Endogenous Response to the Interbank Loan 'Network Tax': Stability and Efficiency[*]

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Abstract

The purpose of this paper is to study a simple application of a tax levied on the bank's contribution to contagion risk. We find that such a regime may change the trade-off between liquidity coinsurance and counterparty risk that motivates the endogenous formation of the financial network in the first place, potentially leading to a less connected architecture. Furthermore, if that regime bestows the weight of the levy on both borrower and lender it can shift the system towards safer grounds. However, a safer network may be less efficient due to the conjugation of countervailing effects that must be assessed when the objective is welfare maximisation. Since we model bank interactions as a network formation game, we are able to provide an account of the rationale behind these effects, which can be determinant to the design process of the policy function.

Keywords: Financial Network, Regulation, Counterparty Risk, Liquidity Coinsurance.

JEL Classification Numbers: D85, G18, G21.

1 Introduction

The turmoil in the financial markets that had its roots in the 2007-2009 US subprime crisis prompted government action all around the world motivated by contagion concerns, leaving a heavy bill for

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the tax payers to pick up. The assertion that government intervention is unavoidable in the midst of a full blown financial crisis, moved the focus of regulation proposals towards *“measures to reduce and address the fiscal costs of future financial failures”* (IMF; 2010, p. 2). One of those measures is to involve the banks in the cost bearing of the financial system’s rescue. This prompts the question of how to define this contribution. The purpose of this paper is to study a simple application of a tax levied on the bank’s contribution to contagion risk, with a special emphasis on the system’s connectedness.

The network structure of the interbank market is a crucial point to contagion since it provides information on how the failure of a bank may spillover to its neighbours, potentially leading to the widespread collapse of the system. Imagine that bank A is connected to bank B that goes bankrupt and suppose also that this bankruptcy brings with it the bankruptcy of A. Now, suppose that C is not connected neither to A nor to B, then the impact on C is considerably different comparing to the case where a connection exists with A and/or B. Fundamentally the network of connections is important to study how contagion can spread once an initial event triggers a cascade of bankruptcies.

We aim to study the impact that the levy, ’network tax’, has on the endogenous formation of the network. We are able to do so by modelling the tax as an exogenous shock to the environment that forces the system to adjust to it. To this end, we build a network formation game, which to the best of our knowledge is a novel approach in the analysis of such a tax. In order to do so, we build on the model of Castiglionesi and Navarro (2010), in which several banks with different levels of capitalisation may establish credit lines (i.e., connections) among each other. The assets chosen by the banks depend on their level of capitalisation. Poorly capitalised banks (peripheral) gamble with their customers deposits to obtain private benefits, whereas well capitalised banks invest in safe projects. When well capitalised banks (core) decide whether or not to establish credit lines with less capitalised banks they do so based on a trade-off between liquidity coinsurance (i.e., insurance against idiosyncratic liquidity shocks) and exposure to credit risk (i.e., exposure to a potential interbank loss). A contributory regime that charges banks according to their exposure to contagion risk changes the trade-off that motivates the formation of the network in the first place. Now, banks that contribute more to the risk of contagion are assigned a heavier tax regime, which has the potential to shift their risky activities towards safer grounds (i.e., the tax forces banks to re-evaluate their investment decisions to avoid the fiscal burden - re-evaluation effect). Furthermore, since we assume that the burden of the tax is divided equally between the lender and the borrower, safer banks are forced to internalise the costs of connecting to risky neighbours. This makes the conditions under which a safer bank is willing to connect to a peripheral neighbour more demanding, potentially leading to a less connected network.

Although a less connected network may be safer (via the reduction in the contagion probability and the reallocation of coinsurance towards safer banks - counterparty risk and reallocation effects), it may also be less efficient (via the loss in coinsurance opportunities - coinsurance effect). Thus, a potential trade-off between stability and efficiency may emerge. Stability here can be measured by the probability of contagion and consequently of government intervention, i.e., the lower the number of risky links the higher is the system's stability. Efficiency in this context means the extent to which the benefits of coinsurance can be accrued by the participants in the banking system. More importantly, given our definition of the welfare function, these benefits are not restricted to the shareholders. In the end of the day, the conjugation of the variety of effects triggered by the tax leads to a non-monotonic relation between the tax and total welfare, underscoring the importance of understanding what motivates the formation of the network. Therefore, any attempt to design a policy function that fails to take this into account may not be optimal and welfare reducing.

The paper proposes a three bank model, in which each bank is composed of depositors and shareholders/investors. Depositors have the right to withdraw a conditional promised return, whereas shareholders are entitled the residual value after the banks' portfolio is liquidated and depositors are repaid in the final period, i.e., depositors are senior creditors with respect to all other (even with respect to interbank claims). Shareholders (that also play the role of managers) define banks' investment decisions. There are three alternatives: (i) a safe asset; (ii) a risky asset that yields the same expected return of the safe asset if it succeeds and nothing if it fails, but entitles the shareholders to a private benefit and; (iii) liquidity used as a precautionary measure to buffer an idiosyncratic liquidity shock taking place in the interim period. To say that an investment is safe has, in this context, a very particular meaning. An asset is taken to be risky if its return is uncertain even if it is refinanced in the interim period.

Since whatever the bank's illiquid investment decision might be it needs to be refinanced in the interim period, banks can establish relationships among themselves in order to obtain liquidity coinsurance. These connections that take the form of conditional reciprocal credit lines, i.e, the financial network is an undirected one. When deciding their 'neighbours', banks trade-off the benefit of coinsurance with the potential exposure to contagion if the neighbouring bank invests in the risky asset, i.e., if it gambles. Since investors have limited liability, the investment in the risky asset occurs when the bank is poorly capitalised.

The literature on financial networks 'exploded' in the aftermath of the recent financial crisis. Most of the papers written on this issue analyse contagion effects (for a survey see Allen and Babus (2009)) and only few of them take into account the endogenous formation of the network (e.g., Babus (2006), Leitner (2005) and Castiglionesi and Navarro (2010)). The importance of endogenous network formation is related to the Lucas' critique (Lucas; 1976), i.e., when designing a policy its makers should take into account if the system can adjust to the policy, it will do just that. Acharya et al. (2010) presents a model where banks internalise their systemic contribution if they are taxed on it, but the focus is on the portfolio allocation decision and not on the formation of the interbank loan network. The paper closest to ours is Krahnen and Bluhm (2010), who use a numerical model, that takes into account the interconnections among banks (interbank claims and portfolio selection decisions), to determine the optimal systemic risk charge based on the Shapley Value (Shapley; 1952) (other papers that use a similar approach are Tarashev et al. (2009), Staum (2009) and Tarashev and Drehmann (2011)). Although they also explain the intuition of how banks can adjust to the tax, they take the network structure as given which differentiates their work from ours. The importance of modelling this issue as a network formation game is to inform policy-makers on how their decisions will actually affect the behaviour they are trying to regulate, which can be a crucial factor in their success. We stress the importance of the trade-off that motivates the formation of the network in the definition of the optimal policy function.

The paper is organised as follows. The next section introduces the main ingredients needed to assess the impact of the tax. Section 3 deals with this impact on the re-formation of the interbank loan network and discusses some policy implications. Section 4 concludes.

2 The Model

The model used in this paper follows closely the one proposed by Castiglionesi and Navarro (2010), that suggests as a motivation for banks to form links with each other the benefit of liquidity coinsurance. However, as undercapitalised banks' shareholders are assumed to derive a private benefit from gambling, a trade-off emerges in the network formation game. On the one hand, banks have the benefit of insuring against their liquidity shocks when they establish links with other banks. On the other hand, forming a link exposes a bank to the risk of a gambling neighbour if the gambling asset defaults. The setup we are using ignores the issue of capital transfers which is heavily studied in the base model. There are three main differences between the basic setup used here to analyse the 'network tax' and the one presented in Castiglionesi and Navarro (2010): (i) the liquidity coinsurance function depends now on the entire network and not only on the number of direct neighbours; (ii) the counterparty risk factor is a function that depends on the network and not on a constant as in the baseline model and; (iii) the total welfare function includes now the expected costs of government intervention.

2.1 Basic Setup

There are three banks. The events play out in five dates $t \in \{0,1,2,3,4\}$. A continuum of consumers endowed with a monetary unit at $t = 0$ lie in a region contained in the set $N = \{1,2,3\}$. In each region there is a representative bank, where the consumers deposit their early endowments and withdraw them only in the final date to consume. Each bank i is also endowed in the initial period with capital, denoted by $e_i \in [0, \bar{e}]$ with $e = \{e_1, e_2, e_3\}$, owned by its shareholders. The economy is thus represented by the pair (N, e). After the endowments are realised and the financial network is chosen (at $t = 0$), banks choose the projects that they invest in (at $t = 1$), offer contingent deposit contracts to consumers (at $t = 2$), illiquid assets receive an idiosyncratic liquidity shock prompting their refinancing at the cost of losing the returns if this requirement is not met (at $t = 3$) and finally returns materialise and agents consume (at $t = 4$).

Banks have two illiquid investment opportunities: (i) a 'safe' project, denoted by b, that yields an expected return $\overline{R} > 1$ if it is refinanced in the advent of the liquidity shock and; (ii) a 'gambling' project, denoted by g, that yields $\overline{R}$ with probability α and 0 with probability $(1-\alpha)$ plus a certain private benefit bestowed on the shareholders denoted by $B > 0$. The difference between project b and g is that even if g is refinanced it can have a null return with positive probability. Furthermore, banks can also invest in liquidity to self-insure against the idiosyncratic shock.

The financial network, i.e., the set of sets of neighbouring banks is denoted by $K = \underset{i \in N}{\cup} K_i$ with $K_i \subseteq N$ and $k_i = \#K_i$. Similarly, the set of gambling neighbours is denoted by $G_i \subseteq K_i$ with $g_i = \#G_i$. As in (Castiglionesi and Navarro; 2010) we only consider undirected networks[1], i.e., credit lines are reciprocal conditional on the realisation of the shock.

2.2 Network Structure and Liquidity Coinsurance

The liquidity shock determines the amount needed to refinance the illiquid asset. If the bank cannot refinance it, then the return is lost. Therefore, in order to avoid losing the return, banks wish to establish connections (e.g. credit lines) with other banks that receive a negatively correlated shock. Let equation (1) denote the probability of a bank being able to refinance its own illiquid investments:

$$\varphi(k_i|K) = \frac{1}{2} + \sum_{j \in K_i} \sum_{l=0}^{k_j - 1} \frac{C_l^{k_j - 1}}{2^{k_j + 1}} \frac{1}{l+1} - \frac{k_i - 1}{2^3}. \tag{1}$$

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[1] If we were dealing with a directed network, i.e., a link would mean non reciprocal credit lines, the trade-off that would motivate the formation of a single link would be completely different form the present case since a link would only provide liquidity insurance for one of the banks.

with[2] $\varphi'(k_i|K) > 0$ and $\varphi''(k_i|K) < 0$

Equation (1), only valid for the three banks case, makes the probability of coinsurance dependent on the entire network (unlike the form assumed in Castiglionesi and Navarro (2010)), reflecting that even if banks are all connected amongst themselves there may be not enough liquidity available to fulfil all the needs. Therefore, when a liquidity endowed neighbour is linked with two neighbours in need of liquidity, it is assumed that the liquidity needs of each bank are satisfied with probability $\frac{1}{2}$.

The highly combinatorial nature of this problem is reflected in the terms $\sum_{j\in K_i}\sum_{l=0}^{k_j-1}\frac{C_l^{k_j-1}}{2^{k_j+1}}\frac{1}{l+1}$ and $\frac{k_i-1}{2^N}$. The first term, calculates the probability of coinsurance given the number of neighbour's neighbours, i.e., if a bank i is connected to bank j that has a negatively correlated shock the resulting increase in the probability of coinsurance depends on the number of the remaining j's neighbours (other than i, i.e, $k_j - 1$) that also receive a negatively correlated shock with j ($l \in \{0,...,k_j-1\}$). This way of calculating the marginal gains of coinsurance leads to double counting issues, which are corrected by subtracting the final term.

The idea is to calculate de probability of lending given the set of neighbours' neighbours. The absence of negative liquidity shock in half of the states of nature, i.e, a bank that receives a low liquidity shocks has a liquidity surplus so it does not need to go to the interbank loan market to refinance its assets, accounts for the initial term $\frac{1}{2}$. However, the probability of coinsurance is not relevant *per se*. The important factor is the effect of coinsurance in the expected payoff that is given by $f(k_i|K) = 1 + \varphi(k_i|K)$.

Assuming perfect competition, depositors receive the full advantage from liquidity insurance, such that the depositors' expected payoff in bank i is given by: $D_i = [1+\varphi(k_i|K)]R$, with R being the expected autarky return. Since investment in liquidity precludes the bank from achieving the maximum profitability from the safe asset, we have $R < \bar{R}$[3]. Both high (ω_H) and low (ω_L) shocks are i.i.d. (i.e., there is aggregate uncertainty) and occur with the same probability $\left(\frac{1}{2}\right)$.

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[2] The signs of the first and second derivatives indicate that an additional link increases the probability of coinsurance albeit at a decreasing rate.

[3] The proof of this can be found in Castiglionesi and Navarro (2010, p. 40), where the authors find that $R = \frac{1}{2}\left[(1-\gamma)\bar{R}+\omega_H\right]$ with $\gamma(=\frac{\omega_H-\omega_L}{2})$ and ω_H denoting the optimal choice of liquidity (when the liquidity shocks are neither too high or too low) and the high liquidity shock, respectively.

2.3 Network Structure and Counterparty Risk

At $t = 1$, banks make their investment decisions. Let $s_i \in \{b, g\}$ be the project chosen by bank i. Taking the network, K, and the strategy profile, $s = \{s_i\}_{i \in N}$, as given, let $p_i(K, s)$ denote the probability that bank i does not go bankrupt, i.e., is able to repay the amount contractualised to its depositors. Assuming:

$$p_i(K,s) = \begin{cases} \prod_{j \in K_i} \pi_j(s_j) & if \quad s_i = b \\ \alpha \prod_{j \in K_i} \pi_j(s_j) & if \quad s_i = g \end{cases}, \tag{2}$$

with $\pi_j(s_j) = \begin{cases} 1 & if \quad s_j = b \\ \eta_i(\alpha, K) & if \quad s_j = g \end{cases}$.

The formulation chosen for p_i reflects that the bankruptcy probability is higher for a bank if it gambles, *ceteris paribus*.

The function $\eta_i(\alpha, K)$ can be interpreted as the risk exposure to the gambling counterparties, i.e., it is the probability that a loss in the interbank loan will be borne by bank i. This probability depends on two other probabilities: (i) lending to a gambling neighbour (denoted by event LGN), given by equation (3) and; (ii) the gambling lender defaults (denoted by event GLD), which is by definition $(1-\alpha)$.

$$prob(LGN|K) = \sum_{j \in g_i} \left[\left(\frac{1}{2}\right)^N \left(\sum_{l=0}^{k_j - 1} \frac{1}{1+l} \right) 2^{N-(k_j+1)} \right] - \frac{g_i - 1}{2^N}. \tag{3}$$

From equation (3), follows that the probability of lending to a gambling bank: (a) increases with the number of gambling neighbours, holding the neighbours' neighbours constant and; (b) decreases with the number of number of neighbours' neighbours, holding the lender's links constant.

Combining (i) and (ii) we can derive equation (4), which reflects the counterparty risk:

$$\eta_i(\alpha, K) = 1 - prob(LGN|K) \times (1 - \alpha). \tag{4}$$

Writing η_i has a function of the current network structure allows for the counterparty risk to be structurally traced back to its origin.

2.4 Expected Payoffs and Investment Project Choice

Since there are two types of agents in the economy, the expected payoff must be determined for shareholders and depositors. As it was assumed that the depositors receive the total gain of coinsurance[4], then their payoff is given by: $M_i(K,s) = p_i(K,s) D_i = p_i(K,s) f(k_i|K) R$. Another assumption is that $\alpha R \geq 1$, which implies that even if the gambling bank is in autarky consumers will accept the deposit contract.

The equity stake has the following expected payoff:

$$m_i(K,e_i,s) = \begin{cases} \eta_i(\alpha,K)\left[(1+e_i) f(k_i|K) R - D_i\right], & if \quad s_i = b \\ \alpha\eta_i(\alpha,K)\left[(1+e_i) f(k_i|K) R - D_i\right] + B & if \quad s_i = g \end{cases}.$$

The shareholder's investment decision rationale as the network formation process are presented after introducing the tax, which is the aim of the next section.

3 'Network Tax'

The purpose of this paper is to study a simple application of a 'network tax', i.e., a tax that is levied based on the banks' contribution to the risk of contagion. The motivation comes from the meeting of the G-20 in April 2010 that delved on how should the banks contribute in a fair and substantial manner to future rescues of the financial system.

For the purpose of this paper we are mainly interested in the following topic[5]:

> *"**Rate of the levy:** A uniform rate has the benefit of ease of implementation, but it does not contribute to reducing riskiness and systemicness. A risk-adjusted rate could be designed to address the contribution to systemic risk. Ideally, the rate would vary according to the size of the systemic risk externality, e.g., based on a network model which would take into account all possible channels of contagion."* (IMF; 2010, p. 12).

In this section, we are going to show that network models are not only important to determine this rate, but they can also provide an account on the changes triggered by the tax that can be determinant when the policy objective goes beyond financial stability.

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[4] If this assumption were to be relaxed, one would expect a shift towards a safer system since the increase in shareholders' payoff would make the gambling project less attractive.

[5] This document was brought to our attention through the specialised website www.financialnetworkanalysis.com maintained by *Kimmo Soramäki*.

Although the model presented in Castiglionesi and Navarro (2010) does not take into account systemic effects, it does provide a simple structure that allows the study of what changes are to be expected when the links established become subject to a levy and how that affects the risk of direct contagion. Given this limited definition of risk of contagion, the investment decision will be the fundamental factor in the delimitation of the base of the tax, i.e., two banks each investing in the safe asset should not be levied since their connection does not imply any contagion risk.

To simplify the analysis, let us assume that capital is perfectly observable (both for banks and for the regulator) and consequently that investment decisions are also perfectly foreseeable.

3.1 Tax Formulation

The fundamental issue involved in the definition of the tax is who should pay it. Should the full burden of the levy be supported by the gambling borrower? Or should the lender also contribute?

The purpose of such a contribution is to provide governments with the means to recapitalise the financial system if dire times were to present themselves (again). Therefore, if the burden were to be bestowed only to a fraction of those who benefit from the rescue that could lead to a free-rider problem. Such a tax could be used to minimise the underestimation of risk when a lender is connecting itself to an institution that is implicitly guaranteed by government intervention (i.e., *too-something-to-fail*[6]). In this paper we will assume that whenever there is a gambling bank involved in the link the tax must be paid by both parties.

The tax formulation addressed in the paper is a very general one given by a function $\tau_i(x)$, where x denotes those links that involve contagion risk and therefore should be levied on bank i (i.e., if a core bank establishes a link with a peripheral bank then it has to pay the tax on that link, whereas a gambling bank will always pay the tax in every link it establishes since it is the source of contagion risk). The only restriction we impose is that the tax collected be increasing in the potential exposure to risk, i.e., $\tau_i'(x) > 0$.

3.2 Impact on the Payoffs

For the sake of simplicity, let us assume that the full weight of the tax is bestowed on the shareholders. Therefore, their payoffs are now given by:

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[6] As a great variety of such implicit guarantees have been studied in the literature, including here only the classic too-big-to-fail issue would be a restricting view. Therefore, designating this issue by too-something-to-fail encompasses other concepts as the ones found in Acharya and Yorulmazer (2007) and in Markose et al. (2010).

$$m_i(K,e_i,s,\tau) = \begin{cases} p_i(K,s)\left[(1+e_i)f(k_i|K)R - D_i\right] - \tau_i(g_i), & if \quad s_i = b \\ p_i(K,s)\left[(1+e_i)f(k_i|K)R - D_i\right] + B - \tau_i(k_i) & if \quad s_i = g \end{cases}.$$

3.3 Impact on Banks' Investment Decisions

In this model, the decision to invest in the safe asset or to 'gamble' is crucially determined by the equity that investors/shareholders hold. Since their liability is limited, i.e., losses are limited to the equity stake while gains are not, then shareholders will gamble when the bank is undercapitalised. Therefore, the tax will have an impact on investment decisions. Since we have assumed that gambling banks are *per se* a source of risk, the decision to gamble brings with it a heavier tax regime than the one allowed by the alternative investment opportunity.

Following closely Castiglionesi and Navarro (2010), the bank will invest in the safe asset if and only if

$$\eta_i(\alpha,K)f(k_i|K)Re_i - \tau_i(g_i) \geq \alpha\eta_i(\alpha,K)f(k_i|K)Re_i + B - \tau_i(k_i) \Leftrightarrow$$

$$\eta_i(\alpha,K)f(k_i|K)Re_i \geq \alpha\eta_i(\alpha,K)f(k_i|K)Re_i + B + \tau_i(g_i) - \tau(k_i)$$

which implies,

$$e_i \geq \frac{B}{(1-\alpha)f(k_i|K)R\eta_i(\alpha,K)} - \frac{\tau_i(k_i) - \tau_i(g_i)}{(1-\alpha)f(k_i|K)R\eta_i(\alpha,K)} = I^\star(\alpha,K,\tau). \tag{5}$$

Note that if there was no tax, equation (5) would be equal to the one presented in Castiglionesi and Navarro (2010, p. 11).

From equation (5) stems the concept of *Investment Nash Equilibrium after Levy*[7] *(INEL) :*

An allocation (K,e,s,τ) is an INEL for a given economy (N,e), with $e = (e_i)_{i \in N}$, if $m_i(K,e_i,s,\tau) \geq m_i(K,e_i,(s_{-i},\widetilde{s_i}),\tau) \quad \forall\, i \in N$, with $\widetilde{s_i} \in \{b,g\}$. In other words, a INEL exists when given the financial network, the capital level and the contributory regime, the current portfolio allocation provides the better response to other banks portfolio choices. An allocation (K,e,s,τ) is an INEL for a given economy if and only if $\quad \forall \quad i \in N$

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[7] This definition is very close to the one of Investment Nash Equilibrium presented in Castiglionesi and Navarro (2010, p. 11).

$$s_i = \begin{cases} b, & if \quad e_i \geq I^{\star}(\alpha, K, \tau) \\ g, & if \quad e_i < I^{\star}(\alpha, K, \tau) \end{cases}$$

Taking into account that $\tau'(x) \geq 0$ and $G_i \subseteq K_i$ such that $k_i \geq g_i$, then the minimum level of capital that motivates the investment in the safe asset with the tax (as given by equation (5)) is lower than under a null tax. The banks that invest in the safe asset are denote henceforth by core banks and those that invest in the gambling asset by peripheral.

3.4 Impact on Network Formation

Since the 'network tax' changes the trade-off that motivates the formation of the network, it is here that the fundamental impact of the 'network tax' hits.

Before the introduction of the levy, the trade-off consisted in exchanging the benefit of the increase of the probability of coinsurance with the cost of exposure to the risk of failure induced by a gambling neighbour. Now, there is an added cost independent of the nodes' investment decision.

Definition 1[8] *An allocation after levy (K, e, s, τ) is pairwise stable (PSL) if the following conditions are met:*

1. *For all $i \in K_j$ (and consequently $j \in K_i$): $m_i(K, e, s, \tau) \geq m_i(K \setminus ij, e, \tilde{s}, \tau)$ and $m_j(K, e, s, \tau) \geq m_j(K \setminus ij, e, \tilde{s}, \tau)$ for all allocations $(K \setminus ij, e, \tilde{s}, \tau)$ that are INEL;*
2. *For all $i \notin K_j$ (and consequently $j \notin K_i$): if there is an INEL $(K \cup ij, e, \tilde{s}, \tau)$ such that $m_i(K, e, s, \tau) < m_i(K \cup ij, e, \tilde{s}, \tau)$, then $m_j(K, e, s, \tau) > m_j(K \cup ij, e, \tilde{s}, \tau)$.*

The first condition states that nodes can deviate unilaterally if the addition of the link is not beneficial to them. The second, expresses the idea that if there is a link that has not been established that is beneficial to one of the nodes, then it must be that the link would reduce the surplus of the other node.

Definition 2[9] *An allocation after levy (K, e, s, τ) is a decentralised equilibrium (DEL) if it is an INEL and PSL.*

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[8] Note that this definition is very close to definition 2 in Castiglionesi and Navarro (2010, p. 24)

[9] Note that this definition is very close to definition 3 in Castiglionesi and Navarro (2010, p. 24)

3.4.1 Core-Core Relations

Castiglionesi and Navarro (2010, p. 24) postulate in proposition 7 that the network exhibits a core-periphery structure, i.e., all core banks are connected amongst themselves. A core-periphery structure is one where the core banks establish connections amongst themselves and the peripheral banks can be connected amongst themselves and with the core depending on the factors that motivate the formation of the network. There is empirical evidence that the financial network *"shows power laws in the degree distribution" Boss et al. (2004, p. 678)*, which supports this view that a small number of banks having a large number of connections (core banks) and a large number of banks being connected with a small number of counterparties (peripheral banks).

To see whether this property still holds, let us start by analysing the interconnectedness of the core. In the context of core-core relations, $p_i(K,s) = 1$ and $m_i(K,e_i,s,\tau) = e_i[1+\varphi(k_i|K)]R$. Since indirect contagion is ruled out, establishing a link with a core bank does not pose any risk of contagion and, consequently is not taxed. Therefore, the original argument establishing that banks in the core were all connected to each other remains valid.

3.4.2 Relations involving Peripheral Nodes

The effect of the tax is only visible on relations involving at least one gambling bank as that is the only source of risk of contagion in the model. The next proposition summarises the results:

Proposition 1 *The tax changes the trade-off that motivates the formation of the network by adding a cost of forming a risky link. Therefore, the conditions for the establishment of links with peripheral nodes are more stringent with the introduction of the levy, i.e., the maximum counterparty risk accepted decreases with the tax as follows:* $\alpha \geq 1 - \frac{f(k_i|K) - f(k_i-1|K\setminus ij)}{\Lambda} + \frac{\tau_i(k_i) - \tau_i(k_i-1)}{e_i R \Lambda}$, *with* $\Lambda = prob(LGN|K) f(k_i|K) - prob(LGN|K\setminus ij) f(k_i-1|K\setminus ij)$ *being the difference in lending probabilities adjusted coinsurance benefits of the link.*

Proof.

The condition establishing the optimality of a link being created between a core bank and a peripheral one is given by:

$$\eta_i(\alpha,K)\, e_i f(k_i|K) R - \tau_i(g_i) \geq \eta_i(\alpha,K\setminus ij)\, e_i f(k_i|K\setminus ij) R - \tau_i(g_i-1).$$

which implies,

$$\alpha \geq 1 - \frac{f(k_i|K) - f(k_i - 1|K \setminus ij)}{\Lambda} + \frac{\tau_i(g_i) - \tau_i(g_i - 1)}{e_i R \Lambda}. \tag{6}$$

Comparing equation (6) with the one that would be obtained in the absence of the tax, i.e., the maximum counterparty risk accepted by a core bank when deciding to connect to an additional gambling neighbour is lower as the incorporation of the tax affects the trade-off that motivates the formation of the network.

The condition establishing the optimality of a connection being created between a peripheral bank and a core one is given by:

$$\alpha\eta_i(\alpha,K)\,e_i f(k_i|K)\,R + B - \tau_i(k_i) \geq \alpha\eta_i(\alpha,K\setminus ij)\,e_i f(k_i|K\setminus ij)\,R + B - \tau_i(k_i - 1).$$

which implies,

$$\alpha\eta_i(\alpha,K)\,e_i R\left[f(k_i|K) - f(k_i - 1|K\setminus ij)\right] \geq \tau_i(k_i) - \tau_i(k_i - 1). \tag{7}$$

In its turn, equation (7) tells us that since now the gambling bank has to pay tax whenever it creates a link, even connecting to a core bank is a decision that involves a cost-benefit analysis.

In its turn, the condition establishing the optimality of a link being established between two peripheral banks is given by:

$$\alpha\eta_i(\alpha,K)\,e_i f(k_i|K)\,R + B - \tau_i(k_i) \geq \alpha\eta_i(\alpha,K\setminus ij)\,e_i f(k_i|K\setminus ij)\,R + B - \tau_i(k_i - 1).$$

which implies,

$$\eta_i(\alpha,K)\,e_i f(k_i|K)\,R - \eta_i(\alpha,K\setminus ij)\,e_i f(k_i|K\setminus ij)\,R \geq \tfrac{1}{\alpha}\left[\tau_i(k_i) - \tau_i(k_i - 1)\right] \Rightarrow$$

$$\eta_i(\alpha,K)\,e_i f(k_i|K)\,R - \eta_i(\alpha,K\setminus ij)\,e_i f(k_i|K\setminus ij)\,R \geq \tau_i(k_i) - \tau_i(k_i - 1) \Rightarrow$$

$$\alpha \geq 1 - \frac{f(k_i|K) - f(k_i - 1|K \setminus ij)}{\Lambda} + \frac{\tau_i(k_i) - \tau_i(k_i - 1)}{e_i R \Lambda}. \tag{8}$$

Comparing equation (8), that is quite similar to equation (6), with the one that would be obtained in the absence of the tax, i.e., the maximum counterparty risk accepted by a peripheral bank when deciding to connect to an additional gambling neighbour is lower as the incorporation of the tax affects the trade-off that motivates the formation of the network. ■

Subsection 3.4.1 tells us that the completely connected core property still holds after introducing the tax. What remains to be answered is what are the effects one expects to observe when there are different types (core-periphery vs periphery-periphery) of links in the network. To do so, let us focus on a complete network comprised by a core bank and two peripherals.

Proposition 2 *In a complete network comprised by a core bank and two peripherals, the core bank values more its connections than the peripheral ones, as long as* $\frac{e_C}{e_P} \geq \frac{\Lambda_{PP}}{\Lambda_{CP}}$*, where* e_C *and* e_P *are the capital levels of the core bank and the least capitalised gambling bank, respectively.*

Proof.

The intuition for this proof is the following: since peripheral banks are less capitalised the gains of coinsurance are lower compared to the ones obtained by a core bank that relies solely on the investment in the asset. To do so, we analyse under what conditions a link between two gambling neighbours (peripheral link - PP) and a link between the core bank and one of the peripherals (core-peripheral link - CP) is formed. If the conditions for establishing the peripheral link are more demanding than the ones required by the core-peripheral link, then the core bank values more establishing the additional risky link than a gambling bank in the given network.

To confirm this, let us look at equations (6) and (8). Removing the tax out of the picture, the verification is straightforward since the benefits of coinsurance are the same and known in both links and given the assumed network the probabilities involved are known:

$$\Lambda_{CP} = prob_{CP}(A|K)f(k_i|K) - prob_{CP}(A|K \setminus ij)f(k_i - 1|K \setminus ij) = 1.75 \times 0.25 - 1.6875 \times 0.1875.$$

$$\Lambda_{PP} = prob_{PP}(A|K)f(k_i|K) - prob_{PP}(A|K \setminus ij)f(k_i - 1|K \setminus ij) = 1.75 \times 0.125.$$

Note that these values are parameter independent, they stem solely from the network structure and the coinsurance function assumptions. The calculation of these values is a straightforward exercise given equations (1) and (3).

Since $\Lambda_{PP} > \Lambda_{CP} \Rightarrow \alpha^{\star}_{CP} < \alpha^{\star}_{PP}$, we have that without the tax the marginal value of the CP link is higher than the value of the PP link, i.e., the core bank is more willing to accept counterparty risk (lower α) in order to benefit from coinsurance.

With the tax, the verification of this proposition requires the establishment of a sufficient condition. Since the number of risky links subject to the levy before and after the establishment of both links under review is the same, the difference lies in the denominator of the final term in equations (6) and (8).

There are two countervailing effects here. In one hand, the marginal effect of coinsurance per unit of capital invested is higher for the peripheral node (as long as α is high enough) when forming the link with the other gambling node, i.e., $\Lambda_{PP} > \Lambda_{CP}$. However, on the other hand, by definition the gambling node has less capital. Therefore, the sufficient condition that guarantees the validity of this proposition is: $\Lambda_{CP} e_C \geq \Lambda_{PP} e_P \Leftrightarrow \frac{e_C}{e_P} \geq \frac{\Lambda_{PP}}{\Lambda_{CP}}$, where e_C and e_P are the capital levels of the core bank and the least capitalised gambling bank, respectively. ■

Corollary 1 *In a complete network comprised by a core bank and two peripherals, the first link to be disconnected due to the tax is the one established between the two peripheral banks, as long as* $\frac{e_C}{e_P} \geq \frac{\Lambda_{PP}}{\Lambda_{CP}}$*, where* e_C *and* e_P *are the capital levels of the core bank and the least capitalised gambling bank, respectively.*

Proof.

Proposition 1 shows that the conditions for establishing links that involve at least one gambling node become more demanding with the tax. Proposition 2 shows that, under a sufficient condition, the CP link is more valuable than the PP link. Therefore, it follows naturally that, under the same sufficient condition, the first link to be disconnected in the assumed network is the PP link. ■

3.5 Welfare

The main purpose of this paper is to argue that the tax affects the financial network and, thus affects total welfare. This requires the definition of a measure of welfare. Such a function must express: (i) the sum of the expected payoff earned by depositors and shareholders; (ii) the transfers from the shareholders to the deposit insurance fund and; (iii) the expected costs of government intervention. Building on the function presented in (Castiglionesi and Navarro; 2010, p. 11), we propose the following welfare function:

$$\mathbf{W} = \sum_{i=1}^{n} \left[M_i\left(K,s\right) + m_i\left(K,e_i,s,\tau\right) + \tau_i\left(base_i\right) \right] - D.I.\left(\alpha,K\right). \tag{9}$$

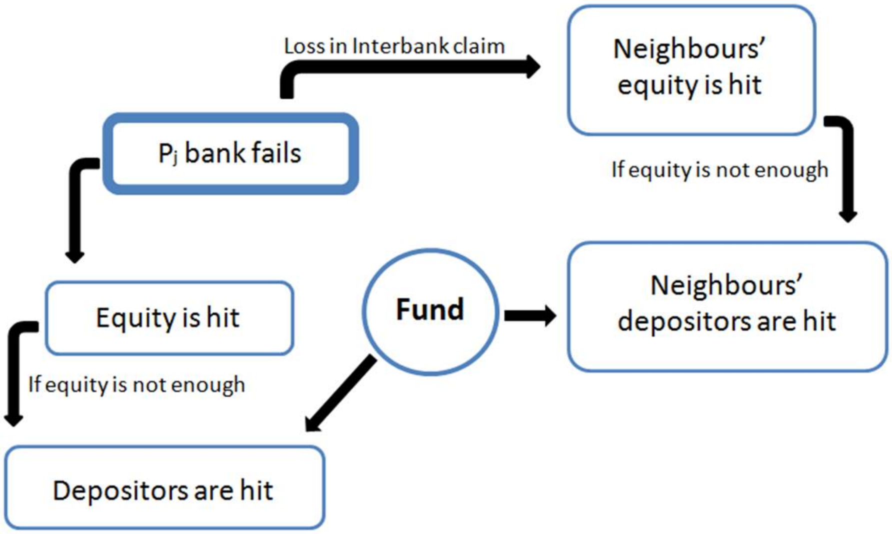
where $base_i$ denotes the number of links that are taxed in the case of bank i, $D.I.\left(\alpha,K\right)$ are the expected costs of government intervention given the financial network and α. Note that in a world with three banks, there is a single interbank loan that can be subject to default since there is only liquidity to satisfy at most one shock. Since deposits are contingent on the shock, default can only occur when bank has excess liquidity to supply to its neighbours. Conditional on this excess liquidity, the amount promised to depositors is the sum of the amount of deposits invested in the illiquid asset, $(1-\gamma)R$, plus the excess liquidity $\gamma - \omega_L$.

The probabilities of the depositors sustaining a loss depend on the type of bank they are in. Those with their endowment deposited in core banks are only subject to a loss in a possible interbank claim and are partially covered by their bank's capital, i.e., $\mathbb{C}_1 = (1-\alpha)\sum_{i\in K\setminus\{G\}}\left[prob_i(LGN|K)(\frac{\omega_H-\omega_L}{2} - e_iR)\right]$. However, for the other ones, they can sustain losses even if there is no loss in the interbank claim. This happens because the assets in which gambling banks invest are subject to default risk on their own. Therefore, there are three circumstances that lead to a loss in deposits:

- There is an interbank loss but the own asset does not default, i.e., $\mathbb{C}_2 = (1-\alpha)\,\alpha\sum_{i\in G}\left[prob_i(LGN|K)(\frac{\omega_H-\omega_L}{2} - e_iR)\right]$;
- There is no interbank loss (given that there is excess liquidity that occurs with probability $\frac{1}{2}$) but the own asset defaults, i.e., $\mathbb{C}_3 = \frac{1}{2}\left(1-\alpha\right)\left(1-\gamma\right)R\sum_{i\in G}\left[1 - prob_i(LGN|K)\right]$;
- There is a loss in both assets, i.e., $\mathbb{C}_4 = \left(1-\alpha\right)^2\left[(1-\gamma)R + \gamma - \omega_L\right]\sum_{i\in G} prob_i(LGN|K)$.

Summing all of these components we find the expected costs of deposit insurance, i.e., $D.I.\left(\alpha,K\right) = \mathbb{C}_1 + \mathbb{C}_2 + \mathbb{C}_3 + \mathbb{C}_4$.

Figure 1: Bankruptcy Mechanism with Solvency Fund



The tax is created to build a fund used to repay depositors affected by the original bankruptcy. In those states of nature where contagion arises the corresponding payoffs must be supplemented by government transfers that use the fund's resources, as depicted in figure 1.

There are three main effects in action when the tax increases (decreases): (i) coinsurance effect - the less (more) connected network reduces the benefits of coinsurance leading to a lower (higher) total welfare; (ii) counterparty risk effect - the less (more) connected network reduces the probability of lending to a risky link, which in its turn reduces the expected costs of government intervention, leading to a higher (lower) total welfare and; (iii) reallocation effect - the less (more) connected network allows a reallocation of coinsurance from the peripheral nodes to the core (core to periphery), as stated by corollary 1, leading to a higher (lower) total welfare. There are two additional effects: (i) the tax exerts a discrete effect, i.e., unless the tax leads to a broken link, its effects are neutral in terms of total welfare since it comprises a transfer from shareholders to the fund and; (ii) re-evaluation effect - if the tax is high enough the peripheral banks will change their investment decision, as stated by equation (5), thus becoming core banks. It is also worth noting that since the counterparty risk effect and the reallocation effect move in the same direction it is not possible to differentiate between them.

Focusing on the region where banks maintain their investment decisions unaltered, we can see that there is a trade-off between stability and efficiency. Stability here can be measured by the probability of contagion and consequently of government intervention. In the context of this model, the lower the number of risky links the higher is the system's stability. Note that financial networks tend to exhibit the 'robust-yet-fragile' property, i.e., up to a point the number of connections increases the system's stability, but when that tipping point is surpassed the increasing connectivity of the system leads to increasing instability. Therefore, since the stabilising effect of a tax only occurs when the tipping point is surpassed, we are focusing on the region where there is an economic fundamental for the tax. Efficiency in this context means the extent to which the benefits of coinsurance can be accrued by the participants. More importantly, given our definition of the welfare function, these benefits are not restricted to the shareholders. The existence of depositors that benefit from the degree of coinsurance allowed by the financial network opens the door for the analysis of the effects such a tax introduces in the real economy, which is closely related with the network-based financial accelerator (see Delli Gatti et al. (2010)) and will be pursued in future research.

Note that the welfare function can only be evaluated after the network is formed, which is dependent on the choice of the tax formulation. The next subsection provides clues to this selection process.

3.6 Selection of the Tax Formulation

The main contribution of this paper is to show that the selection of the tax formulation is affected by the endogenous formation of the network, unlike in Krahnen and Bluhm (2010) where the network is taken to be stationary when the policy function is defined. To do so, we propose an iterative algorithm were the network is taken to be stationary only locally for the purpose of evaluating total welfare. Then as the algorithm iterates over all proposed values the network is re-formed.

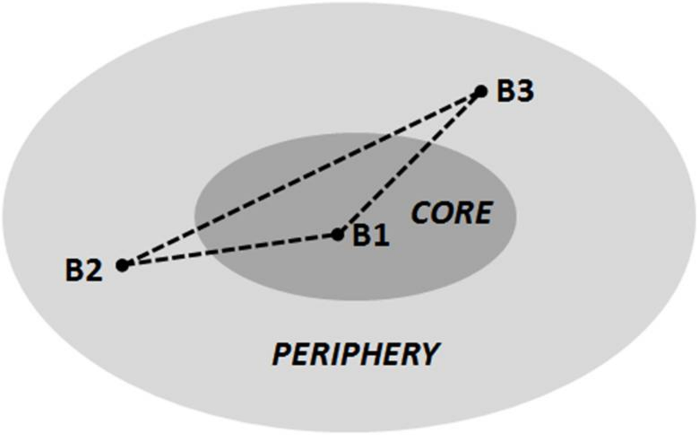
3.6.1 Algorithm - Steps

1. Guess $\tau(x)$, with $x = 1, ..., n-1$;
2. Check all networks that can be formed given $\tau(x)$, i. e., what banks lie in the core/periphery;
3. Check if the network is Pairwise Stable;
4. Evaluate total welfare;
5. Go back to 1. until all proposed values delimited by the policymaker are analysed.

Although this algorithm does provide a comparison mechanism among different policy functions, and consequently can be used to find the optimal policy function, the main objective here is to show how the tax affects the network structure conditional on the incentives that motivate the formation of the network in the first place. Also, it allows to observe under what conditions the dominance of all the effects described before takes place.

3.6.2 An Example

Figure 2: Complete Network



To exemplify the mechanic of the model, let us analyse under what conditions the complete network is a decentralised equilibrium without tax.

A DEL requires an allocation (K, e, s) to be an INEL and to be PSL. Starting by the INE requirement we take the network as given and determine the capital thresholds that separate the investment decisions as given by equation (5). Assuming the following parametrisation we get the thresholds presented in table (1):

Parameter	Values
$\mathbf{e}$	$[0.041 \quad 0.02 \quad 0.02]$
α	0.8
R	1.5
B	0.02

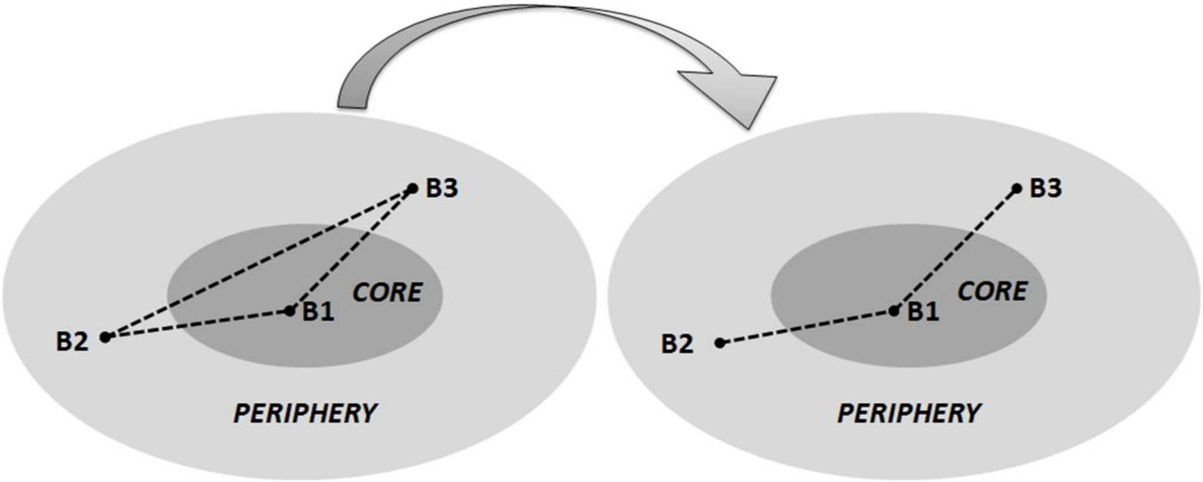
Table 1: Capital Thresholds

	$k_i = 0$	$k_i = 1$	$k_i = 2$
$g_i = 0$	0.0667	0.0395	0.0381
$g_i = 1$	-	0.041	0.0391
$g_i = 2$	-	-	0.0401

Given the capital vector assumed, the allocation is an INEL. To see whether an allocation is PSL we need to check each link formed according to the condition on equation (10):

$$\eta_i(\alpha, K) f(k_i|K) \geq \eta_i(\alpha, K \setminus ij) f(k_i - 1|K \setminus ij). \tag{10}$$

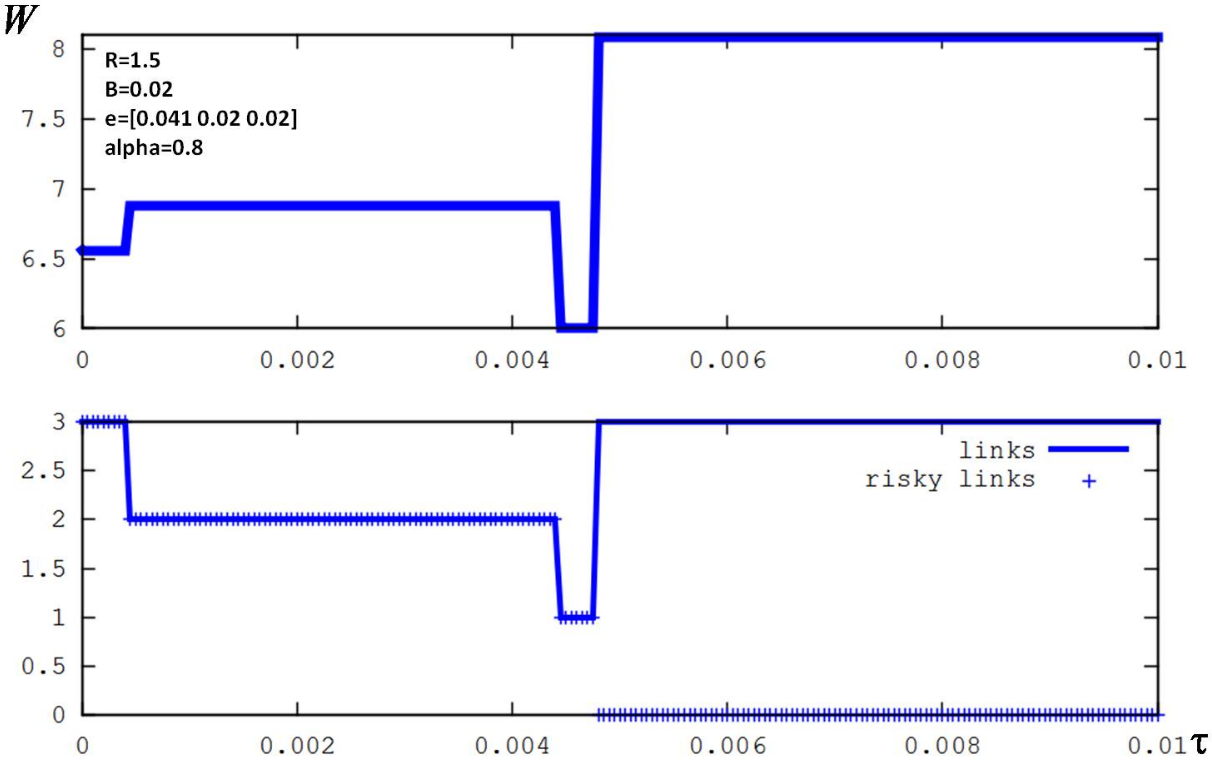
Figure 3: Peripheral Link Elimination



For the purpose of exemplification we only analyse the link established between the two peripheral banks, as shown in Figure 3. The process is the same for the rest of the links. From equation (10) we have:

$\alpha \geq 1 - \frac{f(k_i|K) - f(k_i - 1|K \backslash ij)}{prob(A|K) f(k_i|K) - prob(A|K \backslash ij) f(k_i - 1|K \backslash ij)} \Leftrightarrow \alpha \geq 1 - \frac{1.75 - 1.6875}{0.125 \times 1.75 - 0 \times 1.6875} \Leftrightarrow \alpha \geq 0.71429$. Therefore, the link between the peripheral banks is established if and only if the counterparty risk is low enough.

Figure 4: Welfare Effects



To illustrate how the algorithm works, we relax the assumption of a null tax that was assumed hitherto in this example. First, all possible networks are enumerated[10] and then they are restricted according to the constraints in steps 3 and 4. Once the set of feasible networks is defined, total welfare was evaluated for each policy function. For ease of exposition, we start by assuming a flat tax and analyse under what parametrisation the effects described before dominate.

In figure 4, the counterparty risk/reallocation effects dominate at first, i.e., the elimination of the PP link leads to an increase in welfare at first. However, as the tax increases one of the remaining CP links can no longer be sustained, depriving the core bank from the benefits of coinsurance (also note that $\varphi'' < 0$). This happens until the tax hits a critical level that motivates the peripheral banks to change their investment decisions, leading to a complete network comprised strictly of core banks.

The fundamental point of this analysis is that such a tax has a non-monotonic effect in the financial network and consequently in welfare. Therefore, the choice of the policy function must take into account how these effects affect the network, which can only be grasped with the understanding of how the network forms (or how it re-forms).

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[10] To do so, the methodology proposed in Ruskey and Williams (2009) was used. The translation into c code was conducted by Prof. Paulo S. A. Sousa (INESC-Porto, Portugal) for the project Fique and Sousa (2009) funded by FCT grant no. BII/UNI/4089/EEI/2008 and then translated by Fique, J.P. for the use of this paper.

3.7 Policy Implications

The main policy implication of the paper is that understanding the motivation on how the financial network forms is invaluable in the determination of the optimal policy function. The key issue is to determine what are the dominant effects when the tax changes.

Also, the initial proposition on the introduction of the tax is that it has the potential to make the network less connected when it involves gambling nodes. During a period where the policy focus is to restart lending such a tax can be an obstacle in the accomplishment of the main goal. Furthermore, since the network may exhibit the above mentioned 'robust-yet-fragile' property, the tax may also impair the efforts of increasing the system's stability.

4 Conclusion

The main contribution of this paper is to show that a contributory regime based on exposure to contagion risk can affect the network structure and brings with it a variety of welfare effects. We find that the conjugation of these effects leads to a non-monotonic relation between the tax and total welfare, potentially leading to a trade-off between stability and efficiency. Therefore, any attempt to design a policy function that fails to take this into account may not be optimal and welfare reducing. The existence of depositors that benefit from the degree of coinsurance allowed by the financial network opens the door for the analysis of the effects that such a tax introduces in the real economy, which is left for future research.

Our analysis also suggests that during a period where the policy focus is to restart lending in the interbank market such a tax can be an obstacle in the accomplishment of the main goal. Furthermore, since the network may exhibit the above mentioned 'robust-yet-fragile' property, the tax may also impair the efforts of increasing the system's stability.

The basic setup has some limitations though, as it only incorporates the effects of direct contagion precluding a broader analysis of systemic risk which is determinant to a more complete analysis of this 'network tax'. Also, links do not necessarily correspond to interbank exposures, they take the form of credit lines that may or may not be used conditional on the nodes connected receiving a negatively correlated liquidity shock, so our proposal to tax them is only a first approximation. However, the analysis carried out through this paper remains valid in the case of actual interbank claims, since our point is that the tax changes the trade-off between liquidity coinsurance and counterparty risk that motivates the formation of the network in the first place. Although the base model has some limitations, it does provide a simple structure that allows the study of what changes are to be expected when the links established become subject to a levy and how that affects the reformation of the network after the contributory regime is established.

Our main contribution points towards the importance of the interconnectedness between the financial network and the real economy. Therefore, future research would include a better understanding of this interconnectedness, preferably in a dynamic environment where the transition process can be studied. Furthermore, our analysis suggests that asymmetric information would bring with it serious complications, further enquiry should also approach this issue.

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