The Term Structure of Interbank Risk

Damir Filipović and Anders B. Trolle
Ecole Polytechnique Fédérale de Lausanne and Swiss Finance Institute

Abstract

We use the term structure of spreads between rates on interest rate swaps indexed to LIBOR and overnight indexed swaps to infer a term structure of interbank risk. We develop a dynamic term structure model with default risk in the interbank market that, in conjunction with information from the credit default swap market, allows us to decompose the term structure of interbank risk into default and non-default components. On average, from August 2007 to January 2011, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample and is correlated with various measures of market-wide liquidity. Further out the term structure, the default component is the dominant driver of interbank risk throughout the sample period. These results hold true in both the USD and EUR markets and are robust to different model parameterizations and measures of interbank default risk. The analysis has implications for monetary and regulatory policy as well as for pricing, hedging, and risk-management in the interest rate swap market.

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CONTAINS ONLINE APPENDIX

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"The age of innocence – when banks lent to each other unsecured for three months or longer at only a small premium to expected policy rates – will not quickly, if ever, return".
Mervin King, Bank of England Governor, 21 October 2008

1 Introduction

A major issue in the recent financial crisis was the increase in interbank risk and the resulting stress in the interbank market. This had important real effects since the interbank market is critical for banks' liquidity management and plays an important role in the implementation and transmission of monetary policy. In addition, interbank rates, such as LIBOR, are important benchmarks for a wide variety of fixed income products including interest rate swaps, interest rate futures, mortgages, loan agreements, and saving accounts. Not surprisingly, therefore, there has recently been a large number of theoretical and empirical studies on interbank risk. Existing papers, however, focus on short term interbank risk. In contrast, in this paper we study the *term structure* of interbank risk, which we show is driven by multiple factors and provides important new insights on interbank risk.

To understand our approach, consider Figure 1. The solid line shows the spread between 3M LIBOR, which is a reference rate for unsecured interbank borrowing and lending, and the fixed rate on a 3M overnight indexed swap (OIS), which is a common risk-free rate proxy. This money market spread has been used in many recent papers as a measure of interbank risk. It was very small and stable until the onset of the credit crisis in August 2007 and then suddenly increased substantially and became highly volatile. At the same time, since Fall 2009 it has more or less reverted back to pre-crisis levels, except for an increase related to the escalation of the European sovereign debt crisis.

The dotted line shows the spread between the fixed rate on a 5Y regular interest rate swap (IRS) with floating-leg payments indexed to 3M LIBOR, and the fixed rate on a 5Y OIS. We show in the paper that this spread essentially reflects expectations about future 3M LIBOR-OIS spreads and, therefore, provides valuable insights into market participants' perceptions about future interbank risk. As such, we can use IRS-OIS spreads at different maturities to infer a term structure of interbank risk. Importantly, the longer-maturity swap spreads reflect information about interbank risk that is not contained in money market spreads. For instance, prior to the onset of the credit crisis, the term structure of interbank risk was essentially flat with swap spreads only a few basis points higher than money market spreads. Then, at the onset of the financial crisis, swap spreads increased much less than money market spreads, resulting in a strongly downward-sloping term structure of interbank risk, and indicating that market participants expected the extremely elevated levels of interbank risk observed in the money market to be a relatively short-lived phenomenon. Finally, in the most recent time-period, with money market spreads having reverted to pre-crisis levels, swap spreads remain well above pre-crisis levels and significantly higher than money market spreads, implying an upward-sloping term structure of interbank risk, and indicating that market participants expect interbank risk to increase in the future (or require a large risk premium for bearing future interbank risk).

Our paper makes both theoretical and empirical contributions to the literature on interbank risk. We first provide a model for the term structure of interbank risk. We then apply the model to study interbank risk since the onset of the financial crisis, decomposing the term structure of interbank risk into default and non-default components, and studying how these components vary over time.

In our model, a LIBOR-OIS spread can arise for two reasons. The first reason is default risk. LIBOR reflects the rate at which banks that belong to the LIBOR panel can obtain unsecured funding for longer terms (typically 3M or 6M) in the interbank market. An important feature of LIBOR is that panel banks are selected based on their credit quality (as well as the scale of their market activities), and a bank that experiences a significant deterioration in its credit quality will be dropped from the panel and replaced by a bank with superior credit quality. This implies that the credit quality inherent in LIBOR is being refreshed over time. Rather than modeling the funding costs of individual panel banks, we assume a sequence of banks, each of which represents the panel at a given point in time. Then, the current LIBOR reflects the expected future default risk of the bank that represents the current panel.

An OIS is a swap where the floating-leg payments are indexed to a reference rate for unsecured overnight funding, which we assume approximately equals the average cost of unsecured overnight funding for LIBOR panel banks. Then, the reference rate reflects the default risk of the bank that represents the current panel, and the fixed OIS rate reflects the expected default risks of the respective banks that represent the future panels. Due to the potential for refreshment of the LIBOR panel, this is lower than the expected future default risk of the bank that represents the current panel, and the fixed OIS rate, therefore, is below the corresponding LIBOR. It also follows that the default component of the LIBOR-OIS spread is increasing in the maturity of LIBOR, since there is an increasing risk of credit quality deterioration of the bank that initially represents the panel relative to the respective banks that represent the future panels.

Second, a LIBOR-OIS spread can arise due to factors not directly related to default risk – primarily liquidity risk. For instance, banks may be reluctant to provide term loans in the interbank market because they fear that they may not themselves be able to raise funds, or may do so only at elevated rates, if they are hit by an adverse liquidity shock. This precautionary motive for hoarding cash is modeled by Allen, Carletti, and Gale (2009) and Acharya and Skeie (2010), among others. Alternatively, banks may have a speculative motive for hoarding cash in that they anticipate possible fire-sales of assets by other financial institutions hit by liquidity shocks, in which cases the return on holding cash will be high; see, e.g., Acharya, Gromb, and Yorulmazer (2007), Acharya, Shin, and Yorulmazer (2010) and Diamond and Rajan (2010).[1] Whatever the motivation, hoarding of liquidity will reduce the volume of longer term loans and increase the rates on such loans. Rather than modeling these mechanisms directly, we simply posit a "residual" factor that captures the component of the LIBOR-OIS spread that

[1]A recent paper by Gale and Yorulmazer (2011) integrates the precautionary and speculative motive for hoarding cash.

is not due to default risks. To the extent that liquidity risk is correlated with default risk, for instance if liquidity hoarding is more prevalent when aggregate default risk is high, the residual component captures the component of liquidity risk that is orthogonal to default risk.

Since a long-term IRS-OIS spread reflects expectations about future short-term LIBOR-OIS spreads, the term structure of IRS-OIS spreads reflects the term structures of the default and non-default components of the LIBOR-OIS spreads. To identify the default component, we use information from the credit default swap (CDS) market. At each point in time we construct a CDS spread term structure for the representative panel bank as a composite of the CDS spread term structures for the individual panel banks. Assuming that CDS spreads are pure measures of default risk of the underlying entities, the CDS spread term structure for the representative panel bank allows us to identify the process driving the default component of LIBOR-OIS spreads.

More specifically, we develop a general affine model. Depending on the specification, two factors drive the OIS term structure, one or two factors drive the default component of LIBOR-OIS spreads (i.e., the risk of credit quality deterioration of the bank that represents the panel at a given point in time relative to the respective banks that represent the future panels), and one or two factors drive the non-default component of LIBOR-OIS spreads. The model is highly tractable with analytical expressions for LIBOR, OIS, IRS, and CDS.

We apply the model to study interbank risk from the onset of the financial crisis in August 2007 until January 2011. We utilize a panel data set consisting of term structures of OIS rates, rates on interest rate swaps indexed to 3M as well as 6M LIBOR, and CDS spreads – all with maturities up to 10Y. The model is estimated by maximum likelihood in conjunction with the Kalman filter.

We first conduct a specification analysis, which shows that a specification with two factors driving the OIS term structure, two factors driving the default component of the LIBOR-OIS spread, and one factor driving the non-default component of the LIBOR-OIS spread has a good fit to the data. We then use this specification to decompose the term structure of interbank risk into default and non-default components. We find that, on average, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample, while further out the term structure, the default component is the dominant driver of interbank risk throughout the sample period.[2]

To understand the determinants of the non-default component of interbank risk, we regress the residual factor on a number of illiquidity proxies for the fixed-income market including the spread between the 3M OIS rate and the 3M Treasury bill rate (Krishnamurthy (2010)), the yield spread

[2]In principle, our analysis also allows us to determine how much of the variation in the term structure of interbank risk is due to variation in risk premia. In practice, however, risk premia are quite imprecisely estimated given the short sample, so we refrain from making too strong statements in this regard. It does seem, however, that participants in the interbank market require a premium for bearing exposure to both default and non-default (liquidity) risk.

between 10Y Refcorp bonds and 10Y Treasury notes (Longstaff (2004)), the "noise" measure recently introduced by Hu, Pan, and Wang (2010), and the notional amount of Treasury delivery fails reported by primary dealers used by Fleckenstein, Longstaff, and Lustig (2011) and others as a measure of disruptions in fixed income market liquidity. The factor is significantly related to all four illiquidity measures and jointly they explain a large fraction (about 70 percent) of the variation in the factor. This suggests that the non-default component is strongly related to market illiquidity.

We conduct a variety of robustness test which show that the results hold true for alternative model parameterizations and measures of interbank default risk. By using CDS spreads to identify the default component of interbank risk, our approach is reminiscent of Longstaff, Mithal, and Neis (2005), Blanco, Brennan, and Marsh (2005), and Ang and Longstaff (2011), among others, who use CDS spreads as pure measures of default risk. However, a number of recent papers, including Buhler and Trapp (2010) and Bongaerts, de Jong, and Driessen (2011), have found that CDS spreads may be affected by liquidity effects.[3] Since we mostly use CDS contracts written on large financial institutions, which are among the most liquid contracts in the CDS market, and since we aggregate individual CDS spreads, which reduces the effect of idiosyncratic noise in the individual CDS spreads, we believe it is reasonable to use the composite CDS spreads to infer the default risks of the representative panel banks. Nevertheless, we also consider two alternative measures of default risks that correct for possible liquidity effects. First, we measure default risk by 90 percent of the composite CDS spreads, which, given the results in Buhler and Trapp (2010), seems to be a reasonable lower bound on the default component of CDS spreads. And, second, we measure default risk by composite CDS spreads constructed solely from the banks with the most liquid CDS contracts. None of these alternative measures substantially change the decomposition of the term structure interbank risk.

Throughout, we also report results for the EUR market. Not only does this serve as an additional robustness test, but this market is interesting in its own right. First, by several measures, the market is even larger than the USD market. Second, the main shock to the interbank market in the second half of our sample emanated from the Eurozone with its sovereign debt crisis. And, third, the structure of the EUR market is such that the reference overnight rate in an OIS exactly matches the average cost of unsecured overnight funding of EURIBOR (the EUR equivalent of LIBOR) panel banks providing a robustness check of this assumption. Interestingly, results for the EUR market are very similar to those of the USD market.

Our analysis has several practical applications. First, the framework could be a valuable tool for central banks and regulatory authorities, as it provides market expectations about future stress in interbank markets. In addition, the decomposition into default and non-default (liquidity) components can help guide appropriate policy responses (recapitalization of banks, termination/introduction of

[3]While it is possible that CDS spreads are also affected by counterparty risk, Arora, Gandhi, and Longstaff (2009) find that this effect is minimal, which is consistent with the widespread use of collateralization and netting agreements.

central bank lending facilities, etc).[4]

Second, the model has implications for pricing, hedging, and risk-management in the interest rate swap market. Since the onset of the credit crisis, market participants have been exposed to significant *basis risk*: Swap cash flows are indexed to LIBOR but, because of collateral agreements, are discounted using rates inferred from the OIS market. Furthermore, swap portfolios at most financial institutions are composed of swap contracts indexed to LIBOR rates of various maturities creating another layer of basis risks. Our model provides a useful framework for managing overall interest rate risk and these basis risks in an integrated way.

Our paper is related to Collin-Dufresne and Solnik (2001), who study the term structure of spreads between corporate bonds and interest rate swaps, and Liu, Longstaff, and Mandell (2006), Johannes and Sundaresan (2007), and Feldhutter and Lando (2008), who study the term structure of spreads between interest rate swaps and Treasuries. Our paper has a different focus and also has the methodological advantage of not using bonds, the prices of which were heavily influenced by liquidity issues during the financial crisis. By only considering swap contracts, we expect liquidity to be less of an issue and to be more uniform across instruments leading to a clean decomposition of the term structure of interbank risk.

A number of papers have analyzed the 3M LIBOR-OIS spread and attempted to decompose it into default and liquidity components. These papers include Schwartz (2010), Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008), Michaud and Upper (2008), and Eisenschmidt and Tapking (2009). They all study the early phase of the financial crisis before the collapse of Lehman Brothers and find, with the exception of Taylor and Williams (2009), that liquidity was a key driver of interbank risk during this period. We find a similar result for the short end of the term structure of interbank risk. However, at the longer end of the term structure of interbank risk, default risk appears to have been the dominant driver even during the early phase of the financial crisis, underscoring the importance of taking the entire term structure into account when analyzing interbank risk.[5]

Several papers such as Bianchetti (2009), Fujii, Shimada, and Takahashi (2009), Henrard (2009), and Mercurio (2009, 2010) have developed pricing models for interest rate derivatives that take the stochastic swap-OIS spread into account. These models are highly reduced-form in that spreads between OIS and swaps indexed to different LIBOR rates are modeled independently of each other and also not decomposed into different components. In contrast, we provide a unified model for all such

[4]From a regulatory standpoint, the framework could also prove helpful in determining the right discount curve for the valuation of long-term insurance liabilities, where discount factors are typically allowed to include a liquidity risk component but not a default risk component.

[5]Smith (2010) studies LIBOR-OIS spreads of maturities up to 12M within a dynamic term structure model and attributes the most of the variation in spreads to variation in risk-premia. A somewhat problematic aspect of her analysis is that the default component of LIBOR-OIS spreads is identified by the spread between LIBOR and repo rates, which clearly contains a significant liquidity component during much of the period.

spreads making it possible to aggregate the risks of large swap portfolios and analyze their underlying determinants.

The rest of the paper is organized as follows: Section 2 describes the market instruments. Section 3 describes the model for the term structure of interbank risk. Section 4 discusses the data and the estimation approach. Section 5 presents the results. Section 6 considers a variety of robustness tests. Section 7 concludes, and several appendices contain additional information.

2 Market instruments

We describe the market instruments that we use in the paper. We first consider the basic reference rates and then a variety of swap contracts that are indexed to these reference rates.

2.1 Reference rates

A large number of fixed income contracts are tied to an interbank offered rate. The main reference rate in the USD-denominated fixed income market is the USD London Interbank Offered Rate (LIBOR), while in the EUR-denominated fixed income market it is the European Interbank Offered Rate (EURIBOR).[6] Both LIBOR and EURIBOR are trimmed averages of rates submitted by sets of banks. In the case of LIBOR, each contributor bank bases its submission on the question *at what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size.* In the case of EURIBOR, the wording is slightly different and each contributor bank submits *the rates at which euro interbank term deposits are being offered within the euro zone by one prime bank to another.* Therefore, LIBOR is an average of the rates at which banks believe they can obtain unsecured funding, while EURIBOR is an average of the rates at which banks believe a prime bank can obtain unsecured funding. This subtle difference becomes important when quantifying the degree of default risk inherent in the two rates. Both rates are quoted for a range of terms, with 3M and 6M being the most important and most widely followed. In the following, we let $L(t, T)$ denote the $(T-t)$-maturity LIBOR or EURIBOR rate that fixes at time t.[7]

[6]LIBOR is managed by the British Bankers' Association, while EURIBOR is managed by the European Banking Federation. There also exists a EUR LIBOR, although this rate has not received the same benchmark status as EURIBOR.

[7]During the credit crisis, there was some concern about the integrity of LIBOR and whether certain banks engaged in strategic behavior to signal information about their credit quality or influence LIBOR to benefit positions in LIBOR-linked instruments. However, a Bank of International Settlements study by Gyntelberg and Wooldridge (2008) finds that "if there were any attempts to manipulate fixings during the recent turbulence, trimming procedures appear to have minimised their impact". In addition, there are strict governance mechanisms in place that should identify anomalous rates. In the paper, we therefore take the rates as representing actual funding costs.

For both LIBOR and EURIBOR, contributor banks are selected based on their credit quality and the scale of their market activities. During our sample period, the LIBOR panel consisted of 16 banks, while the EURIBOR panel was significantly larger and consisted of 42 banks.[8] An important feature of both panels is that they are reviewed and revised periodically. A bank that experiences a significant deterioration in its credit quality (and/or its market share) will be dropped from the panel and be replaced by a bank with superior credit quality.

An increasing number of fixed income contracts are tied to an index of overnight rates. In the USD market, the benchmark is the effective Federal Funds (FF) rate, which is a transaction-weighted average of the rates on overnight unsecured loans of reserve balances held at the Federal Reserve that banks make to one another. In the EUR market, the benchmark is the Euro Overnight Index Average (EONIA) rate, computed as a transaction-weighted average of the rates on all overnight unsecured loans in the interbank market initiated by EURIBOR panel banks. Therefore, in the EUR market, the benchmark overnight rate reflects the average cost of unsecured overnight funding of panel banks. We assume that the same holds for the USD market, although the set of banks from which the effective Federal Funds rate is computed does not exactly match the LIBOR panel.[9]

For the sake of convenience we will from now on use “LIBOR” as a generic term for an interbank offered rate, comprising both LIBOR and EURIBOR, whenever there is no ambiguity.

2.2 Pricing collateralized contracts

Swap contracts between major financial institutions are virtually always collateralized to the extent that counterparty risk is negligible. In this section, we provide the generic pricing formula of collateralized cashflows that we will use below to price swap contracts.[10] Consider a contract with a contractual nominal cashflow X at maturity T. Its present value at $t < T$ is denoted by $V(t)$. We assume that the two parties in the contract agree on posting cash-collateral on a continuous marking-to-market basis. We also assume that, at any time $t < T$, the posted amount of collateral equals 100% of the contract's present value $V(t)$. The receiver of the collateral can invest it at the risk-free rate $r(t)$ and has to pay an agreed rate $r_c(t)$ to the poster of collateral. The present value thus satisfies the following integral equation

$$V(t) = E_t^Q \left[e^{-\int_t^T r(s)ds} X + \int_t^T e^{-\int_t^u r(s)ds} \left(r(u) - r_c(u) \right) V(u) \, du \right], \qquad (1)$$

[8]After the end of our sample period, the USD LIBOR panel was expanded to 20 banks and the EURIBOR panel was expanded to 44 banks.

[9]Participants in the Federal Funds market are those with accounts at Federal Reserve Banks, which include US depository institutions, US branches of foreign banks, and government-sponsored enterprises.

[10]Similar formulas have been derived in various contexts by Johannes and Sundaresan (2007), Fujii, Shimada, and Takahashi (2009), and Piterbarg (2010).

where $E_t^Q \equiv E^Q[\cdot \mid \mathcal{F}_t]$ denotes conditional expectation under the risk neutral measure Q.[11] It is shown in Appendix A that this implies the pricing formula

$$V(t) = E_t^Q \left[e^{-\int_t^T r_c(s)ds} X \right]. \tag{2}$$

For $X = 1$, we obtain the price of a collateralized zero-coupon bond

$$P_c(t,T) = E_t^Q \left[e^{-\int_t^T r_c(s)ds} \right]. \tag{3}$$

In the sequel, we assume that the collateral rate $r_c(t)$ is equal to an instantaneous proxy $L(t,t)$ of the overnight rate, which we define as

$$r_c(t) = L(t,t) = \lim_{T \to t} L(t,T). \tag{4}$$

In reality, best practice among major financial institutions is daily mark-to-market and adjustment of collateral. Furthermore, cash collateral is the most popular form of collateral, since it is free from the issues associated with rehypothecation and allows for faster settlement times. Finally, FF and EONIA are typically the contractual interest rates earned by cash collateral in the USD and EUR markets, respectively. The assumptions we make above, therefore, closely approximate current market reality.[12]

2.3 Interest rate swaps (IRS)

In a regular interest rate swap (IRS), counterparties exchange a stream of fixed-rate payments for a stream of floating-rate payments indexed to LIBOR of a particular maturity. More specifically, consider two discrete tenor structures

$$t = t_0 < t_1 < \cdots < t_N = T \tag{5}$$

and

$$t = T_0 < T_1 < \cdots < T_n = T, \tag{6}$$

and let $\delta = t_i - t_{i-1}$ and $\Delta = T_i - T_{i-1}$ denote the lengths between tenor dates, with $\delta < \Delta$.[13] At every time t_i, $i = 1, ..., N$, one party pays $\delta L(t_{i-1}, t_i)$, while at every time T_i, $i = 1, ..., n$, the other party pays ΔK, where K denotes the fixed rate on the swap. The swap rate, $IRS_{\delta,\Delta}(t,T)$, is the value of K that makes the IRS value equal to zero at inception and is given by

$$IRS_{\delta,\Delta}(t,T) = \frac{\sum_{i=1}^N E_t^Q \left[e^{-\int_t^{t_i} r_c(s)ds} \delta L(t_{i-1}, t_i) \right]}{\sum_{i=1}^n \Delta P_c(t, T_i)}. \tag{7}$$

[11] Throughout, we assume a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, Q)$, where Q is a risk-neutral pricing measure.

[12] ISDA (2010) is a detailed survey of current market practice. Further evidence for the pricing formula given in this section is provided by Whittall (2010), who reports that the main clearing-house of interbank swap contracts now use discount factors extracted from the OIS term structure to discount collateralized swap cashflows.

[13] In practice, the length between dates will vary slightly depending on the day-count convention. To simplify notation, we suppress this dependence.

In the USD market, the benchmark IRS pays 3M LIBOR floating vs. 6M fixed, while in the EUR market, the benchmark IRS pays 6M EURIBOR floating vs. 1Y fixed. Rates on IRS indexed to LIBOR of other maturities are obtained via basis swaps as discussed below.

2.4 Basis swaps (BS)

In a basis swap (BS), counterparties exchange two streams of floating-rate payments indexed to LIBOR of different maturities, plus a stream of fixed payments. The quotation convention for basis swaps differs across brokers, across markets, and may also have changed over time.[14] However, as demonstrated in the online appendix, the differences between the conventions are negligible. Consider a basis swap in which one party pays the δ_1-maturity LIBOR while the other party pays the δ_2-maturity LIBOR with $\delta_1 < \delta_2$. We use the quotation convention in which the basis swap rate, $BS_{\delta_1,\delta_2}(t,T)$, is given as the difference between the fixed rates on two IRS indexed to δ_2- and δ_1-maturity LIBOR, respectively. That is,

$$BS_{\delta_1,\delta_2}(t,T) \quad = \quad IRS_{\delta_2,\Delta}(t,T) - IRS_{\delta_1,\Delta}(t,T). \tag{8}$$

This convention has the advantage that rates on non-benchmark IRS are very easily obtained via basis swaps.

2.5 Overnight indexed swaps (OIS)

In an overnight indexed swaps (OIS), counterparties exchange a stream of fixed-rate payments for a stream of floating-rate payments indexed to a compounded overnight rate (FF or EONIA). More specifically, consider the tenor structure (6) with $\Delta = T_i - T_{i-1}$.[15] At every time T_i, $i = 1, ..., N$, one party pays ΔK, while the other party pays $\Delta \overline{L}(T_{i-1}, T_i)$, where $\overline{L}(T_{i-1}, T_i)$ is the compounded overnight rate for the period $[T_{i-1}, T_i]$. This rate is given by

$$\overline{L}(T_{i-1}, T_i) = \frac{1}{\Delta} \left(\prod_{j=1}^{K_i} (1 + (t_j - t_{j-1}) L(t_{j-1}, t_j)) - 1 \right), \tag{9}$$

where $T_{i-1} = t_0 < t_1 < \cdots < t_{K_i} = T_i$ denotes the partition of the period $[T_{i-1}, T_i]$ into K_i business days, and $L(t_{j-1}, t_j)$ denotes the respective overnight rate. As in Andersen and Piterbarg (2010, Section 5.5), we approximate simple by continuous compounding and the overnight rate by the instantaneous rate $L(t,t)$ given in (4), in which case $\overline{L}(T_{i-1}, T_i)$ becomes

$$\overline{L}(T_{i-1}, T_i) = \frac{1}{\Delta} \left(e^{\int_{T_{i-1}}^{T_i} r_c(s) ds} - 1 \right). \tag{10}$$

[14] We thank Fabio Mercurio for discussions about basis swap market conventions.

[15] In contrast to an IRS, an OIS typically has fixed-rate payments and floating-rate payments occurring at the same frequency.

The OIS rate is the value of K that makes the OIS value equal to zero at inception and is given by

$$OIS(t,T) = \frac{\sum_{i=1}^{n} E_t^Q \left[e^{-\int_t^{T_i} r_c(s)ds} \Delta \overline{L}(T_{i-1}, T_i) \right]}{\sum_{i=1}^{n} \Delta P_c(t,T_i)} = \frac{1 - P_c(t,T_n)}{\sum_{i=1}^{n} \Delta P_c(t,T_i)}. \quad (11)$$

In both the USD and EUR markets, OIS payments occur at a 1Y frequency, i.e. $\Delta = 1$. For OISs with maturities less than one year, there is only one payment at maturity.

2.6 The IRS-OIS spread

Combining (7) and (11), a few computations yield

$$IRS_{\delta,\Delta}(t,T) - OIS(t,T) = \frac{\sum_{i=1}^{N} E_t^Q \left[e^{-\int_t^{t_{i-1}} r_c(s)ds} P_c(t_{i-1}, t_i) \delta \big(L(t_{i-1}, t_i) - OIS(t_{i-1}, t_i) \big) \right]}{\sum_{i=1}^{n} \Delta P_c(t,T_i)}. \quad (12)$$

This equation shows that the spread between the rates on, say, a 5Y IRS indexed to δ-maturity LIBOR and a 5Y OIS reflects (risk-neutral) expectations about future δ-maturity LIBOR-OIS spreads during the next 5 years.[16] To the extent that the LIBOR-OIS spread measures short-term interbank risk, the IRS-OIS spread reflects expectations about future short-term interbank risks – more specifically, about short-term interbank risks among the banks that constitute the LIBOR panel at future tenor dates, which may vary due to the periodic updating of the LIBOR panel. Consequently, we will refer to the term structure of IRS-OIS spreads as *the term structure of interbank risk*.

2.7 Credit default swaps (CDS)

In a credit default swap (CDS), counterparties exchange a stream of coupon payments for a single default protection payment in the event of default by a reference entity. As such, the swap comprises a premium leg (the coupon stream) and a protection leg (the contingent default protection payment). More specifically, consider the tenor structure (5) and let τ denote the default time of the reference entity.[17] The present value of the premium leg with coupon rate C is given by

$$V_{\text{prem}}(t,T) = C\, I_1(t,T) + C\, I_2(t,T),$$

[16] Note that (12) only holds true if the fixed payments are made with the same frequency in the two swaps, which is the case in the EUR market but not in the USD market. For the more general case, suppose that the payments in the OIS are made on the tenor structure $t = T'_0 < T'_1 < \cdots < T'_{n'} = T$, with $\Delta' = T'_i - T'_{i-1}$. Then one can show that (12) holds with $OIS(t_{i-1}, t_i)$ replaced by $w(t) OIS(t_{i-1}, t_i)$, where $w(t) = \frac{\sum_{i=1}^{n} \Delta P_c(t,T_i)}{\sum_{i=1}^{n'} \Delta' P_c(t,T'_i)}$. In the USD market, where $\Delta = 1/2$ and $\Delta' = 1$, $w(t)$ is always very close to one and (12) holds up to a very small approximation error.

[17] CDS contracts are traded with maturity dates falling on one of four roll dates, March 20, June 20, September 20, or December 20. At initiation, therefore, the actual time to maturity of a CDS contract will be close to, but rarely the same, as the specified time to maturity. Coupon payments are made on a quarterly basis coinciding with the CDS roll dates.

where $C\,I_1(t,T)$ with

$$I_1(t,T) = E_t^Q \left[\sum_{i=1}^{N} e^{-\int_t^{t_i} r_c(s)ds} (t_i - t_{i-1}) 1_{\{t_i < \tau\}} \right] \tag{13}$$

is the value of the coupon payments prior to default time τ, and $C\,I_2(t,T)$ with

$$I_2(t,T) = E_t^Q \left[\sum_{i=1}^{N} e^{-\int_t^{\tau} r_c(s)ds} (\tau - t_{i-1}) 1_{\{t_{i-1} < \tau \le t_i\}} \right] \tag{14}$$

is the accrued coupon payment at default time τ. The present value of the protection leg is

$$V_{\text{prot}}(t,T) = E_t^Q \left[e^{-\int_t^{\tau} r_c(s)ds} \left(1 - R(\tau)\right) 1_{\{\tau \le T\}} \right], \tag{15}$$

where $R(\tau)$ denotes the recovery rate at default time τ. The CDS spread, $CDS(t,T)$, is the value of C that makes the premium and protection leg equal in value at inception and is given by[18]

$$CDS(t,T) = \frac{V_{\text{prot}}(t,T)}{I_1(t,T) + I_2(t,T)}.$$

3 Modeling the term structure of interbank risk

We describe our model for the term structure of interbank risk. We first consider the general framework and then specialize to a tractable model with analytical pricing formulas.

3.1 The general framework

Rather than modeling the funding costs of individual panel banks, we assume a sequence of banks each of which represents the panel at a given point in time. More specifically, we assume the extended doubly stochastic framework provided in Appendix B below, where for any $t_0 \ge 0$, the default time of the bank that represents the panel at t_0 is modeled by some random time $\tau(t_0) > t_0$. This default time admits an nonnegative intensity process $\lambda(t_0, t)$, for $t > t_0$, with initial value $\lambda(t_0, t_0) = \Lambda(t_0)$.

In view of the doubly stochastic property (34), the time t_0-value of an unsecured loan with notional 1 to this representative bank over period $[t_0, T]$ equals

$$B(t_0,T) = E_{t_0}^Q \left[e^{-\int_{t_0}^{T} r(s)ds} 1_{\{\tau(t_0) > T\}} \right] = E_{t_0}^Q \left[e^{-\int_{t_0}^{T} (r(s) + \lambda(t_0,s))ds} \right]. \tag{16}$$

Note that here we assume zero recovery of interbank loans, which is necessary to keep the subsequent affine transform analysis tractable. Absent market frictions, the $(T - t_0)$-maturity LIBOR rate $L(t_0,T)$ satisfies $1 + (T - t_0)L(t_0,T) = 1/B(t_0,T)$.

In practice, LIBOR may be affected by factors not directly related to default risk. For instance, banks may hoard cash for precautionary reasons as in the models of Allen, Carletti, and Gale (2009)

[18] While these "par spreads" are quoted in the market, since 2009 CDS contracts have been executed with a standardized coupon and an upfront payment to compensate for the difference between the par spread and the coupon. However, our CDS database consists of par spreads throughout the sample period.

and Acharya and Skeie (2010), or for speculative reasons as in the models of Acharya, Gromb, and Yorulmazer (2007), Acharya, Shin, and Yorulmazer (2010), and Diamond and Rajan (2010). Either way, the volume of longer term interbank loans decrease and the rates on such loans increase beyond the levels justified by default risk. We allow for a non-default component in LIBOR by setting

$$L(t_0, T) = \frac{1}{T - t_0} \left(\frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T), \tag{17}$$

where $\Xi(t_0, T)$ is a multiplicative residual term that satisfies

$$\lim_{T \to t_0} \Xi(t_0, T) = 1.$$

It follows from (4) that the collateral rate $r_c(t_0)$ becomes

$$r_c(t_0) = \lim_{T \to t_0} \frac{1}{T - t_0} \left(\frac{1}{B(t_0, T)} - 1 \right) \Xi(t_0, T) = -\frac{d}{dT} B(t_0, T)|_{T=t_0} = r(t_0) + \Lambda(t_0). \tag{18}$$

Combining (11) (in the case of a single payment) and (17), we get the following expression for the LIBOR-OIS spread

$$L(t_0, T) - OIS(t_0, T) = \frac{1}{T - t_0} \left(\left[\frac{1}{B(t_0, T)} - \frac{1}{P_c(t_0, T)} \right] + \left[\left(\frac{1}{B(t_0, T)} - 1 \right) (\Xi(t_0, T) - 1) \right] \right). \tag{19}$$

The first bracketed term in (19) is the default component. Due to the periodic refreshment of the credit quality of the LIBOR panel, the time-t_0 expected future default risk of the bank that represents the panel at time t_0, $\lambda(t_0, t)$, is larger than the time-t_0 expected initial default risk of the respective banks that represent the future panels, $\lambda(t, t) \equiv \Lambda(t)$. From (16) and (3) in conjunction with (18) it follows that $B(t_0, T) > P_c(t_0, T)$, which implies that the default component is positive. The second bracketed term in (19) is the non-default component, which is positive provided that $\Xi(t_0, T) > 1$.

For the analysis, we also need expressions for the CDS spreads of the bank that represents the panel at t_0. The factors $I_1(t_0, T)$ and $I_2(t_0, T)$ in the present value of the premium leg given in (13) and (14) become

$$I_1(t_0, T) = \sum_{i=1}^{N} (t_i - t_{i-1}) E_{t_0}^Q \left[e^{-\int_{t_0}^{t_i} r_c(s) ds} 1_{\{t_i < \tau(t_0)\}} \right] = \sum_{i=1}^{N} (t_i - t_{i-1}) E_{t_0}^Q \left[e^{-\int_{t_0}^{t_i} (r_c(s) + \lambda(t_0, s)) ds} \right] \tag{20}$$

and[19]

$$\begin{aligned} I_2(t_0, T) &= \sum_{i=1}^{N} E_{t_0}^Q \left[e^{-\int_{t_0}^{\tau(t_0)} r_c(s) ds} (\tau(t_0) - t_{i-1}) 1_{\{t_{i-1} < \tau(t_0) \leq t_i\}} \right] \\ &= \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} (u - t_{i-1}) E_{t_0}^Q \left[e^{-\int_{t_0}^{u} (r_c(s) + \lambda(t_0, s)) ds} \lambda(t_0, u) \right] du. \end{aligned} \tag{21}$$

In line with the assumption of zero recovery of interbank loans in the derivation of (16), we shall assume zero recovery for the CDS protection leg. Its present value (15) thus becomes

$$V_{\text{prot}}(t_0, T) = E_{t_0}^Q \left[e^{-\int_{t_0}^{\tau(t_0)} r_c(s) ds} 1_{\{\tau(t_0) \leq T\}} \right] = \int_{t_0}^{T} E_{t_0}^Q \left[e^{-\int_{t_0}^{u} (r_c(s) + \lambda(t_0, s)) ds} \lambda(t_0, u) \right] du. \tag{22}$$

[19]Here we use the fact that, using the terminology of Appendix B below, $e^{-\int_{t_0}^{u} \lambda(t_0, s) ds} \lambda(t_0, u)$ is the $\mathcal{F}_\infty \vee \mathcal{H}_{t_0}$-conditional density function of $\tau(t_0)$, see e.g. Filipović (2009, Section 12.3).

3.2 An affine factor model

We now introduce an affine factor model for $r(t)$, the intensities $\Lambda(t_0)$ and $\lambda(t_0, t)$, and the residual $\Xi(t_0, T)$. We assume that the risk-free short rate, $r(t)$, is driven by a two-factor Gaussian process[20]

$$\begin{aligned} dr(t) &= \kappa_r(\gamma(t) - r(t))\, dt + \sigma_r\, dW_r(t) \\ d\gamma(t) &= \kappa_\gamma(\theta_\gamma - \gamma(t))\, dt + \sigma_\gamma \left(\rho\, dW_r(t) + \sqrt{1-\rho^2}\, dW_\gamma(t)\right), \end{aligned} \tag{23}$$

where $\gamma(t)$ is the stochastic mean-reversion level of $r(t)$, and ρ is the correlation between innovations to $r(t)$ and $\gamma(t)$.

For simplicity, we assume that the initial default intensity of the respective representative panel banks, $\Lambda(t_0)$, is constant

$$\Lambda(t_0) \equiv \Lambda.$$

However, it is straightforward to extend the setting to a stochastic $\Lambda(t_0)$. The default intensity process of the bank that represents the panel at t_0, $\lambda(t_0, t)$, is modeled by

$$\lambda(t_0, t) = \Lambda + \int_{t_0}^{t} \kappa_\lambda(\Lambda - \lambda(t_0, s))\, ds + \sum_{j=N(t_0)+1}^{N(t)} Z_{\lambda,j}, \tag{24}$$

where $N(t)$ is a simple counting process with jump intensity $\nu(t)$ and $Z_{\lambda,1}, Z_{\lambda,2}, \dots$ are i.i.d. exponential jump sizes with mean $\frac{1}{\zeta_\lambda}$. That is, we assume that deterioration in credit quality of the bank that represents the panel at time t_0 relative to the respective banks that represent the future panels occurs according to a jump process. In particular, the bank that represents the panel at time t_0 will represent the panel on a random time horizon until the first jump of $\lambda(t_0, t)$. Between jumps, we allow for $\lambda(t_0, t)$ to mean-revert towards Λ.[21] The intensity of credit quality deterioration, $\nu(t)$, evolves according to either a one-factor square-root process

$$d\nu(t) = \kappa_\nu(\theta_\nu - \nu(t))\, dt + \sigma_\nu \sqrt{\nu(t)}\, dW_\nu(t), \tag{25}$$

or a two-factor square-root process

$$\begin{aligned} d\nu(t) &= \kappa_\nu(\mu(t) - \nu(t))\, dt + \sigma_\nu \sqrt{\nu(t)}\, dW_\nu(t) \\ d\mu(t) &= \kappa_\mu(\theta_\mu - \mu(t))\, dt + \sigma_\mu \sqrt{\mu(t)}\, dW_\mu(t), \end{aligned} \tag{26}$$

where $\mu(t)$ is the stochastic mean-reversion level of $\nu(t)$.

[20]The model is equally tractable with $r(t)$ being driven by a two-factor square-root process. While this may seem more appropriate given the low interest rate environment during much of the sample, we found that the fit to the OIS term structure is slightly worse with this specification. Nevertheless, the decomposition of the term structure of interbank risk is almost identical for two specifications.

[21]In Section 6 below, we explore an alternative specification, where deterioration in credit quality is permanent.

Finally, the multiplicative residual term, $\Xi(t_0,T)$, is modeled by

$$\frac{1}{\Xi(t_0,T)} = E^Q_{t_0}\left[e^{-\int_{t_0}^T \xi(s)ds}\right], \tag{27}$$

where $\xi(t)$ evolves according to either a one-factor square-root process

$$d\xi(t) = \kappa_\xi(\theta_\xi - \xi(t))\,dt + \sigma_\xi\sqrt{\xi(t)}\,dW_\xi(t), \tag{28}$$

or a two-factor square-root process

$$\begin{aligned} d\xi(t) &= \kappa_\xi(\epsilon(t) - \xi(t))\,dt + \sigma_\xi\sqrt{\xi(t)}\,dW_\xi(t) \\ d\epsilon(t) &= \kappa_\epsilon(\theta_\epsilon - \epsilon(t))\,dt + \sigma_\epsilon\sqrt{\epsilon(t)}\,dW_\epsilon(t), \end{aligned} \tag{29}$$

where $\epsilon(t)$ is the stochastic mean-reversion level of $\xi(t)$.

In the following, we will use the notation $\mathbb{A}(X,Y,Z)$ to denote a specification where $r(t)$, $\nu(t)$, and $\xi(t)$ are driven by X, Y, and Z factors, respectively. We analyze three progressively more complex model specifications: $\mathbb{A}$(2,1,1), where the state vector is given by (23), (25), and (28), $\mathbb{A}$(2,2,1), where the state vector is given by (23), (26), and (28), and $\mathbb{A}$(2,2,2), where the state vector is given by (23), (26), and (29). All specifications have analytical pricing formulas for LIBOR, OIS, IRS, and CDS. These formulas, along with sufficient admissability conditions on the parameter values, are derived in Appendix C.

For the empirical part, we also need the dynamics of the state vector under the objective probability measure $P \sim Q$. Given our relatively short sample period, we assume a parsimonious market price of risk process

$$\Gamma(t) = \left(\Gamma_r, \Gamma_\gamma, \Gamma_\nu\sqrt{\nu(t)}, \Gamma_\mu\sqrt{\mu(t)}, \Gamma_\xi\sqrt{\xi(t)}, \Gamma_\epsilon\sqrt{\epsilon(t)}\right)^\top \tag{30}$$

such that $dW(t) - \Gamma(t)\,dt$ becomes a standard Brownian motion under P with Radon–Nikodym density process[22]

$$\frac{dP}{dQ}|_{\mathcal{F}_t} = \exp\left(\int_0^t \Gamma(s)^\top dW(s) - \frac{1}{2}\int_0^t \|\Gamma(s)\|^2\,ds\right).$$

4 Data and estimation

We estimate the model on a panel data set that covers the period starting with the onset of the credit crisis on August 09, 2007 and ending in January 12, 2011. We do not include the pre-crisis period, given that a regime switch in the perception of interbank risk appear to have occurred at the onset of the crisis, see Figure 1.

[22]We charge no explicit premium for the jump intensity and size risk of $\lambda(t_0,t)$ in (24).

4.1 Interest rate data

The interest rate data is from Bloomberg. We collect daily OIS rates with maturities 3M, 6M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, and 10Y.[23] We also collect daily IRS and BS rates with maturities of 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, and 10Y as well as 3M and 6M LIBOR and EURIBOR rates. The rates on OIS, IRS, and BS are composite quotes computed from quotes that Bloomberg collects from major banks and inter-dealer brokers.

In the USD market, the benchmark IRS is indexed to 3M LIBOR (with fixed-rate payments occurring at a 6M frequency), and the rate on an IRS indexed to 6M LIBOR is obtained via a BS as

$$IRS_{6M,6M}(t,T) = IRS_{3M,6M}(t,T) + BS_{3M,6M}(t,T) \tag{31}$$

Conversely, in the EUR market, the benchmark IRS is indexed to 6M EURIBOR (with fixed-rate payments occurring at a 1Y frequency), and the rate on an IRS indexed to 3M EURIBOR is obtained via a BS as

$$IRS_{3M,1Y}(t,T) = IRS_{6M,1Y}(t,T) - BS_{3M,6M}(t,T) \tag{32}$$

In the paper, we focus on the spreads between rates on IRS and OIS with the same maturities. Therefore, for each currency and on each day in the sample, we have two spread term structures given by

$$SPREAD_{\delta}(t,T) = IRS_{\delta,\Delta}(t,T) - OIS(t,T), \tag{33}$$

for $\delta = 3\text{M}$ or $\delta = 6\text{M}$ and $\Delta = 6\text{M}$ (1Y) in the USD (EUR) market

Table 1 shows summary statistics of the data. For a given maturity, interest rate spreads are always increasing in the tenor (the maturity of the LIBOR rate to which an IRS is indexed). This is consistent with the idea that a 6M LIBOR loan contains more default and liquidity risk than two consecutive 3M LIBOR loans. For a given tenor, the mean and volatility of spreads decrease with maturity. While the mean spreads are similar across the two markets, spread volatility tends to be higher in the USD market.

4.2 CDS spread data

The CDS data is from Markit, which is the leading provider of CDS quotes. Markit collects quotes from major market participants and constructs daily composite quotes. Since data supplied by Markit is widely used for marking-to-market CDS contracts, its quotes are closely watched by market participants. For each bank in the LIBOR and EURIBOR panels, we collect daily spread term structures for CDS

[23] In Bloomberg, there is no USD 7Y OIS rate. Also, the time series for the USD 10Y OIS rate starts July 28, 2008.

contracts written on senior obligations. The term structures consist of 6M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, and 10Y maturities.

Tables 2 and 3 shows summary statistics for the CDS spreads of the constituents of the LIBOR and EURIBOR panels, respectively.[24] The tables also show the currency of the CDS contracts[25], the size of the banks' balance sheets as reported in the 2009 annual reports, a measure of liquidity of the CDS contracts, and the date from which CDS data is available in the Markit database.

Our measure of liquidity is the average daily trading volume in terms of notional as reported by the Depository Trust and Clearing Corporation (DTCC), a global repository that records the details of virtually all CDS trades in the global market. The data covers the period from June 20, 2009 to March 19, 2011 (data was not available prior to this period) and only includes trading activity that involves a transfer of risk between market participants. Also, the data only covers the top 1000 reference entities (in terms of the notional of outstanding contracts) and some banks, particularly from the EURIBOR panel, are not covered (or only covered during parts of the period, in which case we also do not report numbers).

We see that the LIBOR panel mainly consists of very large banks with significant trading activity in their CDS contracts, although it also includes some medium-sized banks for which the CDS contracts are traded less actively. For the EURIBOR panel, there is a larger cross-sectional dispersion of the size of the member banks and the trading activity in their CDS contracts, which is natural given that the panel consists of significantly more banks than the LIBOR panel.

4.3 Measures of interbank default risk

To measure interbank default risk, we initially assume that CDS spreads are pure measures of the default risk of the underlying entities. At each point in time we construct a CDS spread term structure for the representative panel bank as a composite of the CDS spread term structures for the individual panel banks. In doing so, we are careful to match as closely as possible the default risk inherent in LIBOR and EURIBOR.

[24] As mentioned in Section 2.1, the EURIBOR panel consisted of 42 banks during our sample period. Three of the smaller panel banks – Bank of Ireland, Banque et Caisse d'Epargne de l'Etat, and Confederacion Espanola de Cajas de Ahorros – were not in the Markit database.

[25] For European-based banks, the CDS contracts in the database supplied by Markit are denominated in EUR and subject to the Modified-Restructuring (MR) clause. For US-based banks, the CDS contracts are denominated in USD and subject to the MR clause until Dec 31, 2008 and the No-Restructuring (XR) clause, thereafter. Finally, for Japan-based banks, the CDS contracts are denominated in JPY until Dec 31, 2008 and USD, thereafter, and are subject to the Complete-Restructuring (CR) clause. Even though the currency denomination differs across CDS contracts, the CDS spreads are expressed as a rate and are, therefore, free of units of account.

The LIBOR panel

As discussed in Section 2.1, LIBOR is a trimmed mean of the rates at which banks estimate they can obtain unsecured funding for a given term. Since the submitted rates depend on the banks' own default risks, LIBOR itself presumably reflects a trimmed mean of the default risks of the panel banks. Therefore, we measure of the default risk of the bank that represents LIBOR at a given point in time by aggregating the CDS spreads of the individual LIBOR panel banks in the same way that LIBOR is computed from the submitted rates, namely by removing the top and bottom 25 percent of spreads and computing a simple average of the remaining spreads. The resulting default risk measure is denoted CDS_{TrMean}.

The EURIBOR panel

As also discussed in Section 2.1, EURIBOR is a trimmed mean of the rates at which banks estimate a prime bank (not necessarily themselves) can obtain unsecured funding for a given term. While the notion of a prime bank is ambiguous, we interpret it as a representative bank among the panel. Since the median rather than the mean seems to be the appropriate statistics in this case, it is plausible that the submitted rates reflect what each bank perceives is the median default risk in the panel. Being a trimmed mean of the submitted rates, EURIBOR itself also reflects the median default risk in the panel. Therefore, we measure the default risk of the bank that represents EURIBOR at a given point in time by taking the median of the CDS spreads of the individual EURIBOR panel banks. We denote this default risk measure CDS_{Median}.

Liquidity issues

The assumption that CDS spreads are pure measures of default risk is made in several papers, including Longstaff, Mithal, and Neis (2005), Blanco, Brennan, and Marsh (2005), and Ang and Longstaff (2011). However, a number of recent papers have found that CDS spreads may be affected by liquidity effects. For instance, Buhler and Trapp (2010) find that, on average, 95 percent of the observed mid CDS spread is due to default risk, while the remaining is due to liquidity risk and the correlation between default and liquidity risk. This implies that the premium due to liquidity is earned by the seller of default protection and that CDS spreads are upward-biased measures of default risk. Similar results are reached by Bongaerts, de Jong, and Driessen (2011) and others.

In the case of LIBOR, liquidity may be less of an issue since the panel mainly consists of banks with relatively liquid CDS contracts, and since we use a trimmed mean of the individual CDS spreads, which reduces the effect of idiosyncratic noise at the level of the individual spreads. In the case of EURIBOR, where there is larger cross-sectional dispersion in the liquidity of the banks' CDS contracts and where we work with a median spread, liquidity issues may be more important.

For both panels, we consider two alternative measures of default risks that correct for possible liquidity effects. First, we measure default risk by 90 percent of the composite CDS spreads, corresponding to a situation where protection sellers earn a significant liquidity premium. Given the results in Buhler and Trapp (2010), this is likely to be a lower bound on the default component of CDS spreads. Second, we measure default risk by constructing the composite CDS spreads as described above, but only using data from those banks where the average daily notional of CDS transactions are larger than 50 million USD equivalent.[26] In each market, the two alternative default risk measures are denoted CDS_{LIQ1} and CDS_{LIQ2}.

iTraxx Senior Financials index

As an alternative to computing composite CDS spreads from the panel constituents, for the EUR market we also consider the iTraxx Senior Financials CDS index. This index is quoted directly in the market and tracks the spreads on CDS contracts written on senior obligations of 25 large European financial institutions. The index tends to be more liquid than the individual contracts, but clearly both its construction and the fact that some of the underlying institutions are not part of EURIBOR[27] makes it an imperfect measure of the default risk inherent in EURIBOR. Also, it is only available for maturities of 5Y and 10Y. Nevertheless, it serves as an interesting robustness test.

Table 1 shows summary statistics of the composite CDS spreads. On average, the level of CDS spreads increase with maturity, while CDS spread volatility decrease with maturity. In the USD market, we have, on average, $CDS_{LIQ1} < CDS_{TrMean} < CDS_{LIQ2}$, while in the EUR market, we have, on average, $CDS_{LIQ1} < CDS_{LIQ2} < CDS_{Median} < CDS_{iTraxx}$. The magnitudes of CDS_{TrMean} in the USD market and CDS_{Median} in the EUR market are rather similar despite the EURIBOR panel being composed of significantly more banks than the LIBOR panel.

Our main sets of results will be based on the original default risk measures, while in Section 6 we investigate the sensitivity of the results to taking possible CDS liquidity effects into account.

4.4 Maximum-likelihood estimation

We estimate the specifications using maximum-likelihood in conjunction with Kalman filtering. Due to the non-linearities in the relationship between observations and state variables, we apply the non-linear

[26]To put these numbers into perspective, we computed summary statistics for the trading activity among the top 1000 reference entities that were not sovereigns. On a quarterly basis, the median varies between 15.0 and 20.8 million USD, while the mean varies between 25.0 and 33.6 million USD. With a cutoff of 50 million USD, we are clearly focusing on the most liquid segment of the CDS market.

[27]For instance, for the iTraxx series 14, launched in September 2010, 14 of the 25 financial institutions were also members of the EURIBOR panel.

unscented Kalman filter, which is found by Christoffersen et al. (2009) to have very good finite-sample properties in the context of estimating dynamic term structure models with swap rates. Details on the estimation approach are provided in Appendix D.

For identification purposes, we fix ζ_λ at 10, corresponding to a mean jump size in the default intensity of the bank that represents the panel at t_0, $\lambda(t_0, t)$, of 1000 bp. This corresponds to a significant deterioration in the credit quality of the representative panel bank in the event of a shock.

Furthermore, in a preliminary analysis, we find that it is difficult to reliably estimate the initial default intensity of the representative panel bank, Λ. Its value is not identified from the OIS term structure and, in the absence of very short-term CDS rates for the representative panel banks, is also hard to pin down from the CDS term structures. From (4) and (18), we have that Λ is the difference between the instantaneous proxy of the overnight interbank rate, $L(t, t)$, and the truly risk-free rate, $r(t)$. Therefore, one can get an idea about the magnitude of Λ, by examining the spread between FF or EONIA rates and overnight repo rates, which are virtually riskfree due to the practice of overcollateralization of repo loans; see, e.g., Longstaff (2000). During our sample, overnight general collateral (GC) repo rates for Treasury, MBS, and Agency securities were, on average, 15 bp, 2 bp, and 5 bp below the FF rate. In interpreting these numbers, one should keep in mind that the Treasury GC repo rate at times included a significant convenience yield due to periodic very large demand for Treasury collateral during the credit crisis, making this rate a less accurate proxy for the risk-free rate than MBS and Agency GC repo rates.[28] In the EUR market, the overnight GC repo rate for government bonds was, on average, 1 bp below the EONIA rate. Overall, this suggests that there is very little default risk in the market for overnight interbank deposits. In the following, we fix Λ at 5 bp, but reasonable variations in the value of Λ do not change our results.

5 Results

5.1 Maximum-likelihood estimates

Table 5 displays parameter estimates and their asymptotic standard errors.[29] The estimates are strikingly similar across the two market and we, therefore, focus on the USD estimates.

[28]Indeed, several times during our sample, the Treasury GC repo rate spikes down by more than a hundred basis points only to spike up again a few days later. The MBS and Agency GC repo rates are much less susceptible to such downward spikes. As observed by Hordahl and King (2008) "As the available supply of Treasury collateral dropped, those market participants willing to lend out Treasuries were able to borrow cash at increasingly cheap rates. At times, this effect pushed US GC repo rates down to levels only a few basis points above zero".

[29]It is straightforward to verify that for all the specifications, the parameter values satisfy the sufficient admissability conditions in Lemma C.4 in the Appendix.

Under the risk-neutral measure, the long-run mean of the intensity of credit quality deterioration, $\nu(t)$, varies between 0.23 and 0.46, depending on the specification. This implies an expected jump arrival time between approximately 2 and 5 years. Obviously the expected jump arrival times are to be put in relation to our assumed mean jump size of 1000 bp and zero recovery (or 100% loss rate). An increase in either the mean jump size or the recovery rate would lead to an increase in the expected jump arrival time.[30]

In the $\mathbb{A}(2,2,1)$ and $\mathbb{A}(2,2,2)$ specifications, $\nu(t)$ is relatively volatile and displays fast mean-reversion towards $\mu(t)$, which in turn is less volatile and has much slower mean-reversion. Hence, $\nu(t)$ captures transitory shocks to the intensity of credit quality deterioration, while $\mu(t)$ captures more persistent shocks. In the $\mathbb{A}(2,1,1)$ specification, the speed of mean-reversion and volatility lie between those of $\nu(t)$ and $\mu(t)$ in the more general specifications. Although rather imprecisely estimated, the market prices of risk Γ_ν and Γ_μ are negative in all specifications. This implies that under the physical measure, the long-run mean of credit quality deterioration[31] is lower than the risk-neutral long-run mean, indicating that market participants require a premium for bearing exposure to default risk.[32] Between jumps, the reversion of the default intensity towards Λ occurs relatively fast under the risk-neutral measure, with κ_λ estimated between 1.82 and 2.26.

In all specifications, the residual factor, $\xi(t)$, is very volatile, exhibits very fast mean-reversion, and has a long-run mean of essentially zero. In the $\mathbb{A}(2,2,2)$ specifications, $\xi(t)$ is mean-reverting towards $\epsilon(t)$ which is less volatile and has slower mean-reversion. Hence, $\xi(t)$ captures transitory shocks to the non-default component, while $\epsilon(t)$ captures moderately persistent shocks. The market prices of risk Γ_ξ and Γ_ϵ are mostly negative, although not uniformly so, but very imprecisely estimated. If anything, this indicates that market participants require a premium for bearing exposure to the non-default risk factors, such as liquidity risk.

5.2 State variables

Figure 2 displays the state variables for the three specifications estimated on USD data. The corresponding figure for the EUR market is similar and available in the online appendix. It is interesting to see the reaction of the state variables to the three most important shocks to the interbank market during the sample period: the Bear Stearns near-bankruptcy on March 16, 2008, the Lehman Brothers

[30]As a rule of thumb, either doubling the assumed mean jump size or halving the expected loss rate is nearly the same as doubling the expected jump arrival time.

[31]In the $\mathbb{A}(2,1,1)$ specification, the long-run mean under the physical measure is given by $\theta_\nu\kappa_\nu/(\kappa_\nu - \sigma_\nu\Gamma_\nu)$, while in the $\mathbb{A}(2,2,1)$ and $\mathbb{A}(2,2,2)$ specifications, it is given by $\theta_\mu\kappa_\nu\kappa_\mu/\left[(\kappa_\nu - \sigma_\nu\Gamma_\nu)(\kappa_\mu - \sigma_\mu\Gamma_\mu)\right]$.

[32]The long-run mean under the physical measure varies between 0.06 and 0.16, depending on the specification. This translates into an expected jump arrival time between approximately 6 and 17 years. It is a common finding in the literature that default risk carries a risk premium; see, e.g., Driessen (2005), Berndt et al. (2008), and Pan and Singleton (2008).

bankruptcy on September 15, 2008, and the escalation of the European sovereign debt crisis often marked by the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. The figure shows that $\nu(t)$ increases leading up to the Bear Stearns near-default but quickly decreases after the take-over by J.P. Morgan. If anything, the opposite is true of $\xi(t)$. Immediately following the Lehman default, $\xi(t)$ spikes while $\nu(t)$ increases more gradually and does not reach its maximum until March 2009. Finally, with the escalation of the European sovereign debt crisis, $\nu(t)$ increases while $\xi(t)$ does not react. These dynamics hold true regardless of the model specification and suggest that an increase in the risk of credit quality deterioration was the main factor driving interbank risk around the first and third episode, while an increase in risk factors not directly related to default risk was the main driver in the aftermath of the Lehman default.

5.3 Specification analysis

For each of the model specifications, we compute the fitted OIS rates, interest rate spreads, and CDS spreads based on the filtered state variables. For each day in the sample and within each category – OIS, $SPREAD_{3M}$, $SPREAD_{6M}$, and CDS – we then compute the root mean squared pricing errors (RMSEs) of the available rates or spreads, thereby constructing time series of RMSEs.

The first three rows of Panel A in Table 6 display the means of the RMSE time series in the USD market. The next two rows report the mean difference in RMSEs between two model specifications along with the associated t-statistics. Given that all specifications have two factors driving the OIS term structure, they obviously produce almost the same fit to OIS rates. However, they differ significantly in their fit to interest rate spreads and CDS rates. $\mathbb{A}(2,2,1)$ has a significantly better fit than $\mathbb{A}(2,1,1)$ to the CDS term structure, with the mean RMSE decreasing from 11.6 bp to 6.6 bp. It also appears to trade off a statistically significant better fit to the term structure of swap spreads indexed to 6M LIBOR, for a statistically insignificant worse fit to the term structure of swap spreads indexed to 3M LIBOR. $\mathbb{A}(2,2,2)$ improves upon $\mathbb{A}(2,2,1)$ with a statistically significant better fit to the term structures of CDS rates and swap spreads indexed to 6M LIBOR and a marginally statistically significant better fit to the term structure of swap spreads indexed to 3M LIBOR. Economically, however, the improvement of $\mathbb{A}(2,2,2)$ over $\mathbb{A}(2,2,1)$ is modest (about 0.5 bp in terms of average RMSEs) and we do not expect more elaborate models to perform much better.

Panel B in Table 6 display the results for the EUR market, which are similar to those obtained for the USD market.[33] In general, the model tends to have a slightly better fit to the EUR data than the USD data. This is also apparent from Table 5 where, for each specification, the estimated variance on the pricing errors is smaller for the EUR market.[34]

[33]The main difference appear to be that for the EUR market, the more elaborate specifications generate an improvement in the fit to the OIS term structure, which is statistically significant if still economically small.

[34]In our model, we assume that the OIS reference rate equals the average cost of unsecured overnight funding

Since we value parsimony, in the following we use the $\mathbb{A}(2,2,1)$ specification to decompose the term structure of interbank risk into default and non-default components. In Section 6, we investigate the sensitivity of the decomposition to the choice of model specification.

5.4 Decomposing the term structure of interbank risk

We measure the default component as the hypothetical swap spread that would obtain if default risk were the only risk factor in the interbank market. This is computed by setting the residual term to one, $\Xi(t_0, T) = 1$. The non-default component is then given by the difference between the fitted swap spread and the default induced swap spread.

Table 7 displays, for each maturity, the means and standard deviations of the time-series of the two components. Consider first the USD market. Panel A1 shows the decomposition of swap spreads indexed to 3M LIBOR. At the short end of the term structure, the default components is, on average, slightly smaller than the non-default component. As maturity increase, the default component, on average, decrease relatively little and in fact increase for maturities beyond 4 years. On the other hand, the non-default component, on average, decrease rapidly with maturity. The upshot is that, as maturity increases, default increasingly becomes the dominant component. Panel A2 shows the decomposition of swap spreads indexed to 6M LIBOR. Both the default and non-default components are larger, on average. However, the default component increases relatively more than the non-default component. Otherwise, the overall pattern is the same, with default increasingly becoming the dominant component as maturity increases.

Panels B1 and B2 display the corresponding results for the EUR market. At the short end of the term structure, the default component constitute a slightly larger fraction of spreads than is the case for the USD market, while the opposite is true at the long end of the term structure. But broadly the results are similar to the USD market.

Another observation from Table 7 is that both components are very volatile, particularly at the short end of the term structure. For the USD market, Figure 3 displays the time-series of the default and non-default components of the 3M and 6M LIBOR-OIS spreads (Panels A and B) and the 5Y swap spreads indexed to 3M or 6M LIBOR (Panels C and D). Consider first the money market spreads. Prior to the Lehman default, the default component constitutes a relatively small part of spreads, except for

for LIBOR panel banks, which implies that the LIBOR-OIS spread goes to zero as maturity goes to zero. In principle, an interesting out-of-sample test of the model is the extent to which very short term LIBOR-OIS spreads implied by the model correspond to those observed in the data. In practice, however, very short-term LIBOR-OIS spreads are extremely noisy and display little correlation with longer term spreads. For instance, in the USD market, the shortest LIBOR maturity is overnight and the correlation between changes in the overnight LIBOR-FF spread and changes in the 3M and 6M LIBOR-OIS spreads are 0.08 and -0.01, respectively. One would need to add additional factors to the model to capture the largely idiosyncratic behavior at the very short end of the spread curve.

a brief period around the Bear Stearns near-default. In the aftermath of the Lehman default, the non-default component increases rapidly but then declines, while the default component increases gradually. The result is that by March 2009 and for the rest of the sample period, including the European sovereign debt crisis, spreads are almost exclusively driven by the default component. Consider next the 5Y swap spreads. Clearly, default is an overall more important component. Even prior to the Lehman default, the default component is the dominant driver of spreads. Immediately after the Lehman default both the default and non-default components increase after which the default component gradually becomes the exclusive driver of spreads.

Figure 4 displays the time-series of the decomposition of the spreads in the EUR market. The main difference compared to the USD market is that from mid-2009 until the escalation of the EUR sovereign debt crisis, the non-default component re-emerges as a driver of spreads, particularly at the short end of the term structure.

5.5 Understanding the residual component

In our model, $\xi(t)$ and $\epsilon(t)$ are residual factors that capture the component of interbank risk that is orthogonal to default risk. It is tempting to think of these factors as capturing liquidity risk. To see if this interpretation is justified, we regress $\xi(t)$ inferred from the $\mathbb{A}(2,2,1)$ specification on various illiquidity proxies. Given the over-the-counter nature of interbank borrowing and lending, we do not have illiquidity measures that are specific to this market.[35] Instead, we use four measures of illiquidity at the level of the broader fixed income market. The first measure, also discussed in Krishnamurthy (2010), is the spread between the 3M OIS rate and the 3M Treasury bill rate. Both rates are virtually free of default risk, but Treasury bills are arguably the most liquid debt market instrument making the spread a good illiquidity proxy.[36] The second measure, first analyzed by Longstaff (2004), is the yield spread between 10Y Refcorp bonds and 10Y off-the-run Treasury notes. Refcorp bonds have the same default risk as Treasuries, but lower liquidity. The third measure is the "noise" measure recently introduced by Hu, Pan, and Wang (2010) – henceforth HPW – which is a daily aggregate of Treasury price deviations from "fair-value". The final measure is the weekly sum of the notional amount of Treasury delivery fails reported by primary dealers and published by the Federal Reserve Bank of New York, which Fleckenstein, Longstaff, and Lustig (2011) and others argue is a good proxy for disruptions in fixed income market liquidity.

Table 8 displays results from univariate and multivariate regressions of $\xi(t)$ on the illiquidity mea-

[35] There does exist a trading platform for EUR interbank deposits, *e*-Mid. However, the maturities of the traded deposits are almost exclusively overnight (see Angelini, Nobili, and Picillo (2009)), while we are interested in liquidity measures for longer term deposits.

[36] Admittedly, OIS rates are not a completely risk-free rate due to the indexation to the unsecured overnight rate. But as argued in Section 4.4, the default risk component is very small.

sures. Since the HPW noise measure is only available until December 31, 2009, we run the regressions using data up to this date. We run the regressions on daily data, except when including the Treasury settlement fails, in which cases we run the regressions on weekly data.[37] Consider first the USD market (Panel A of Table 8). In the univariate regressions, the coefficients on all the illiquidity measures are positive. The OIS-Tbill spread has limited explanatory power with a marginally statistically significant coefficient and an R^2 of 0.07. The Refcorp-Treasury spread has somewhat higher explanatory power, with a statistically significant coefficient and an R^2 of 0.18. Finally, the HPW noise measure and the amount of Treasury fails both have high explanatory power, with strongly statistically significant coefficients and R^2s of 0.59 and 0.41, respectively. In the multivariate regressions, the coefficients on the OIS-Tbill and Refcorp-Treasury spreads become insignificant, while the coefficients on the HPW noise measure and the amount of Treasury fails remain positive and significant, and the R^2s increase to about 0.70.

Consider next the EUR market (Panel B of Table 8). The results are generally consistent with those of the USD market, although the explanatory power of the illiquidity measures are lower. This is not surprising, given that all our illiquidity measures are USD based. In the multivariate regressions, only the HPW noise measure remains statistically significant, and the R^2s are around 0.36.

Taken together, we believe the results lend support to the conjecture that the non-default component of interbank risk is strongly related to market illiquidity.

6 Robustness tests

The decomposition in Section 5 is based on the $\mathbb{A}(2,2,1)$ specification along with the CDS_{TrMean} and CDS_{Median} measures of interbank default risk in the USD and EUR markets, respectively. In this section, we investigate the robustness of our results to alternative model specifications as well as interbank default risk measures that correct for possible liquidity effects in CDS spreads. Throughout, we focus on the USD market and the swap spread term structure indexed to 3M LIBOR. Conclusions for the spread term structure indexed to 6M LIBOR and for the EUR market are very similar.[38]

[37] We convert daily time-series to weekly by summing up the daily observations over the week. This matches the construction of the time-series of Treasury delivery fails.

[38] Results for the EUR market are available in the online appendix. Only in the case where we use CDS_{iTraxx} to measure interbank default risk does the results differ noticeable from the baseline results. To some extent, this is due to the imperfect overlap between the set of underlying institutions in the iTraxx index and the EURIBOR panel. But to a large extent it is due to the fact that the index is only available for maturities of 5Y and 10Y leading to a less accurate identification of the term structure of interbank default risk.

6.1 Alternative model specifications

To investigate the sensitivity to alternative model specifications, we redo the interbank risk decomposition using the $\mathbb{A}(2,1,1)$ and $\mathbb{A}(2,2,2)$ specifications along with the original measure of interbank default risk. These results are reported in Panels A and B in Table 9. In both cases, the decomposition is very similar to that using the $\mathbb{A}(2,2,1)$ (Panel A1 in Table 7). One difference is that for the $\mathbb{A}(2,1,1)$ specification, the default component at the short end of the term structure is somewhat less volatile, which is not surprising given the one-factor nature of the risk of credit quality deterioration.

Figures 5 and 6 show the time-series of the decomposition at the short and long end (the 5Y point) of the spread term structure, respectively. In both figures, Panel A corresponds to the $\mathbb{A}(2,1,1)$ specification, while Panel B corresponds to the $\mathbb{A}(2,2,2)$ specification. Comparing with Panels A and C in Figure 3 confirms that the decompositions are very similar to that of the $\mathbb{A}(2,2,1)$ specification.

In all the specifications considered so far, we allow $\lambda(t_0, t)$ to mean-revert towards Λ between jumps. We also explore the effect of assuming that shocks to the credit quality of the bank that represents the panel at time t_0 are permanent, which corresponds to setting $\kappa_\lambda = 0$ in (24). We reestimate the $\mathbb{A}(2,2,1)$ specification subject to this constraint. A likelihood ratio test strongly rejects the constraint.[39] More importantly, the restricted specification has a significantly worse fit to the data with the average RMSE for $SPREAD_{3M}$, $SPREAD_{6M}$, and CDS_{TrMean} increasing to 13.36 bp, 11,72 bp, and 8.55 bp, respectively. Consequently, we do not consider that specification in greater detail.

6.2 Alternative measures of interbank default risk

To investigate the sensitivity to possible liquidity effects in the CDS market, we reestimate the $\mathbb{A}(2,2,1)$ specification with the two liquidity-corrected default risk measures CDS_{LIQ1} and CDS_{LIQ2} described in Section 4.3. Panels C and D in Table 9 report the decomposition of the spread term structure in these two cases. At the short end of the term structure, the decomposition using CDS_{LIQ1} (Panel C) on average attributes a slightly smaller fraction of interbank risk to default risk compared with the original decomposition, while the the decomposition using CDS_{LIQ2} (Panel D) on average attributes a slightly larger fraction of interbank risk to default risk. This is consistent with the fact that, on average, $CDS_{LIQ1} < CDS_{TrMean} < CDS_{LIQ2}$. Further out the term structure, the differences are very small.

Again, Figures 5 and 6 show the time-series of the decomposition at the short and long end of the spread term structure, with Panel C corresponding to CDS_{LIQ1} and Panel D corresponding to CDS_{LIQ2}. Comparing again with Panels A and C in Figure 3 underscores the robustness of the decomposition to reasonable assumptions about liquidity effects in the CDS market.

[39]Note that in our setting a likelihood-ratio test is only approximate, since the QML/Kalman filter estimation approach is not consistent.

7 Conclusion

In this paper, we contribute to the rapidly growing literature on the interbank market by studying the term structure of interbank risk. We follow most existing studies by measuring interbank risk by the spread between a LIBOR rate and the rate on an overnight indexed swap (OIS) of identical maturity. We show that the spread between the fixed rate on a long-term interest rate swap indexed to, say, 3M LIBOR, and a similar long-term OIS reflects the risk-neutral expectations about future 3M LIBOR-OIS spreads. This allows us to infer a term structure of interbank risk from swap spreads of different maturities. We develop a dynamic term structure model with default risk in the interbank market that, in conjunction with information from the credit default swap market, allows us to decompose the term structure of interbank risk into default and non-default components. We apply the model to study interbank risk from the onset of the financial crisis in August 2007 until January 2011. We find that, on average, the fraction of total interbank risk due to default risk increases with maturity. At the short end of the term structure, the non-default component is important in the first half of the sample and is correlated with various measures of market-wide liquidity. Further out the term structure, the default component is the dominant driver of interbank risk throughout the sample period. These results hold true in both the USD and EUR markets and are robust to different model parameterizations and measures of interbank default risk. we also discuss potential applications of the model framework within monetary and regulatory policy as well as the pricing, hedging, and risk-management in the interest rate swap market.

A Proof of (2)

Discounting the integral equation (1) gives

$$e^{-\int_0^t r(s)ds}V(t) = E_t^Q\left[e^{-\int_0^T r(s)ds}X + \int_t^T e^{-\int_0^u r(s)ds}\left(r(u) - r_c(u)\right)V(u)\,du\right].$$

Hence

$$M(t) = e^{-\int_0^t r(s)ds}V(t) + \int_0^t e^{-\int_0^u r(s)ds}\left(r(u) - r_c(u)\right)V(u)\,du$$

is a Q-martingale. We obtain

$$d\left(e^{-\int_0^t r(s)ds}V(t)\right) = -\left(r(t) - r_c(t)\right)\left(e^{-\int_0^t r(s)ds}V(t)\right)dt + dM(t).$$

Integration by parts then implies

$$\begin{aligned}
d\left(e^{-\int_0^t r_c(s)ds}V(t)\right) &= d\left(e^{\int_0^t (r(s)-r_c(s))ds}e^{-\int_0^t r(s)ds}V(t)\right) \\
&= e^{-\int_0^t r(s)ds}V(t)e^{\int_0^t (r(s)-r_c(s))ds}\left(r(t) - r_c(t)\right)dt \\
&\quad + e^{\int_0^t (r(s)-r_c(s))ds}\left(-\left(r(t) - r_c(t)\right)\left(e^{-\int_0^t r(s)ds}V(t)\right)dt + dM(t)\right) \\
&= e^{\int_0^t (r(s)-r_c(s))ds}dM(t).
\end{aligned}$$

Hence $e^{-\int_0^t r_c(s)ds}V(t)$ is a Q-martingale, and since $V(T) = X$ we conclude that

$$e^{-\int_0^t r_c(s)ds}V(t) = E_t^Q\left[e^{-\int_0^T r_c(s)ds}X\right],$$

which proves (2).

B Extended doubly stochastic framework

Here, we briefly recap and extend the standard doubly stochastic framework for modeling default times in our setting.[40] The main aspect of our extension is the fact that we can incorporate arbitrarily many default times in one framework. Thereto, we assume that the filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, Q)$ carries an i.i.d. sequence of standard exponential random variables $\varepsilon(t_0) \sim \text{Exp}(1)$, for $t_0 \geq 0$, which are independent of $\mathcal{F}_\infty$. For every $t_0 \geq 0$, we let $\lambda(t_0, t)$ be a nonnegative $\mathcal{F}_t$-adapted intensity process with the property

$$\int_{t_0}^t \lambda(t_0, s)\,ds < \infty$$

for all finite $t \geq t_0$. We then define the random time

$$\tau(t_0) = \inf\left\{t > t_0 \mid \int_{t_0}^t \lambda(t_0, s)\,ds \geq \varepsilon(t_0)\right\} > t_0.$$

[40]Standard references are Duffie and Singleton (2003) and Lando (2004).

Note that $\tau(t_0)$ is not an $\mathcal{F}_t$-stopping time but becomes a stopping time with respect to the enlarged filtration $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t$ where $\mathcal{H}_t = \vee_{t_0 \geq 0} \sigma \left(H(t_0, s) \mid s \leq t \right)$ is the filtration generated by all $\tau(t_0)$-indicator processes $H(t_0, t) = 1_{\{\tau(t_0) \leq t\}}$. The $\mathcal{G}_t$-stopping times $\tau(t_0)$ are then $\mathcal{F}_t$-doubly stochastic in the sense that

$$E^Q \left[Y \, 1_{\{\tau(t_0) > T\}} \mid \mathcal{G}_{t_0} \right] = E^Q_{t_0} \left[Y \, e^{-\int_{t_0}^T \lambda(t_0, s) \, ds} \right] \tag{34}$$

for all $\mathcal{F}_T$-measurable nonnegative random variables Y, see e.g. Filipović (2009, Lemma 12.2).

C Pricing formulas for the affine model

In this section we derive the pricing formulas for the affine model used in this paper. It is evident from the system of stochastic differential equations composed of (23), (26), and (29), that the partial state vectors $(r(t), \gamma(t))^\top$, $(\nu(t), \mu(t), \lambda(t_0, t))^\top$, and $(\xi(t), \epsilon(t))^\top$ form independent autonomous affine jump-diffusion processes. Hence the subsequent exponential-affine expressions (35), (37), (41) follow directly from the general affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2), and the fact that $r_c(t) = r(t) + \Lambda$, see (18). The following formulas are for the full $\mathbb{A}(2, 2, 2)$ model. The nested versions, $\mathbb{A}(2, 2, 1)$ and $\mathbb{A}(2, 1, 1)$, are obtained by setting the respective model parameters, $\kappa_\epsilon, \theta_\epsilon, \sigma_\epsilon$ and $\kappa_\mu, \theta_\mu, \sigma_\mu$, equal to zero, and setting $\epsilon(t) \equiv \theta_\xi$ and $\mu(t) \equiv \theta_\nu$, respectively.

Lemma C.1. *The time t price of the collateralized zero-coupon bond maturing at T equals*

$$\begin{aligned} P_c(t, T) &= E^Q_t \left[e^{-\int_t^T r_c(s) ds} \right] \\ &= \exp \left[A(T - t) + B_r(T - t) r(t) + B_\gamma(T - t) \gamma(t) \right] \end{aligned} \tag{35}$$

where the functions A and $B = (B_r, B_\Lambda)^\top$ solve the system of Riccati equations

$$\begin{aligned} \partial_\tau A(\tau) &= \frac{\sigma_r^2}{2} B_r(\tau)^2 + \rho \sigma_r \sigma_\gamma B_r(\tau) B_\gamma(\tau) + \frac{\sigma_\gamma^2}{2} B_\gamma(\tau)^2 + \kappa_\gamma \theta_\gamma B_\gamma(\tau) - \Lambda \\ \partial_\tau B_r(\tau) &= -\kappa_r B_r(\tau) - 1 \\ \partial_\tau B_\gamma(\tau) &= -\kappa_\gamma B_\gamma(\tau) + \kappa_r B_r(\tau) \\ A(0) &= 0, \quad B(0) = 0. \end{aligned} \tag{36}$$

Lemma C.2. *The time t_0-value of an unsecured loan with notional 1 in (16) equals*

$$\begin{aligned} B(t_0, T) &= E^Q_{t_0} \left[e^{-\int_{t_0}^T (r(s) + \lambda(t_0, s)) ds} \right] \\ &= P_c(t_0, T) \exp \left[C(T - t_0) + D_\nu(T - t_0) \nu(t_0) + D_\mu(T - t_0) \mu(t_0) + D_\lambda(T - t_0) \Lambda \right] \end{aligned} \tag{37}$$

where the functions C and $D = (D_\nu, D_\mu, D_\lambda)^\top$ solve the system of Riccati equations

$$
\begin{aligned}
\partial_\tau C(\tau) &= \kappa_\mu \theta_\mu D_\mu(\tau) + \kappa_\mu \Lambda D_\lambda(\tau) + \Lambda \\
\partial_\tau D_\nu(\tau) &= \frac{\sigma_\nu^2}{2} D_\nu(\tau)^2 - \kappa_\nu D_\nu(\tau) + \frac{D_\lambda(\tau)}{\zeta_\lambda - D_\lambda(\tau)} \\
\partial_\tau D_\mu(\tau) &= \frac{\sigma_\mu^2}{2} D_\mu(\tau)^2 - \kappa_\mu D_\mu(\tau) + \kappa_\nu D_\nu(\tau) \\
\partial_\tau D_\lambda(\tau) &= -\kappa_\lambda D_\lambda(\tau) - 1 \\
C(0) &= 0, \quad D(0) = 0.
\end{aligned}
\tag{38}
$$

Proof. We write

$$
\begin{aligned}
B(t_0, T) &= E_{t_0}^Q \left[e^{-\int_{t_0}^T (r_c(s) - \Lambda + \lambda(t_0, s)) ds} \right] \\
&= E_{t_0}^Q \left[e^{-\int_{t_0}^T r_c(s) ds} \right] E_{t_0}^Q \left[e^{-\int_{t_0}^T (\lambda(t_0, s) - \Lambda) ds} \right].
\end{aligned}
$$

Now the claim follows from the general affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2). Note that $D_\lambda(\tau) < 0$ for all $\tau > 0$. Hence the rational function on the right hand side of the equation for $\partial_\tau D_\nu(\tau)$ is well defined and derived by

$$
\begin{aligned}
\int_0^\infty \left(e^{D_\lambda(\tau)\xi} - 1 \right) \zeta_\lambda \, e^{-\zeta_\lambda \xi} \, d\xi &= \zeta_\lambda \int_0^\infty e^{-(\zeta_\lambda - D_\lambda(\tau))\xi} \, d\xi - 1 \\
&= \frac{\zeta_\lambda}{\zeta_\lambda - D_\lambda(\tau)} - 1 \\
&= \frac{D_\lambda(\tau)}{\zeta_\lambda - D_\lambda(\tau)}.
\end{aligned}
$$

□

We obtain the following exponential affine expression for the $(T - t_0)$-maturity LIBOR rate $L(t_0, T)$.

Corollary C.3. *The $(T - t_0)$-maturity LIBOR rate given in (17) equals*

$$
\begin{aligned}
L(t_0, T) = \frac{1}{T - t_0} & \left(P_c(t_0, T)^{-1} \exp\left[-C(T - t_0) - D_\nu(T - t_0)\nu(t_0) - D_\mu(T - t_0)\mu(t_0) - D_\lambda(T - t_0)\Lambda \right] - 1 \right) \\
& \times \exp\left[-E(T - t_0) - F_\xi(T - t_0)\xi(t_0) - F_\epsilon(T - t_0)\epsilon(t_0) \right]
\end{aligned}
\tag{39}
$$

with $C(T - t_0)$ and $D(T - t_0)$ given in Lemma C.2, and where the functions E and $F = (F_\xi, F_\epsilon)^\top$ solve the Riccati equations

$$
\begin{aligned}
\partial_\tau E(\tau) &= \kappa_\epsilon \theta_\epsilon F_\epsilon(\tau) \\
\partial_\tau F_\xi(\tau) &= \frac{\sigma_\xi^2}{2} F_\xi(\tau)^2 - \kappa_\xi F_\xi(\tau) - 1 \\
\partial_\tau F_\epsilon(\tau) &= \frac{\sigma_\epsilon^2}{2} F_\epsilon(\tau)^2 - \kappa_\epsilon F_\epsilon(\tau) + \kappa_\xi F_\xi(\tau) \\
E(0) &= 0, \quad F(0) = 0.
\end{aligned}
\tag{40}
$$

Proof. In view of (27) and the affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2), the multiplicative residual term is given by

$$\frac{1}{\Xi(t_0,T)} = \exp\left[E(T-t_0) + F_\xi(T-t_0)\xi(t_0) + F_\epsilon(T-t_0)\epsilon(t_0)\right] \tag{41}$$

where the functions E and $F = (F_\xi, F_\epsilon)^\top$ solve the Riccati equations (40). The corollary now follows from (17) and Lemma C.2. □

In view of (7) we also need a closed form expression for

$$I = E_t^Q\left[e^{-\int_t^T r_c(s)ds}(T-t_0)L(t_0,T)\right]$$

for time points $t \leq t_0 < T$. Using the tower property of conditional expectations we calculate

$$\begin{aligned}
I &= E_t^Q\left[e^{-\int_t^{t_0} r_c(s)ds}\, E_{t_0}^Q\left[e^{-\int_{t_0}^T r_c(s)ds}\right](T-t_0)L(t_0,T)\right] \\
&= E_t^Q\left[e^{-\int_t^{t_0} r_c(s)ds} P_c(t_0,T)(T-t_0)L(t_0,T)\right] \\
&= \Big(E_t^Q\left[e^{-\int_t^{t_0} r_c(s)ds}\exp\left[-C(T-t_0) - D_\nu(T-t_0)\nu(t_0) - D_\mu(T-t_0)\mu(t_0) - D_\lambda(T-t_0)\Lambda\right]\right] \\
&\quad\quad - E_t^Q\left[e^{-\int_t^{t_0} r_c(s)ds} P_c(t_0,T)\right]\Big) \\
&\quad\quad \times E_t^Q\left[\exp\left[-E(T-t_0) - F_\xi(T-t_0)\xi(t_0) - F_\epsilon(T-t_0)\epsilon(t_0)\right]\right] \\
&= \Big(P_c(t,t_0)e^{-C(T-t_0)-D_\lambda(T-t_0)\Lambda} E_t^Q\left[\exp\left[-D_\nu(T-t_0)\nu(t_0) - D_\mu(T-t_0)\mu(t_0)\right]\right] - P_c(t,T)\Big) \\
&\quad \times E_t^Q\left[\exp\left[-E(T-t_0) - F_\xi(T-t_0)\xi(t_0) - F_\epsilon(T-t_0)\epsilon(t_0)\right]\right].
\end{aligned}$$

The conditional expectations on the right hand side of the last equality can easily be obtained in closed form using the affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2).

It remains to be checked whether the above conditional expectations are well defined. Sufficient admissibility conditions on the model parameters are provided by the following lemma, the proof of which is in the online appendix.

Lemma C.4. (i) *Suppose $\kappa_\lambda \geq 0$, and define*

$$\Theta_\nu = \sqrt{\kappa_\nu^2 + 2\frac{\sigma_\nu^2}{\zeta_\lambda\kappa_\lambda + 1}}, \tag{42}$$

$$C_\nu = \frac{\frac{2}{\zeta_\lambda\kappa_\lambda+1}\left(e^{\Theta_\nu(T-t_0)} - 1\right)}{\Theta_\nu\left(e^{\Theta_\nu(T-t_0)} + 1\right) + \kappa_\nu\left(e^{\Theta_\nu(T-t_0)} - 1\right)},$$

$$\Theta_\mu = \sqrt{\kappa_\mu^2 + 2\sigma_\mu^2\kappa_\nu C_\nu}, \tag{43}$$

$$C_\mu = \frac{2\kappa_\nu C_\nu\left(e^{\Theta_\mu(T-t_0)} - 1\right)}{\Theta_\mu\left(e^{\Theta_\mu(T-t_0)} + 1\right) + \kappa_\mu\left(e^{\Theta_\mu(T-t_0)} - 1\right)}.$$

If

$$\kappa_\nu > \frac{1}{2}\sigma_\nu^2 C_\nu \tag{44}$$

and

$$\begin{aligned}\kappa_\mu \geq{}& \sigma_\mu^2 C_\mu e^{-\frac{\kappa_\mu}{2}\tau^*} \\ &+ \frac{4\kappa_\nu^2\sigma_\mu^2}{\kappa_\mu\sigma_\nu^2}\left({}_2F_1\left(1, \frac{\kappa_\mu}{2\kappa_\nu}; \frac{\kappa_\mu + 2\kappa_\nu}{2\kappa_\nu}; \frac{\left(\sigma_\nu^2 C_\nu - 2\kappa_\nu\right) e^{\kappa_\nu\tau^*}}{\sigma_\nu^2 C_\nu}\right)\right. \\ &\left. -e^{-\frac{\kappa_\mu}{2}\tau^*}\,{}_2F_1\left(1, \frac{\kappa_\mu}{2\kappa_\nu}; \frac{\kappa_\mu + 2\kappa_\nu}{2\kappa_\nu}; \frac{\sigma_\nu^2 C_\nu - 2\kappa_\nu}{\sigma_\nu^2 C_\nu}\right)\right)\end{aligned} \tag{45}$$

where ${}_2F_1$ denotes the Gauss hypergeometric function and

$$\tau^* = \frac{1}{\kappa_\nu}\log\max\left\{\frac{\left(2\kappa_\nu - \sigma_\nu^2\frac{\kappa_\mu^2}{2\kappa_\nu\sigma_\mu^2}\right)\frac{2\kappa_\nu\sigma_\mu^2}{\kappa_\mu^2}C_\nu}{2\kappa_\nu - \sigma_\nu^2 C_\nu}, 1\right\}, \tag{46}$$

then

$$E^Q\left[\exp\left[-D_\nu(T-t_0)\nu(t_0) - D_\mu(T-t_0)\mu(t_0)\right]\right] < \infty.$$

(ii) *Define*

$$\begin{aligned}\Theta_\xi &= \sqrt{\kappa_\xi^2 + 2\sigma_\xi^2}, \\ C_\xi &= \frac{2\left(e^{\Theta_\xi(T-t_0)} - 1\right)}{\Theta_\xi\left(e^{\Theta_\xi(T-t_0)} + 1\right) + \kappa_\xi\left(e^{\Theta_\xi(T-t_0)} - 1\right)}, \\ \Theta_\epsilon &= \sqrt{\kappa_\epsilon^2 + 2\sigma_\epsilon^2\kappa_\xi C_\xi}, \\ C_\epsilon &= \frac{2\kappa_\xi C_\xi\left(e^{\Theta_\epsilon(T-t_0)} - 1\right)}{\Theta_\epsilon\left(e^{\Theta_\epsilon(T-t_0)} + 1\right) + \kappa_\epsilon\left(e^{\Theta_\epsilon(T-t_0)} - 1\right)}.\end{aligned}$$

If conditions (44) *and* (45) *hold for $C_\nu, \kappa_\nu, \sigma_\nu, C_\mu, \kappa_\mu, \sigma_\mu$ replaced by $C_\xi, \kappa_\xi, \sigma_\xi$, $C_\epsilon, \kappa_\epsilon, \sigma_\epsilon$, respectively, then*

$$E^Q\left[\exp\left[-F_\xi(T-t_0)\xi(t_0) - F_\epsilon(T-t_0)\epsilon(t_0)\right]\right] < \infty.$$

Remark C.5. *Note that $\tau^* = 0$ if and only if $\frac{\kappa_\mu^2}{2\kappa_\nu\sigma_\mu^2} \geq C_\nu$. In this case,* (45) *reads as $\kappa_\mu \geq \sigma_\mu^2 C_\mu$, which is automatically satisfied as is shown at the end of the proof of Lemma C.4.*

For the CDS coupon rate calculations, we need the respective exponential affine expressions for (20), (21) and (22). For $I_1(t_0,T)$ we obtain

$$I_1(t_0,T) = \sum_{i=1}^{N}(t_i - t_{i-1})e^{-(t_i-t_0)\Lambda}B(t_0,t_i). \tag{47}$$

In both formulas for $I_2(t_0,T)$ and $V_{\text{prot}}(t_0,T)$ the following expression shows up

$$J(t_0,u) = E_{t_0}^Q\left[e^{-\int_{t_0}^u (r_c(s)+\lambda(t_0,s))ds}\lambda(t_0,u)\right].$$

Lemma C.6. *We have*

$$J(t_0,u) = \left(g(u-t_0) + h_\nu(u-t_0)\nu(t_0) + h_\mu(u-t_0)\mu(t_0) + h_\lambda(u-t_0)\Lambda\right)e^{-(u-t_0)\Lambda}B(t_0,u)$$

where the functions g and $h = (h_\nu, h_\mu, h_\lambda)^\top$ solve the linear inhomogeneous system of ordinary differential equations

$$\begin{aligned}
\partial_\tau g(\tau) &= \kappa_\mu \theta_\mu h_\mu(\tau) + \kappa_\lambda \Lambda h_\lambda(\tau) \\
\partial_\tau h_\nu(\tau) &= \sigma_\nu^2 D_\nu(\tau) h_\nu(\tau) - \kappa_\nu h_\nu(\tau) + \frac{\zeta_\lambda h_\lambda(\tau)}{(\zeta_\lambda - D_\lambda(\tau))^2} \\
\partial_\tau h_\mu(\tau) &= \sigma_\mu^2 D_\mu(\tau) h_\mu(\tau) - \kappa_\mu h_\mu(\tau) + \kappa_\nu h_\nu(\tau) \\
\partial_\tau h_\lambda(\tau) &= -\kappa_\lambda h_\lambda(\tau) \\
g(0) &= 0, \quad h(0) = (0,0,1)^\top.
\end{aligned} \tag{48}$$

and where the functions $D = (D_\nu, D_\mu, D_\lambda)^\top$ are given in Lemma C.2.

Proof. We first decompose $J(t_0, u) = P_c(t_0, u) I(t_0, u)$ with

$$I(t_0, u) = E_{t_0}^Q \left[e^{-\int_{t_0}^u \lambda(t_0,s)ds} \lambda(t_0, u) \right],$$

which we can compute by differentiating the respective moment generating function[41]:

$$I(t_0, u) = \frac{d}{dv} E_{t_0}^Q \left[e^{-\int_{t_0}^u \lambda(t_0,s)ds} e^{v\lambda(t_0,u)} \right] |_{v=0}. \tag{49}$$

The affine transform formula in Duffie, Filipović, and Schachermayer (2003, Section 2), gives us

$$\begin{aligned}
&E_{t_0}^Q \left[e^{-\int_{t_0}^u \lambda(t_0,s)ds} e^{v\lambda(t_0,u)} \right] \\
&\qquad = \exp\left[G(u - t_0, v) + H_\nu(u - t_0, v)\nu(t_0) + H_\mu(u - t_0, v)\mu(t_0) + H_\lambda(u - t_0, v)\Lambda \right]
\end{aligned}$$

where the functions G and $H = (H_\nu, H_\mu, H_\lambda)^\top$ solve the system of Riccati equations

$$\begin{aligned}
\partial_\tau G(\tau, v) &= \kappa_\mu \theta_\mu H_\mu(\tau, v) + \kappa_\lambda \Lambda H_\lambda(\tau, v) \\
\partial_\tau H_\nu(\tau, v) &= \frac{\sigma_\nu^2}{2} H_\nu(\tau, v)^2 - \kappa_\nu H_\nu(\tau, v) + \frac{H_\lambda(\tau, v)}{\zeta_\lambda - H_\lambda(\tau, v)} \\
\partial_\tau H_\mu(\tau, v) &= \frac{\sigma_\mu^2}{2} H_\mu(\tau, v)^2 - \kappa_\mu H_\mu(\tau, v) + \kappa_\nu H_\nu(\tau, v) \\
\partial_\tau H_\lambda(\tau, v) &= -\kappa_\lambda H_\lambda(\tau, v) - 1 \\
G(0, v) &= 0, \quad H(0, v) = (0, 0, v)^\top.
\end{aligned} \tag{50}$$

Hence from (49) we obtain

$$\begin{aligned}
I(t_0, u) &= (g(u - t_0) + h_\nu(u - t_0)\nu(t_0) + h_\mu(u - t_0)\mu(t_0) + h_\lambda(u - t_0)\Lambda) \\
&\quad \times \exp\left[G(u - t_0, 0) + H_\nu(u - t_0, 0)\nu(t_0) + H_\mu(u - t_0, 0)\mu(t_0) + H_\lambda(u - t_0, 0)\Lambda \right]
\end{aligned}$$

[41] Note that the change of order of differentiation and expectation is justified by dominated convergence. Indeed, it follows from Duffie, Filipović, and Schachermayer (2003, Theorem 2.16) that $E_{t_0}^Q \left[e^{-\int_{t_0}^u \lambda(t_0,s)ds} e^{v\lambda(t_0,u)} \right]$ is finite for all v in some neighborhood of zero.

where $g(\tau) = \frac{d}{dv}G(\tau, v)|_{v=0}$, and $h = (h_\nu, h_\mu, h_\lambda)^\top$ is given by $h(\tau) = \frac{d}{dv}H(\tau, v)|_{v=0}$. Note that $G(\tau, 0) = C(\tau) - \tau\Lambda$ and $H(\tau, 0) = D(\tau)$, see Lemma C.2. Differentiating both sides of the system (50) in v at $v = 0$ shows that the functions g and h solve the linear inhomogeneous system of ordinary differential equations (48). Thus the lemma is proved. □

D Maximum likelihood estimation

D.1 The state space form

We cast the model in state space form, which consists of a measurement equation and a transition equation. The measurement equation describes the relationship between the state variables and the OIS rates, interest rate spreads, and CDS spreads, while the transition equation describes the discrete-time dynamics of the state variables.

Let X_t denote the vector of state variables. While the transition density of X_t is unknown, its conditional mean and variance is known in closed form, since X_t follows an affine diffusion process. We approximate the transition density with a Gaussian density with identical first and second moments, in which case the transition equation becomes

$$X_t = \Phi_0 + \Phi_X X_{t-1} + w_t, \quad w_t \sim N(0, Q_t), \tag{51}$$

with Φ_0, Φ_X, and Q_t given in closed form.[42]

The measurement equation is given by

$$Z_t = h(X_t) + u_t, \quad u_t \sim N(0, \Omega), \tag{52}$$

where Z_t is the vector of OIS rates, interest rate spreads, and CDS spreads observed at time t, h is the pricing function, and u_t is a vector of iid. Gaussian pricing errors with covariance matrix Ω. To reduce the number of parameters in Ω, we follow usual practice in the empirical term structure literature in assuming that the pricing errors are cross-sectionally uncorrelated (that is, Ω is diagonal), and that the same variance, σ^2_{err}, applies to all pricing errors.

D.2 The unscented Kalman filter

If the pricing function were linear $h(X_t) = h_0 + HX_t$, the Kalman filter would provide efficient estimates of the conditional mean and variance of the state vector. Let $\hat{X}_{t|t-1} = E_{t-1}[X_t]$ and $\hat{Z}_{t|t-1} = E_{t-1}[Z_t]$

[42] Approximating the true transition density with a Gaussian, makes this a QML procedure. While QML estimation has been shown to be consistent in many settings, it is in fact not consistent in a Kalman filter setting since the conditional covariance matrix Q_t in the recursions depends on the Kalman filter estimates of the volatility state variables rather than the true, but unobservable, values; see, e.g., Duan and Simonato (1999). However, simulation results in several papers have shown this issue to be negligible in practice.

denote the expectation of X_t and Z_t, respectively, using information up to and including time $t-1$, and let $P_{t|t-1}$ and $F_{t|t-1}$ denote the corresponding error covariance matrices. Furthermore, let $\hat{X}_t = E_t[X_t]$ denote the expectation of X_t including information at time t, and let P_t denote the corresponding error covariance matrix. The Kalman filter consists of two steps: prediction and update. In the prediction step, $\hat{X}_{t|t-1}$ and $P_{t|t-1}$ are given by

$$\hat{X}_{t|t-1} = \Phi_0 + \Phi_X \hat{X}_{t-1} \tag{53}$$

$$P_{t|t-1} = \Phi_X P_{t-1} \Phi_X' + Q_t, \tag{54}$$

and $\hat{Z}_{t|t-1}$ and $F_{t|t-1}$ are in turn given by

$$\hat{Z}_{t|t-1} = h(\hat{X}_{t|t-1}) \tag{55}$$

$$F_{t|t-1} = H P_{t|t-1} H' + \Omega. \tag{56}$$

In the update step, the estimate of the state vector is refined based on the difference between predicted and observed quantities, with $\hat{X}_t = E_t[X_t]$ and P_t given by

$$\hat{X}_t = \hat{X}_{t|t-1} + W_t(Z_t - \hat{Z}_{t|t-1}) \tag{57}$$

$$P_t = P_{t|t-1} - W_t F_{t|t-1} W_t', \tag{58}$$

where

$$W_t = P_{t|t-1} H' F_{t|t-1}^{-1} \tag{59}$$

is the covariance between pricing and filtering errors.

In our setting, the pricing function is non-linear for all the instruments included in the estimation, and the Kalman filter has to be modified. Non-linear state space systems have traditionally been handled with the extended Kalman filter, which effectively linearizes the measure equation around the predicted state. However, in recent years the unscented Kalman filter has emerged as a very attractive alternative. Rather than approximating the measurement equation, it uses the true non-linear measurement equation and instead approximates the distribution of the state vector with a deterministically chosen set of sample points, called "sigma points", that completely capture the true mean and covariance of the state vector. When propagated through the non-linear pricing function, the sigma points capture the mean and covariance of the data accurately to the 2nd order (3rd order for true Gaussian states) for any nonlinearity.[43]

More specifically, a set of $2L+1$ sigma points and associated weights are selected according to the

[43] For comparison, the extended Kalman filter estimates the mean and covariance accurately to the 1st order. Note that the computational costs of the extended Kalman filter and the unscented Kalman filter are of the same order of magnitude.

following scheme

$$\begin{array}{rclrcll}\hat{\mathcal{X}}^0_{t|t-1} & = & \hat{X}_{t|t-1} & w^0 & = & \frac{\kappa}{L+\kappa} & \\ \hat{\mathcal{X}}^i_{t|t-1} & = & \hat{X}_{t|t-1}+\left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i & w^i & = & \frac{1}{2(L+\kappa)} & i=1,...,L \\ \hat{\mathcal{X}}^i_{t|t-1} & = & \hat{X}_{t|t-1}-\left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i & w^i & = & \frac{1}{2(L+\kappa)} & i=L+1,...,2L,\end{array} \tag{60}$$

where L is the dimension of $\hat{X}_{t|t-1}$, κ is a scaling parameter, w^i is the weight associated with the i'th sigma-point, and $\left(\sqrt{(L+\kappa)P_{t|t-1}}\right)_i$ is the i'th column of the matrix square root. Then, in the prediction step, (55) and (56) are replaced by

$$\hat{Z}_{t|t-1} = \sum_{i=0}^{2L} w^i h(\hat{\mathcal{X}}^i_{t|t-1}) \tag{61}$$

$$F_{t|t-1} = \sum_{i=0}^{2L} w^i (h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})(h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})' + \Omega. \tag{62}$$

The update step is still given by (57) and (58), but with W_t computed as

$$W_t = \sum_{i=0}^{2L} w^i (\hat{\mathcal{X}}^i_{t|t-1} - \hat{X}_{t|t-1})(h(\hat{\mathcal{X}}^i_{t|t-1}) - \hat{Z}_{t|t-1})' F^{-1}_{t|t-1}. \tag{63}$$

Finally, the log-likelihood function is given by

$$\log L = -\frac{1}{2}\log 2\pi \sum_{t=1}^{T} N_t - \frac{1}{2}\sum_{t=1}^{T}\log|F_{t|t-1}| - \frac{1}{2}\sum_{t=1}^{T}(Z_t - \hat{Z}_{t|t-1})' F^{-1}_{t|t-1}(Z_t - \hat{Z}_{t|t-1}), \tag{64}$$

where T is the number of observation dates, and N_t is the dimension of Z_t.

	Maturity								
	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
	Panel A: USD market								
OIS	1.17	1.17	1.26	1.63	2.06	2.42	2.72		$3.17^{\dagger}$
	(1.48)	(1.43)	(1.35)	(1.21)	(1.12)	(1.03)	(0.96)		(0.55)
$SPREAD_{3M}$	58.7		51.2	43.8	39.0	35.4	32.5		$28.7^{\dagger}$
	(57.5)		(34.6)	(23.2)	(17.2)	(14.0)	(11.9)		(8.2)
$SPREAD_{6M}$		79.1	70.0	58.0	50.8	45.8	41.9		$38.1^{\dagger}$
		(57.4)	(42.7)	(28.2)	(20.8)	(16.9)	(14.2)		(7.7)
CDS_{TrMean}		67.8	70.2	78.7	85.3	93.4	99.1	102.1	104.8
		(46.5)	(44.9)	(41.2)	(37.9)	(37.0)	(35.9)	(34.5)	(33.3)
CDS_{LIQ1}		61.1	63.2	70.9	76.8	84.1	89.2	91.9	94.3
		(41.9)	(40.4)	(37.1)	(34.1)	(33.3)	(32.3)	(31.0)	(30.0)
CDS_{LIQ2}		78.7	82.9	91.2	98.8	106.2	113.1	114.6	116.6
		(55.1)	(53.4)	(48.4)	(45.0)	(43.0)	(42.0)	(40.7)	(39.2)
	Panel B: EUR market								
OIS	1.91	1.93	2.00	2.21	2.45	2.67	2.85	3.14	3.44
	(1.67)	(1.65)	(1.58)	(1.38)	(1.23)	(1.13)	(1.02)	(0.87)	(0.74)
$SPREAD_{3M}$	58.7		49.6	43.0	39.6	36.0	34.3	32.0	29.9
	(35.6)		(21.7)	(15.2)	(12.2)	(11.3)	(10.0)	(8.6)	(7.4)
$SPREAD_{6M}$		73.5	66.3	55.9	50.6	45.7	43.1	39.6	36.2
		(36.2)	(24.4)	(16.1)	(13.0)	(12.9)	(11.8)	(10.5)	(9.2)
CDS_{Median}		70.5	72.9	81.3	88.6	95.8	102.3	104.8	107.3
		(43.0)	(40.5)	(37.7)	(35.9)	(35.2)	(34.8)	(34.4)	(33.9)
CDS_{LIQ1}		63.4	65.6	73.2	79.7	86.2	92.1	94.3	96.6
		(38.7)	(36.4)	(33.9)	(32.3)	(31.7)	(31.3)	(31.0)	(30.5)
CDS_{LIQ2}		64.9	67.8	76.1	83.8	90.9	97.4	99.6	102.1
		(39.3)	(38.6)	(36.2)	(35.2)	(35.0)	(35.3)	(34.9)	(34.5)
CDS_{iTraxx}							104.0		109.0
							(39.0)		(37.2)

Notes: The table shows means and, in parentheses, standard deviations of the time series. $SPREAD_{3M}$ denotes the difference between the fixed rates on an IRS indexed to 3M LIBOR/EURIBOR and an OIS with the same maturity. $SPREAD_{6M}$ denotes the difference between the fixed rates on an IRS indexed to 6M LIBOR/EURIBOR and an OIS with the same maturity. CDS_{TrMean} and CDS_{Median} are the CDS spread term structures for the representative LIBOR and EURIBOR panel banks, respectively. CDS_{LIQ1}, and CDS_{LIQ2} are the CDS spread term structures corrected for possible liquidity effects as described in the main text. CDS_{iTraxx} is the iTraxx Senior Financials CDS index. OIS rates are measured in percentages, while interest rate spreads and CDS spreads are measured in basis points. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011, except those marked with † which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

Table 1: Summary statistics

Bank	Currency	Mean CDS	Std CDS	Balance	Liquidity	Start date
Bank of America	USD	136	62	2230	203	09-Aug-2007
Bank of Tokyo Mitsubishi	USD	71	28	1619	17	09-Aug-2007
Barclays	EUR	111	46	2227	117	09-Aug-2007
Citigroup	USD	201	122	1857	155	09-Aug-2007
Credit Suisse	EUR	108	40	997	71	06-May-2008
Deutsche Bank	EUR	95	32	2151	159	09-Aug-2007
HSBC	EUR	75	29	2364	38	09-Aug-2007
J. P. Morgan Chase	USD	92	36	2032	174	09-Aug-2007
Lloyds TSB	EUR	122	58	925	58	09-Aug-2007
Rabobank	EUR	77	39	871	—	09-Aug-2007
Royal Bank of Canada	USD	75	36	606	—	09-Aug-2007
Societe Generale	EUR	93	34	1467	66	09-Aug-2007
Norinchukin Bank	USD	85	41	630	2	09-Aug-2007
RBS	EUR	134	53	2739	117	09-Aug-2007
UBS	EUR	120	62	1296	81	09-Aug-2007
WestLB	EUR	118	41	347	16	09-Aug-2007

Notes: The table displays data on the banks that are members of the LIBOR panel. For each bank, it shows the currency of the CDS contracts, the mean and standard deviation of the 5Y CDS spread in basis points per annum, the size of the balance sheet in billion USD equivalent as reported in the 2009 annual report, the average daily notional of CDS transactions in million USD equivalent as reported by the Depository Trust and Clearing Corporation, and the date from which the 5Y CDS contract is available in the Markit database.

Table 2: LIBOR panel

Bank	Currency	Mean CDS	Std CDS	Balance	Liquidity	Start date
Erste Bank	EUR	186	70	202	8	11-Aug-2008
Raiffeisen Zentralbank	EUR	174	93	148	8	09-Aug-2007
Dexia Bank	EUR	216	103	578	14	09-Aug-2007
KBC	EUR	143	76	324	3	09-Aug-2007
Nordea	EUR	74	31	508	——	09-Aug-2007
BNP-Paribas	EUR	71	27	2058	72	09-Aug-2007
Societe Generale	EUR	93	34	1024	66	09-Aug-2007
Natixis	EUR	161	76	449	5	09-Aug-2007
Credit Agricole	EUR	95	37	1557	74	09-Aug-2007
CIC	EUR	92	34	236	——	09-Aug-2007
Landesbank Berlin	EUR	108	32	144	——	14-Aug-2007
Bayerische Landesbank	EUR	101	30	339	6	09-Aug-2007
Deutsche Bank	EUR	95	32	1501	159	09-Aug-2007
WestLB	EUR	118	41	242	16	09-Aug-2007
Commerzbank	EUR	88	29	844	91	09-Aug-2007
DZ Bank	EUR	106	32	389	——	09-Aug-2007
Genossenschaftsbank	EUR	134	19	68	——	31-Oct-2008
Norddeutsche Landesbank	EUR	99	29	239	——	09-Aug-2007
Landesbank Baden-Wurttemberg	EUR	107	33	412	——	09-Aug-2007
Landesbank Hessen-Thuringen	EUR	111	31	170	——	09-Aug-2007
National Bank of Greece	EUR	342	286	113	——	09-Aug-2007
Allied Irish Banks	EUR	297	265	174	20	09-Aug-2007
Intesa Sanpaolo	EUR	83	40	625	117	09-Aug-2007
Monte dei Paschi di Siena	EUR	105	57	225	84	09-Aug-2007
Unicredit	EUR	121	42	929	114	21-May-2008
ING Bank	EUR	91	35	1164	35	09-Aug-2007
RBS	EUR	145	20	1912	117	20-Aug-2010
Rabobank	EUR	77	39	608	——	09-Aug-2007
Caixa Geral De Depositos	EUR	164	132	121	——	09-Aug-2007
Banco Bilbao Vizcaya Argentaria	EUR	114	62	535	133	09-Aug-2007
Banco Santander	EUR	109	52	1111	156	24-Aug-2007
La Caixa	EUR	178	87	272	——	09-Aug-2007

Notes: Continued in Table 4

Table 3: EURIBOR panel

Bank	Currency	Mean CDS	Std CDS	Balance	Liquidity	Start date
Barclays	EUR	111	46	1554	117	09-Aug-2007
Danske Bank	EUR	87	44	416	9	09-Aug-2007
Svenska Handelsbanken	EUR	67	29	207	5	09-Aug-2007
UBS	EUR	120	62	904	81	09-Aug-2007
Citigroup	USD	201	122	1296	155	09-Aug-2007
J.P. Morgan Chase	USD	92	36	1418	174	09-Aug-2007
Bank of Tokyo Mitsubishi	USD	71	28	1224	17	09-Aug-2007

Notes: The table displays data on the banks that are members of the EURIBOR panel. For each bank, it shows the currency of the CDS contracts, the mean and standard deviation of the 5Y CDS spread in basis points per annum, the size of the balance sheet in billion EUR equivalent as reported in the 2009 annual report, the average daily notional of CDS transactions in million USD equivalent as reported by the Depository Trust and Clearing Corporation, and the date from which the 5Y CDS contract is available in the Markit database.

Table 4: EURIBOR panel (cont.)

	USD market			*EUR market*		
	$\mathbb{A}(2,1,1)$	$\mathbb{A}(2,2,1)$	$\mathbb{A}(2,2,2)$	$\mathbb{A}(2,1,1)$	$\mathbb{A}(2,2,1)$	$\mathbb{A}(2,2,2)$
κ_r	0.1885 (0.0143)	0.2282 (0.0205)	0.3122 (0.1322)	0.2526 (0.0068)	0.2461 (0.0042)	0.2235 (0.0031)
σ_r	0.0055 (0.0004)	0.0054 (0.0003)	0.0054 (0.0003)	0.0053 (0.0003)	0.0053 (0.0003)	0.0051 (0.0002)
κ_γ	0.4667 (0.0278)	0.3864 (0.0303)	0.2836 (0.1234)	0.4703 (0.0156)	0.4663 (0.0097)	0.4566 (0.0076)
θ_γ	0.1340 (0.0031)	0.1304 (0.0022)	0.1263 (0.0020)	0.0503 (0.0003)	0.0514 (0.0003)	0.0603 (0.0009)
σ_γ	0.2251 (0.0180)	0.1814 (0.0166)	0.1285 (0.0547)	0.0336 (0.0026)	0.0415 (0.0029)	0.0808 (0.0044)
ρ	-0.2115 (0.2365)	-0.2286 (0.1771)	-0.1694 (0.1965)	-0.2564 (0.0831)	-0.3198 (0.0813)	-0.2712 (0.1319)
κ_ν	0.3268 (0.0020)	2.0977 (0.0493)	2.1843 (0.0697)	0.2603 (0.0019)	2.6835 (0.0528)	2.8773 (0.0627)
σ_ν	0.3925 (0.0085)	0.6418 (0.0577)	0.5602 (0.0537)	0.3082 (0.0069)	0.6845 (0.0488)	0.5489 (0.0396)
κ_μ		0.0499 (0.0067)	0.0340 (0.0072)		0.0156 (0.0046)	0.0152 (0.0079)
θ_ν or θ_μ	0.2326 (0.0019)	0.3844 (0.0258)	0.4634 (0.0635)	0.2162 (0.0012)	0.6196 (0.1268)	0.6285 (0.2555)
σ_μ		0.2549 (0.0074)	0.2643 (0.0074)		0.2049 (0.0047)	0.2144 (0.0056)
κ_λ	2.2595 (0.0149)	2.1878 (0.0113)	1.8242 (0.0159)	2.0701 (0.0086)	1.8452 (0.0083)	1.4773 (0.0083)
κ_ξ	7.1547 (0.2180)	6.2883 (0.1114)	7.2128 (0.1187)	6.0240 (0.2739)	5.7965 (0.1982)	6.7061 (0.1504)
σ_ξ	13.9675 (0.4648)	12.2294 (0.2364)	14.0501 (0.2538)	11.7578 (0.5793)	11.4319 (0.4121)	13.1116 (0.3151)
κ_ϵ			1.3112 (0.0594)			0.4801 (0.0252)
θ_ξ or θ_ϵ	0.0000 (0.0002)	0.0010 (0.0005)	0.0000 (0.0001)	0.0000 (0.0001)	0.0027 (0.0008)	0.0001 (0.0002)
σ_ϵ			1.9325 (0.1089)			0.6564 (0.0415)
Γ_r	-0.1545 (0.1468)	-0.1885 (0.1640)	-0.3023 (0.2932)	-0.0764 (0.1345)	0.0143 (0.0546)	-0.0409 (0.0372)
Γ_γ	-0.2349 (0.1198)	-0.2111 (0.1372)	-0.1731 (0.1160)	-0.1324 (0.0821)	-0.1511 (0.1133)	-0.1191 (0.0727)
Γ_ν	-0.3725 (0.3613)	-0.5631 (0.5913)	-0.7185 (0.5389)	-0.5773 (0.4445)	-0.4501 (0.5041)	-0.4010 (0.2847)
Γ_μ		-0.4549 (0.1865)	-0.6938 (0.4232)		-0.1193 (0.1229)	-0.0798 (0.1276)
Γ_ξ	0.0207 (0.0688)	-0.0372 (0.0827)	-0.1474 (0.4261)	-0.0511 (0.1011)	0.0794 (0.2725)	0.0272 (0.1239)
Γ_ϵ			-0.1251 (0.1939)			-0.2624 (0.8710)
σ_{err} (bp)	10.4255 (0.0235)	8.6173 (0.0223)	8.1734 (0.0216)	10.0982 (0.0207)	8.1173 (0.0166)	7.3956 (0.0154)
$\log L \times 10^{-4}$	-10.0436	-9.5909	-9.4830	-11.1765	-10.5827	-10.3603

Notes: The sample period is August 09, 2007 to January 12, 2011. Outer-product standard errors are in parentheses. For identification purposes, we fix ζ_λ at 10 (corresponding to a mean jump size of 1000 bp) and Λ at 5 bp. σ_{err} denotes the standard deviation of pricing errors.

Table 5: Maximum-likelihood estimates

	OIS	$SPREAD_{3M}$	$SPREAD_{6M}$	CDS
		Panel A: USD market		
$\mathbb{A}$(2,1,1)	7.14	7.65	7.63	11.55
$\mathbb{A}$(2,2,1)	7.06	8.12	6.99	6.62
$\mathbb{A}$(2,2,2)	7.02	7.65	6.37	6.19
$\mathbb{A}$(2,2,1)-$\mathbb{A}$(2,1,1)	-0.09^{*}	0.47	-0.63^{**}	-4.93^{***}
	(-1.71)	(0.99)	(-2.51)	(-4.95)
$\mathbb{A}$(2,2,2)-$\mathbb{A}$(2,2,1)	-0.04	-0.47^{*}	-0.62^{***}	-0.43^{***}
	(-0.79)	(-1.78)	(-5.72)	(-3.26)
		Panel B: EUR market		
$\mathbb{A}$(2,1,1)	6.16	7.66	8.23	11.77
$\mathbb{A}$(2,2,1)	5.93	7.83	7.16	7.07
$\mathbb{A}$(2,2,2)	5.59	7.13	6.22	6.34
$\mathbb{A}$(2,2,1)-$\mathbb{A}$(2,1,1)	-0.23^{***}	0.17	-1.06^{***}	-4.70^{***}
	(-8.12)	(0.63)	(-4.04)	(-4.62)
$\mathbb{A}$(2,2,2)-$\mathbb{A}$(2,2,1)	-0.34^{***}	-0.70	-0.95^{***}	-0.73^{***}
	(-3.36)	(-1.57)	(-4.08)	(-4.18)

Notes: The table reports means of the root mean squared pricing error (RMSE) time-series of OIS rates, interest rate spreads and CDS spreads. $SPREAD_{3M}$ denotes the difference between the fixed rates on an IRS indexed to 3M LIBOR/EURIBOR and an OIS with the same maturity. $SPREAD_{6M}$ denotes the difference between the fixed rates on an IRS indexed to 6M LIBOR/EURIBOR and an OIS with the same maturity. Units are basis points. T-statistics, corrected for serial correlation up to 50 lags using the method of Newey and West (1987), are in parentheses. $*$, $**$, and $***$ denote significance at the 10%, 5%, and 1% levels, respectively. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011,

Table 6: Comparing model specifications

	Maturity								
	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
				Panel A1: $SPREAD_{3M}$, USD market					
Default	28.1		25.2	24.0	23.8	23.9	24.1		28.6†
	(26.8)		(17.2)	(12.5)	(10.7)	(9.8)	(9.2)		(5.8)
Non-default	33.4		20.4	10.6	7.2	5.5	4.5		1.8†
	(45.2)		(27.3)	(14.1)	(9.5)	(7.3)	(6.0)		(3.5)
				Panel A2: $SPREAD_{6M}$, USD market					
Default		45.9	43.1	40.9	40.5	40.7	41.0		48.6†
		(39.7)	(30.1)	(21.8)	(18.5)	(16.8)	(15.7)		(10.0)
Non-default		38.3	29.6	15.6	10.6	8.1	6.7		2.9†
		(53.2)	(40.5)	(21.2)	(14.3)	(10.9)	(8.9)		(5.4)
				Panel B1: $SPREAD_{3M}$, EUR market					
Default	28.6		24.2	22.5	22.1	22.1	22.2	22.7	23.4
	(23.1)		(13.7)	(10.0)	(8.8)	(8.2)	(7.9)	(7.5)	(7.2)
Non-default	30.5		21.9	11.7	8.0	6.2	5.1	3.9	3.0
	(34.1)		(22.8)	(12.0)	(8.2)	(6.3)	(5.1)	(3.8)	(2.8)
				Panel B2: $SPREAD_{6M}$, EUR market					
Default		46.7	42.4	39.3	38.5	38.5	38.7	39.4	40.8
		(34.0)	(24.7)	(17.9)	(15.6)	(14.5)	(13.8)	(13.1)	(12.5)
Non-default		34.1	31.0	17.3	11.9	9.2	7.6	5.8	4.5
		(36.1)	(31.2)	(17.1)	(11.7)	(8.9)	(7.3)	(5.4)	(4.0)

Notes: The table shows the decomposition of the spread term structures using the $\mathbb{A}(2,2,1)$ specification and the CDS_{TrMean} and CDS_{Median} measures of interbank default risk in the USD and EUR markets, respectively. Each spread is decomposed into a default and a non-default component and the table displays means and, in parentheses, standard deviations of the time-series of the two components. $SPREAD_{3M}$ and $SPREAD_{6M}$ denote the spread term structures indexed to 3M and 6M LIBOR/EURIBOR, respectively. Units are basis points. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011, except those marked with † which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

Table 7: Decomposition of the term structure of interbank risk

OIS-Tbill	RefCorp-Treasury	HPW noise	Fails	adj. R^2
		Panel A: USD market		
0.016^{*} (1.804)				0.065
	0.025^{**} (2.380)			0.184
		0.748^{***} (5.925)		0.591
			16.859^{***} (7.112)	0.409
0.010^{*} (1.768)	-0.017 (-1.126)	0.967^{***} (4.275)		0.684
0.007 (0.998)	-0.017 (-0.993)	0.904^{***} (3.183)	4.290^{**} (2.166)	0.715
		Panel B: EUR market		
0.006^{**} (2.129)				0.046
	0.008 (1.280)			0.077
		0.241^{***} (3.072)		0.296
			5.267^{***} (3.503)	0.188
0.004 (1.299)	-0.007 (-0.951)	0.328^{***} (2.984)		0.361
0.003 (0.941)	-0.007 (-0.858)	0.330^{**} (2.295)	0.587 (0.415)	0.366

Notes: The table reports results from regressing $\xi(t)$ inferred from the $\mathbb{A}(2,2,1)$ specification on four illiquidity measures: the 3M OIS-Tbill spread, the 10Y Refcorp-Treasury yield spread, the Hu, Pan, and Wang (2010) noise measure, and the weekly sum of the notional amount of Treasury settlement fails (average of failure to deliver and failure to receive) reported by primary dealers. The first three measures are in basis points, while the forth measure is in USD billions. In each panel, the regressions are run with daily data, except the fourth and sixth regressions involving Treasury settlement fails, which are run with weekly data (summing up the daily observations over the week). T-statistics, corrected for serial correlation up to 22 lags in the daily regressions (4 lags in the weekly regressions) using the method of Newey and West (1987), are in parentheses. $*$, $**$, and $***$ denote significance at the 10%, 5%, and 1% levels, respectively. Each time series consists of 600 daily observations (or 125 weekly observations) from August 09, 2007 to December 31, 2009,

Table 8: The non-default component and liquidity

	Maturity						
	3M	1Y	2Y	3Y	4Y	5Y	10Y
			Panel A: $\mathbb{A}(2,1,1)$, CDS_{TrMean}				
Default	25.1 (17.2)	25.1 (15.3)	25.0 (13.2)	25.0 (11.5)	24.9 (10.2)	24.9 (9.0)	26.8† (5.2)
Non-default	35.8 (45.2)	21.3 (26.4)	10.9 (13.5)	7.4 (9.2)	5.7 (7.0)	4.6 (5.7)	1.9† (3.4)
			Panel B: $\mathbb{A}(2,2,2)$, CDS_{TrMean}				
Default	27.8 (25.2)	23.3 (15.7)	21.5 (11.2)	21.0 (9.5)	21.1 (8.7)	21.3 (8.2)	25.7† (5.2)
Non-default	31.9 (45.1)	22.3 (27.2)	13.8 (14.8)	10.1 (10.2)	7.9 (7.9)	6.5 (6.4)	2.9† (3.8)
			Panel C: $\mathbb{A}(2,2,1)$, CDS_{LIQ1}				
Default	25.8 (24.9)	24.5 (16.9)	24.0 (12.6)	24.1 (10.9)	24.3 (9.9)	24.6 (9.3)	29.0† (5.9)
Non-default	35.7 (46.2)	22.0 (28.0)	11.3 (14.4)	7.7 (9.7)	5.9 (7.4)	4.8 (6.1)	1.9† (3.6)
			Panel D: $\mathbb{A}(2,2,1)$, CDS_{LIQ2}				
Default	31.0 (28.5)	26.2 (17.8)	24.0 (12.5)	23.3 (10.5)	23.1 (9.5)	23.1 (8.9)	27.1† (5.6)
Non-default	31.7 (43.4)	19.7 (26.8)	10.2 (13.8)	7.0 (9.4)	5.4 (7.2)	4.4 (5.9)	1.7† (3.4)

Notes: The table shows alternative decompositions, for the USD market, of the spread term structure indexed to 3M LIBOR, $SPREAD_{3M}$. Each spread is decomposed into a default and a non-default component and the table displays means and, in parentheses, standard deviations of the time-series of the two components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{TrMean} measure of interbank default risk. Panels C and D display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1} and CDS_{LIQ2} measures of interbank default risk, respectively. Units are basis points. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011, except those marked with † which consist of 643 daily observations from July 28, 2008 to January 12, 2011.

Table 9: Alternative decomposition of the term structure of USD interbank risk

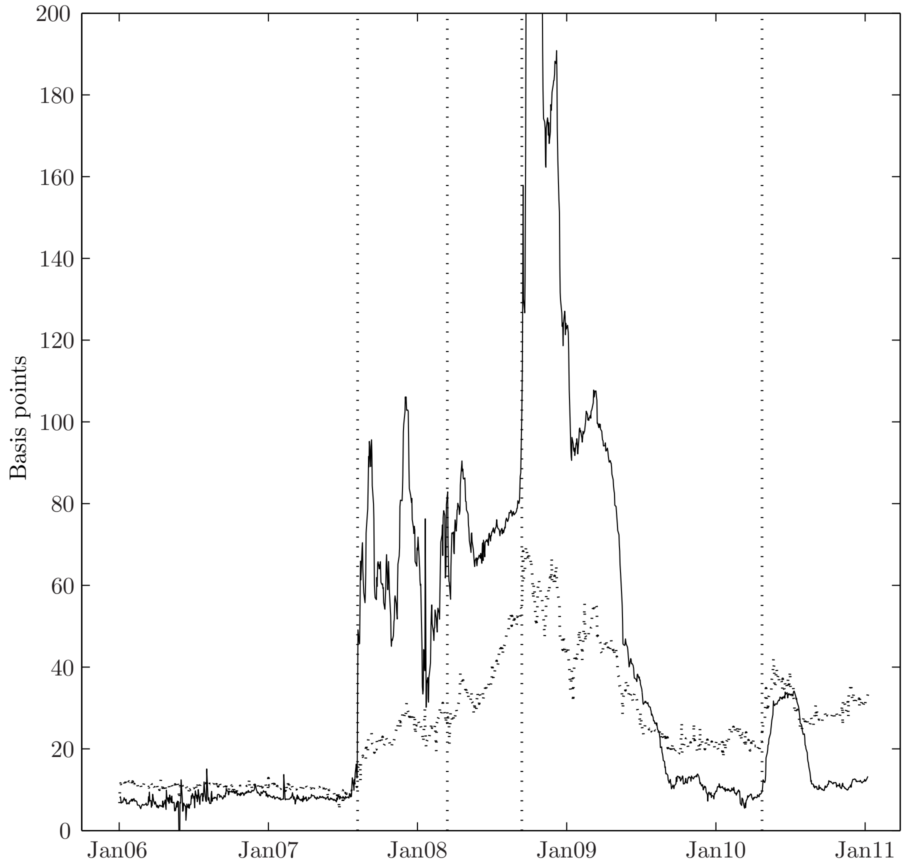

Figure 1: Money market and swap market spreads

The figures shows time-series of the spread between 3M LIBOR and the 3M OIS rate (solid line) and the spread between the rate on a 5Y interest rate swap indexed to 3M LIBOR and the 5Y OIS rate (dotted line). Note that the 3M LIBOR-OIS spread reached a maximum 366 basis points on October 10, 2008. The vertical dotted lines mark the beginning of the financial crisis on August 9, 2007, the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Both time series consists of 1313 daily observations from January 02, 2006 to January 12, 2011.

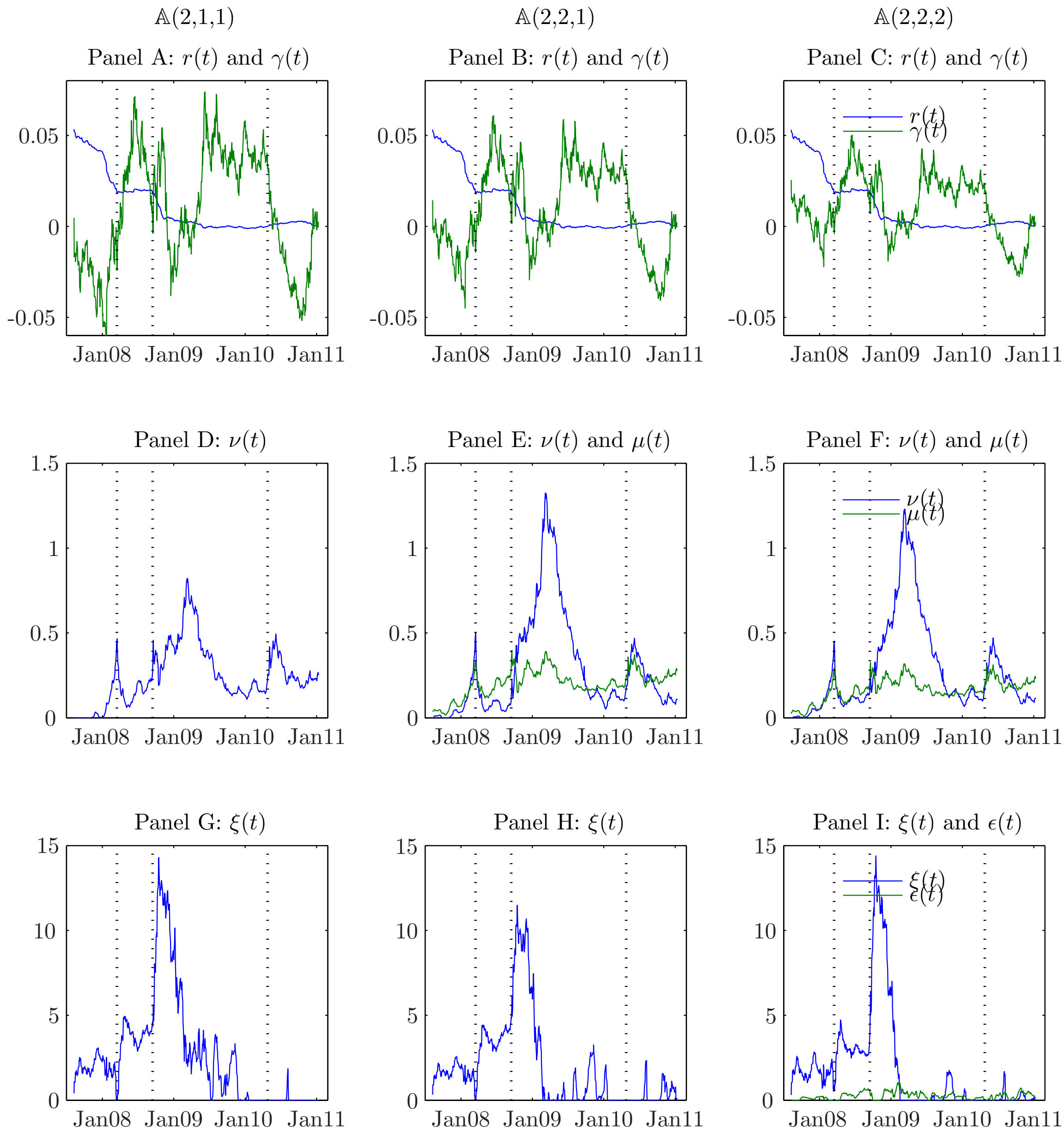

Figure 2: State variables, USD

The figure shows the state variables for the three model specifications estimated on USD data The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

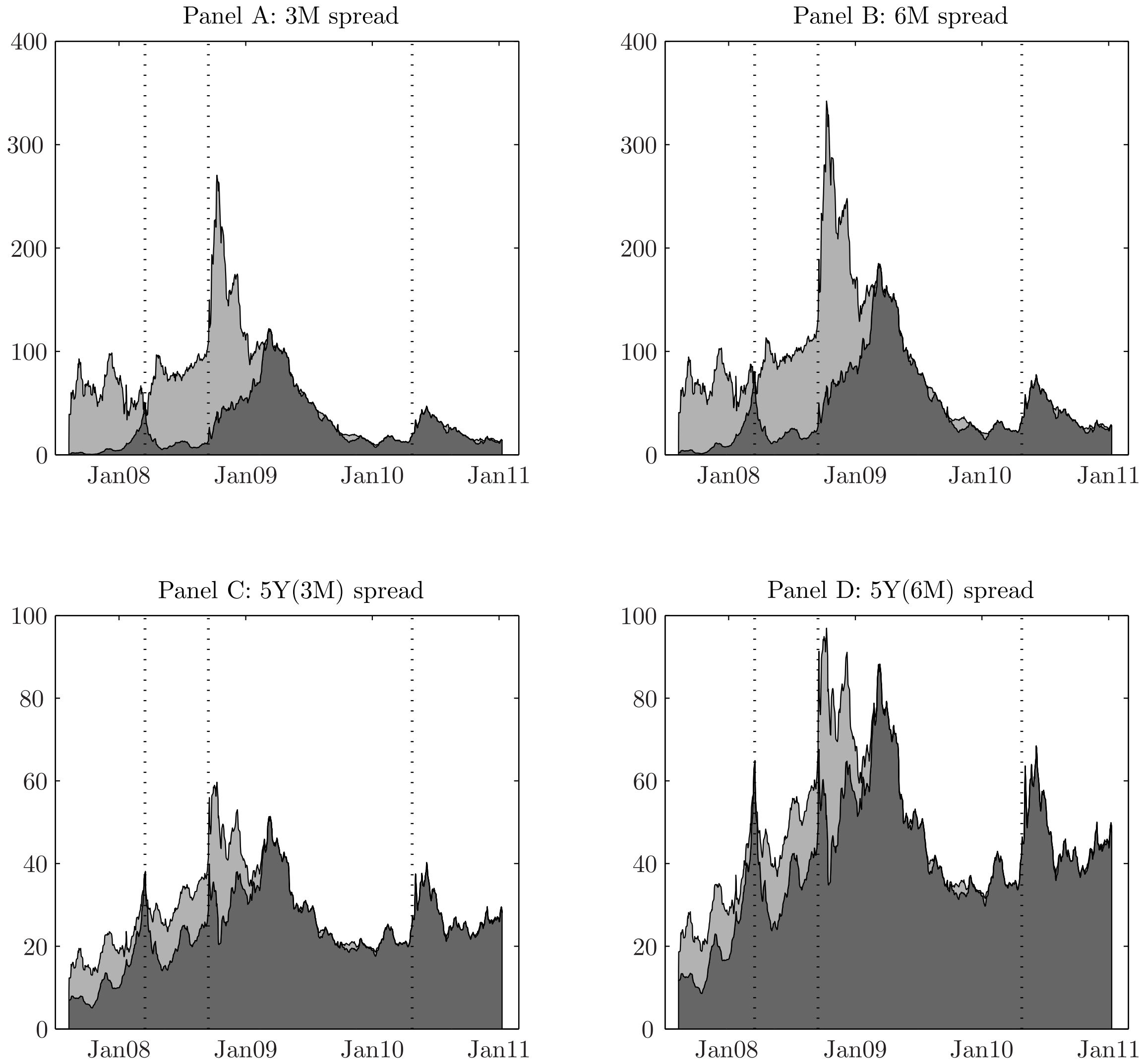

Figure 3: Decomposition of USD interbank risk

Decomposing USD interbank risk into default (dark-grey) and non-default (light-grey) components using the $\mathbb{A}(2,2,1)$ specification and the CDS_{TrMean} measure of interbank default risk. Panels A and B display decompositions of the 3M and 6M LIBOR-OIS spread, respectively. Panels C and D display decompositions of the 5Y IRS-OIS spread indexed to 3M and 6M LIBOR, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

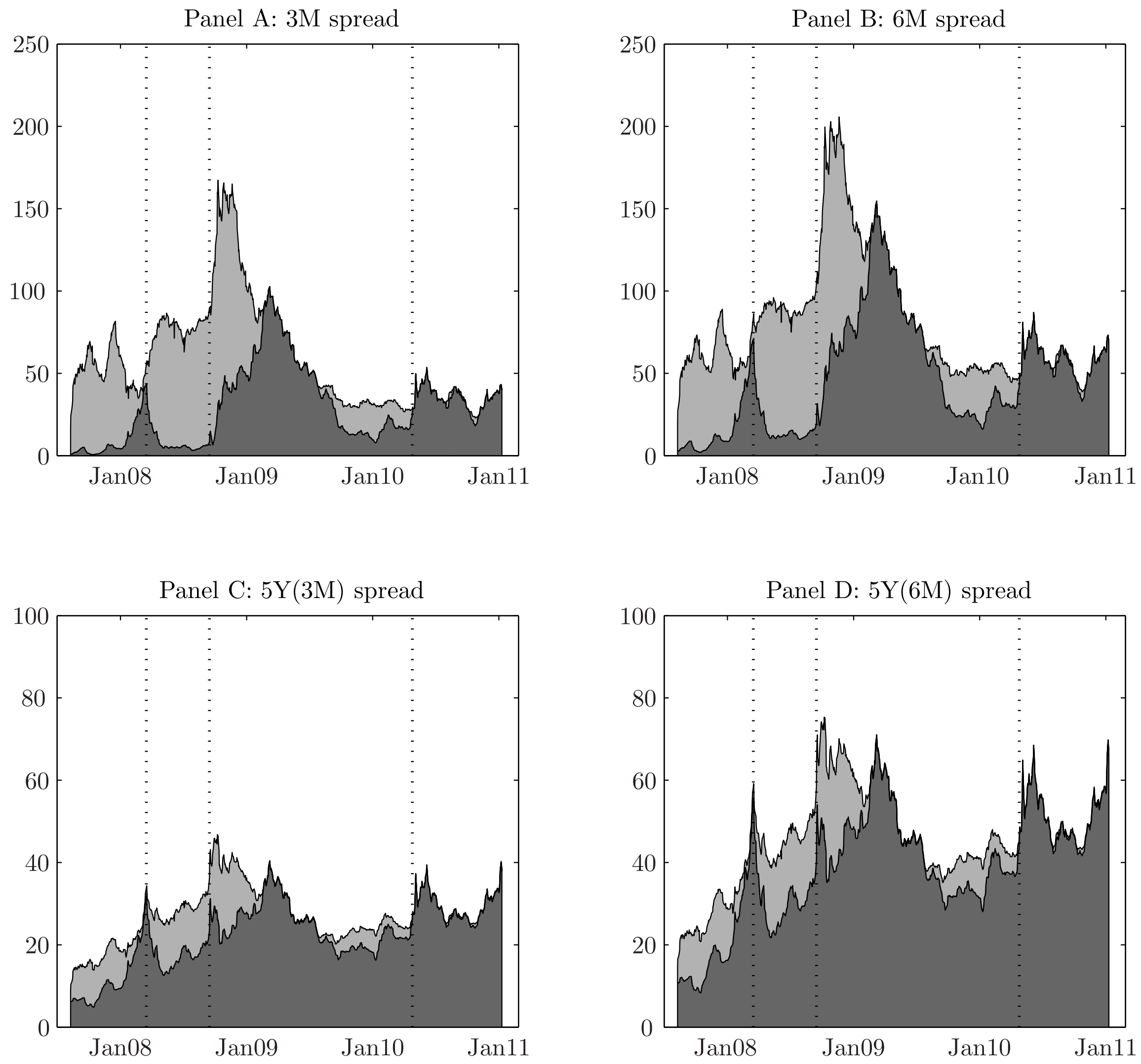

Figure 4: Decomposition of EUR interbank risk

Decomposing EUR interbank risk into default (dark-grey) and non-default (light-grey) components using the $\mathbb{A}(2,2,1)$ specification and the CDS_{Median} measure of interbank default risk. Panels A and B display decompositions of the 3M and 6M EURIBOR-OIS spread, respectively. Panels C and D display decompositions of the 5Y IRS-OIS spread indexed to 3M and 6M EURIBOR, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

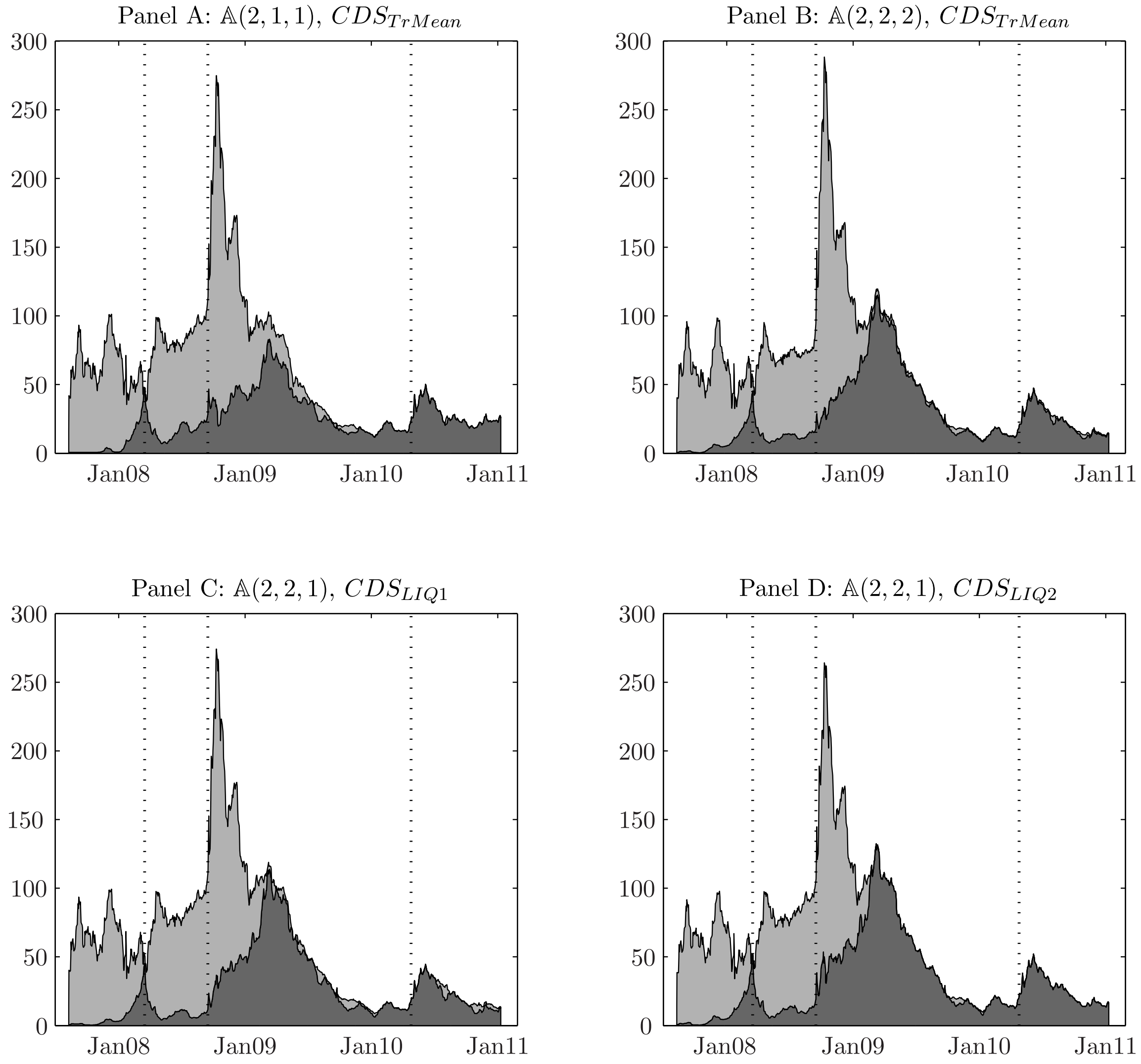

Figure 5: Alternative decompositions of USD interbank risk, 3M horizon

Alternative decompositions of the 3M LIBOR-OIS spread into default (dark-grey) and non-default (light-grey) components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{TrMean} measure of interbank default risk. Panels C and D display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1} and CDS_{LIQ2} measures of interbank default risk, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

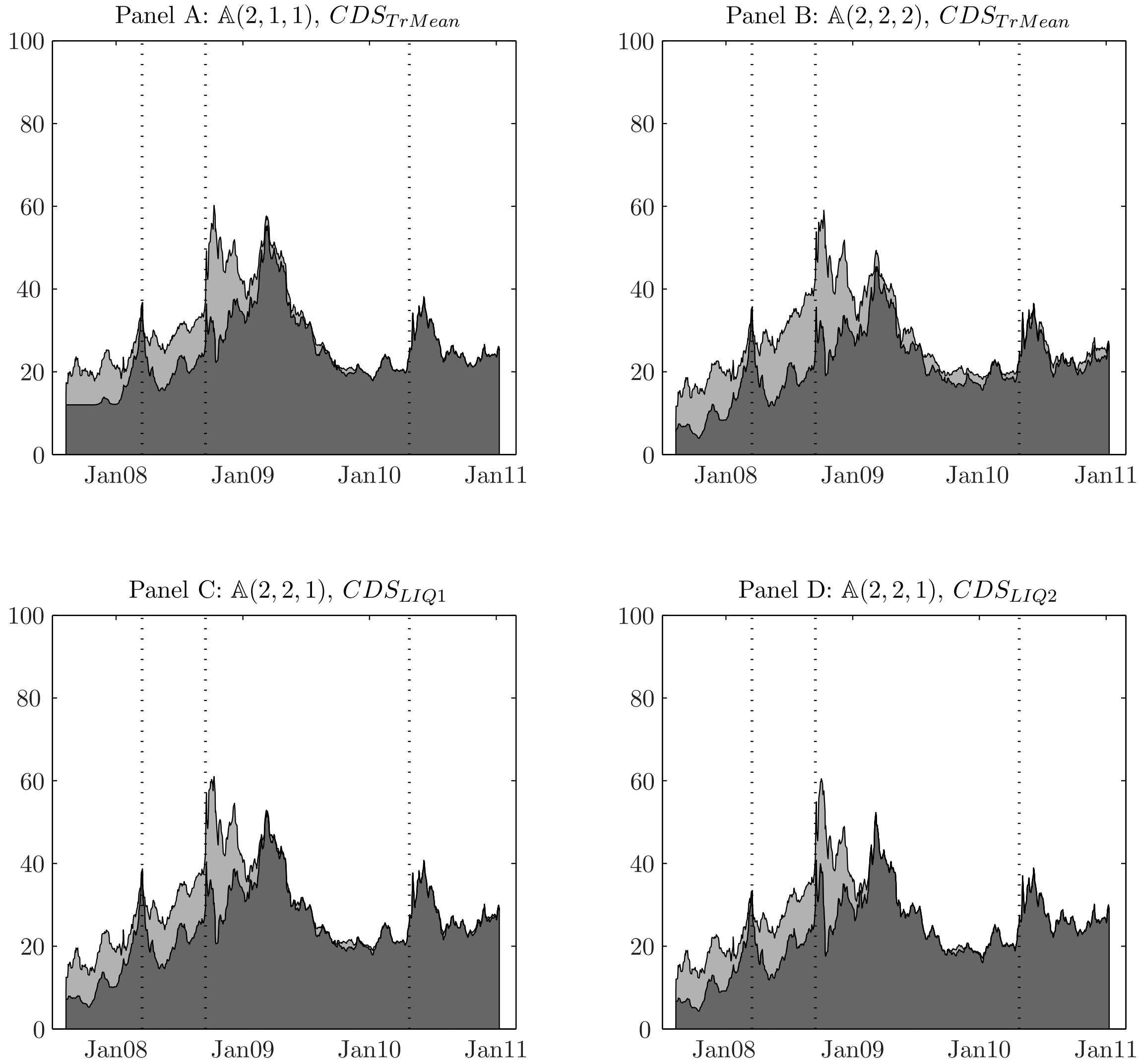

Figure 6: Alternative decompositions of USD interbank risk, 5Y horizon

Alternative decompositions of the 5Y IRS-OIS spread indexed to 3M LIBOR into default (dark-grey) and non-default (light-grey) components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{TrMean} measure of interbank default risk. Panels C and D display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1} and CDS_{LIQ2} measures of interbank default risk, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

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Online Appendix to "The Term Structure of Interbank Risk"

Damir Filipović and Anders B. Trolle

Ecole Polytechnique Fédérale de Lausanne and Swiss Finance Institute

E Proof of Lemma C.4

For the proof of Lemma C.4 we first recall a fundamental comparison result for ordinary differential equations, which is a special case of a more general theorem proved by Volkmann (1972):

Lemma E.1. *Let $R(\tau, v)$ be a continuous real map on $\mathbb{R}_+ \times \mathbb{R}$ and locally Lipschitz continuous in v. Let $p(\tau)$ and $q(\tau)$ be differentiable functions satisfying*

$$\begin{aligned}\partial_\tau p(\tau) &\leq R(\tau, p(\tau))\\ \partial_\tau q(\tau) &= R(\tau, q(\tau))\\ p(0) &\leq q(0).\end{aligned}$$

Then we have $p(\tau) \leq q(\tau)$ for all $\tau \geq 0$.

We only prove the first part of Lemma C.4, the proof of the second part being similar.[44] It follows from Duffie, Filipović, and Schachermayer (2003, Theorem 2.16), see also Filipović (2009, Theorem 10.3), that the affine transform formula

$$E^Q\left[\exp\left[u_\nu \nu(\tau) + u_\mu \mu(\tau)\right]\right] = \exp\left[\phi(\tau, u) + \psi_\nu(\tau, u)\nu(0) + \psi_\mu(\tau, u)\mu(0)\right]$$

holds, and the expectation on the left hand side is finite in particular, for $u = (u_\nu, u_\mu)^\top \in \mathbb{R}^2$ if $\phi(\tau, u)$, $\psi_\nu(\tau, u)$ and $\psi_\mu(\tau, u)$ are finite solutions of the corresponding system of Riccati equations

$$\begin{aligned}\partial_\tau \phi(\tau, u) &= \kappa_\mu \theta_\mu \psi_\mu(\tau, u) + \Lambda\\ \phi(0, u) &= 0\\ \partial_\tau \psi_\nu(\tau, u) &= \frac{\sigma_\nu^2}{2}\psi_\nu(\tau, u)^2 - \kappa_\nu \psi_\nu(\tau, u)\\ \psi_\nu(0, u) &= u_\nu\\ \partial_\tau \psi_\mu(\tau, u) &= \frac{\sigma_\mu^2}{2}\psi_\mu(\tau, u)^2 - \kappa_\mu \psi_\mu(\tau, u) + \kappa_\nu \psi_\nu(\tau, u)\\ \psi_\mu(0, u) &= u_\mu.\end{aligned} \tag{E.1}$$

It is thus enough to show that both, $\psi_\nu(\tau, u^*)$ and $\psi_\mu(\tau, u^*)$, are finite for all $\tau \geq 0$ and for $u^* := (-D_\nu(T - t_0), -D_\mu(T - t_0))^\top$.

We first provide a bound for u^*. Note that $D_\lambda(\tau)$, defined as solution of (38), is given by $D_\lambda(\tau) = -\frac{1}{\kappa_\lambda}\left(1 - e^{-\kappa_\lambda \tau}\right)$. This implies $-\frac{1}{\kappa_\lambda} \leq D_\lambda(\tau) \leq 0$ and thus

$$-\frac{1}{\zeta_\lambda \kappa_\lambda + 1} \leq \frac{D_\lambda(\tau)}{\zeta_\lambda - D_\lambda(\tau)} \leq 0.$$

[44] Indeed, after replacing $\frac{1}{\zeta_\lambda \kappa_\lambda + 1}$ by 1 in (E.2), and in the definition of $p_\nu(\tau)$, Θ_ν and C_ν below, the proof of the second part of Lemma C.4 is literally the same.

In view of (38) and Lemma E.1 we conclude that $p_\nu(\tau) \leq D_\nu(\tau) \leq 0$ where $p_\nu(\tau)$ solves the Riccati differential equation

$$\begin{aligned} \partial_\tau p_\nu(\tau) &= \frac{\sigma_\nu^2}{2} p_\nu(\tau)^2 - \kappa_\nu p_\nu(\tau) - \frac{1}{\zeta_\lambda \kappa_\lambda + 1} \\ p_\nu(0) &= 0. \end{aligned} \tag{E.2}$$

The explicit solution of (E.2) is well known to be

$$p_\nu(\tau) = -\frac{\frac{2}{\zeta_\lambda \kappa_\lambda + 1}\left(e^{\Theta_\nu \tau} - 1\right)}{\Theta_\nu\left(e^{\Theta_\nu \tau} + 1\right) + \kappa_\nu\left(e^{\Theta_\nu \tau} - 1\right)}$$

where Θ_ν is defined in (42), see e.g. Filipović (2009, Lemma 10.12). We thus obtain the estimate

$$0 \leq -D_\nu(T - t_0) \leq -p_\nu(T - t_0) = C_\nu. \tag{E.3}$$

Similarly, in view of (38), (E.3) and Lemma E.1, we infer that $p_\mu(\tau) \leq D_\mu(\tau) \leq 0$ where $p_\mu(\tau)$ solves the Riccati equation

$$\begin{aligned} \partial_\tau p_\mu(\tau) &= \frac{\sigma_\mu^2}{2} p_\mu(\tau)^2 - \kappa_\mu p_\mu(\tau) - \kappa_\nu C_\nu \\ p_\mu(0) &= 0. \end{aligned} \tag{E.4}$$

Again, the explicit solution of (E.4) is readily available, see e.g. Filipović (2009, Lemma 10.12):

$$p_\mu(\tau) = -\frac{2\kappa_\nu C_\nu\left(e^{\Theta_\mu \tau} - 1\right)}{\Theta_\mu\left(e^{\Theta_\mu \tau} + 1\right) + \kappa_\mu\left(e^{\Theta_\mu \tau} - 1\right)}$$

where Θ_μ is defined in (43). Moreover, it follows by inspection that $p_\mu(\tau) \downarrow P_1$ as $\tau \to \infty$ for the left critical point

$$P_1 = \frac{\kappa_\mu - \sqrt{\kappa_\mu^2 + 2\sigma_\mu^2 \kappa_\nu C_\nu}}{\sigma_\mu^2}$$

of the differential equation (E.4), and we obtain the estimates

$$0 \leq -D_\mu(T - t_0) \leq -p_\mu(T - t_0) = C_\mu \leq -P_1. \tag{E.5}$$

Next, we give a priori bounds on $\psi_\nu(\tau, u^*)$ and $\psi_\mu(\tau, u^*)$. Denote by $P_2 = \frac{2\kappa_\nu}{\sigma_\nu^2}$ the right critical point of the homogeneous Riccati differential equation (E.1) for $\psi_\nu(\tau, u)$, and denote by

$$q_\nu(\tau) = \frac{2\kappa_\nu C_\nu}{\left(2\kappa_\nu - \sigma_\nu^2 C_\nu\right) e^{\kappa_\nu \tau} + \sigma_\nu^2 C_\nu} \tag{E.6}$$

the solution of (E.1) for $\psi_\nu(\tau, u)$ with initial condition $u_\nu = C_\nu$, see e.g. Filipović (2009, Lemma 10.12). It then follows from Lemma E.1 and by inspection that

$$0 \leq \psi_\nu(\tau, u^*) \leq q_\nu(\tau) \text{ and } q_\nu(\tau) \downarrow 0 \text{ for } \tau \to \infty \text{ if } C_\nu < P_2, \tag{E.7}$$

which is (44).

Now suppose that (44) holds, that is, $C_\nu < P_2$. Combining (E.7) with (E.1), (E.5) and Lemma E.1 implies

$$0 \le \psi_\mu(\tau, u^*) \le q_\mu(\tau)$$

where $q_\mu(\tau)$ solves the time-inhomogeneous Riccati equation

$$\begin{aligned}\partial_\tau q_\mu(\tau) &= \frac{\sigma_\mu^2}{2} q_\mu(\tau)^2 - \kappa_\mu q_\mu(\tau) + \kappa_\nu q_\nu(\tau) \\ q_\mu(0) &= C_\mu.\end{aligned}$$

If $\sigma_\mu = 0$ then obviously $q_\mu(\tau)$ is finite for all $\tau \ge 0$, and there is nothing left to prove. So from now on we assume that $\sigma_\mu > 0$ and $\kappa_\mu \ge 0$. Since there is no closed form expression for $q_\mu(\tau)$ available in general, we are going to control q_μ from above by a time-inhomogeneous linear differential equation. Hereto note the elementary fact that

$$\frac{\sigma_\mu^2}{2} x^2 - \kappa_\mu x + \kappa_\nu q_\nu(\tau) \le -\frac{\kappa_\mu}{2} x + \kappa_\nu q_\nu(\tau) \text{ for all } 0 \le x \le \frac{\kappa_\mu}{\sigma_\mu^2}.$$

Hence, by Lemma E.1, the solution f of

$$\begin{aligned}\partial_\tau f(\tau) &= -\frac{\kappa_\mu}{2} f(\tau) + \kappa_\nu q_\nu(\tau) \\ f(0) &= C_\mu\end{aligned}$$

dominates q_μ, that is, $0 \le q_\mu(\tau) \le f(\tau)$ for all $\tau \ge 0$, if

$$f(\tau) \le \frac{\kappa_\mu}{\sigma_\mu^2} \tag{E.8}$$

for all $\tau \ge 0$.

We now claim that (E.8) holds for any fixed $\tau \ge 0$ if and only if

$$\int_0^\tau e^{\frac{\kappa_\mu}{2} s} \left(\kappa_\nu q_\nu(s) - \frac{\kappa_\mu^2}{2\sigma_\mu^2} \right) ds \le \frac{\kappa_\mu}{\sigma_\mu^2} - C_\mu. \tag{E.9}$$

Indeed, f can be represented by the variation of constants formula

$$f(\tau) = e^{-\frac{\kappa_\mu}{2}\tau} C_\mu + \int_0^\tau e^{-\frac{\kappa_\mu}{2}(\tau - s)} \kappa_\nu q_\nu(s)\, ds. \tag{E.10}$$

Hence (E.8) is equivalent to

$$C_\mu + \int_0^\tau e^{\frac{\kappa_\mu}{2} s} \kappa_\nu q_\nu(s)\, ds \le \frac{\kappa_\mu}{\sigma_\mu^2} e^{\frac{\kappa_\mu}{2}\tau}. \tag{E.11}$$

The right hand side of (E.11) can be rewritten as

$$\frac{\kappa_\mu}{\sigma_\mu^2} e^{\frac{\kappa_\mu}{2}\tau} = \frac{\kappa_\mu}{\sigma_\mu^2} \left(e^{\frac{\kappa_\mu}{2}\tau} - 1 \right) + e^{\frac{\kappa_\mu}{2}\tau} = \frac{\kappa_\mu^2}{2\sigma_\mu^2} \int_0^\tau e^{\frac{\kappa_\mu}{2} s}\, ds + \frac{\kappa_\mu}{\sigma_\mu^2}.$$

Plugging this in (E.11) and rearrange terms yields (E.9).

In view of (E.7) we infer that the maximum of the left hand side of (E.9) is attained at $\tau = \tau^*$ where

$$\tau^* = \inf\left\{\tau \geq 0 \mid \kappa_\mu^2 - 2\sigma_\mu^2 \kappa_\nu q_\nu(\tau) \geq 0\right\} < \infty,$$

and which by (E.6) can be written as in (46). Hence the bound (E.8) holds for all $\tau \geq 0$ if and only if (E.8) holds for $\tau = \tau^*$. This again is equivalent to (45), since the integral in (E.10) can be expressed as[45]

$$\int_0^\tau e^{-\frac{\kappa_\mu}{2}(\tau-s)} \kappa_\nu q_\nu(s)\, ds = \frac{4\kappa_\nu^2}{\kappa_\mu \sigma_\nu^2} \left({}_2F_1\left(1, \frac{\kappa_\mu}{2\kappa_\nu}; \frac{\kappa_\mu + 2\kappa_\nu}{2\kappa_\nu}; \frac{\left(\sigma_\nu^2 C_\nu - 2\kappa_\nu\right) e^{\kappa_\nu \tau}}{\sigma_\nu^2 C_\nu}\right) \right.$$
$$\left. - e^{-\frac{\kappa_\mu}{2}\tau} {}_2F_1\left(1, \frac{\kappa_\mu}{2\kappa_\nu}; \frac{\kappa_\mu + 2\kappa_\nu}{2\kappa_\nu}; \frac{\sigma_\nu^2 C_\nu - 2\kappa_\nu}{\sigma_\nu^2 C_\nu}\right)\right)$$

where ${}_2F_1$ is the Gauss hypergeometric function. Finally, note that $\tau^* = 0$ if and only if $\frac{\kappa_\mu^2}{2\kappa_\nu \sigma_\mu^2} \geq C_\nu$. In this case, (E.5) implies

$$\sigma_\mu^2 C_\mu \leq \sqrt{\kappa_\mu^2 + 2\sigma_\mu^2 \kappa_\nu C_\nu} - \kappa_\mu \leq \sqrt{2\kappa_\mu^2} - \kappa_\mu \leq \kappa_\mu,$$

so that (45) automatically holds. This finishes the proof of Lemma C.4.

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[45]We obtained the integral formula from the computational software program Mathematica.

F Comparing alternative quotation convention for basis swaps

As discussed in Section 2.4, there is no universally accepted quotation convention for basis swaps. The two most common conventions are the following. In the first convention (I), and the one we use in the paper, the cash flow in a basis swap is the difference between the cash flows in two IRS indexed to different floating rates. In the case of a 3M/6M basis swap and with $\delta = 3\text{M}$, this implies that one party pays $\delta L(t-\delta, t)$ quarterly, while the other party pays $2\delta L(t-2\delta, t)$ semi-annually, with the fixed spread payments made semi-annually in the USD market and annually in the EUR market. Given that our model has an analytical solution to an IRS, it also has an analytical solution to a basis swap defined according to this convention.

In the second convention (II), all payments occur at the frequency of the longer floating rate with the shorter floating rate paid compounded. In the case of a 3M/6M basis swap, this implies for both markets that on a semi-annual basis one party pays $\delta L(t-2\delta, t-\delta)(1+\delta L(t-\delta, t)) + \delta L(t-\delta, t)$ plus a fixed spread, while the other party pays $2\delta L(t-2\delta, t)$. If we assume that payments are made on tenor structure (5) with $t_i = t_{i-1} + 2\delta$, the basis swap rate according to this convention is given by

$$\begin{aligned} BS_{\delta,2\delta}(t,T) \quad = \quad & \frac{1}{\sum_{i=1}^{N} 2\delta P_c(t,t_i)} \left(\sum_{i=1}^{N} E_t^Q \left[e^{-\int_t^{t_i} r_c(s)ds} \Big(2\delta L(t_{i-1}, t_i) - \right.\right. \\ & \left.\left. \Big(\delta L(t_{i-1}, t_{i-1}+\delta)(1+\delta L(t_{i-1}+\delta, t_i)) + \delta L(t_{i-1}+\delta, t_i) \Big) \Big) \right] \right), \end{aligned} \tag{F.1}$$

We now quantify the difference between the two market conventions. For a given parameter set and state vector, we compute 3M/6M basis swap rates implied by conventions (I) analytically, and by convention (II) via simulation.[46] We consider both markets and for each market two state vectors: the mean state vector and the state vector on the day of the widest 1Y basis swap rate. Table F.1 shows the spread term structures implied by convention (I). It also shows the differences between the spread term structures implied by convention (II) and (I) along with the standard errors of the simulated basis swap rates in parentheses. On a typical day, the differences between the spreads implied by the two conventions are very small both in absolute and relative terms. Even on the day of the widest 1Y basis swap rate, the differences between the spreads remain very small in relative terms. We conclude that our results do not depend on the choice of basis swap convention.

[46] In principle, one can compute basis swap rates for convention (II) analytically as well. However, the expressions are fairly involved and as the spreads can be simulated very accurately using a low number of simulations, we opt for this approach.

	Maturity						
	1Y	2Y	3Y	4Y	5Y	7Y	10Y
				Panel A: USD market, typical day			
(I)	22.087	15.449	12.525	10.864	9.791	8.496	7.482
(II)-(I)	−0.125	−0.118	−0.121	−0.126	−0.131	−0.143	−0.168
	(0.003)	(0.004)	(0.005)	(0.006)	(0.007)	(0.007)	(0.008)
				Panel B: USD market, day of widest 1Y spread			
(I)	39.629	23.089	16.881	13.865	12.114	10.179	8.759
(II)-(I)	−0.601	−0.347	−0.265	−0.233	−0.217	−0.208	−0.219
	(0.008)	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.008)
				Panel C: EUR market, typical day			
(I)	14.546	12.793	11.862	11.211	10.681	9.788	8.687
(II)-(I)	−0.215	−0.202	−0.203	−0.207	−0.209	−0.213	−0.210
	(0.002)	(0.003)	(0.004)	(0.005)	(0.005)	(0.007)	(0.008)
				Panel D: EUR market, day of widest 1Y spread			
(I)	36.655	26.557	22.032	19.435	17.684	15.310	12.996
(II)-(I)	−0.836	−0.583	−0.482	−0.431	−0.400	−0.361	−0.322
	(0.004)	(0.005)	(0.006)	(0.007)	(0.008)	(0.010)	(0.011)

Notes: In this table, we asses the difference between the 3M/6M basis swap rates implied by market conventions (I) and (II). The former can be computed analytically within our model, while the latter is computed by simulation. For the simulation, we use 2000 paths (1000 plus 1000 antithetic) and the basis swap rate implied by convention (I) as a very efficient control variate. In each panel, the first line shows the term structure of basis swap rates implied by convention (I). The second line shows the difference between the term structure of basis swap rates implied by conventions (II) and (I), with standard errors of the simulated basis swap rates reported in parentheses. Panels A and C show results using the mean state vector, while Panels B and D show results using the state vector on the day of the widest 1Y basis swap rate (October 14, 2008 in USD and October 13, 2008 in EUR) We use the $\mathbb{A}(2,2,1)$ specification, with parameter estimates reported in Table 5. Basis swap rates are reported in basis points.

Table F.1: Impact of differences in market convention for basis swaps

G Additional tables and figures

Table G.1 shows alternative decompositions of the term structure of EUR interbank risk. Figure G.1 displays the state variables for the three model specifications estimated on EUR data. Figures G.2 and G.3 show time series of the alternative decompositions of EUR interbank risk at the short end and long end of the term structure, respectively.

	Maturity							
	3M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
				Panel A: $\mathbb{A}(2,1,1)$, CDS_{Median}				
Default	24.9 (14.1)	24.8 (12.9)	24.6 (11.4)	24.5 (10.2)	24.3 (9.2)	24.2 (8.3)	24.1 (7.0)	23.9 (5.6)
Non-default	32.0 (34.9)	21.2 (21.7)	11.0 (11.2)	7.5 (7.6)	5.7 (5.8)	4.7 (4.7)	3.5 (3.5)	2.6 (2.6)
				Panel B: $\mathbb{A}(2,2,2)$, CDS_{Median}				
Default	29.3 (21.8)	22.4 (12.5)	19.8 (9.0)	19.1 (7.9)	19.0 (7.3)	19.1 (7.0)	19.5 (6.6)	20.2 (6.3)
Non-default	28.2 (33.7)	22.0 (22.0)	15.0 (13.0)	12.0 (9.7)	10.2 (7.9)	8.9 (6.7)	7.0 (5.2)	5.3 (3.9)
				Panel C: $\mathbb{A}(2,2,1)$, CDS_{LIQ1}				
Default	26.6 (22.3)	23.7 (13.6)	22.7 (10.2)	22.5 (9.0)	22.6 (8.5)	22.8 (8.1)	23.3 (7.7)	24.1 (7.3)
Non-default	32.2 (34.8)	22.7 (22.9)	12.0 (12.0)	8.2 (8.2)	6.3 (6.3)	5.2 (5.1)	3.9 (3.8)	3.0 (2.8)
				Panel D: $\mathbb{A}(2,2,1)$, CDS_{LIQ2}				
Default	26.4 (22.4)	22.8 (13.3)	21.4 (9.9)	21.2 (8.8)	21.3 (8.3)	21.4 (8.0)	22.0 (7.7)	22.8 (7.4)
Non-default	31.9 (34.0)	23.3 (22.7)	12.7 (12.0)	8.9 (8.2)	7.1 (6.3)	6.0 (5.2)	4.8 (3.9)	4.0 (2.9)
				Panel E: $\mathbb{A}(2,2,1)$, CDS_{iTraxx}				
Default	15.6 (13.9)	17.1 (9.1)	17.8 (7.4)	18.1 (6.9)	18.3 (6.7)	18.4 (6.6)	18.5 (6.6)	18.6 (6.6)
Non-default	40.8 (37.3)	29.1 (23.2)	16.7 (12.4)	12.5 (8.6)	10.6 (6.7)	9.5 (5.6)	8.4 (4.2)	7.7 (3.2)

Notes: The table shows alternative decompositions, for the EUR market, of the spread term structure indexed to 3M LIBOR, $SPREAD_{3M}$. Each spread is decomposed into a default and a non-default component and the table displays means and, in parentheses, standard deviations of the time-series of the two components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{Median} measure of interbank default risk. Panels C, D, and E display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1}, CDS_{LIQ2}, and CDS_{iTraxx} measures of interbank default risk, respectively. Units are basis points. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

Table G.1: Alternative decompositions of the term structure of EUR interbank risk

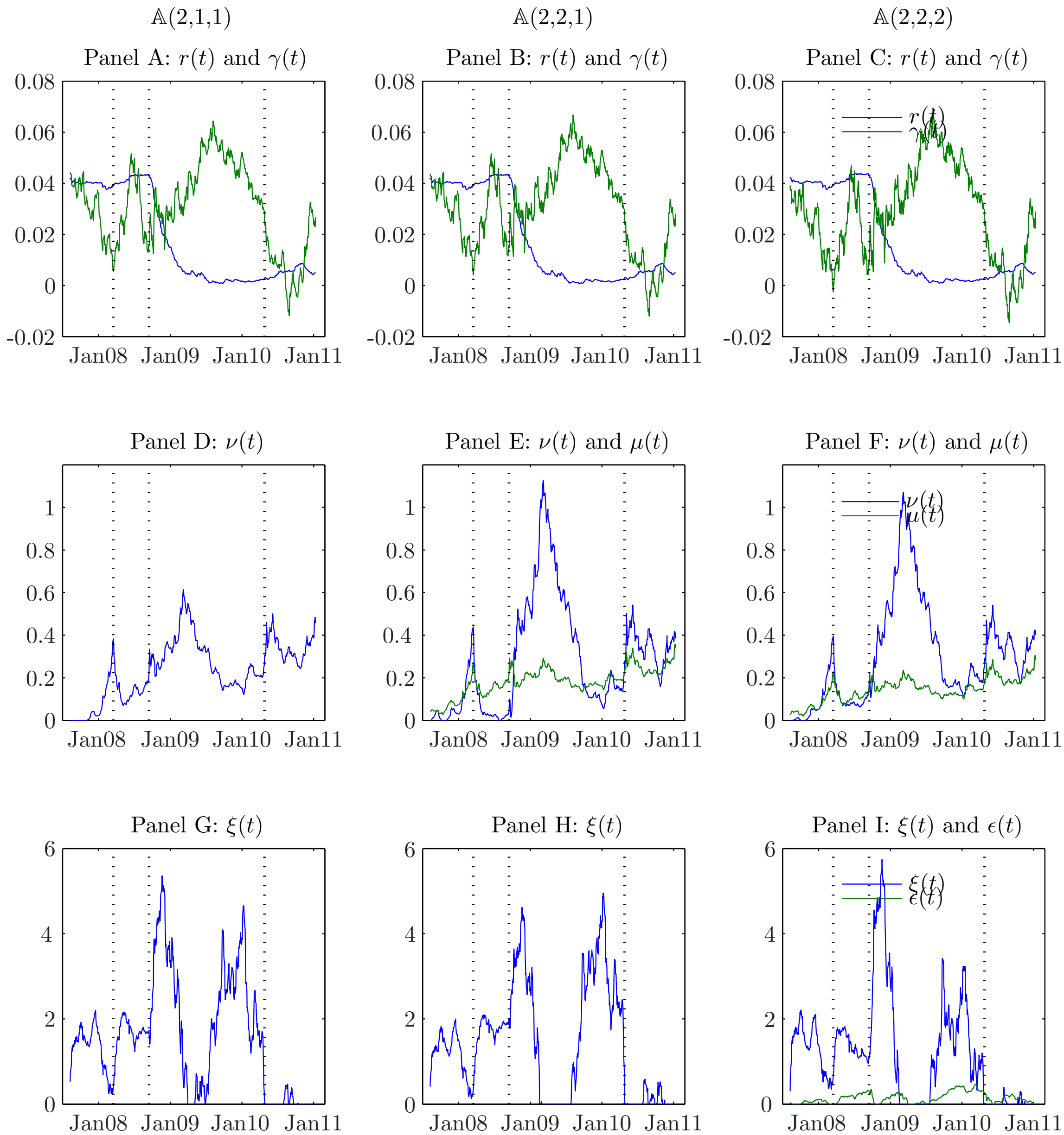

Figure G.1: State variables, EUR

The figure shows the state variables for the three model specifications estimated on EUR data The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

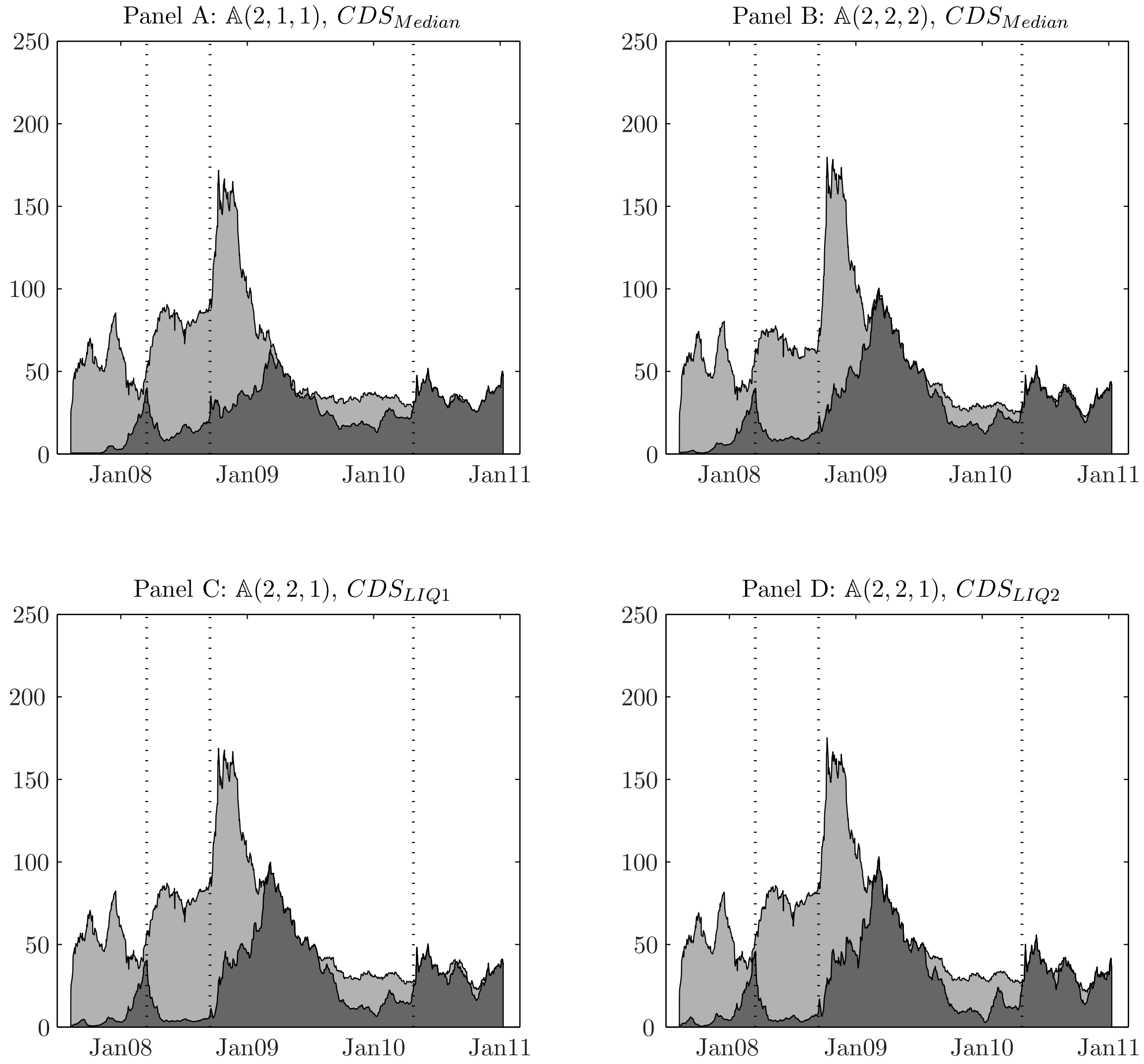

Figure G.2: Alternative decompositions of EUR interbank risk, 3M horizon

Alternative decompositions of the 3M LIBOR-OIS spread into default (dark-grey) and non-default (light-grey) components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{Median} measure of interbank default risk. Panels C and D display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1} and CDS_{LIQ2} measures of interbank default risk, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.

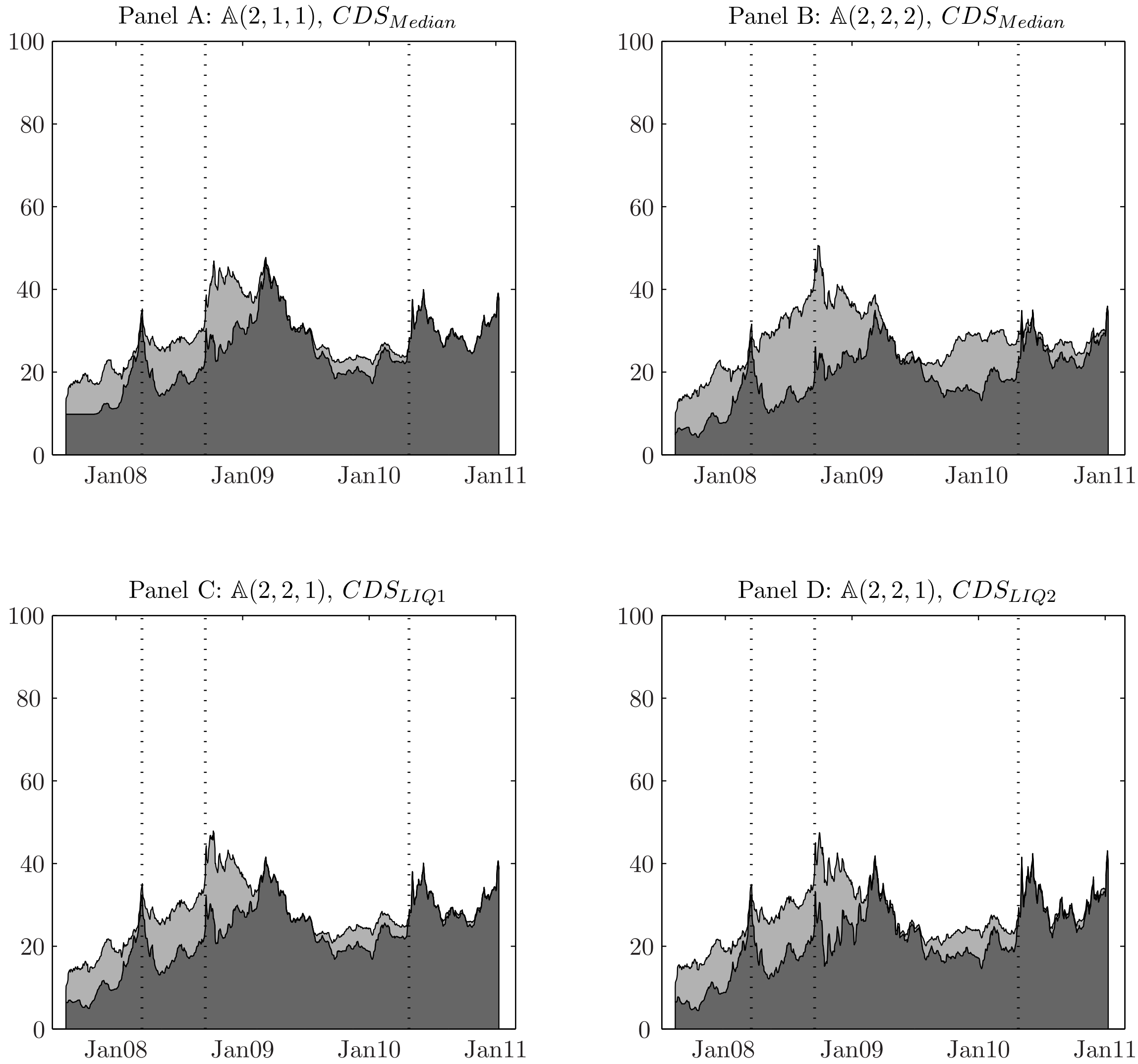

Figure G.3: Alternative decompositions of EUR interbank risk, 5Y horizon

Alternative decompositions of the 5Y IRS-OIS spread indexed to 3M LIBOR into default (dark-grey) and non-default (light-grey) components. Panels A and B display results using the $\mathbb{A}$(2,1,1) and $\mathbb{A}$(2,2,2) specifications, respectively, combined with the CDS_{Median} measure of interbank default risk. Panels C and D display results using the $\mathbb{A}$(2,2,1) specification combined with the CDS_{LIQ1} and CDS_{LIQ2} measures of interbank default risk, respectively. Units are basis points. The vertical dotted lines mark the sale of Bear Stearns to J.P. Morgan on March 16, 2008, the Lehman Brothers bankruptcy filing on September 15, 2008, and the downgrade of Greece's debt to non-investment grade status by Standard and Poor's on April 27, 2010. Each time series consists of 895 daily observations from August 09, 2007 to January 12, 2011.