other: First, I consider the population of investors (Inv): There is a share $\alpha \in [0,1]$ of trusting investors (T), and a share $(1-\alpha)$ of sophisticated investors (S). Second, I consider the population of rating agencies (CRAs): There is a share $\beta \in [0,1]$ of honest CRAs (H), and a share $(1-\beta)$ of inflating CRAs (I). The population space consists of all possible states (α, β) within the square (0,0), (1,0), (1,1), (0,1).

There are other market participants that are not explicitly modeled, for example the issuers providing the investment opportunities, and the regulator possibly admitting new CRAs or banning active CRAs due to underperformance. Thus, I concentrate on the interplay between investors and rating agencies, while taking the remainder of the market as exogenous.

2.3 Strategic Interaction and Payoffs

Unless otherwise noted, I assume that individuals are programmed to follow one pure strategy. As a result of the realized payoffs, they may switch their behavior towards another pure strategy, unsuccessful market participants may leave the market, or new market entrants can observe and imitate the most successful behavior. These are possible economic explanations for the population shares being changing over time, dependent on the realized payoffs.

For each interaction, the CRA receives a random investment, which is good with probability λ. That means, it is drawn out of the market that is composed of a share λ of good investments. An interaction takes place in a random pairing of one investor (type T or S) and one CRA (type H or I) The players cannot recognize each other's types. Depending on the players' types, they receive payoffs as given in the following.

The CRA charges a fee $\Phi \geq 0$ *from the issuer* for giving the rating. The fee is only received for a good rating. This assumption follows Bolton et al. (2011) and can be interpreted as a reduced-form modeling of ratings shopping, as the issuer will then move on and hope to find another agency who gives him the good rating. One could even argue that the issuer's behavior, namely the choice to accept and pay only for good ratings, is modeled here in a very simple form. Effectively, an investment without rating is equivalent to an investment rated as bad in my model. An honest CRA truthfully reports the type of the investment. Thus, if the investment is bad, it cannot sell the rating to the issuer and does not receive a fee. An inflating CRA, in contrast, reports always "good" and receives the fee. On the investor side, trusting investors buy all investments that are rated good. In contrast, sophisticated investors spend a cost $C \geq 0$ to verify the CRA's work. If they meet an inflating CRA with a bad investment rated as good, they don't buy it, *and* they cause reputation costs $\rho \geq 0$ for the CRA. Here, I make a strong assumption that lying CRAs can immediately be recognized and punished. A more realistic assumption would be that such behavior can only be detected with some delay, if at all. However, the assumption is consistent with the previous assumption that investments turn out to be good or bad (without any uncertainty due to overlapping realizations of the outcomes) immediately after making the investment decision. A related critical assumption is that the sophisticated investors still spend monitoring costs to observe the CRAs' behavior, although they can perfectly verify the quality of the investments themselves. It can be motivated by assuming that only by the combination of the information they receive from the CRA and their own verification efforts, the sophisticated investors are able to make this perfect judgment of the investments.

Now I state the payoffs for the investors. First, consider the trusting investors. If they are meeting an honest CRA, they receive a good rating for a good investment, which occurs with probability λ. In this case they invest and receive a net payoff of R. If the investment is bad, the investors are warned, as the CRA refuses to give a good rating, so they do not invest and receive zero payoff. Together, the expected payoff is

$$V_{TH} = \lambda R.$$

Against an inflating CRA, they receive a net payoff of R for a good investment, which occurs with probability λ. However, if the investment is bad, which occurs with probability

$(1-\lambda)$, the CRA still gives a good rating. The trusting investors invest, and consequently receive a net payoff of (-1). Together, their expected payoff is

$$V_{TI} = \lambda R + (1-\lambda)(-1).$$

The resulting expected payoff for trusting investors is

$$\Pi_T^{Inv} = \beta V_{TH} + (1-\beta) V_{TI} = \lambda R - (1-\beta)(1-\lambda).$$

Second, consider the sophisticated investors. If they are meeting an honest or inflating CRA, they receive the same payoff, namely

$$V_{SH} = V_{SI} = \lambda R - C.$$

In either case, they spend the cost C to verify the CRA's work. Thus they manage to invest only in the good investment, which occurs with probability λ. The resulting expected payoff for sophisticated investors is the same, namely

$$\Pi_S^{Inv} = \beta V_{SH} + (1-\beta) V_{SI} = \lambda R - C.$$

The resulting payoffs are summarized in Table 1. The average payoff in the population of investors is

$$\bar{\Pi}^{Inv} = \alpha \Pi_T^{Inv} + (1-\alpha) \Pi_S^{Inv}.$$

Considering the rating agencies, I first state the payoffs for the honest CRAs. They only give a good rating and receive the fee if they observe a good investment, which occurs with probability λ. On the other hand, they are never punished for inflating ratings. Their expected payoff against both trusting and sophisticated investors is therefore

$$X_{HT} = X_{HS} = \lambda \Phi.$$

Thus, the resulting expected payoff for honest CRAs is the same, namely

$$\Pi_H^{CRA} = \alpha X_{HT} + (1-\alpha) X_{HS} = \lambda \Phi. \tag{1}$$

Second, consider the inflating CRAs. If they are meeting a trusting investor, they receive

$$X_{IT} = \Phi.$$

Table 1: Investors' Payoffs.

Investor / CRA	honest	inflating	expected
trusting	$V_{TH} = \lambda R$	$V_{TI} = \lambda R + (1-\lambda)(-1)$	$\Pi_T^{Inv} = \lambda R - (1-\beta)(1-\lambda)$
sophisticated	$V_{SH} = \lambda R - C$	$V_{SI} = \lambda R - C$	$\Pi_S^{Inv} = \lambda R - C$

Table 2: CRAs' Payoffs.

Investor / CRA	honest	inflating
trusting	$X_{HT} = \lambda \Phi$	$X_{IT} = \Phi$
sophisticated	$X_{HS} = \lambda \Phi$	$X_{IS} = \lambda \Phi + (1-\lambda)(\Phi - \rho)$
expected	$\Pi_H^{CRA} = \lambda \Phi$	$\Pi_I^{CRA} = \Phi - (1-\alpha)(1-\lambda)\rho$

As they always give good ratings, they are always paid by the issuers, regardless of the quality of the investment. Against a sophisticated investor, however, their expected payoff is

$$X_{IS} = \lambda \Phi + (1-\lambda)(\Phi - \rho).$$

While the issuer still pays them the fee regardless of the quality of the investment, they are punished whenever they rate a bad investment as good, which happens with probability $(1-\lambda)$, and meet a sophisticated investor. The resulting expected payoff for inflating CRAs is

$$\Pi_I^{CRA} = \alpha X_{IT} + (1-\alpha) X_{IS} = \Phi - (1-\alpha)(1-\lambda)\rho. \quad (2)$$

The payoffs are summarized in Table 2. The average payoff in the population of CRAs is

$$\bar{\Pi}^{CRA} = \beta \Pi_H^{CRA} + (1-\beta)\Pi_I^{CRA}.$$

3 Analysis

3.1 Evolutionary Dynamics

Given the payoff structure, Weibull (1997) shows that the corresponding replicator dynamics can be derived as

$$\frac{\partial \alpha}{\partial t} = \dot{\alpha} = \alpha(\Pi_T^{Inv} - \bar{\Pi}^{Inv}) \quad \text{and} \quad \frac{\partial \beta}{\partial t} = \dot{\beta} = \beta(\Pi_H^{CRA} - \bar{\Pi}^{CRA}).$$

This means that the growth rate $\dot{\alpha}/\alpha$ of the trusting investors' population share equals the difference between the trusting investors' current payoff and the current average payoff in the investor population. If trusting investors perform better than average (i.e., better than sophisticated investors), their share is growing. If they perform worse, their share is shrinking. The opposite holds for the share $(1-\alpha)$ of sophisticated investors, respectively, and an analogous mechanism is at work in the CRA population. These dynamics can be transformed into

$$\dot{\alpha} = \alpha(1-\alpha)\underbrace{(\Pi_T^{Inv} - \Pi_S^{Inv})}_{\Delta\Pi^{Inv}} \quad \text{with} \quad \Delta\Pi^{Inv} = C - (1-\beta)(1-\lambda) \tag{3}$$

and

$$\dot{\beta} = \beta(1-\beta)\underbrace{(\Pi_H^{CRA} - \Pi_I^{CRA})}_{\Delta\Pi^{CRA}}, \quad \text{with} \quad \Delta\Pi^{CRA} = (1-\lambda)((1-\alpha)\rho - \Phi). \tag{4}$$

Again, this allows the following interpretation: When trusting investors perform relatively better than sophisticated investors, then the trusting investors' share α in the investor population is increasing. Similarly, when honest CRAs perform relatively better than inflating CRAs, then the honest CRAs' share β in the CRA population is increasing.

3.2 Stationary Regions

My next step is to derive stationary regions. These refer to states in which there is no movement in either α or β direction, or neither.

3.2.1 Fixed Lines

There is no movement in α direction, if $\dot{\alpha} = 0$ in (3). This is the case if either $\alpha = 0$ or $\alpha = 1$ (on the margins of the population space), or

$$\Delta\Pi^{Inv} = 0 \Leftrightarrow \beta = \beta^* := 1 - \frac{C}{1-\lambda}. \tag{5}$$

Similarly, there is no movement in β direction, if $\dot{\beta} = 0$ in (4). This is the case if either $\beta = 0$ or $\beta = 1$ (again, on the margins of the population space), or

$$\Delta\Pi^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := 1 - \frac{\Phi}{\rho}. \tag{6}$$

For states above or below the fixed lines, I observe from (3) that $\Delta\Pi^{Inv}$ is increasing in β. Therefore, I have for $\beta > \beta^*$ (<) a movement $\dot{\alpha} \geq 0$ (≤ 0). Similarly, I observe from (4) that $\Delta\Pi^{CRA}$ is decreasing in α. Likewise, I have for $\alpha > \alpha^*$ (<) a movement $\dot{\beta} \leq 0$ (≥ 0).

3.2.2 Fixed Points

If there is no movement in either direction, then the corresponding state is a fixed point in the dynamics. From the required condition $\dot{\alpha} = \dot{\beta} = 0$, I derive the corners of the population space as fixed points, as well as the interior fixed point (α^*, β^*). To analyze the properties of the interior fixed point, I follow the method in Schuster et al. (1981). For that purpose, I first write the payoffs for investors and CRAs as matrices $\bar{A}$ and $\bar{B}$, respectively. From Table 1, I have

$$\bar{A} = \begin{pmatrix} \lambda R & \lambda R - (1-\lambda) \\ \lambda R - C & \lambda R - C \end{pmatrix}$$

for the investor payoffs. Schuster et al. (1981) first transform the payoff matrix into one with zeros in the diagonal by subtracting a constant from each column, which does not affect the dynamics. In my case, this leads to

$$A = \begin{pmatrix} 0 & a_{12} \\ a_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & C - (1-\lambda) \\ -C & 0 \end{pmatrix}.$$

Similarly, I write the CRA's payoffs from Table 2 as

$$\bar{B} = \begin{pmatrix} \lambda\Phi & \lambda\Phi \\ \Phi & \lambda\Phi + (1-\lambda)(\Phi - \rho) \end{pmatrix}.$$

The matrix results from transposing Table 2, as it should represent the population that receives the payoffs as the column player. Again, transformation leads to

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & (1-\lambda)(\rho - \Phi) \\ (1-\lambda)\Phi & 0 \end{pmatrix}.$$

From these matrices, Schuster et al. (1981) derive the interior fixed point as

$$(\alpha^*, \beta^*) = (\frac{b_{12}}{b_{12}+b_{21}}, \frac{a_{12}}{a_{12}+a_{21}}),$$

which corresponds to the results (5) and (6) above. For an interior fixed point at (α^*, β^*), I require $\alpha^*, \beta^* \in (0,1)$. From (5) and (6), this means that $0 < 1 - \frac{C}{1-\lambda} < 1$ and $0 < 1 - \frac{\Phi}{\rho} < 1$, or reformulated,

$$0 < C < 1 - \lambda \tag{7}$$

and

$$0 < \Phi < \rho. \tag{8}$$

If the fixed point is indeed in the interior of the population space, Schuster et al. (1981) derive that it can either be a saddle or a center. If $a_{12}b_{12} > 0$, it is a saddle. In contrast, if $a_{12}b_{12} < 0$, it is a center. For an interior fixed point in my model,

$$a_{12}b_{12} = (\underbrace{C - (1-\lambda)}_{<0 \text{ see (7)}})(1-\lambda)\underbrace{(\rho - \Phi)}_{>0 \text{ see (8)}} < 0$$

holds, i.e., it is a center. This means, as explained by Schuster et al. (1981), that the orbits will spiral periodically around the fixed point, keeping their radius and speed constant.

Depending on the parametrization, there may or may not exist an interior fixed point. This results in several interesting cases, which will be analyzed in the following.

3.3 Comparison with Two-Player Game Theory

Related to my model is a two-player game between one investor and one CRA, with the payoffs given in Tables 1 and 2. I show the game in normal form, which is the combination

Table 3: Two-Player Game in Normal Form.

Investor / CRA	honest	inflating
trusting	λR, $\lambda\Phi$	$\lambda R - (1-\lambda)$, Φ
sophisticated	$\lambda R - C$, $\lambda\Phi$	$\lambda R - C$, $\lambda\Phi + (1-\lambda)(\Phi - \rho)$,

of the two payoff matrices, in Table 3. As usually done in two-player game theory, the first and second entry represent the payoffs for the investor and the CRA, respectively.

Throughout the evolutionary dynamics, I assume that for each interaction, there is a random draw of one investor and one CRA with built-in types out of their respective populations. In contrast, for the current section I assume that both the investor and the CRA may choose their optimal strategies. They can either choose a pure strategy, i.e., choose one of their two respective actions with certainty, or they may choose a mixed strategy, which is to randomize their actions. In the latter case, the investor chooses to be trusting with probability α, and the CRA chooses to be honest with probability β.

If the conditions (7) and (8) hold, there is no Nash equilibrium in pure strategies. For a Nash equilibrium in mixed strategies, each player randomizes such that the other player is indifferent between the available strategies. this means that the investor chooses α such that

$$\Delta\Pi^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := 1 - \frac{\Phi}{\rho}.$$

Similarly, the CRA chooses β such that

$$\Delta\Pi^{Inv} = 0 \Leftrightarrow \beta = \beta^* := 1 - \frac{C}{1-\lambda}.$$

These solutions for α and β are the same as those derived as coordinates of the interior fixed point in the evolutionary dynamics in (6) and (5), respectively.

If the conditions (7) and (8) do not hold, there are four possible Nash equilibria in pure strategies: If $C > 1-\lambda$, the Nash equilibrium in pure strategies is "trusting/inflating". If $\Phi > \rho$, it is "sophisticated/inflating". If $\Phi = 0$, it is "trusting/honest". If $C = 0$, it is "sophisticated/honest".

As I will show in the following section, the outcomes of the evolutionary dynamics are similar to the outcomes of the two-player game for the different cases. If conditions

Table 4: Base Case Parameter Values.

share of good investments	$\lambda = 0.5$
net payoff upon investment	$R = 1.1$
verification cost	$C = 0.2$
fee charged by CRA	$\Phi = 1$
reputation cost	$\rho = 1.4$

(7) and (8) hold, I will show that the outcome is an interesting cyclic dynamic behavior around the fixed point. For the other cases, I show that the outcomes of the two-player game are also reached similarly if populations of investors and CRAs interact over time. Moreover, I will discuss in Section 5 how the outcomes change if the model's parameters are dependent on the number of CRAs in the market.

4 Results for an arbitrary number of CRAs

4.1 Base Case: Interior Fixed Point

As the base case, I define the situation with one interior fixed point at (α^*, β^*), i.e., $\alpha^*, \beta^* \in (0, 1)$. To satisfy (7) and (8) derived in the previous section, I choose the share of good investments as $\lambda = 0.5$, the verification cost borne by sophisticated investors as $C = 0.2$, the fee charged by the CRA to the issuer for a good rating as $\Phi = 1$, and the CRA's reputation cost if caught lying as $\rho = 1.4$. The net payoff upon investment is chosen as $R = 1.1$. Note that the latter does not affect the location of (α^*, β^*), and thus the final outcome. However it has an impact on the speed of adjustment in the dynamics. The parameter values are summarized in Table 4. According to (5) and (6), the resulting interior fixed point is located at $(\alpha^*, \beta^*) = (0.29, 0.6)$.

The resulting dynamics are visualized in Figure 1. The arrows indicate the vectors $(\dot{\alpha}, \dot{\beta})$, i.e., the direction and speed of development of the shares in each population for given current shares of trusting investors (α) and honest CRAs (β). Also shown are the fixed lines on the margins of the population space, and the two interior fixed lines.

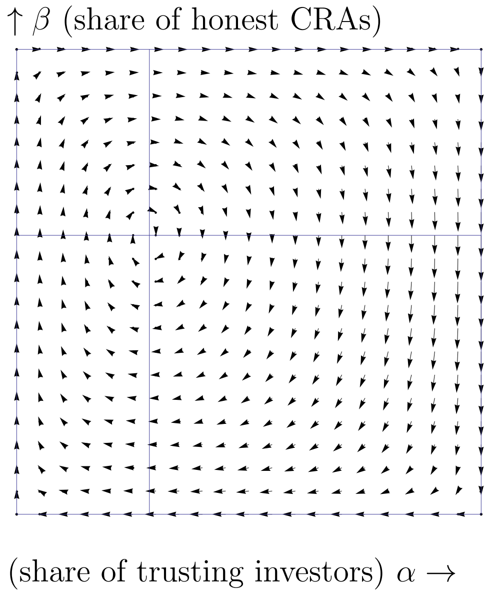


Figure 1: Base Case. Vector Field for $\alpha^*, \beta^* \in (0,1)$. Parameter values: $\lambda = 0.5, R = 1.1, C = 0.2, \Phi = 1, \rho = 1.4$

The latter separate four different regimes. Dependent on the current investor sophistication level, either the honest or inflating CRAs are more successful. For example, start in the middle of the population space, at (0.5,0.5). This point lies in the lower right quadrant of the population space. Then the share of trusting investors (α) is higher than on the fixed line (α^*). As a consequence, the inflating CRAs are the more successful ones, and their share is growing.[2] Also, the current state has a level of honest CRAs (β) that is below the fixed line (β^*). Therefore the trusting investors often happen to put their money in bad investments, and therefore the sophisticated investors are better off in such a situation and can improve their market share. Together, as the arrows are indicating, the market moves towards less honest (and more inflating) CRAs and less trusting (and more sophisticated) investors.

Once the fixed line at α^* is crossed, the lower left quadrant of the population space is entered. Now the share of trusting investors is still shrinking, because of the high market share of inflating CRAs. However, now there are enough sophisticated investors in the market to make reputation costs more important for the CRAs than rating fees. Therefore the honest CRAs make more profit now, and they consequently improve their

[2]Such a state is in line with the findings by Bolton et al. (2011) that "the presence of more trusting investors [...] give CRAs incentives to inflate the quality of investments".

market share.

When crossing the fixed line at β^*, yet another regime is reached in the upper left quadrant of the population space. Now the share of honest CRAs is high enough that it does not pay off anymore for the individual investor to invest in the monitoring of the CRAs. Therefore the trusting investors now perform better than the sophisticated ones and gain in market share. Still, there are enough sophisticated investors in the market to make the honest CRAs better off than the inflating ones, and thus the honest CRAs' market share is further growing.

In the next regime transition, the fixed line at α^* is crossed again, and the upper right quadrant of the population space is entered. Here, the trusting investors are still becoming more numerous. Due to the low share of the inflating CRAs, it is still safe to buy all investments rated as good, rather than investing in monitoring. However, the trusting investors have already become such a big group, that it is again beneficial for the CRAs to inflate ratings, as they can do so with little risk of being caught. Therefore the inflating CRAs gain market share on expense of the honest ones. Finally, the last transition over the fixed line at β^* leads again into the lower right quadrant of the population space, where I started the investigation.

As previously derived, the fixed point (α^*, β^*) is a center in the dynamics. This means that the clockwise cycles of movement in the population that are displayed in Figure 1 will spiral periodically around the fixed point. These cycles of movement are consistent with evidence of both CRAs' and investors' behavior varying over the business cycle. Related is the prediction of Bolton et al. (2011) that "ratings inflation is more likely in boom times when investors have lower incentives to perform due diligence, as the ex-ante quality of investments is then higher." In the language of my model, this corresponds to a lower share of sophisticated investors in the market. The effect of the ex-ante investment quality will be analyzed in Section 4.3.

The conclusion for the base case is depending on the current regime of the market. It pays off temporarily for CRAs to be honest, but only if there are enough sophisticated investors in the market, who make reputation loss a real threat for them. Otherwise, ratings inflation is the best strategy for CRAs. My result for the base case is consistent with other theories, e.g., Skreta and Veldkamp (2009) and Bolton et al. (2011), who suggest that

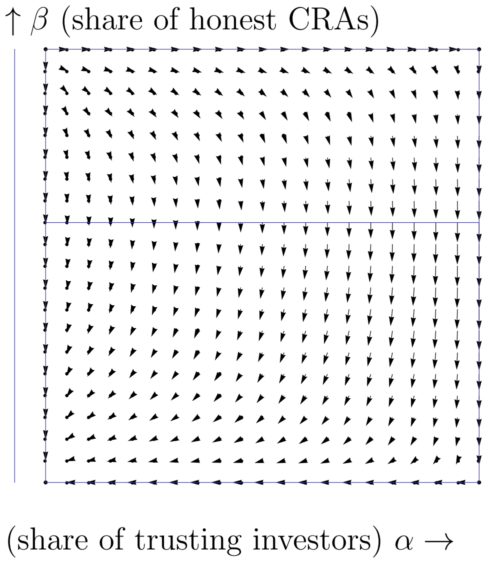


Figure 2: High Fees (Relative to Reputation Costs). Vector Field for $\alpha^* < 0, \beta^* \in (0, 1)$. Parameter values: $\lambda = 0.5, R = 1.1, C = 0.2, \Phi = \mathbf{1.5}, \rho = 1.4$

ratings inflation is most severe for complex investment products, i.e., products exceeding the investor sophistication level, and more trusting investors. However, I emphasize that even for the base case of my model, "strategic honesty" can pay off temporarily.

Apart from the base case, I highlight several other interesting cases in the following. These occur for parameter constellations in which one or both of the coordinates (α^*, β^*) lie outside of the population space. Interestingly, other outcomes will be reached for these cases, including stable equilibria in which the competitive market for credit ratings will be served exclusively by honest CRAs.

4.2 Outcome: Sophisticated Investors and Inflating CRAs

I begin with describing the socially least desirable equilibrium. It occurs if $\rho < \Phi$, which means that the inflating CRA, if caught, faces reputation costs below the fees earned. Then it follows from (6) that $\alpha^* < 0$. The resulting dynamics are visualized in Figure 2, given that $\beta^* \in (0, 1)$.

The resulting equilibrium is that the honest CRAs die out, and trusting investors die out as well. From an investor point of view, it pays off to collect information oneself instead of listening to the CRAs. The sophisticated investors can thus avoid to buy the bad

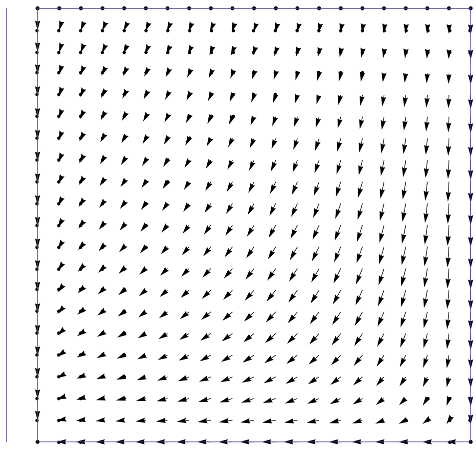


Figure 3: High Fees and Zero Monitoring Costs. Vector Field for $\alpha^* < 0, \beta^* = 1$. Parameter values: $\lambda = 0.5, R = 1.1, C = \mathbf{0}, \Phi = \mathbf{1.5}, \rho = 1.4$

investments. Still, the issuers pay fees to the rating agencies. As these fees are higher than the reputation costs, it is still worth for the rating agencies to produce useless information and get paid for it.

Interestingly, the resulting equilibrium is the same if, in addition to $\rho < \Phi$ (implying $\alpha^* < 0$), I assume $C = 0$, i.e., sophisticated investors can observe the inflating behavior of CRAs at zero monitoring costs. Then it follows from (5) that $\beta^* = 1$. The resulting dynamics are visualized in Figure 3. In that case, sophisticated investors outperform the trusting ones even more. This is visible if starting at a point in the population space with a high share of honest CRAs. In Figure 2, the share of trusting investors is then initially still increasing, while the honest CRAs die out. In Figure 3, however, it never pays off to be a trusting investor. Thus, their share starts decreasing even for the highest shares of honest CRAs. Only if there are 100% honest CRAs, the corresponding state on the $\beta^* = 1$ line is a fixed point. Then, sophisticated and trusting investors show the same performance. However, such a fixed point is unstable. If only a tiny share of inflating CRAs enters the market, they will be able to gain market share whenever they meet trusting investors, and thus make the latter die out in the end. The outcome for $C = 0$ is a bit more socially desirable than for $C > 0$, because in the first case, at least the sophisticated investors do not spend money monitoring the CRAs' behavior, namely the production of useless

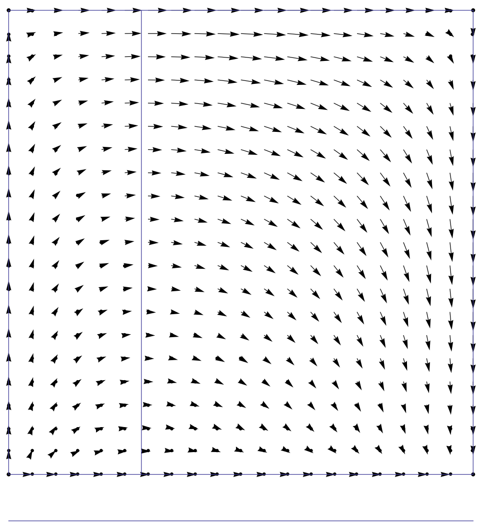


Figure 4: High Monitoring Costs (Relative to Share of Bad Investments). Vector Field for $\alpha^* \in (0,1), \beta^* < 0$. Parameter values: $\lambda = 0.5, R = 1.1, C = \mathbf{.55}, \Phi = 1, \rho = 1.4$

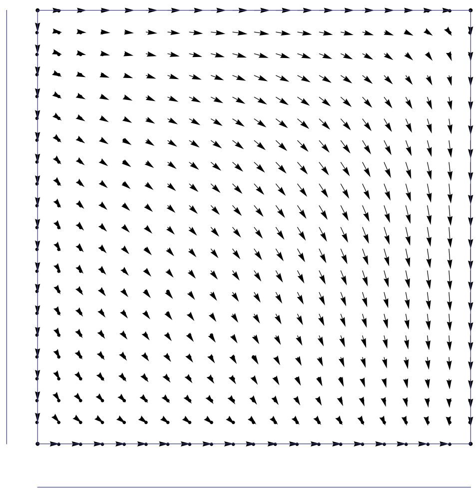
information. However, the issuers still pay fees to an obviously unproductive business.

A possible critique at this point is that even if there are no trusting investors on the market ($\alpha = 0$), the CRAs still receive fees from the issuers. Instead, the issuers might want to make sure that they do not pay for the valueless information delivered to the sophisticated investors, who are able to verify the work of the CRAs and the quality of the investments. I tackle this critique in the Appendix, Section A. There I assume that the CRAs only receive fees if they meet trusting investors and report a good rating to them.

4.3 Outcome: Trusting Investors and Inflating CRAs

Given there are costs to monitor the behavior of the CRAs, but the final outcome is that honest CRAs die out anyway, one could think that it would be more efficient to have only trusting investors remaining, so there is no waste of money for monitoring the CRAs. Indeed, such an outcome is reached if $C > 1 - \lambda$, meaning that the monitoring costs are high relative to the share of bad investments. Then it follows from (5) that $\beta^* < 0$. The resulting dynamics are visualized in Figure 4, given that $\alpha^* \in (0,1)$.

↑ β (share of honest CRAs)



(share of trusting investors) $\alpha \rightarrow$

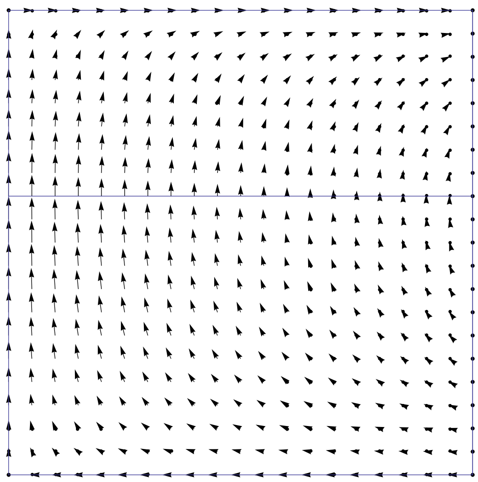
Figure 5: High Monitoring Costs and High Fees. Vector Field for $\alpha^*, \beta^* < 0$. Parameter values: $\lambda = 0.5, R = 1.1, C = \mathbf{.55}, \Phi = \mathbf{1.5}, \rho = 1.4$

In the resulting equilibrium, sophisticated investors die out, because monitoring does not pay off. This is a situation similar to what Bolton et al. (2011) associate with boom times. If there is a high ex-ante quality of investments on the market, then trusting investors perform better than sophisticated ones.

Again, there is a second, similar case. In addition to $C > 1 - \lambda$ (implying $\beta^* < 0$), consider $\rho < \Phi$, which means that the inflating CRA, if caught, faces reputation costs below the fees earned. Then it follows from (6) that also $\alpha^* < 0$. The resulting dynamics are visualized in Figure 5.

The final outcome is the same in either case. In the first case, i.e., $\alpha^* \in (0, 1)$, the share of honest CRAs might increase initially, if there are sufficiently many sophisticated investors. Then there is temporarily enough cost of reputation loss when inflating ratings, such that it pays off to be an honest CRA. However, in the long run, both sophisticated investors and honest CRAs will die out. In the second case, i.e., $\alpha^* < 0$, the reputation costs are too little relative to fees earned. Then the honest CRAs start dying out right away, from any starting point in the population space.

↑ β (share of honest CRAs)



(share of trusting investors) $\alpha \rightarrow$

Figure 6: Zero Fees. Vector Field for $\alpha^* = 1, \beta^* \in (0, 1)$. Parameter values: $\lambda = 0.5, R = 1.1, C = 0.2, \Phi = \mathbf{0}, \rho = 1.4$

4.4 Outcome: Trusting Investors and Honest CRAs

Now I present the most desirable outcome. It leads to a stable equilibrium, in which CRAs are honest and provide the best possible information, whereas investors are trusting and do not have to verify the work of the CRAs. Such an outcome is reached when $\Phi = 0$, which means that the CRA does not receive any fees. Then it follows from (6) that $\alpha^* = 1$. The resulting dynamics are visualized in Figure 6, given that $\beta^* \in (0, 1)$.

The result confirms the intuition that the "issuer pays" model is inherently wrong, if ratings shopping is possible and the CRAs effectively only receive fees for good ratings. My model suggests that the threat of sophisticated investors monitoring and possibly punishing the CRAs is effective. In the final outcome, however, no resources have to be spent on the monitoring, and therefore all investors become trusting, because there will be no inflating CRAs in the market, as long as the CRAs' income is not driven by whether they issue good or bad ratings. So the conclusion from this case is, similar to Camanho et al. (2010), that the main problem in the market for ratings and the first thing to abolish is the "issuer pays" model. Also Bolton et al. (2011) come to the conclusion that "upfront fees (as in the Cuomo plan) accompanied by enforcing automatic disclosure of ratings and oversight of analytical standards will minimize distortions from conflicts of interest and

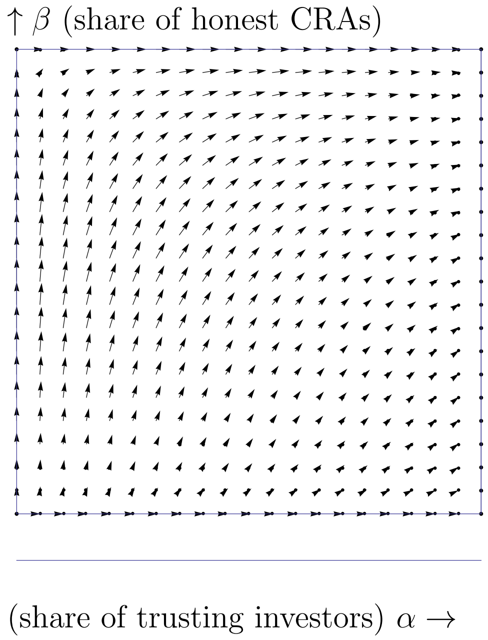


Figure 7: Zero Fees and High Monitoring Costs. Vector Field for $\alpha^* = 1, \beta^* < 0$. Parameter values: $\lambda = 0.5, R = 1.1, C = \mathbf{.55}, \Phi = \mathbf{0}, \rho = 1.4$

shopping."

A similar situation occurs if, in addition to $\Phi = 0$ (implying $\alpha^* = 1$), I consider the case $C > 1-\lambda$. This means that the monitoring costs exceed the share of bad investments. Then it follows from (5) that $\beta^* < 0$. The resulting dynamics are visualized in Figure 7.

Again, the only difference between the two cases is the path on which the final outcome is reached. For $\beta^* \in (0, 1)$, Figure 6 shows that for a low initial share of honest CRAs, there is an advantage for the sophisticated investors, as the monitoring costs are relatively low compared to the share of bad investments. In contrast, for $\beta^* < 0$, there are rather few bad investments on the market, so monitoring does not pay off for any initial state in the population space. In consequence, Figure 7 shows that the sophisticated investors start dying out right away. As a remark, it is possible that the $\alpha = 1$ margin is reached before $\beta = 1$. This means that the sophisticated investors have already died out, while there are still inflating CRAs in the market. Then there is no further change, as either type of CRA shows the same performance. Both receive no fees, and there is no punishment for inflating ratings. But even an invading sophisticated investor would die out again, because the monitoring costs still exceed the benefit of avoiding the bad investments.

Another related case occurs if $\rho \to \infty$, i.e., reputation costs are very high. Then it

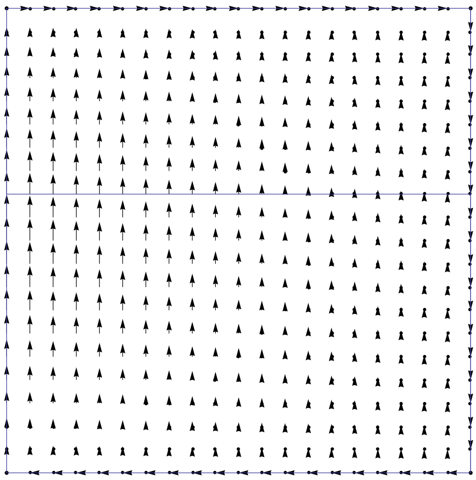


Figure 8: Very High Reputation Costs. Vector Field for $\alpha^* \to 1, \beta^* \in (0,1)$. Parameter values: $\lambda = 0.5, R = 1.1, C = 0.2, \Phi = 1, \rho = \mathbf{1'000'000}$ (looks the same for $\beta^* < 0$)

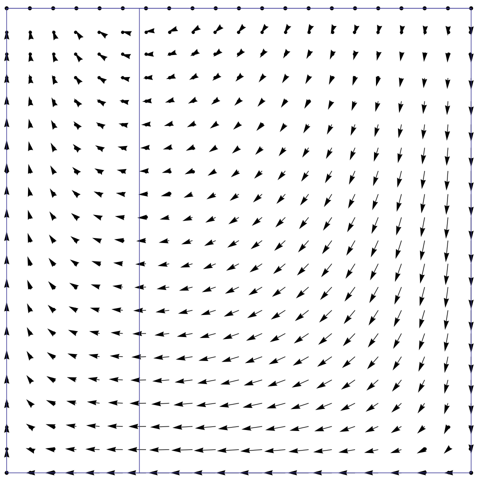
follows from (6) that $\alpha^* \to 1$. The resulting dynamics are visualized in Figure 8 for a very high $\rho = 1'000'000$, while still $\beta^* \in (0,1)$.

Similarly, the inflating CRAs die out. Once there are only honest CRAs remaining, it does not pay off anymore to be a sophisticated investor and monitor the CRAs. Therefore the final outcome is again one in which there are only trusting investors and honest CRAs. However, now the equilibrium is unstable. Once an inflating CRA manages to invade the market, it will collect more fees than the honest ones, without a risk of being punished (as long as the sophisticated investors remain extinct). Therefore the market might move on the $\alpha = 1$ line all the way to another equilibrium in (1,0), i.e., with 100% inflating CRAs and still only trusting investors. Again, this is not stable, as a single sophisticated investor invading will outperform the trusting ones, and thus the market might move towards (0,0). The same holds for the other two margins of the population space, neither corner is a stable equilibrium.

4.5 Outcome: Sophisticated Investors and Honest CRAs

Finally, consider the case $C = 0$, i.e., sophisticated investors can monitor the CRAs at zero cost. Then it follows from (5) that $\beta^* = 1$. The resulting dynamics are visualized in

↑ β (share of honest CRAs)



(share of trusting investors) $\alpha \rightarrow$

Figure 9: Zero Monitoring Costs. Vector Field for $\alpha^* \in (0,1), \beta^* = 1$. Parameter values: $\lambda = 0.5, R = 1.1, C = \mathbf{0}, \Phi = 1, \rho = 1.4$

Figure 9, given that $\alpha^* \in (0,1)$.

The resulting equilibrium is that trusting investors die out, and so do inflating CRAs. Independent of the initial population shares, the sophisticated investors are the more successful ones and therefore survive, whereas the inflating CRAs will finally be driven out of the market and all CRAs will be honest. The lesson from this case could be that the burden of monitoring the CRAs' performance should be taken over by a regulator, instead of the individual investors. After all, the sophisticated investors bear the cost of monitoring individually, but the trusting investors can free-ride on the benefits of these monitoring efforts. From a welfare perspective and within the framework of the model, the outcome in the current section is equally good as the one in Section 4.4. In the current section, the outcome still involves monitoring effort, however, the effort is costless by assumption. So in either section, the outcome is a market with 100% honest CRAs, and without monitoring cost.

5 Effect of competition and number of CRAs

The framework of Evolutionary Game Theory does not require to explicitly specify the number of market participants. It allows for an arbitrary number of participants in the populations of both investors and CRAs. I see it as a reasonable assumption to regard the number of investors as uncountably large. In the following, the focus will therefore be on the number of CRAs and their effect on the outcome of the game. First I present the special cases of a monopoly CRA and a duopoly of CRAs. Then I turn back to the oligopoly case and present a way to explicitly model the effect of the number of CRAs on the market.

5.1 Monopoly CRA

First I assume, as stated in Section 2.3, that even a monopoly CRA is programmed to follow a pure strategy, i.e. it cannot freely choose the most advantageous strategy. In that case, I have either $\beta = 0$ or $\beta = 1$ in the language of the model. This means, the monopoly CRA is either inflating or honest, but there will be no change of its behavior over time. From the point of view of the investors, this situation is equivalent to facing a population of many CRAs that follow all the same strategy.

Second I relax the mentioned assumption from Section 2.3, and I ask instead, what would be the optimal behavior of a CRA that can strategically be either inflating or honest as a best response to the population of investors it is facing? Still, I assume that the population of investors consists of uncountably many individuals, each of them programmed to follow a pure strategy of being either sophisticated or trusting.[3] Thus, the monopoly CRA meets an honest investor in each interaction with probability α, and a sophisticated investor with probability $(1 - \alpha)$. Correspondingly, its expected payoffs are Π_H^{CRA} and Π_I^{CRA} as given in (1) and (2), respectively. Obviously, the resulting optimal strategy is to be honest if $\Pi_H^{CRA} > \Pi_I^{CRA}$ (i.e., $\Delta\Pi^{CRA} > 0$) and be inflating otherwise. If the investors' population shares are adjusting clockwise, like in the base case, then the

[3] As a link to traditional game theory, consider a game between a monopoly CRA and a single investor. Then the solution is a Nash equilibrium in mixed strategies, with the investor being trusting with probability α^*, and the CRA being honest with probability β^*.

prescribed behavior for the monopoly CRA is a bang-bang strategy of jumping back and forth between $\beta = 0$ and $\beta = 1$, while the investors' shares are smoothly moving back and forth in a small region around α^*. In the other cases with corner solutions, the investors' share α might move monotonically into one direction, with the monopoly CRA's strategy switching when α^* is crossed.

5.2 Duopoly of CRAs

As in the last section, I first assume that either CRA in duopoly is programmed to a pure strategy. If both follow the same strategy, then there will be a constant $\beta = 0$ or $\beta = 1$ situation for the CRAs, which from the point of view of the investors is again equivalent to facing a population of many CRAs, or a monopoly CRA, following the same strategy. If the two CRAs follow different strategies, then β and $(1 - \beta)$, respectively, are the market shares of the one that is honest and inflating, respectively. Their shares develop over time as derived earlier for arbitrary many CRAs, and also for the investors, it makes no difference to facing a population of many CRAs, other things equal. However, in the next section I will discuss how the number of CRAs might influence the model's parameters and thus also be relevant for the investors.

Second, what if each CRA can choose the optimal strategy? Note that there is no direct interaction between the CRAs. Therefore, for each CRA interacting with the investors, the optimal strategies are just the same as derived above for the monopoly case. On the other hand, from the investor point of view it does not make a difference how many CRAs there are actively choosing, they will experience the same bang-bang strategies no matter which CRA they are meeting.

5.3 Oligopoly of CRAs

As shown in the previous two sections, the number of CRAs as such does not make an important difference for the dynamics on the market, given that other things are equal. However, the latter is a strong and questionable assumption. On the contrary, it is very reasonable to assume that the model's parameters are affected by the market structure. In the following, I allow the verification cost C_N, the fee Φ_N charged by the CRA, and the reputation cost ρ_N, to be dependent on the number of CRAs N, as indicated by the subscript.

5.3.1 Assumptions

I hypothesize that the verification cost C_N and the reputation cost ρ_N should be increasing in the number of CRAs N, while the fee Φ_N charged by the CRA should be decreasing. The motivation is that it takes more effort, i.e., higher verification costs, for the sophisticated investors to monitor the CRAs' work, if they have to cover a market with more participants. On the other hand, such a market provides more alternatives, therefore the CRAs' risk to lose business to competitors and thus their reputation costs are higher with more competitors. While I assume that a monopoly CRA can charge the highest fees for its services, increasing competition drives down the fees. As a simple specification of the three functions that satisfies the mentioned hypotheses, I choose

$$C_N = C_1 \cdot N \tag{9}$$

given a verification cost $C_1 > 0$ for the monopoly CRA. Moreover, I choose

$$\rho_N = \rho_2(N - 1) \tag{10}$$

given a reputation cost $\rho_2 > 0$ for the CRAs in duopoly. This implies zero reputation cost for the monopoly CRA. The motivation is that for example for regulatory reasons, the issuers need at least one rating in any case. Therefore there is no outside option, even if the CRA is known to be cheating.[4] Finally, I choose

$$\Phi_N = \frac{\Phi_1}{N} \tag{11}$$

given a fee $\Phi_1 > 0$ charged by the monopoly CRA. The fee is thus approaching zero for a perfectly competitive market with high N.[5]

[4] If the issuers even need more than one rating for regulatory reasons, then the reputation cost will remain zero as long as there are not more CRAs than ratings needed.

[5] An alternative specification would be $\Phi_N = (\Phi_1 - MC)/N + MC$, ensuring that on a perfectly competitive market the fees charged still equal the CRAs' marginal costs MC.

5.3.2 Analysis

I start with rewriting the fixed lines (5) and (6) as

$$\beta^* = 1 - \frac{C_N}{1-\lambda} \quad \text{and} \quad \alpha^* = 1 - \frac{\Phi_N}{\rho_N},$$

respectively, to account for the dependence on the number of CRAs N. Similarly, the conditions for an interior fixed point (7) and (8) become

$$0 < C_N < 1 - \lambda \quad \text{and} \quad 0 < \Phi_N < \rho_N,$$

respectively. Recalling that C_N should be increasing in N, it is possible (given that $C_1 < 1 - \lambda$) that there is a switch from the base case towards the case with $\beta^* < 0$, resulting in trusting investors and inflating CRAs, as described in Section 4.3. According to (9), this happens when

$$N > \frac{1-\lambda}{C_1}.$$

Interestingly, the switch does not necessarily happen when switching from monopoly to duopoly or from duopoly to oligopoly. Depending on the magnitude of the verification cost relative to the average quality of investments in the market, one market structure could be prevailing even for a market of several CRAs, and then the market entry of one more CRA could suddenly cause a switch of the market structure. Here I see a relevant contribution to the existing literature, for example Bolton et al. (2011). I point out that it is not enough to compare monopoly and duopoly markets to answer the general question what is the effect of more competition.

Now I analyze the behavior for perfect competition, i.e. when the number of CRAs N approaches infinity. From (9), (10), and (11), I have

$$C_N \to \infty, \quad \rho_N \to \infty, \quad \text{and} \quad \Phi_N \to 0 \quad \text{for} \quad N \to \infty.$$

In this case, $\alpha^* \to 1$ and $\beta^* < 0$, which corresponds to the case resulting in trusting investors and honest CRAs, as described in Section 4.4.

For a finite number of CRAs, I focus on the assumptions that ρ_N should be increasing and Φ_N should be decreasing in N. Thus, for large N, I have $\Phi_N < \rho_N$, which corresponds to the base case. But if $\Phi_1 > \rho_1$, which is the case for my assumption of $\Phi_1 > 0$ and $\rho_1 = 0$, there will be switch of the market structure at some specific N. For the monopoly case and $\Phi_1 > \rho_1$, I have $\alpha^* < 0$, and the market structure is resulting in sophisticated investors and inflating CRAs, as described in Section 4.2. The critical N, at which the market switches to the base case, satisfies

$$\Phi_N < \rho_N \Leftrightarrow \frac{\Phi_1}{N} < \rho_2(N-1) \Leftrightarrow N > \frac{1}{2}(1+\sqrt{1+4\frac{\Phi_1}{\rho_2}}) \geq 1.$$

Again, the switch in market structure does not necessarily happen when switching from monopoly to duopoly or from duopoly to oligopoly. Depending on the relation between reputation cost and fees, it could be that monopoly and duopoly show the same behavior, i.e. do not leave room for honest CRAs, and the switch happens when a third or fourth CRA enters the market.

Note that my specification $\rho_1 = 0$ leads to a special situation for the monopoly CRA. The solution for α^* from (6) is not valid. Instead, (4) yields

$$\Delta\Pi^{CRA}(\rho = 0) = -(1-\lambda)\Phi,$$

which is negative for all α, if $\Phi > 0$, and zero for all α, if $\Phi = 0$. Thus, for positive fees the honest CRAs die out, or in a monopoly in which the CRA is allowed to choose its action, it will be inflating. The case $\Phi = \rho = 0$ is not reached in my definitions above: in the monopoly situation, the only case with $\rho = 0$, I assume that the monopoly CRA still receives positive fees.

Summarizing, there are the following possible transitions dependent on the number of CRAs N: For the monopoly case ($N = 1$) with zero reputation costs, I have the market structure resulting in sophisticated investors and an inflating CRA. This may remain the same for small numbers of CRAs, e.g., in duopoly. Then, when $N > \frac{1}{2}(1+\sqrt{1+4\frac{\Phi_1}{\rho_2}})$, the reputation costs exceed the fees and there is a switch to the base case. For even higher N, it becomes too expensive for sophisticated investors to monitor the CRAs, relative to the risk of receiving a bad investment once in a while. Therefore there is a switch from the base case towards the case with trusting investors and inflating CRAs, when $N > \frac{1-\lambda}{C_1}$.[6]

[6]If $\frac{1-\lambda}{C_1} < \frac{1}{2}(1+\sqrt{1+4\frac{\Phi_1}{\rho_2}})$, then the intermediate area with the base case is disappearing. There is a switch from the case with sophisticated investors and inflating CRAs towards the case with trusting investors and inflating CRAs, when $N > \frac{1-\lambda}{C_1}$, and then a further switch between the two subcases of Section 4.3, with the same resulting outcome, when $N > \frac{1}{2}(1+\sqrt{1+4\frac{\Phi_1}{\rho_2}})$.

Finally, for $N \to \infty$, the fees become minimal and reputation costs are huge. Thus there are no more incentives for ratings inflation, and the case resulting in trusting investors and honest CRAs is reached.

6 Conclusion

First, I draw a conclusion on policy implications of my work, without explicitly addressing the number of CRAs on the market. It is based on the analysis of the desirable cases in Section 4, i.e., those in which the honest CRAs survive in the long run. From Section 4.4, I conclude that it is essential to find an alternative solution to the "issuer pays" model, and particularly to prevent that rating agencies can achieve higher revenues by issuing good ratings. From Section 4.5, I conclude that the monitoring of CRAs' performance and their possible punishment should rather be done (even more so) by regulators, rather than individual investors. If these issues can be solved, then the market for credit ratings will function well, independent of the size of the CRA market.

Second, I conclude from Section 5 that a perfectly competitive CRA market can prevent incentives for ratings inflation and is thus beneficial for ratings quality, which is in line with conventional wisdom and the regulators' view, but not with most other recent research on CRAs and competition. For small numbers of CRAs, e.g., monopoly, duopoly, or maybe a market with three or four agencies, I have a market structure resulting in sophisticated investors and inflating CRAs. For intermediate CRA market sizes, one might observe a cyclic change of market shares as described in my base case. If the CRA market is even larger, trusting investors will dominate, but CRAs still behave inflating. Only for a perfectly competitive CRA market, on which ratings inflation creates only little fees, but prohibitively high reputation costs, I predict both trusting investors and honest CRAs to survive in the long run.

References

Becker, B. and T. Milbourn (2011). How did increased competition affect credit ratings? *Journal of Financial Economics 101*, 493–514.

Bolton, P., X. Freixas, and J. Shapiro (2011, March). The credit ratings game. Journal of Finance, forthcoming.

Camanho, N., P. Deb, and Z. Liu (2010, March). Credit rating and competition. Working Paper.

Doherty, N. A., A. Kartasheva, and R. D. Phillips (2009, February). Information effect of entry into credit ratings market: The case of insurers' ratings. Working Paper.

He, J., J. Qian, and P. E. Strahan (2010, December). Credit ratings and the evolution of the mortgage-backed securities market. Working Paper.

Hörner, J. (2002). Reputation and competition. *American Economic Review 92*(3), 644–663.

Klein, B. and K. B. Leffler (1981). The role of market forces in assuring contractual performance. *Journal of Political Economy 89*(4), 615–641.

Mathis, J., J. McAndrews, and J.-C. Rochet (2009). Rating the raters: Are reputational concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics 56*, 657–674.

Schuster, P., K. Sigmund, J. Hofbauer, and R. Wolff (1981). Selfregulation of behaviour in animal societies. part 2: Games between two populations without selfinteraction. *Biological Cybernetics 40*, 9–15.

Skreta, V. and L. Veldkamp (2009). Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics 56*, 678–695.

Stolper, A. (2009). Regulation of credit rating agencies. *Journal of Banking & Finance 33*(7), 1266–1273.

Weibull, J. W. (1997). *Evolutionary Game Theory*. Cambridge, MA: MIT Press.

A Alternative specification of payoffs

In this section, I tackle the critique that so far, even if there are no trusting investors on the market ($\alpha = 0$), the CRAs still receive fees from the issuers. From now on, I assume that the CRAs only receive fees if they meet trusting investors and report a good rating to them. In that way, the issuers make sure that they do not pay for the valueless information delivered to the sophisticated investors, who are able to verify the work of the CRAs and the quality of the investments. So in my alternative specification, the issuers effectively become an active third population in the game and negotiate fees dependent on the expected benefits. The payoffs to the investors are not affected by my alternative specification of payoffs.

A.1 Payoffs

First, I state the new payoffs for the honest CRAs. They still only give a good rating if they observe a good investment, which occurs with probability λ. Whether they receive the fee depends on the type of investor they meet. If it is a trusting investor, they do, so

$$X_{HT} = \lambda\Phi.$$

However, if they meet a sophisticated investor, they do not receive the fee, and

$$X_{HS} = 0.$$

Thus, the resulting expected payoff for honest CRAs is

$$\Pi_H^{CRA} = \alpha X_{HT} + (1-\alpha)X_{HS} = \alpha\lambda\Phi.$$

It might be hard to motivate that the fees paid by the issuers can be conditioned on the individual investors that the CRAs meet, i.e., that $X_{HT} \neq X_{HS}$. An alternative interpretation is that the fee Π_H^{CRA} is paid by the issuers according to their expectation how many trusting investors the CRAs will meet on average. The dynamics only require Π_H^{CRA}, not the individual payoffs. So an equivalent specification would be that the CRAs receive Π_H^{CRA} in each interaction, regardless of the type of investor they meet.

Table 5: CRAs' Payoffs, Alternative Specification.

Investor / CRA	honest	inflating
trusting	$X_{HT} = \lambda\Phi$	$X_{IT} = \Phi$
sophisticated	$X_{HS} = 0$	$X_{IS} = -(1-\lambda)\rho$
expected	$\Pi_H^{CRA} = \alpha\lambda\Phi$	$\Pi_I^{CRA} = \alpha\Phi - (1-\alpha)(1-\lambda)\rho$

Second, consider the inflating CRAs. If they are meeting a trusting investor, they receive

$$X_{IT} = \Phi,$$

like in the original specification. As they always give good ratings, they are always paid by the issuers, regardless of the quality of the investment. Against a sophisticated investor, however, their expected payoff is

$$X_{IS} = -(1-\lambda)\rho.$$

Here the issuer does not pay the fee, as the sophisticated investors can judge the investment quality themselves. Still, the CRAs are punished whenever they rate a bad investment as good, which happens with probability $(1-\lambda)$. The resulting expected payoff for inflating CRAs is

$$\Pi_I^{CRA} = \alpha X_{IT} + (1-\alpha)X_{IS} = \alpha\Phi - (1-\alpha)(1-\lambda)\rho.$$

Similar to the specification for the honest CRAs, one might alternatively interpret the resulting expected payoff Π_I^{CRA} in the following way: the CRAs receive the fee $\alpha\Phi$ for each interaction, independent of the type of investor they meet. The issuers adjust the fee by the expected usefulness of the rating, i.e., the likelihood α that it reaches a trusting investor. On top of that, the CRAs face the cost $(1-\lambda)\rho$ whenever they meet a sophisticated investor.

The payoffs are summarized in Table 5. As in the original specification, the average payoff in the population of CRAs is defined as

$$\bar{\Pi}^{CRA} = \beta\Pi_H^{CRA} + (1-\beta)\Pi_I^{CRA}.$$

A.2 Analysis

For the evolutionary dynamics as in Section 3.1, I need

$$\Delta\Pi^{CRA} = \Pi_H^{CRA} - \Pi_I^{CRA} = (1-\lambda)((1-\alpha)\rho - \alpha\Phi).$$

The corresponding value $\Delta\Pi^{Inv}$ for the investors remains unchanged. Likewise, for the fixed lines as in Section 3.2.1 it holds that β^* remains unchanged. However, the second fixed line becomes

$$\Delta\Pi^{CRA} = 0 \Leftrightarrow \alpha = \alpha^* := \frac{\rho}{\rho + \Phi}.$$

For an interior fixed point, the conditions are now $0 < C < 1 - \lambda$ as in (7), and from $0 < \frac{\rho}{\rho+\Phi} < 1$, I have

$$0 < \Phi \quad \text{and} \quad 0 < \rho.$$

For the extreme cases $\Phi = 0$ and $\rho = 0$, I have $\alpha^* = 1$ and $\alpha^* = 0$, respectively. For the properties of the fixed point, I derive the corresponding matrices as in Section 3.2.2. The investor payoff matrices $\bar{A}$ and A remain unchanged. The CRA's payoffs from Table 5 become

$$\bar{B} = \begin{pmatrix} \lambda\Phi & 0 \\ \Phi & -(1-\lambda)\rho \end{pmatrix}.$$

Transformation leads to

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & (1-\lambda)\rho \\ (1-\lambda)\Phi & 0 \end{pmatrix}.$$

As in Section 3.2.2, I can verify that $\alpha^* = \frac{b_{12}}{b_{12}+b_{21}}$. Next I test whether an interior fixed point in my model, under the alternative specification, is a saddle or a center. Since

$$a_{12}b_{12} = (\underbrace{C - (1-\lambda)}_{<0 \text{ see (7)}})(1-\lambda)\rho < 0$$

holds, it is a center according to Schuster et al. (1981).

A.3 Effect of competition and number of CRAs

Next I repeat the analysis of Section 5 for the alternative specification of payoffs. The fixed lines as functions of the number of CRAs N are now

$$\beta^* = 1 - \frac{C_N}{1-\lambda} \quad \text{and} \quad \alpha^* = \frac{\rho_N}{\rho_N + \Phi_N},$$

respectively. While β^* remains unchanged, α^* reflects the changes derived earlier in this section. Similarly, the conditions for an interior fixed point become

$$0 < C_N < 1 - \lambda, \quad 0 < \Phi_N, \quad \text{and} \quad 0 < \rho_N,$$

respectively. I use the definitions introduced in Section 5 for the verification cost C_N, the fee Φ_N charged by the CRA, and the reputation cost ρ_N. Then in monopoly, $\rho_1 = 0$ and thus $\alpha^* = 0$. If the monopoly CRA can choose its strategy, it will be inflating. Depending on the relation between verification cost and average quality of investments on the market, the sophisticated investors will be the only ones to survive for $C_1 < 1-\lambda$, and the trusting ones for $C_1 > 1-\lambda$, respectively. The expected payoff for the CRA is $\Pi_I^{CRA}(\rho = 0) = \alpha\Phi$. Thus, the monopoly CRA can either not collect any fees, if $\alpha = 0$, and the issuers know there are no trusting investors on the market, or the CRA collects the full fee, if the investment quality is so good that only trusting investors are remaining in the market, who are happy to use the CRA's services despite their bad quality. For perfect competition ($N \to \infty$), I have $\Phi_N \to 0$ and thus $\alpha^* \to 1$. In duopoly and oligopoly with any finite number of CRAs, $\alpha^* \in (0, 1)$ will be an interior solution.