Are Over-the-Counter Derivatives Required for Interbank Hedging? *

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Abstract

In the absence of taxes, imperfect information, and importantly, additional regulation, the answer is “No”. Banks face the classic tradeoff between risk sharing and the incentive to maintain their asset quality. We show that under most economic circumstances, banks’ value will be higher if they hedge with interbank loan contracts as long as the loan repayments can be renegotiated ex-post. Banks optimally create a highly interconnected network with large interbank debt that essentially commits ex-post solvent banks to bailing out all insolvent banks whenever the banking system as a whole is solvent. Standard bank regulation, however, sets constraints on the lending of a bank to any other bank and in the presence of constraints, derivatives usage becomes optimal as banks attempt to create the highly connected network. Our derivatives irrelevance result holds for a large range of liquidation costs, banks’ costs to improve asset quality, weak and strong bankruptcy regimes, small and large banking systems, the presence of unhedgeable background risk, and reserve requirements.

Key Words: Systemic risk; interbank loans; renegotiation; bankruptcy mechanism; optimal risk-sharing network; derivatives irrelevance

JEL Classification: G21, C1, C78, C81, E44

Introduction

In the aftermath of the financial crisis of 2008, the future of the market for over-the-counter derivative (OTCD) contracts is once again being hotly debated. The exponential growth of the market for OTCDs in the past two decades has facilitated the transfer of risk between financial institutions but has created an ongoing concern that it has made banking systems around the world more fragile. Unlike exchange traded contracts, OTCDs are contracts directly between counterparties (as opposed to the exchange) and hence the final settlement on these contracts can be renegotiated depending on the credit conditions of the counterparties.[1] The central question we ask in this paper is if the ban on OTCDs will limit the ability of banks to share risks with each other. Our main result that we develop in this paper is to show that more commonly used simple interbank loans that can also be renegotiated ex-post, are able to provide better hedging and bank value maximizing tools to banks. Overall, banks seeking to purely manage risk will not be worse of with the prohibition of the OTCDs although their profits from trading and other speculative activity may indeed suffer. An important caveat is that the result holds only in the absence of additional regulation. However, following suggestions from the Basel Committee, there are constraints on interbank lending to a single party in most countries that we show can undermine banks' hedging.[2] Banks, in our analysis, optimally respond by using OTCDs in an attempt to replicate the hedges with interbank loans, and in such a setting, the loss of OTCDs indeed limit their risk sharing abilities and lower bank value.

We start by providing the main intuition on the optimality of the renegotiable interbank loan contract in the absence of regulatory constraints. Banks in our model attempt to share the risks of their asset streams by writing interbank loans and two other types of contracts — asset swaps and credit default swaps (CDS). The only benefit of hedging in our model is the reduction in dead weight costs from the liquidation of a failed banks' assets. In the absence of any moral hazard problems, either set of derivatives can provide perfect risk sharing for the banks. However, banks face the classic tradeoff between risk sharing and the incentive to maintain their asset quality: after hedging

[1] For example, in the failure of AIG in 2008, recent commentary (e.g. see the July 1, 2010 *New York Times* article entitled "Figure in A.I.G. Testifies") notes that AIG had in fact renegotiated its due payments on credit default swaps it had written to several large financial institutions, including Goldman Sachs and Societe Generale, and absent government intervention would have paid about $40 billion less to these counterparties.

[2] In the US, for example, a national bank may only lend up to 15% of its capital to one bank (see, e.g, §*32.3, FDIC Law, Regulations, Related Acts - 8000 - Miscellaneous Statutes and Regulations*, which can be viewed at `http://www.fdic.gov/regulations/laws/rules/8000-7400.html#fdic8000lending323`).

they no longer have the incentive to maintain the quality of their asset streams.[3] We show in our analysis that the moral hazard problem is most acute for asset swaps, which reduces banks' exposure to their own assets. Due to the moral hazard problem, banks do not choose perfect hedges, and all derivatives are non-redundant. In this setting, we find that renegotiable interbank loans provide the best spanning possibilities to minimize dead weight liquidation costs. The actual payoffs of the interbank loans are determined ex-post by renegotiation, which can be seen as a way of exact customization of payoffs. The intuition for the spanning is best illustrated with a simple example.

Example 1 *Consider a system with three banks, whose ex-post asset values are* $A_1 = 1.2$, $A_2 = A_3 = 0.95$. *Each bank has deposits of 1. Without any hedging, ex-post bank 1 is solvent and banks 2 and 3 are insolvent. Banks may have hedged ex-ante with either of two hedges: (i) interbank loans that require circular payments of 0.25 (bank 1 pays bank 2, bank 2 pays 3, and bank 3 pays bank 1); or (ii) CDS: each insolvent bank receives a payment of* $0.5 \cdot \max(1 - A_i, 0)$, *from each other bank.*[4] *All interbank payments are subject to bilateral netting. Any bank that does not pay its interbank commitments in full is taken to the bankruptcy court where bankruptcy costs are 100 percent.*

We first show that CDS contracts, even if renegotiated cannot save the two insolvent banks. The ex-post CDS payments from bank 1 to banks 2 and 3 are 0.025 each. Clearly, these payments are insufficient to rescue either bank. Moreover, bank 1 would not agree to pay them any more than the maximum payments due, so renegotiation is ineffective as well.

We next show that with renegotiations, interbank loans will be able to span the payments required to rescue banks 2 and 3. Even bank 1 cannot meet its full obligations on the 0.25 of interbank payments it owes to bank 2, and without renegotiation, it would get taken to the bankruptcy court as well. We will show formally later that the optimal strategy for bank 1 is to make a reduced payment to bank 2 of 0.1, and to accept a payment of 0 from bank 3. Bank 2 in turn would optimally make a reduced payment of 0.05 to bank 3, and both banks would remain solvent and the payoffs of both

[3] The moral hazard problem is an oft cited reason for the deterioration in the quality of mortgages and other lending by banks in the past decade.

[4] Insuring half of the shortfall with each other bank provides full insurance only when both counterparties survive. As we will show later, overinsuring is not always optimal as it reduces the incentives to maintain the quality of the assets. Thompson (2010) finds that the possibility of the insurer's default might solve an adverse selection problem and induce protection buyers to reveal their risk to the insurer. Our model differs by analyzing mutual renegotiable insurance between banks under symmetric information, where both parties face a moral hazard problem to maintain the quality of their assets.

banks would be 0. Offering them even slightly more would make them better off than being liquidated. There are two reasons why the interbank loan is successful in averting liquidations: First, the solvent bank, which itself has a large payment outstanding, is forced to renegotiate or be liquidated itself. Therefore, these loans have a commitment role, and must be large enough so that they will be effective to bind solvent banks to perform the bailout. Second, the circular payments ensure that there is an outstanding due payment from the final proposer to the first proposer, which limits the amount the first proposer can extract and ensures that it minimizes liquidation costs for the entire set of banks.

There are several important aspects of this example that we will build on in the paper. First, to be effective, a highly interconnected hedging network is required that will "span" banks' liquidation risk. Second, banks make endogenous bankruptcy decisions, which are tied to the renegotiations. Each bank will evaluate its alternatives by renegotiating or inducing bankruptcy of its counterparties. Therefore, the start of systemic crises are endogenous equilibrium decisions by banks. Third, the renegotiation solution is efficient for interbank loans, but inefficient for CDS contracts. Finally, bank 1 (the solvent bank) pays out less and is hence better off ex-post with CDS contracts. The depositors of failed banks will have to be paid by deposit insurance. Ex-ante, this will raise the deposit insurance premium for banks hedging with CDS contracts rather than interbank loans, and will determine the choice of contract.

The role of default in customizing the payoffs of securities and increasing the span of existing securities has been studied in Dubey, Geanakopolos, and Shubik (2005). However, we show that the customization is limited in the absence of renegotiations of payoffs. In particular in our model, defaultable interbank loan securities have fairly rigid payoffs even when default is considered so that banks would never choose to use *any* interbank loans if renegotiation was not permitted.[5] In contrast, we show that the renegotiable interbank loan contract is the optimal contract to the joint risk management and asset quality problem faced by banks. In particular, we show that with sufficiently large interbank connections, banks will always choose an ex-post efficient liquidation policy. The liquidation policy without loans, but instead with swaps or CDS contracts is ex-post

[5] We consider this result quite significant since it displays the value of interbank loans in our model relative to the existing literature on systemic risks in a network structure. Much of the existing literature (see e.g. Elsinger, Lehar, and Summer (2006)) measures systemic risk from pure interbank loans and simulates defaults without renegotiation. However, our proposition suggests that if banks did not anticipate renegotiating these loans in periods of distress, they would be unlikely to hedge with pure interbank connections, as assumed in these papers. In addition, we show that the correlation between banks' liquidations will be severely underestimated when renegotiations are not modeled.

inefficient. Moreover, the optimality of interbank loans holds in a wide range of economic settings, which we will discuss next.

To determine the robustness of our result, we consider various alternative economic settings. First, we allow for some unhedgeable background risks on the books of banks. We show that the proportion of unhedgeable risk alters the frequency of required renegotiations, but does not affect the optimality of interbank loans in ensuring successful renegotiations. Next we consider alternative bankruptcy regimes in which the enforcement power of the court is strong or weak. For the settlement of large interbank loans, we show that the bankruptcy regime affects the division of the pie between banks, but does not determine the frequency of bank liquidations, so that ex-ante bank profits are invariant across regimes. We next consider large banking systems, where the number of bank liquidations in the economy affects real activity (and hence the marginal utility of the representative agent) and hence changes the fair price of bank deposit insurance. For large enough banking systems, the increase in liquidation correlation lowers the value of hedging so that derivatives add little to banks' values and may choose to remain unhedged.

It is then reasonable to ask if the renegotiation of interbank loan contracts is a realistic assumption. While there is no empirical study that documents the frequency of renegotiations of these contracts, Roberts and Sufi (2009) provide an empirical analysis of the renegotiation of private credit agreements between US public firms and financial institutions. They report that over 90 percent of long-term debt contracts are renegotiated prior to maturity. The interbank loan contracts that we model are also mostly bilateral contracts between banks, and it would be reasonable to expect similar high rates of renegotiations to manage the costs of defaults of financial institutions (see e.g. Footnote 1).

Related Literatures

The research closest to ours is the work on OTC markets in Duffie, Garleanu, and Pederson (2007) and Duffie, Garleanu, and Pederson (2005), which include search and bargaining as important elements of valuation and hedging in these markets. These papers however only study bilateral bargaining, which do not lead to inefficient liquidations. These papers also do not study the optimal network of interbank hedges. Our model is also related to the bank runs literature starting with Diamond and Dybvig (1983), where the decision to run is coordination failure among depositors. Major extensions of this bank run framework to study systemic risk due to liquidity shocks with general networks are in Allen and Gale (2000) (for a survey see Allen and Babus (2009)). As in our paper, Brusco and Castiglionesi (2007) introduce moral hazard issues to this framework. We extend this literature by including derivatives in addition to interbank loans for risk sharing as well as incorporating a feedback effect from hedging to asset quality. Our analysis is richer than the bank runs literature because we allow for possible renegotiation among solvent banks, as well as partial forgiveness of promised payments from insolvent banks. Finally, Leitner (2005) studies the incentives for banks to bail each other out in a network structure where there is built-in complementarity in banks' investment policies. In our framework both the optimal network (and hence complementarities between banks) and the bailout decision are optimally determined by banks.

There is now also a sizable and growing empirical literature on the systemic risks of banks in a network context. Humphrey (1986), Angelini, Maresca, and Russo (1996), Sheldon and Maurer (1998), Furfine (2003), Degryse and Nguyen (2004), Wells (2002), and Upper and Worms (2004) investigate contagious defaults that result from the hypothetical failure of a single institution. The systemic impact of simultaneous shocks to multiple banks has been studied in Elsinger, Lehar, and Summer (2006), and more recently in Cont, Moussa, and Minca (2010), Gauthier, Lehar, and Souissi (2010), and Billio, Getmansky, Lo, and Pelizon (2010). However, these papers do not permit renegotiation of contracts in the systemic transmission between banks, which as we show has implications for both the likelihood and the correlation of bank liquidations.

Our paper also contributes to the literature on the renegotiation of debt contracts. In most papers a solution to the bargaining game at the time of renegotiation always exists due to the special assumptions made in these papers. Several papers assume that players are able to make "take-it-or-leave-it offers" with exogenous bargaining strengths [see, e.g. Hart and Moore (1998), Garleanu and Zwiebel (2009), and Hackbarth, Hennessy, and Leland (2007)]. Paper such as Bolton and Scharfstein (1996), Rajan and Zingales (1998) and David (2001) endogenize bargaining power using the Shapley value of the game as the solution concept. Our bargaining solution is motivated by the recent work by Maskin (2003), where the sequential random arrival of banks to a bargaining site where not only the division of the pie but the decisions by banks on who to bargain with is endogenously determined. The decision to not bargain is our mechanism for inducing the bankruptcy. The random arrival order takes away any first mover advantage to any given bank.

Interbank Hedging Securities in Different Countries

Banks in different countries and systems are highly connected through exposures from derivatives as well as traditional interbank debt. Using data that summarizes the use of derivatives worldwide provided by the Bank of International Settlements (BIS), Figure 1 shows the dramatic increase in notional amounts of total OTCD and asset swap and credit default swaps (CDS) contracts, the two OTCD contracts that we study in this paper. While notional amounts in derivatives are huge, market values are substantially lower and actual exposures by banks are further reduced through netting agreements and collateral. Using measures of net exposures it can be argued that OTCD markets are of comparable size or smaller than the interbank debt market and more concentrated amongst few big banks (Singh and Aitken (2009)). Data from the BIS suggests that the ratio of gross credit exposure from derivatives is less than a third the size of the interbank market.[6]

Across different banking systems, there are differences in OTCD relative to interbank loan use for hedging. For large US banks, interbank loan and derivative exposure are of equivalent size (see *Board of Governors of the Federal Reserve System, Assets and Liabilities of Commercial Banks in the United States - H.8*, 2010). For banks in EU countries, interbank exposure is much larger than OTCD exposure (see *European Central Bank, EU Banking Sector Stability*, 2010). For the Canadian banking system as analyzed in Gauthier, Lehar, and Souissi (2010) the average bank has 20 times the exposure to interbank debt over OTCDs after netting, which likely results from the more stringent regulations in Canada relative to other countries.

The remainder of the paper is structured as follows: In section 1, we provide the structure of the model and the bankruptcy procedure that settles claims in an interbank system. In section 2, we provide a game theoretic analysis of renegotiations among banks, and in section 3 we study the properties of optimal OTC contracts. Section 4 concludes. An appendix provides the proofs of all propositions.

[6] The gross credit exposure from OTCDs for the banks in the G10 countries and Switzerland was USD 3.5 trillion in June 2010 (see *Bank for International Settlements, Semiannual OTC Derivatives Statistics at End-June*, 2010). For a sample of 43 countries at the same point in time the cross border interbank loans were about USD 15 trillion, about four times larger (see *Bank for International Settlements, Locational banking statistics*, 2010). Including within-country interbank debt (for which we do not have statistics) would make the relative size of the interbank loan market even bigger.

1 The Model

We consider a simple two period model of a banking system. All contracts are written at date 0 and are settled at date 1.

Assumption 1 *There are N identical risk neutral banks, each of which has an 'outside' asset with random value, $\tilde{A}_i = \tilde{B}_i \cdot \tilde{C}_i$. The total asset value is broken up into hedgeable and non-hedgeable components. The first component, $\tilde{B}_i$ is hedgeable and the banks in the system can write over-the-counter derivative (OTCD) contracts on the realized values of$\tilde{B}_i$. Each bank has a senior deposit liability payment due at maturity of L_i. The "outside" equity of each bank before any interbank settlements is $\tilde{e}_i = \tilde{A}_i - L_i$*

For simplicity we assume that the N asset distributions are identically distributed

$$\begin{aligned} B_i &\sim \mathrm{LN}(\mu_0 + \mu_1\, h_i - 0.5\, \zeta\, \sigma^2, \zeta\, \sigma^2), \\ C_i &\sim \mathrm{LN}(-0.5\,(1-\zeta)\sigma^2, (1-\zeta)\, \sigma^2) \end{aligned}$$

where $0 < \zeta < 1$ denotes the proportion of variance that is hedgeable, and the assumption of log-normality has no special purpose expect to ensure that the assets always have positive value. We assume that the correlation between $\tilde{B}_i$ and $\tilde{B}_j$ is ρ, and $\tilde{B}_i$ is uncorrelated with $\tilde{B}_j$ and $\tilde{C}_j$. The term h_i represents the level of effort that each bank can exert to increase the mean of the asset value. We assume that the effort has a cost to each bank of $\gamma\, h_i^2$. The effort is financed by the equity holders and the cost is incurred at date 0. At date 1, this cost is sunk, and hence does not affect settlements.

Assumption 2 *Each bank enters into interbank risk sharing agreements with each other bank, each promising a state contingent payoff of*

$$\begin{aligned} \tilde{l}_{ij} &= a + b\tilde{B}_i + c\max[L_j - \tilde{B}_j, 0], \quad \forall i, j = \mathit{mod}(i+1, N) \\ \tilde{l}_{ij} &= b\tilde{B}_i + c\max[L_j - \tilde{B}_j, 0], \quad \forall i, j \neq \mathit{mod}(i+1, N) \end{aligned}$$

The interbank claims are junior to the deposits, and consist of interbank loans and some OTCDs.

(i) Banks are located on a 'circle' and each bank agrees to pay to the bank on its right a fixed amount a at date 1 in return for a cash at date 0. The payment represents a risky interbank loan since the bank may not be able to repay it in full. The cash raised from the loan from the bank to the right is loaned to the bank at the left and since all banks are ex-ante identical, these cash payments exactly cancel out.

(ii) The component $b\tilde{B}_i$ is the amount of its asset that bank i swaps with bank j in return for the same amount $b\tilde{B}_j$. Notice that the *quid pro quo* exchange arises from the assumption that the banks are ex-ante identical so that the flows have equal expected values.

(iii) The component $c\max[L_j - \tilde{B}_j, 0]$ represents a reciprocal credit default swap (CDS) arrangement with bank j, where bank i pays bank j the shortfall amount that it faces at date 1. Once again since the banks enter into reciprocal CDS agreements, bank i receives $c\max[L_i - \tilde{B}_i, 0]$ from bank j, and the ex-ante premiums cancel by symmetry.

Note that since the ex-ante values of the total payments are identical, entering into such agreements has no impact on the leverage ratios of the banks.

Assumption 3 *There are bilateral netting agreements on all OTCD contracts so that ex-post bank i pays bank j the net amount $l_{ij} - l_{ji}$ when this amount is positive. Otherwise bank j pays bank i the negative of this quantity. We define $d_{ij} = \max(l_{ij} - l_{ji}, 0)$ as the obligation of i to j after netting. The payments l_{ij} are as in Assumption 2.*

It is relevant to note that with the bilateral netting agreement in place, interbank loans are effective in risk management only when they are circular as in Assumption 2 (i) as more general loans between each possible pair i and j would have net payments at date 1 of zero. Swaps and CDS contracts are written between each pair of banks. It is also relevant to note that bilateral netting applies as well to any collateral posted by banks on OTCDs (see, e.g. Stulz (2009)) so that in our setup with ex-ante identical banks, collateral is ineffective in mitigating counterparty credit risk between banks.

The structure of this network of banks for the case where $N = 3$ is displayed in Figure 2. We will study the optimal ex-post settlement policy of the banks of their deposits as well as their interbank claims.

Assumption 4 *At date 1 the N banks attempt to settle all claims. If all banks are solvent ex-post, then all claims are settled in full. Otherwise, the N banks attempt to renegotiate these claims and decide on which banks should be optimally liquidated. If renegotiations break down then we assume* *that a regulator imposes the bankruptcy code of the economy on these banks, which determines how claims are settled.*

For the banking system with interbank claims, the division of assets of each bank in the bankruptcy mechanism poses a simultaneous system of conditions, since the amount each bank can pay the other banks depends on how much it receives from these other banks. We call such a system a *clearing* vector, which we describe in detail in section 1.1 below. We will use the vector $x_{ij}^{\mathcal{F}}$ to denote the final settlement from bank i to bank j for the promised payment of d_{ij} either from the successful renegotiation or from the clearing vector in bankruptcy. The superscript denotes the subgame of the bargaining game that is being played. For the full game, $\mathcal{F} = \mathcal{N}$, and we will simplify the notation by omitting the superscript. We will let $x_i = \sum_{j=1}^{N} x_{ij}$ to be the sum of all interbank payments made by bank i to all counterparties, and let $y_i = \sum_{j=1}^{N} x_{ji}$, be the total interbank payments received by bank i.

Assumption 5 *We assume that all depositors have zero time discount. The deposit is senior to all other claims made by the bank. Each bank purchases fairly priced deposit insurance for its deposits. The deposit insurance premium is determined by*

$$\omega_i = E\left[\mathbf{1}_{\{D_i>0\}}\, M_n\, \max[L_i - (1-\Phi)\tilde{A}_i - y_i), 0]\right], \forall i \in \mathcal{N} \tag{1}$$

where $\mathbf{1}_{\{D_i>0\}}$ is a liquidation indicator for bank i, which takes the value of 1 whenever the assets of the bank are liquidated, M_n for $n = 0, \cdots, N$ is the pricing kernel of the economy when the number of banks defaulting in the economy is n, and a fraction Φ of the assets are lost upon liquidation because of bankruptcy costs. The deposit insurance premium is also financed by the bank's equity holders and the cost is incurred at date 0. As the effort costs above, at date 1, this cost is sunk as well, and hence does not affect settlements.

If the banking system is negligible compared to the size of the economy, then the number of banks liquidated would be unrelated to the state of the macroeconomy. If instead, aggregate output and consumption in the economy is related to the number of bank liquidations, then it is reasonable to assume that the marginal value of a dollar in the deposit insurance pricing function varies with the number of liquidated banks. If the kernel, M_n, increases in n, then ceteris paribus deposit insurance will be priced higher when bank liquidations are more correlated. Such as assumption is consistent with the observation that banks in the financial system are "too big to fail." Our analysis is in partial equilibrium and we do not explicitly model the impact of bank liquidations on the kernel.

Assumption 6 *There are no taxes and information is perfect.*

The assumption is used to simplify our analysis. Differential taxes on interbank loans and other derivatives may indeed be an important driver of the usage of derivatives in different countries. Perfect information is assumed to simplify the multilateral renegotiation between counterparties.

1.1 The Bankruptcy Mechanism

If at date 1, the banks are unable to settle all claims, then the regulator of the economy steps in and determines a clearing vector of payments that each bank in the system makes in lieu of its promised payments. We generalize the seminal analysis of clearing vectors in Eisenberg and Noe (2001) to include liquidation costs. In addition, in the analysis of the renegotiation game we will formulate the clearing vectors when some of the banks' claims are settled and the remaining banks bargain over the remaining claims. We denote the complete set of banks with the set $\mathcal{N} = \{1, ..., N\}$. After some banks leave the game, then the remaining set is $\mathcal{F} = \{1, ., f, .., F\}$. We therefore generalize the notation of Eisenberg and Noe (2001) to include the superscript $\mathcal{F}$ to denote that the clearing vector is conditional on the active banks' in the game.

Let $d_i^{\mathcal{F}} = \sum_{j=1}^{F} d_{ij}^{\mathcal{F}}$, be the total obligations of bank i in the partition $\mathcal{F}$. We define the relative liabilities matrix of the partition $\mathcal{F}$ as $\Pi^{\mathcal{F}}$ with elements

$$
\begin{aligned}
\Pi_{ij}^{\mathcal{F}} &= \frac{d_{ij}^{\mathcal{F}}}{d_i^{\mathcal{F}}} \quad \text{if } \ d_i^{\mathcal{F}} > 0 \\
&= 0 \qquad\qquad \text{otherwise}.
\end{aligned}
$$

Let $p^{\mathcal{F}}$ be the F vector of payments that each bank makes. Then, the vector of clearing payments received by the banks are given by the vector $r^{\mathcal{F}} = (\Pi^{\mathcal{F}})' \cdot p^{\mathcal{F}}$. Then the clearing vector $p^{\mathcal{F}}$ for this banking system must satisfy

$$
p_i^{\mathcal{F}} = \min\left[d_i^{\mathcal{F}}, \max\left(A_i^{\mathcal{F}} - \Phi A_i^{\mathcal{F}} \mathbf{1}_{p_i^{\mathcal{F}} < d_i^{\mathcal{F}}} + r_i^{\mathcal{F}} - L_i, 0\right)\right], \forall i \in \mathcal{F}. \tag{2}
$$

The definition states that either bank i makes its full interbank payment of $d_i^{\mathcal{F}}$, or the regulator will liquidate its assets with a proportional liquidation cost of Φ and these proceeds are used along with the payments that i receives from the other banks to first pay off the deposit holders, and the remaining amount is paid to the other banks in settlement of its interbank claims. This can be written more compactly as

$$p^{\mathcal{F}} = \min\left[d^{\mathcal{F}}, \max\left[A^{\mathcal{F}} - \Phi\, A^{\mathcal{F}}\mathbf{1}_{p^{\mathcal{F}}<d^{\mathcal{F}}} + (\Pi^{\mathcal{F}})' \cdot p^{\mathcal{F}} - L^{\mathcal{F}}, 0\right)\right], \tag{3}$$

where $\max$, $\min$, and $\mathbf{1}$ denote the component wise maximum, minimum, and indicator functions, respectively. The right hand side of this equation can be written as a vector valued mapping $\Psi(\tilde{p})$. The clearing vector is the fixed point of this mapping. Eisenberg and Noe (2001) prove the existence of a fixed point of this mapping, and show that it can be found by the method of successive approximation. These authors also show that the fixed point is unique when there are zero liquidation costs. We instead find a robust set of examples with positive liquidation costs in which there are at least two fixed points. We first provide an example and then an interpretation of the two fixed points as alternative bankruptcy regimes.

Example 2 *(Non Uniqueness of Clearing Vectors) Consider the case of three banks that have ex-post asset values, $A_i = 1.5$ for each i and each bank has deposits of 1. In addition each bank has an interbank liability due of 1 to each bank to its right. Lets assume that $\Phi = 0.5$. Then there are two clearing vectors. In the first, each bank pays the full amount of 1 on its interbank loan and receives 1. In the second, each bank pays 0 and receives 0.*

Let's verify that these are both valid clearing vectors. If each bank receives 1, then it can pay its depositors 1 and still have 1.5 left over, which can then be used to pay the interbank loan. Therefore each bank making full payment is a fixed point. If on the other hand, each bank receives 0 as an interbank payment, then it has only 1.5 to make payments of 2 and hence it must liquidate. Upon liquidation, there is only 0.75 left, which is all used to pay depositors, and nothing is left to pay the interbank loan. Therefore paying 0 is a second clearing vector. Notice the "systemic" risk in this second clearing payment vector: Each bank defaults on its commitments only because it receives nothing on commitments owed to it. We will make this definition more precise below.

The role of a non-zero Φ is important. If $\Phi = 0$ then as in Eisenberg and Noe (2001), we would have a single clearing vector, the first one. For any $\Phi > 0.33$ though, the second clearing vector is also valid. Finally, its worth pointing out that the example is a little extreme because with the second clearing vector all payment vectors are zero. We can construct similar examples where only one or two banks have zero clearing payments.

Motivated by the example and following Elsinger, Lehar, and Summer (2006) we make a distinction between ‘fundamental” defaults, and “contagious” defaults. The default of bank i is called fundamental if bank i is not able to honor its promises under the assumptions that all other banks honor their promises,

$$\sum_{j=1}^{F} \Pi_{ji}^{\mathcal{F}} d_j^{\mathcal{F}} + e_i^{\mathcal{F}} - d_i^{\mathcal{F}} < 0. \tag{4}$$

A contagious default occurs, when bank i defaults only because other banks are not able to keep their promises, i.e.,

$$\sum_{j=1}^{F} \Pi_{ji}^{\mathcal{F}} d_j^{\mathcal{F}} + e_i^{\mathcal{F}} - d_i^{\mathcal{F}} \quad \geq \quad 0 \tag{5}$$

$$\text{but} \tag{6}$$

$$\sum_{j=1}^{F} \Pi_{ji}^{\mathcal{F}} p_j^{\mathcal{F}} + e_i^{\mathcal{F}} - d_i^{\mathcal{F}} \quad < \quad 0. \tag{7}$$

Using these definitions, the defaults in the second clearing vector are all contagious.

We interpret the two different clearing vectors as two distinct bankruptcy regimes. The first we call the “strong” regime, since it implies that all banks pay larger amounts for their interbank commitments, and in turn receive more from other banks. The second is the “weak” regime, in which banks pay out less and receive less on their commitments. Both clearing vectors are ‘fair’ in the sense that limited liability of all equity holders and absolute priority of all claims is maintained in both. The choice of the regime is determined by the enforcement power of the regulator, and its determination is outside the scope of this model. However, we note that unlike the analysis in Eisenberg and Noe (2001) and Elsinger, Lehar, and Summer (2006), we do not assume that banks’ actual payments for their interbank claims are determined completely by the clearing payment vectors. The clearing vector is the value that each bank will pay out if the set of banks jointly fail to renegotiate all claims among themselves, and approach the regulator. In this case, the payoff for bank i under the filtration $\mathcal{F}$ is given by its outside equity value plus all the payments it receives from other banks under bankruptcy as defined in equation (3) minus the face value of its liabilities since equity holders cannot enjoy a payout until the debt is paid off in full:

$$w_i^{\mathcal{F}} = \max(e_i^{\mathcal{F}} + (\Pi^{\mathcal{F}})' \cdot p^{\mathcal{F}} - d_i^{\mathcal{F}}, 0). \tag{8}$$

In the next section we model these renegotiations and then study the implications for recovery rates on the interbank claims in the two bankruptcy regimes.

2 Renegotiation of Interbank OTCD Payments

In this section we provide an analysis of the bargaining game that takes place at date 1 between the N banks if some or all of them fail to make full payments on the interbank OTCD commitments. The banks engage in multilateral renegotiations for the resolution of their claims.

2.1 The Bargaining Protocol

The game starts with nature choosing a bank to become the proposer, who then makes simultaneous take-it-or-leave-it offers to all its counterparties to settle its outstanding claims. If all the offers are accepted, the claims with the proposer as counterparty are eliminated, cash payments are transferred, and the proposer leaves the game. All claims between the remaining banks are unaffected, and nature selects one of the remaining banks to become the new proposer. If at least one counterparty rejects the offer then the predefined bankruptcy process is triggered for all the banks that are still in the game and interbank obligations are settled using the bankruptcy payment vector as described in Section 1.1. The game ends when either there is only one bank left in the game or when the active players go to bankruptcy court.

We impose two restrictions on what offers have to be accepted: A bank that has a claim on the proposer cannot reject an offer in which it would get paid in full and a bank which owes funds to the proposer cannot reject an offer to pay zero. These restrictions are quite intuitive as neither can a creditor bank sue a counterparty when its claim is paid off in full nor can a debtor demand anything beyond complete debt forgiveness. They also ensure that a group of healthy banks can cut loose a severely underfunded bank by paying off all their debt to that bank in full and thus cutting the link so that the underfunded bank's default would not spread through the system.

Banks that are unsuccessful at renegotiations have to follow the rules of the bankruptcy code, in which outside depositors have to be paid first and the remaining assets are shared proportionally amongst creditors using the clearing mechanism discussed in Section 1.1. We summarize these constraints in the following definition of feasible payments:

Definition 1 (Feasibility) *Let d_{ij} be the promised payments of bank i to bank j, $\mathcal{L}$ be the set of banks that get liquidated, then a set x_{ij} of payments from bank i to bank j is feasible if for any bank $i \notin \mathcal{L}$:*

$$0 \leq x_{ij} \leq d_{ij} \text{ and } \sum_j x_{ij} \leq A_i - L_i + \sum_j x_{ji} \tag{9}$$

and for any bank $i \in \mathcal{L}$

$$x_{ij} = \max\left(\frac{d_{ij}}{\sum_j d_{ij}}(A_i(1-\Phi) - L_i + \sum_j x_{ji}), 0\right) \tag{10}$$

Equation (9) ensures that the sum of renegotiated payments can be spanned by the set of outstanding OTCDs. For the banks in liquidation, equation (10) ensures that assets are divided according to the rules of the bankruptcy mechanism as described in Section 1.1.

Before analyzing the bargaining game let us first define an efficient outcome. In our model efficiency can be defined on two levels: here we want to address efficiency of renegotiations for a given set of interbank claims, which we refer to as ex-post efficiency. Later in the paper we will address efficient choice of effort and risk sharing through interbank contracts, which we will refer to ex-ante efficiency.

Definition 2 (Ex-post Efficiency) *A set of interbank contracts is ex-post efficient if it minimizes bankruptcy costs among counterparties under the restriction that any renegotiated payments ex-post are feasible using the set of contracts as in Definition 1.*

After asset values are realized, the social planner will minimize total bankruptcy costs, which are the only dead-weight losses in our model. The question is how much freedom we should give to the social planner to reallocate assets across banks. If one would set no restrictions for the planner, then an efficient outcome can be reached whenever the sum of the outside assets exceeds the sum of deposits whether or not interbank linkages exist or how they are structured. This flexibility is on our point of view unattainable for a planner or bank regulator in reality as it violates basic property rights. The Federal Reserve, for example, cannot expropriate a healthy bank and hand over assets to a needy bank. It can however, as in the case of Long Term Capital Management, allow (and even persuade) banks to renegotiate claims with a troubled institution. We therefore also require the social planner to stay within the framework of contracted obligations. As we will see later, the renegotiations in our model are not always ex-post efficient. Essentially, renegotiations inefficiencies in our model arise from a free-rider problem when one bank owes net interbank payments to two other banks.

Definition 3 (Ex-ante Efficiency) *A set of interbank contracts is ex-ante efficient if it minimizes bankruptcy costs without the restriction of ex-post feasible settlements.*

A set of contracts that is ex-ante efficient will lead to perfectly correlated liquidations among banks, which occur only when there are insufficient resources ex-post in the system, that is$\sum_i e_i < 0$. This is a stronger notion of efficiency, which is still second-best given the incentive problems faced by banks.

It is important to note that the bankruptcy decision is itself endogenous in our model. An important determinant of the bargaining power of each counterparty is the payoff that it will receive in bankruptcy if it is invoked, which is given in equation (8). This payoff sets a lower bound on what any counterparty can get by rejecting an offer and thereby triggering the bankruptcy process.

The bargaining solution that we study in the three player game is subgame perfect. Let the game start with a proposal by bank k to players i and j. When bank i evaluates an offer from proposer k it anticipates the effect that the proposer's offers have in its expected payoff in subsequent subgames. If banks i and j accept bank k's proposal, the latter will leave the game and the following game between i and j is played after adjusting each player's resources for the settlement with k. Additionally, both remaining banks face uncertainty about the order of proposers in the following subgame. Let $v_i^{-k}(e_i)$ be the expected value obtained by bank i in future rounds of bargaining given current resources, e_i, and given that k has left the game. Subgame perfection requires that what bank i gets by accepting k's proposal is at least as much as what i can get by rejecting k's offer and thereby invoking the bankruptcy mechanism. Specifically if k has an obligation towards bank i, i.e. $d_{ki} > 0$ then

$$v_i^{-k}(e_i + x_{ki}) \geq w_i \quad \text{if } x_{ki} < d_{ki} \tag{11}$$

and if k has a claim on bank i, i.e. $d_{ik} > 0$ then

$$v_i^{-k}(e_i - x_{ik}) \geq w_i \quad \text{if } x_{ik} > 0 \tag{12}$$

The conditions in (11) and (12) restrict the applicability of subgame perfection to cases where bank i can in fact invoke bankruptcy, i.e., in subgames that will actually be reached. For example, (11) applies only if bank i is a net creditor relative to k and k does not pay in full. We will see in Example 4 below that a creditor bank can be forced to accept an offer which makes it worse off than under the bankruptcy mechanism. Similarly, (12) applies only if i owes funds to k and k asks for a positive settlement.

Ironically, a more efficient bankruptcy mechanism with lower bankruptcy costs can increase the frequency of liquidations. Lower bankruptcy costs Φ increase lower bounds w that banks receive in bankruptcy and can increase the required offers to exceed the sum of currently available resources and lead to avoidable liquidations.

Before solving the bargaining game, let us define what we understand as successful renegotiations and characterize the solution of the game:

Definition 4 *Renegotiation between players in $\mathcal{F}$ is successful when all parties agree on a settlement $0 \leq x_{ij}^{\mathcal{F}} \leq d_{ij}^{\mathcal{F}}$ and all banks survive.*

The bargaining solution that we consider has several appealing features that make it applicable to the settlement of interbank claims. First, a bank that is deeply insolvent can use the bankruptcy option to its advantage by refusing to accept any partial settlement offers from solvent counterparties that do not make it solvent. Second, the bankruptcy decision is endogenous, and in particular, the first proposer can destroy the bankruptcy option of remaining players. If a bank is deeply insolvent, then the other banks can "cut loose" this bank by either paying debt owed to it in full or accepting zero for the debts owed by this bank. The remaining banks bargain over the remaining resources in the system. We will call an outcome where a weak bank's claims are settled in full but it is still liquidated a partial bargaining solution, which is formally defined here.

Definition 5 *A bargaining solution is complete if no bank rejects the proposal and all banks survive. In a partial solution no bank rejects the proposal and some banks get liquidated.*

2.2 Solving for Equilibrium of the Two Bank Case

We first provide an analysis for the two bank case where we show that the equilibrium of the bargaining game is always efficient. Suppose that bank 1 on net owes 2 a payment of d_{12}. There are no further rounds of bargaining and thus we do not need to form expectations on future payoffs.

Then the subgame perfection conditions in (11) and (12) become:

$$v_2(e_2 + x_{12}) = e_2 + x_{12} \geq \max(e_2 + p_{12}, 0) \qquad \text{if } x_{12} < d_{12} \tag{13}$$

$$v_1(e_1 - x_{12}) = e_1 - x_{12} \geq \max(e_1 - d_{12}, 0) \qquad \text{if } x_{12} > 0, \tag{14}$$

respectively, where $p_{12} = \min(d_{12}, \max(e_1 - A_1\Phi\mathbf{1}_{p_{12}<d_{12}}, 0))$ is the bankruptcy payment vector. We then have to walk through several cases to solve for the equilibrium outcome:

Proposition 1 *Suppose bank 1 owes 2 a payment of d_{12}. Then the outcome is as follows:*

1. *If $d_{12} \leq e_1$, bank 1 pays d_{12} and never gets liquidated. Bank 2 gets liquidated if $e_2 + d_{12} < 0$.*
2. *If $0 \leq e_1 < d_{12}$, the bankruptcy payment vector is $p_{12} = \max(A_1(1-\Phi)-L, 0)$. A successful renegotiation (i.e. no liquidation) will only occur whenever $e_1 + e_2 \geq 0$. If bank 1 proposes first, the settlement $x_{12} = max(p_{12}, -e_2)$. If bank 2 proposes first, $x_{12} = e_1$.*
3. *If $e_1 < 0$, bank 1 pays zero and gets liquidated. In this case, bank 2 gets liquidated if $e_2 < 0$.*

The allocation in the two player game is ex-post efficient.

The intuition for the bargaining equilibrium leading to an efficient liquidation policy is that if the solvent bank decides to bail out the bank in trouble by accepting less than full settlement on the interbank claim from the insolvent bank, then it can fully appropriate the preempted liquidation costs. We shall see in the following subsection that when there are three banks, this result will no longer hold.

2.3 Solving for Equilibrium of the Three Bank Case

To solve for the general game we have to consider two possible ex-post realized network structures: after netting, interbank obligations either form a circle or form a two path structure as illustrated in Figure 3. In the circular structure each bank is symmetric with respect to the network structure as it has one incoming and one outgoing payment so that the order of proposers has no effect on the efficiency of bargaining. In the two-path structure the order of bargaining will determine the dead weight losses in equilibrium. In general the subgame perfect solution of the bargaining game is a solution to a linear program. The exact linear program will depend on which network structure is realized as well as which bank is the initial proposer, and we provide one such program below.

Let us start with the two-path structure and label the banks with two, one and zero obligations as banks 1, 2, and 3, respectively. To keep the illustration brief, here we only formulate the problem explicitly when bank 3 is the first proposer and is due payments of d_{13} from bank 1 and d_{23} from bank 2. We solve similar problems for the other cases with alternative structures and order of proposers. If these payments cannot be made in full, bank 3 can invoke the bankruptcy mechanism to get the payoff w_3. Alternatively, it makes settlement offers of x_{12} and x_{13}. Bank 3's problem can be written as the following linear program, which we will denote as LP_3.

$$\sup_{x_{13},x_{23}} \quad e_3 + x_{13} + x_{23} \tag{15}$$

$$0 \leq x_{13} \leq d_{13} \tag{16}$$

$$0 \leq x_{23} \leq d_{23} \tag{17}$$

$$e_1 - x_{13} > 0 \tag{18}$$

$$e_2 - x_{23} + d_{12} > 0 \tag{19}$$

$$e_1 - x_{13} + e_2 - x_{23} > 0 \tag{20}$$

$$\frac{1}{2}\left(e_2 - x_{23} + \min(d_{12}, e_1 - x_{13})\right) + \frac{1}{2}\left(e_2 - x_{23} + p_{12}^{-3}\right) > w_2 \tag{21}$$

Equations (16) and (17) arise from ex-post feasibility of the payments given the interbank payments due. Equations (18) - (20) ensure that the following subgame between banks 1 and 2 has a solution in which both banks remains solvent as in Section 2.2.[7] Finally, equation (21) ensures subgame perfection by ensuring that the expected payoff of bank 2 in the following two-bank subgame (depending on the order of bidder) will give it a higher payoff than currently declining bank 3's offer and invoking the bankruptcy mechanism. In the event that bank 2 is randomly picked to make the final offer to bank 1, it will extract any remaining surplus from bank 1. Otherwise, bank 2 will get its clearing vector payment, p_{12}^{-3} after bank 3 has left the game. It is useful to note that we do not write an analogous subgame perfection condition for bank 1. We have already ensured that in the following subgame between banks 1 and 2, both banks remain solvent, so that irrespective of the order of bidders, bank 1 will be better off than in bankruptcy, where its value is zero.

Bank 3's full optimization when $d_{13} + d_{23} < e_1$ is then to choose the larger of w_3 and the solution to LP_3. We now illustrate three important features of the bargaining game for the settlement of OTCD payments in the following three subsections:

[7] The strict inequalities rule out degenerate equilibria where an insolvent bank ($e_i < 0$) accepts offers on interbank claims that do not make it on net solvent, but give the equity holders the same payoff as in bankruptcy.

2.3.1 Ex-post Inefficient Bargaining

The following example shows that the solution to the bargaining game can be ex-post inefficient. The example is illustrated in Figure 4.

Example 3 *Let the ex-post asset realizations of the three banks be* $A_1 = 1.8, A_2 = 0.4$*, and* $A_3 = 1$*. In addition, let* $L = 1, d_{12} = d_{13} = 1, d_{23} = 0$*, and* $\Phi = 0.1$

Let us first examine the outcome without renegotiations, i.e. the outcome of the bankruptcy mechanism. Bank 1 cannot honor its debt of $L + d_{12} + d_{13} = 3$ and will get liquidated. After liquidation, $A_1(1 - \Phi) = 1.62$ will be distributed amongst its creditors with $(A_1(1 - \Phi) - L)/2 = 0.31$ going to each of the other banks. Bank 2 will have total assets of $0.4 + 0.31 = 0.71$ which is insufficient to cover its debt of 1 and thus bank 2 will be liquidated as well. Bank 3's equity holders get to keep $A_3 - L + 0.31 = 0.31$. Bank 1 is in fundamental default and bank 2 is in contagious default, because if bank 1 had paid in full, bank 2 would have resources of $A_2 + d_{12} = 0.4 + 1 = 1.4$, which would be sufficient to cover its debt of $L = 1$.

In renegotiations, the minimal offer that bank 2 would accept for a settlement of its claim d_{12} is 0.6, which would enable it to pay off depositors in full, and thus avoid bankruptcy. Suppose now that bank 3 proposes first. Bank 1 would therefore only accept a settlement with bank 3 that would leave it with at least 0.6 so that it can subsequently reach an agreement with 2. But leaving 0.6 with bank 1 limits the payment that bank 3 can get to $A_1 - L - 0.6 = 0.2$, which is less than the 0.31 that bank 3 would get under the bankruptcy mechanism. Bank 3 therefore goes to the bankruptcy court and banks 1 and 2 get liquidated. The liquidation of banks 1 and 2 is inefficient according to our definition 2. Consider an alternative allocation in which all banks survive and the payments are $x^e_{12} = 0.6, x^e_{13} = 0.2$. These payments are feasible according to definition 1 and leave $0, 0$, and 0.2 for banks 1,2, and 3, respectively. The allocation is efficient as bankruptcy costs are zero. However, the efficient solution is not an equilibrium of the bargaining game.

More formally, we solve LP_3 in (15) – (21). First, note that $x_{23} = d_{23} = 0$. Therefore, conditions (18) to (20) simplify to: $0.8 - x_{13} > 0$; $-0.6 - 0 + 1 > 0$; and $0.8 - x_{13} - 0.6 + 0 > 0$. The second condition is satisfied and the other two simplify to $x_{13} < 0.2$. Thus bank 1 is only willing to pay up to 0.2 because otherwise it would get liquidated in the subsequent bargaining game, so that the maximum value for bank 3 under the LP_3 is 0.2. Therefore, bank 3 prefers $w_3 = 0.31$, resulting in inefficient liquidations.

It is also interesting to note that bargaining is efficient when bankruptcy costs are sufficiently high. Consider $\Phi = 0.9$ as an example. Then bank 1 pays zero in liquidation as $(A_1(1-\Phi)-L) < 0$. Then in the efficient allocation x^e as suggested above, bank 3 is strictly better off than in liquidation. It is easy to verify that in this case the three banks will find a bargaining solution whenever $\Phi > 2/9$.

The efficiency of bargaining also depends on the order of proposers. When bank 1 proposes first the outcome is similar. Again bank 2 would ask for 0.6 and bank 3 could get at most 0.2, which is less than its liquidation payoff resulting again in inefficient liquidations. When bank 2 proposes first, however, liquidations can be avoided. In stage 1 of the game, bank 2 demands $x_{12} = 0.8$ from bank 1. To ensure that bank 1 accepts this offer, bank 2 must ensure that bank 1 remains solvent in the following subgame when it settles its outstanding commitment of 1 with bank 3. After bank 2 leaves the game, bank 1 is left with assets of 1, just enough to cover senior depositors. At this stage, $p_{13}^{-2} = 0$, so that bank 3 can no longer obtain a positive payoff from enforcing liquidation. So it accepts a settlement of zero. Note that in this case, bank 3 gets an overall payoff of zero, which is worse that what it would have gotten under the bankruptcy mechanism of the full game.

An important feature of the example is that bank 2 has a very negative equity value, and indeed we show below that having one such bank is necessary for a renegotiation failure. Having a negative equity value puts a lower bound on offers that this bank will accept in any renegotiation since equity holders get paid only if the bank become solvent after renegotiations. In addition, we require that bank 1 cannot meet its interbank payments in full so that renegotiation breakdowns can only occur when both low probability events occur.

Proposition 2 *In the two-path structure when a feasible payment exists to span the required bailout, a necessary condition for a breakdown is that at least one bank has negative equity value and the bankruptcy cost parameter is not too high ($\Phi \leq \Phi^* = 1 - \frac{L}{\max(A_1, A_2, L)}$).*

2.3.2 Not All Players Have Bankruptcy Options

An important feature of our bargaining framework is that counterparties that are paid in full for claims owed to them, or those that are permitted to make a zero payment for claims that they owe are not able to invoke the bankruptcy mechanism. Therefore, players who are lower down in the sequence of proposals are not able to guarantee a lower bound on their payoffs with bankruptcy.

Example 4 *Consider a modification of example 3 with a higher asset value for bank 2, and an additional link from bank 2 to 3:* $A_1 = 1.8, A_2 = 2.1, A_3 = 1, L = 1, d_{12} = d_{13} = d_{23} = 1, \Phi = 0.1.$

As before, bank 1 cannot make all promised interbank payments and in bankruptcy would pay each bank 0.31. Bank 2 is solvent and even under the bankruptcy mechanism can pay in full. Finally in bankruptcy, bank 3 would receive 0.31 from bank 1, full payment from bank 2, and pay its senior depositors in full for a value of $1 + 0.31 + 1 - 1 = 1.31$. It is useful to note that even though all banks are directly connected to each other through an outstanding commitment, bank 3 is a "sink" in the sense that payments only flow into it.

Bank 2 is the first proposer: It will extract 0.8 from bank 1 and make full payment to bank 3. Then bank 1 has just enough left to repay the senior depositors and cannot pay 3. Thus 3 is left with $A_3 - L + d_{23} = 1 - 1 + 1 = 1$, which is less than the payoff under the bankruptcy mechanism of the full game. Note that bank 3 cannot block bank 2's opening offer and invoke bankruptcy as it gets paid in full.

Bank 3 is the first proposer: In this case, bank 2 has the option to refuse payment and hence invoke the bankruptcy mechanism. Thus bank 2's subgame perfection condition in (21) has to be observed. If it rejects the initial offer, bank 2 would get $A_2 - L + p_{12} - d_{23} = 2.1 - 1 + 0.31 - 1 = 0.41$ in bankruptcy. Using (21), bank 2 would only accept the initial offer to pay x_{23} only if its expected continuation payoff satisfies:

$$0.5\ (1.1 - x_{23}\ +\ \min(1, 0 - x_{13}))\ +\ 0.5\left(1.1 - x_{23} + p_{12}^{-3}\right) \geq 0.41. \tag{22}$$

It is easy to check that bank 3's optimal strategy is to minimize the payoff for bank 1 by extracting $x_{13} = e_1 = 0.8$ and setting x_{23} to make (22) bind, which then simplifies to $0.5 \cdot (1.1 - x_{23}) + 0.5 \cdot (1.1 - x_{23}) = 0.41$ or $x_{23} = 0.69$. Since bank 1 will have no resources left, the outcome of the second round of bargaining is $x_{12} = 0$ regardless of the order of proposers. The final payoffs are then 0, 0.41, and 1.49 for banks 1, 2, and 3, respectively.

2.3.3 Partial Bargaining Solutions

Whenever the banks fail to find an efficient bargaining outcome, they can attempt a partial solution by "cutting loose" an extremely weak counterparty. A bank can be cut loose by its counterparty by paying it in full if it is a net creditor or by accepting a payment of zero if its a debtor. In either case, as discussed in Section 2.2, the remaining game is a two player game in which there is a successful renegotiation can be worked out if the players jointly have positive remaining equity value. The following example shows a partial bargaining solution:

Example 5 *To illustrate a partial solution, consider a modification of Example 4 with a very low asset value for bank 2, a higher asset value for bank 1 and a smaller obligation from bank 1 to bank 2:* $A_1 = 2.2, A_2 = 0.3, A_3 = 1, L = 1, d_{12} = 0.5, d_{13} = d_{23} = 1, \Phi = 0.1.$

In this example $e_1 = 1.2$, $e_2 = -0.7$, and $e_3 = 0$. Then a complete bargaining solution is not possible because even if bank1 would pay bank 2 in full and bank 3 would forgive its debt, it could not be saved: $A_2 - L + d_{12} - 0 = 0.3 - 1 + 0.5 < 0$. Thus bank 3, as the first proposer asks for $x_{13} = 0.7$ and $x_{23} = 0$. Bank 2 has to accept and bank 1 accepts as it is left with $A_1 - L - x_{13} = 2.2 - 1 - 0.7 = 0.5$ which is enough to pay off bank 2 in full. Bank 2 is liquidated and bank 3 gets $e_3 + x_{13} + x_{23} = 0.7$. It is easy to verify that 3 is better off than under the bankruptcy mechanism where it would have gotten $e_3 - L + d_{13}/(d_{12} + d_{13})(A_1(1 - \Phi) - L) = 0.6533$.

2.4 The Coase Theorem

Coase argued that as long as bargaining is efficient and property rights are clearly established, externalities will not cause an inefficient allocation of resources. Several other authors (see, e.g., Tirole (1988)) also assert that the bilateral bargaining outcomes are ex-post efficient as long as information is perfect. In contrast, our examples show that a reasonable multilateral bargaining game leads to a solution that is ex-post inefficient.[8] The inefficiency in multilateral bargaining arises due to the free-riding that arises when one party makes a deal with another and attempts to appropriate the resources of the third party, which also has a claim on the pie to be shared. In part, we allow banks to reach partial bargaining solutions to minimize losses from inefficient liquidations. In addition, as we will discuss in Section 3, banks can avoid such inefficient bargaining outcomes with the optimal choice of derivatives, so that the Coase theorem will hold with a very high probability.

[8]There is now a growing literature in economic theory that has provided alternative multilateral bargaining mechanisms where the Coasian conclusion fails to hold. See, e.g., Ray and Vohra (2001), Maskin (2003),Macho-Stadler, Perez-Castrillo, and Wettstein (2007).

3 Optimal OTDC Contracts and Asset Quality Choices by Banks

In this section, we study the optimal choice of interbank OTC contracts in this setting, under the assumption that banks either renegotiate their OTC contracts ex-post and maximize their bargaining values as in Section 2, or do not renegotiate and maximize their profits obtained from the bankruptcy mechanism determined by the regulator as in Section 1.1. One reason for formulating renegotiations is that it potentially mitigates the moral hazard problem, since the benefits of the effort are better captured by the bank in renegotiations, in which it is able to extract greater value from the other banks in the system. In addition, we will see the recovery processes under the two sets of assumptions change the properties of the hedging components and affects their effectiveness in bankruptcy cost reduction, providing incentives, and preventing system runs.

3.1 Banks' Optimization Problems

Let f_i be the three-tuple (a_i, b_i, c_i), which is the set of OTCDs chosen by bank i, and let $h_i(f_i)$ be the optimal effort given that the bank has contracted f. First consider the case without renegotiations. Then, we can write bank i's ex ante profit as

$$\pi_i^{BM} = \max_{f_i} \left[\max_{h_i(f_i)} \left(E[w_i - \omega_i^{BM} - \gamma \cdot h_i^2] \right) \right], \tag{23}$$

where w_i is the ex-post profit of bank i determined by the bankruptcy mechanism defined in equation (8), ω_i^{BM} is the deposit insurance premium in Assumption 5, and the expectation is taken over all realizations of the asset values, $\tilde{A}_i$ in Assumption 1. Individual effort choices and OTC contract choices, are equal across banks since banks asset values have identical distributions and all derivative deals are specified as *quid pro quo* exchanges so that the contract choices are by definition common for all firms. For convenience, we solve the problem in two stages, first choosing the terms of the interbank contracts f_i and then choosing the level of effort h_i conditional on the contracts. Notice that the effort choice has an externality since the OTC contracts partly transfer the benefits of the improved asset stream to increasing the profits and lowering liquidation costs at other banks. Bank i can appropriate these benefits only to the extent that it obtains better recoveries when these other banks have low asset realizations. Therefore, as in any public goods problem, bank i chooses an effort level that maximizes only its personal profit, which is generally lower that the socially optimal level. We formulate the individual bank's problems with renegotiations as

$$\pi_i^R = \max_{f_i} \left[\max_{h_i(f_i)} \left(E[v_i - \omega_i^R - \gamma \cdot h_i^2] \right) \right], \tag{24}$$

where v_i is the value of bank i with renegotiations payments as formulated in Section 2.

Given the lack of explicit closed-form solutions for clearing vectors in bankruptcy, we characterize the optimal contract choices of banks with several numerical examples for the case where there are three banks. We calculate all relevant expectations with Monte-Carlo simulations. We start though with some analytical propositions on optimal OTC risk sharing contract choices of banks.

Proposition 3 *If banks maximize profits without renegotiating settlements on interbank claims ex-post, then a pure interbank loan cannot be the optimal hedging contract.*

The intuition for the result is simply that under the bankruptcy mechanism, bank's equity holders do not get any gain unless they are solvent and can pay off their claims in full. However, with circular pure interbank loans, in states when a bank is solvent, it can never receive more then the face value of the amount it owes. Therefore, such loans can never increase bank profits and are never optimal. It is interesting and important to note that this proposition does not carry over to the case where banks renegotiate their interbank settlements and thus reduce inefficient liquidation costs. As mentioned in Footnote 5, this result shows that the evaluation of systemic risk in the network model literature often ignores the optimally of the network.

We next show a particular OTCD choice, which turns out to be optimal under several of the conditions that we study in this paper and leads to perfect liquidation correlation of banks.

Proposition 4 *If banks maximize profits with ex-post renegotiated settlements, then with a contract with pure interbank loans of $a \geq 2L$, bank liquidations are perfectly correlated, so that either all banks fail when $\sum_{i=1}^{3} e_i < 0$ or all banks survive when $\sum_{i=1}^{3} e_i \geq 0$. Under this contract liquidations are ex-ante efficient.*

The intuition for why the large amount of interbank loans eliminates inefficiencies is that it essentially binds solvent banks to bailing out all insolvent ones (hence avoiding their liquidation) since each bank's outstanding interbank debt exceeds the deposits of the bank's counterparties. The insolvent banks therefore, can use the interbank payment to pay off depositors and depending on the order of proposers in renegotiation pass on the benefits of reduced liquidations to the solvent bank. The circular structure of interbank loans is critical since each bank has only one creditor in the interbank market and there is no free-rider problem. Liquidations only occur when the banking system as a whole in insolvent.

The next proposition provides conditions under which derivatives require no renegotiation.

Proposition 5 *No renegotiations are required when there is no unhedgeable risk ($\zeta = 1$), interbank contractual payments are either only asset swaps or only CDS, and OTCD contracts are such that $b \leq 1/3$ and $c \leq 1/2$.*

The intuition for this result is simply that renegotiations are required only when an insolvent bank owes money to a solvent bank. However, reciprocal asset swaps and CDS contracts each have net payoffs that flow from the stronger bank to the weaker bank. As long as there are no unhedgeable risks, weaker banks will therefore never have to pay money to stronger banks. In the presence of unhedgeable background risks though a bank with a large negative unhedgeable shock and a positive hedgeable shock will owe a net payment on its swaps or CDS contracts to a bank that is stronger, so that these derivatives settlements would require renegotiations. Without, unhedgeable risk, the only case that might require renegotiation is when the strong bank has such a large payment that it would itself become insolvent, and make the weak bank solvent. However, the restrictions that we make on the portfolio of $b \leq 1/3$ or $c \leq 1/2$, imply that such a situation cannot arise either. As we will see in our numerical examples, the constraint on swaps is never binding due to the incentive problem. Banks however may choose very large CDS contracts, which will require renegotiation at maturity.

We end this subsection, with a result that will help in understanding which derivatives choices lead to higher asset quality.

Proposition 6 *In the absence of unhedgeable risk and for positive effort costs ($\gamma > 0$), banks' asset quality and expected profits using large interbank loan contracts, $a \geq 2L$ exceed those than perfect hedging with asset swaps, $b = 1/3$.*

Both choices lead to perfect hedging of bank defaults, however, the interbank loans lead to higher asset quality. This arises because by swapping the banks' equity holders reduce their exposure to their own assets, while with interbank loans, they are still the residual claimants of their entire asset stream.

3.2 Bank Value with Optimal Asset Quality and OTCD Choices – The Base Case

In this and the following subsections we provide some numerical results to shed further light on the effort and contract choices of banks. We choose the parameters of our model to approximate the profiles of Baa-rated banks over a 4-year horizon. We fix the parameters μ_0 and μ_1 so that with the endogenously chosen effort, the banks' asset-to-liabilities ratios are about 1.15. The high leverage is consistent with empirical estimates of leverage for banks (see Berger, DeYoung, Flannery, Lee, and Özde Öztekin (2008)). We choose an asset volatility parameter, σ, and a 4-year time horizon for pricing the OTCD contracts. The σ is chosen so that with the above leverage choice we obtain an endogenously determined bank liquidation probability of 1.24 percent, which is the average historical 4-year cumulative default probability for Baa-rated bonds by Moody's.

We start by discussing banks' optimal choices for the "Base Case", which we define as the case where 70 percent of the risk is hedgeable ($\zeta = 0.7$), the correlation between banks' asset values is 0.1, banks are in the strong bankruptcy regime, and the pricing kernel is flat over the default states (small banking system). The bank's profit for a given risk sharing agreement (a, b, c) is determined by the optimal asset quality from the effort decision given that contract, the benefit from risk sharing (diversification), and the recovery of assets of liquidated firms from renegotiation.

The different hedging choices each trades off the benefits of hedging versus the loss of incentive of banks to maintain asset quality. The incentive problem is most severe for asset swaps where the bank passes on $(1 - (N - 1)\, b)$ of the increase in the mean of $\tilde{A}$ to the other banks, while it still bears the full cost of effort. The optimal effort choice will therefore decrease in b. For interbank loans and CDS contracts this externality is smaller. In the former case the bank's equity holders are the residual claimant of the assets after the interbank loan is paid off, and so they still have a strong incentive to maintain the quality of the assets. This point is illustrated in the top left panels of Figure 5, which plots the optimal effort choice for different amounts of a and b contracted. As reasoned, an individual bank's effort decreases in b but is relatively insensitive to a in this example. For CDS contracts, the interbank liability of the bank depends on the performance of *other banks* and not the bank's own assets, and hence again the adverse effect on effort incentive is smaller than for asset swaps. The top right panel of Figure 5 (notice the different scale) shows small differences in effort choice for interbank debt and CDS contracts.

The bottom left panel of Figure 5 illustrates the bank's profit for different a and b choices holding $c = 0$. The upper and lower surfaces are for the cases with and without renegotiations, respectively. For any given choice of contract values a and b, profits are always higher with renegotiations, because the lowest payoff with renegotiations is in the case where they break down, and then the payoff of banks is as determined by the bankruptcy mechanism (the payoff without renegotiations). The two surfaces only join when banks do not hedge (a=0, b=0). Following the intuition of Proposition 5, the recovery that banks obtain in renegotiations are more important for interbank debt than for asset swaps, and thus we can see that both surfaces are close when interbank debt is zero. In line with Proposition 3 we also see that pure interbank loans are not optimal contracts when there are no renegotiations of OTCD contracts. In fact, profits decline monotonically with an increase in the amount of interbank loans. Even though they do not adversely affect asset quality as much as asset swaps, straight debt contracts due to their inflexible payments without renegotiations are a poor instrument for risk sharing since they lead to a greater incidence of insolvency than the other types of contracts. Asset swaps provide better diversification, because in states in which a bank's asset realization is low, its required payment is low as well, and thus the bank is less likely to be insolvent, and its equity holders can retain positive value. With renegotiations, however, we can see that interbank debt contracts are superior to asset swaps and profits increase in the amount of interbank loans since as we show in Proposition 6, renegotiations allow flexible payments while the nature of the debt contract induces higher asset quality.

Payments on CDS contracts only occur in periods when the other banks are in trouble, and are very effective in reducing liquidation costs and dominate interbank loans that are not renegotiated. However, as seen in the bottom right panel of Figure 5, once we allow renegotiations, interbank loans are the superior hedging choice once again to the extent that banks profits from an optimally chosen large quantity of interbank loans can only be marginally improved by adding CDS contracts. The redundancy of CDS contracts occurs because effectively the payoff on a renegotiated interbank loan contract is similar to that of a CDS contract — solvent bank makes a payment to an insolvent bank so that it can survive. Notice that this ex-post 'replication' happens in our model endogenously due to renegotiations.

Panel A of Table 1 sheds further light on the optimality of interbank loans. The overall optimal hedge portfolio contains interbank loans as well as CDS, however, it is notable that the profit increase relative to using only interbank loans is minuscule, which we shall shortly see is due to the relatively small increase in the span of feasible contract payoffs with swaps. The panel also has the relevant statistics for portfolios with a single type of contract. Even though interbank loans with renegotiation and CDS contracts both induce fairly strong effort levels, banks prefer interbank loans for two main reasons: First, they provide some protection against all risk, while CDS contracts only provide protection against low hedgeable shock outcomes. Second, as shown in Proposition 4, a network with large interbank loans binds solvent banks to bailing out insolvent ones whenever the banking system as a whole is solvent, so that dead weight liquidation costs are minimized. Indeed, banks using pure interbank loans default frequently (16 percent of observations) but are liquidated after renegotiations only rarely (1.25 percent of observations). Default rates for banks using swaps (5.25 percent) or CDS contracts (8.18 percent) are lower, but liquidation rates are much higher at 4.15 and 7.54 percent, respectively.

Without the moral hazard problem, the most effective hedging strategy would be for the banks to each swap a third of their assets to every other bank in the system. This hedging strategy would perfectly hedge banks fluctuations in banks' asset values and would lead to perfectly correlated liquidations, which would only occur when the banking system as a whole is insolvent. However, the moral hazard problem prevents banks from using this simple swapping strategy, which then leads to incomplete risk sharing with swaps. We study the relative hedging effectiveness of the three contracts further in Figure 6.

The relatively flat surfaces in the two panels show the liquidation probability due to an overall insolvent banking system ($\sum_i e_i < 0$), while the spiked surfaces show the liquidation probability when the system in aggregate is solvent, but there is incomplete risk sharing. The left panel shows that as banks use more swaps, the banking system has higher aggregate insolvency risk due to a reduction in asset quality. This kind of insolvency is relatively insensitive to the amount of interbank loans or CDS contracts used. Overall, the flat surfaces isolate the impact of deterioration of asset quality on the liquidation probability.

The spiked surfaces show the liquidation probability that arises due to lack of risk sharing in the system. Quite strikingly, the left panel shows that these liquidations spike when low amounts of either swaps, or more surprisingly, interbank loans are used. The right panel shows that CDS contracts, even if used in large amounts, are not able to provide adequate risk sharing. Proposition 4 shows that with a large amount of interbank loans, solvent banks are committed to bailing out insolvent ones and all liquidations are ex-ante efficient. For the optimally chosen contract, as seen in Table 1, the proportion of liquidations that are ex-ante inefficient (those where $\sum e_i > 0$) is only 2.4 percent, but significantly higher at 80 and 93.5 percent for swaps and CDS contracts, respectively, which arise as these contracts cannot provide the funds required to rescue insolvent banks, which we discuss further below.

There are three reasons why swaps and CDS contracts fail in risk hedging. First, they only provide the required funds to save the banks when their insolvency arises from a large hedgeable shock. The contracts are written on the values of only the hedgeable part of bank's asset values, and fail to *span* the unhedgeable risks. Second, these contracts have some counterparty risk because each bank purchases insurance from the two other banks in the system. If either of these counterparties does not have sufficient resources to fully honor their commitment, the insuring bank's hedges are ineffective. In part, the banks mitigate this risk by purchasing excess insurance (in the case of CDS contracts $c \geq 0.5)$ so that shortfalls of funds from one bank may be met by the excess insurance purchased from the other bank.[9] Finally, unlike interbank loans, these contracts do not provide the incentive for solvent banks to always bail out the insolvent banks. Interbank loans provide an efficient liquidation policy because solvent banks not only provide adequate funds to keep insolvent banks alive, but because there is also a claim from insolvent banks to solvent banks, they are able to able to expropriate any assets that these banks have. This creates the incentive for the solvent banks to minimize the dead weight losses from liquidation. Swap and CDS contracts payments after netting only flow from the solvent to the insolvent banks and hence limits their ability to expropriate all the insolvent banks' assets. In Section 3.4, we will isolate the counterparty and incentive effects in the model without unhedgeable risk (spanning problems).

3.2.1 Breaking Down the Value Creation of OTCDs

To shed further light on the role of the three types of contracts on optimal choices we investigate the overall profitability as well as the marginal impact of each type of contract on each bank's profits through three different channels in our model:

$$\begin{aligned}
\text{Marginal renegotiation recovery impact} &= \pi^{R}(f,h(f)) - \pi^{BM}(f,h(f)) && (25)\\
\text{Marginal risk sharing impact} &= \pi^{BM}(f,h(f)) - \pi^{BM}(0,h(f)) && (26)\\
\text{Marginal asset quality impact} &= \pi^{BM}(0,h(f)) - \pi^{BM}(0,h(0)) && (27)
\end{aligned}$$

[9] In the case where there is no unhedgeable risk, having $c < 0.5$ has no effect on its liquidation probability, since the sum of the payments from its counterparties will not cover the shortfall of the bank's assets below its liabilities. Having $c = 1$ will provide the bank with adequate protection even if one counterparty defaults. With some unhedgeable risk as in the baseline case, the liquidation probability declines smoothly in c, as the unhedgeable shock partly diversifies the hedgeable shock realizations.

The marginal renegotiation recovery impact is the difference between the banks' profits with and without renegotiations using the optimal OTCD contract f and the banks optimal effort $h(f)$ given that contract. The marginal risk sharing impact is the difference between the bank's profit with the optimal OTCDs and effort, and with no hedging but holding the effort at the optimal level. Both profits are computed without renegotiations. Finally, the marginal asset quality impact is is the difference between the profit assuming no hedging, but using the optimal effort with optimal OTCDs and without hedging. Note that the three impacts sum up to the total difference in profit given contract choice f, $\pi^R(f, h(f))$, and the profit without hedging, $\pi^{BM}(0, h(0))$. In Figure 7 we show the overall profit of the three types of contracts if held in isolation (left panels) and the marginal impacts for each contract (right panels).

In the top panel we immediately confirm our earlier finding that bank's profits increase (decrease) in the amount of interbank debt with (without) renegotiations. The right panel shows the marginal impacts. As seen interbank bank does not provide an adverse asset quality incentive so that the marginal impact on asset quality is very small. The other two impacts are striking and in opposite directions. Due to their fixed payments, the interbank loans have a negative impact on risk sharing forcing more defaults with larger amounts of debt. However, this negative impact is more than fully compensated for by the large positive impact from recovery in renegotiations, as all banks write down required payments and bail out all troubled counterparties for large enough amounts of interbank debt. It is also worthwhile noticing that the benefits of interbank debt hold independent of the source of risk, thus providing some insurance against unhedgeable risk as we will further explore in section 3.3.

The middle panels of Figure 7 show that asset swaps have the opposite tradeoff. With more of their assets swapped, banks have better risk sharing but maintain lower asset quality by expending less effort. Overall profits are hump shaped in b as the risk sharing effect dominates for small b, but eventually the lower asset quality lowers profits. Since swaps only need renegotiation when there are large unhedgeable shocks, the impact on renegotiation recovery is low, which can be seen by the small gap for the profits with and without renegotiation in the left panel.

The bottom panels study the three impacts on CDS contracts, and we notice immediately that neither impact is large. We have already shown that renegotiations are not frequently required for these contracts, and that asset quality is also not adversely impacted since the payoffs depend on the performance of the bank's counterparties rather than their own. The only surprising finding is that the impact on risk sharing is also small. It is useful to recall that these contracts provide banks the shortfall in their deposit payments relative to their hedgeable assets and hence hedge liquidation risk. However, with 30 percent unhedgeable risk, any negative realization of the unhedgeable shock to the bank will make the shortfall larger than the protection obtained from the CDS contracts and thus reduce the risk sharing benefit. Without any unhedgeable risk, the risk sharing benefit of CDS contracts is much larger, but is effective only when c is larger than 0.5, so that the bank's shortfall is hedged.

3.2.2 The Impact of OTCDs on Credit Risk and Systemic Risk in the Base Case

We now address the key questions of this paper, which are the effects of OTCDs on credit and systemic risk. We define systemic risk as the large scale breakdown of financial intermediation or, in the context of our model, the occurrence of two or especially three banks defaults. In Figure 8, we study the simulated frequency of bank liquidations and expected liquidation losses for three cases: with no interbank hedging, with interbank hedging but no renegotiations, and hedging with renegotiations with the optimal contract, which contains a large amount of interbank loans. In the left panels we show two sets of bars for each case: the left bar denotes the frequency of liquidations, while the right shows the frequency of defaults by banks on interbank payments, some of which are renegotiated and do not lead to liquidations. In comparing the bars though, it is useful to note that some cases of one default could be followed by two or even three liquidations due to systemic spillovers, and hence the left bar can in principle be higher than the right bar for a given number on the horizontal axis. It is also useful to note that without any interbank hedging the two bars are identical (top panel) as defaults always imply liquidations. It is intuitive that for a given asset quality, hedging and renegotiations should both lower credit risk and systemic risk. However, we study these effects in a setting where asset quality depends on hedging choices so that, the impact on both credit and systemic risk is potentially ambiguous.

The graph illustrates that banks have an incentive to build a fragile network to facilitate ex-post renegotiations. The benchmark case, with no OTCDs, in the top panels, shows that the probability of at least one liquidation is about 20%, however the probability of three defaults is negligible. Moving to the middle panel, the case of hedging with OTCDs and no renegotiations, we find that that the probability of at least one liquidation declines to about 10 percent, however the probability of three liquidations increase somewhat. All incidences of three liquidations are from systemic spillovers and not from fundamental defaults at all three banks simultaneously. Therefore, the use of OTCDs without renegotiations indeed seem to increase systemic risk, although they lower credit risk. Finally moving to the bottom panel, the probability of a default declines further to about six percent, none of which lead to liquidations (see Proposition 4). However, in this case, the probability of two and specially three liquidations increases sharply relative to the other cases. The right panel shows histograms of dead weight liquidation costs, and these show similar patterns as the frequency of liquidations.

The high liquidation correlation in the bottom panels are consistent with a banking system that is highly interconnected, very resilient to defaults of a single institution, but prone to systemic crises. These crises occur with low probability but spread throughout the system. The efficient liquidation policy with large interbank loans ensures that the incidences of three liquidations occur when the banking system is insolvent in aggregate and not due to an avoidable cascade or coordination problem in bailouts. It is useful to note that the banking system is small in the baseline case since the pricing kernel of the economy does not depend on the number of banks liquidated. We will study the issues for large banking systems separately in Section 3.6 below.

3.3 All Unhedgeable Risk

The base case has 30 percent unhedgeable risk. In this subsection we study the optimal hedging strategy when all the risk is unhedgeable by setting $\zeta = 0$ in Assumption 1 keeping all other parameters the same as in the base case. The results are in Panel B of Table 1. With this assumption, the payoffs of swaps and CDS contracts are always zero, however, interbank loans are just as effective as in the case with some hedgeable risk. The promised payment on interbank debt is not state-contingent, but the renegotiated payoff is again ex-post customized to the needs of the counterparties, and the contract is able to induce high asset quality and low liquidation rates for banks. The liquidation rates, systemic risk, and bank value are all quite similar to the base case, which itself has a large amount of interbank debt.

3.4 All Hedgeable Risk

Alternatively, if all the risk is hedgeable ($\zeta = 1$), then the payoffs of CDS and swap contracts are more likely to span banks' hedging needs. However, as seen in Panel C of Table 1, interbank loans that can be renegotiated are still the superior choice for the joint provision of incentives to manage asset quality and hedging even though their relative net benefit is smaller than in the base case. Quite strikingly though, the liquidation probability with swaps is 50 percent higher than interbank loans, and double that for CDS contracts. The differences are smaller than in the base case, since there are no spanning problems. However, the inability of swaps and CDS contracts are due to the counterparty risk and bailout incentive effects that we discussed at the end of section 3.2. For CDS contracts, banks chose $c = 0.8$ to mitigate the counterparty risk so that most of the rise in liquidation probability arises from the lack of incentives for the solvent bank to bail out the insolvent banks. For swaps, the counterparty risk is small as well, since bank's required payments are a proportion of their asset values, which they can meet.

The overall value from hedging also depends on the asset quality induced in the system, and once again, interbank loans and CDS perform similarly in this regard. Overall, interbank loans are again the optimal choice, and asset swaps, which are are the most inefficient in inducing high asset quality, are overall the least preferred contract choice in this setting.

3.5 The Weak Bankruptcy Regime.

In this subsection we study the impact of OTCDs on both systemic and credit risks in a weak bankruptcy regime, with and without renegotiations. We recall that the weak regime results from choosing the fixed point of the bankruptcy mechanisms with the smallest payments. We start off by showing that the banks' payoff with optimally chosen interbank debt contracts is independent of the bankruptcy regime or the proportion of hedgeable risk.

Proposition 7 *Banks' ex-ante profits under pure interbank loans of $a \geq 2\,L$ do not depend on the fraction of unhedgeable risk or the bankruptcy regime.*

If interbank loans are large enough ($a > 2L$) then banks are only liquidated when the banking system is insolvent in aggregate, and thus dead weight losses are independent of the bankruptcy regime. Overall, as can be seen in Panel D of Table 1 we see that bank profits and contract choices under the weak regime differ only slightly from the strong regime and have all the same qualitative features, in particular the optimality of large amounts of interbank loans. What explains this surprising result with a radically different bankruptcy regime? Indeed, there are substantial differences between clearing payments under the strong and the weak regime as it is illustrated in example 2. However, Proposition 4 still holds, which implies that renegotiations are successful whenever the banking system is solvent in aggregate. In the weak regime, the default probability for interbank loans is substantially higher at 23.5 percent compared to 16.4 percent in the base case. However, the liquidation rates are very similar. The different clearing payments in bankruptcy therefore only affect the ex-post splitting of the overall pie, but since banks are ex-ante identical, their ex-ante values are the same in two regimes. The overall optimal contract has more CDS than in the strong regime, however, the incremental bank profit by adding CDS contracts is very small.

3.6 Large Banking Systems

In all cases examined so far we have assumed that the banking system is large so that the pricing kernel in the economy is unrelated to the number of insolvent banks. We now consider cases where the pricing kernel (marginal utility of the representative agent) for deposit insurance is increasing in the number of liquidated banks. Clearly, deposit insurance will be priced higher with this assumption because with hedging liquidations are more correlated so that the probability of multiple liquidations increases.

Optimal hedging strategies and bank profits are given in Table 2 for two different assumptions for the kernel while holding all other parameters as in the base case. Panel B of this table has a larger banking system than panel A. The major change in panel A relative to the base case is that the banks pay higher premiums for deposit insurance, which lowers the benefits of hedging. The optimal hedging strategy still has a large amount of interbank loans that keeps ex-ante liquidations very low and the liquidation probability is also fairly similar to the base case. In panel B, the kernel value is raised more steeply when all banks in the system are liquidated, which would be the case when the banks as a whole provide credit for a major proportion of output produced in the economy. As we see bank value with hedging is very similar to that without hedging. By hedging, the liquidation probability is lower relative to the case without hedging, however, liquidations are more correlated so that the deposit insurance premium of banks is higher as well. Of the three hedging strategies, banks chose only asset swaps, but the incremental value of using these contracts relative to interbank loans or no hedging is minuscule.

3.7 Constrained Contracts

So far we find that straight interbank debt contracts interbank debt contracts are optimal under a wide range of economic settings. The key driving result is Proposition 4, which shows that large interbank loan contracts are able to span the payments required to rescue insolvent banks in renegotiations whenever the banking system as a whole is solvent. But while the interbank debt market is huge, in practice we do not see interbank obligations as large as suggested in our model.

One possible explanation is that regulations prevent banks from implementing the optimal contract. Following suggestions from the Basel Committee, lending to a single counter-party is restricted to a certain fraction of bank capital in most countries. In the US, for example, a national bank may only lend up to 15% of its capital to one bank. We now study the optimal hedging contracts with this added constraint.

Figure 9 analyzes the effect of the regulatory constraint. In each panel, the x-axis plots the upper bound $\bar{a}$, on the amount of interbank debt. The left panel shows a "pecking order" for the different hedging choices in our baseline case. As the upper bound is lowered from around 0.25 (the unconstrained choice for interbank loans), first banks use CDS in addition to interbank loans. Only, when the upper bound is lowered to around 0.12, do swaps become part of the optimal mix of hedges used by banks. It is useful to note that the the CDS usuage is very high (around 0.9) as banks attempt to replicate the highly interconnected system created using interbank debt. The right panel shows the profits and effort choice in the constrained system. With constraints, bank profits cannot be as high despite the large derivative use due to the limitations in hedging discussed in Section 3.2 and overall approximately a third to a half of the overall value of hedging is lost depending on the exact upper bound.

In Table 3, we examine the effect on constraints in the various economic settings analyzed above. The results are very similar to the base case above and the same pecking order emerges. Interbank loans are used to the limit, except in the case of the very large banking system, when they are not used even for the unconstrained case. When all the risk in unhedgeable, banks use no other derivatives, but otherwise use some swaps and very large CDS contracts. In all cases, bank profits are lower and liquidation probabilities are higher relative to the unconstrained case, which shows again the optimality of renegotiated interbank loan contracts.

3.8 Reserve Requirements

In our analysis so far we have assumed that banks are not required to hold a portion of their assets in cash or other safe assets, as is required in many banking systems around the world. In the US for example, banks are required to hold 10 percent of their deposits as reserves. Incorporating such a regulation is fairly straightforward in our model. Results on banks' hedging choices and profits in the base case with the reserve requirements are shown in Panel E Table 1. We do indeed find that reserves can be used a cushion for bad outcomes in banks' asset values, so that liquidation rates are a bit lower, and bank profits are slightly higher than in Panel A. More importantly, among the alternative hedging strategies, interbank loans are still the optimal choice, and conserve the most bank value with hedging.

4 Conclusion

In this paper we develop a derivatives irrelevance result that applies to banks that face a joint risk management and moral hazard problem. Our central argument is that in the absence of regulations, renegotiable interbank loan contracts will provide banks as good or better hedging opportunities than most standard derivatives. With a sufficiently highly connected network of interbank loans, banks' liquidation policies are efficient since solvent banks are committed to bailing out insolvent banks, or themselves face insolvency. The optimality of renegotiable interbank loans is shown to hold in a large set of economic settings, which are explicitly stated in the introduction. The renegotiation possibility is critical for the optimality of these loans as our analysis shows that banks would never use interbank loans for hedging that could not be renegotiated.

The banking system with this optimal contract would be one with low liquidation rates for banks, but a very high correlation of bank liquidation. The high correlation may be the reason underlying restrictions that regulators in most countries have placed on the amount of interbank borrowing/lending to any institution in the banking system. Our analysis shows that the liquidation correlation could also be controlled by having fairly priced deposit insurance, which explicitly charges banks a higher insurance premium for failing simultaneously. In the presence of restrictions on interbank lending however, our analysis implies that derivatives usage becomes value increasing for banks. Indeed our analysis suggests that the growth of derivatives could be driven by the regulations imposed on interbank lending to make banks less likely to collapse simultaneously.

Appendix

Proof of Proposition 1
Let us analyze the three cases:

1. Bank 1 has enough resources to pay its debt in full and by Assumption 4 it thus pays d_{12}. Bank 2 has a value of $e_2 + d_{12}$ and fails if $e_2 + d_{12} < 0$. Liquidation of bank 2 is ex-post efficient according to definition 2 because there exists no feasible payment according to definition 1 that would save bank 2.

2. Using equation (8), we start by specifying the reservation values of the two banks when they attempt to renegotiate the interbank payment. Bank 1 cannot pays its interbank debt in full and under the bankruptcy mechanism it would get liquidated giving it a payoff $w_1 = 0$. Its payment in bankruptcy to bank 2 is $p_{12} = \max(A_1 2(1 - \Phi) - L, 0)$, which would give bank 2 a net payoff of $w_2 = \max(e_2 + p_{12}, 0)$.

   If $e_1 + e_2 < 0$, then clearly, no successful renegotiation is possible, so we now assume that $e_1 + e_2 \geq 0$. Suppose that bank 1 proposes first. In this case, it has to make bank 2 at least as well off as under the bankruptcy mechanism, i.e. $e_2 + x_{12} \geq w_2$. Therefore bank 1 has to offer at least $x_{12} = w_2 - e_2 = \max(p_{12}, -e_2)$, which it can only afford if $e_1 \geq -e_2$ or $e_1 + e_2 \geq 0$. When 2 proposes first, is needs to make bank 1 at least as well off as under the bankruptcy mechanism, that is, $e_1 - x_{12} \geq w_1$. Since bank 1 gets zero in bankruptcy, bank 2 extracts everything ($x_{12} = e_1$) and itself survives whenever $e_2 + x_{12} = e_1 + e_2 > 0$. The allocation is ex-post efficient because banks only get liquidated when there are insufficient overall resources in the banking system, i.e. $e_1 + e_2 < 0$.

3. Bank 1 cannot pay its depositors even when it pays zero to bank 2. It enters bankruptcy and gets liquidated. The clearing payment of bank 1 under the bankruptcy mechanism, p_{12}, is zero because senior depositors get paid first. Bank 2's payoff under the bankruptcy mechanism is $w_2 = \max(e_2, 0)$. Bank 2 gets liquidated whenever $e_2 < 0$. The only feasible payment for bank 1 is zero and therefore no other payment exists that could save a bank from liquidation. Liquidations are therefore ex-post efficient.

This completes the proof. ■

Proof of Proposition 2

Assume that a feasible solution exists such that all banks survive and start with bank 3 as proposer. Renegotiations are only successful when no bank is better off invoking the bankruptcy mechanism. In the first part of the proof we will show that when all $e_i > 0$ the proposer can make a take-it-or-leave-it offer of the clearing vector payments to the other banks. These banks will accept these offers as they will be at least as well off as under the bankruptcy mechanism and allows all banks to survive. The positive outside equity ensures that all banks survive even when clearing payments are zero, e.g. because of high liquidation costs.

Formally, note that the simplex LP_3 defines the set of offers that leave banks 1 and 2 better off than in bankruptcy, and hence will be accepted by these banks. We will show that when all e_i's are positive, the offers $\underline{x}_{13} = p_{13}, \underline{x}_{23} = p_{23}$ are in the simplex, and in addition, bank 3 is at least as well off as under bankruptcy. These offers set a lower bound on the payoff bank 3 can attain in the game when it bids first.

We start with the observation that the clearing payment from 1 to 2 in the subgame after 3 has left the game is equal to the clearing vector under the full game, i.e. $p_{12}^{-3} = p_{12}$. The reason is that in the subgame bank 1 is left with resources of $A_1 - L - p_{13}$, which result in a clearing vector of $p_{12}^{-3} = A_1(1 - \Phi) - L - p_{13}$, which equals p_{12} because by the definition of clearing vectors it has to hold that $A_1(1 - \Phi) - L = p_{13} + p_{12}$.

We now verify that $\underline{x}_{13}, \underline{x}_{23}$ is in the simplex LP_3 and leave bank 3 better off than in bankruptcy. Equations (16) and (17) hold by the definition of the clearing vectors. Since clearing payments can never exceed a bank's equity value it is easy to show that conditions (18) to (20) hold. Equation (21) reduces to:

$$\frac{1}{2}\left(e_2 - p_{23} + \min(d_{12}, e_1 - p_{13})\right) + \frac{1}{2}\left(e_2 - p_{23} + p_{12}\right) \geq e_2 + p_{12} - p_{23}$$

or

$$\min(d_{12}, e_1 - p_{13}) \geq p_{12} \tag{28}$$

Consider two cases: (i) if $e_1 - p_{13} \leq d_{12}$ then equation (28) reduces to $e_1 - p_{12} - p_{13} \geq 0$ which is always satisfied because bank 1 cannot pay more that e_1 under the bankruptcy mechanism. (ii) if $e_1 - p_{13} > d_{12}$ then (28) simplifies to $d_{12} \geq p_{12}$, which is also always satisfied because clearing payments cannot exceed the face value of obligations. The offer $\underline{x}_{13}, \underline{x}_{23}$ is therefore in the simplex of LP_3 and banks 1 and 2 will optimally accept it. Under this proposal, bank 3 receives $e_3 + p_{13} + p_{23}$ which equals its payoff under the bankruptcy mechanism, u_3, as defined in equation (8). Bank 3 therefore has no incentive to invoke the bankruptcy mechanism.

A similar logic applies when other banks propose first. We again construct analogous lower bounds on the payoff of the initial proposer by setting its offers equal to the clearing vector payments. Following the logic presented above it is easy to show that payments under bankruptcy mechanism once a proposer has left the game equal the payments under bankruptcy mechanism of the full game, i.e. $p_{ij}^{-k} = p_{ij}$, and the remainder of the proof is similar.

In the second part of the proof, we show that when a feasible bargaining solution exists, it can be attained whenever bankruptcy costs are sufficiently high. We require Φ to be high enough so that banks 1 and 2 pay zero if they become insolvent under the bankruptcy mechanism, i.e. $\max(A_1, A_2)(1 - \Phi) \leq L$ implies that $p_{12} = p_{13} = 0$ and that $p_{23} = 0$ if 2 is insolvent under the bankruptcy mechanism. The lower bound on bankruptcy costs that assures zero clearing vector payments for insolvent banks is thus $\Phi^* = 1 - \frac{L}{\max(A_1, A_2)}$. The intuition of the remaining proof is that banks will accept any nonnegative payments from renegotiations when their alternative is to receive zero payments under the bankruptcy mechanism.

We now show for $\Phi > \Phi^*$, any feasible solution proposed by bank 3 under which all banks survive is in LP_3, and is hence accepted by the other banks. Definition 1 of feasible payments simplifies to

$$0 \leq x_{ij} \leq d_{ij} \tag{29}$$

$$\sum_j x_{ij} \leq e_i + \sum_i x_{ij}. \tag{30}$$

For bank 1 equation (30) reduces to $x_{12} + x_{13} \leq e_1$ which ensures that $x_{13} < e_1$ and condition (18) holds. For bank 2 equation (30) reduces to $x_{23} \leq e_2 + x_{12}$. Since $x_{12} \leq d_{12}$, condition (19) must hold. Adding equation (30) for banks 1 and 2 yields $x_{12} + x_{13} + x_{23} \leq e_1 + e_2 + x_{12}$, which proves inequality (20). The SP constraint (21) is only relevant when bank 1 cannot pay in full, i.e. $p_{12} < d_{12}$:

$$\frac{1}{2}\left(e_2 - x_{23} + \min(d_{12}, e_1 - x_{13})\right) + \frac{1}{2}\left(e_2 - x_{23} + p_{12}^{-3}\right) \geq e_2 - d_{23} + p_{12}$$

which reduces to

$$d_{23} - x_{23} + \frac{1}{2}\min(d_{12}, e_1 - x_{13}) + \frac{1}{2}p_{12}^{-3} \geq p_{12}. \tag{31}$$

Equation (30) for bank 1 ensures that $x_{12} + x_{13} \leq e_1$ and since all x_{ij} are non-negative, the third term on the left hand side in equation (31) must be non-negative. The payment of 1 to 2 in the subgame, p_{12}^{-3}, like all payments under the bankruptcy mechanism, is non-negative as well. From the assumption $\Phi > \Phi^*$ we know that p_{12} is zero. Equation (31) then holds as long as $x_{23} \leq d_{23}$, which is true because of equation (29). Thus the feasible allocation is in the simplex of LP_3 and thus banks 1 and 2 are not worse off by accepting the feasible solution than by invoking the bankruptcy mechanism.

To complete the proof we have to ensure that bank 3 is at least as well off with renegotiations that invoking bankruptcy, which is equivalent to

$$x_{13} + x_{23} \geq p_{13} + p_{23}. \tag{32}$$

To validate condition (32) we have to analyze three cases: If bank 1 is solvent under the bankruptcy mechanism. i.e. $A_1 - L - d_{12} - d_{13} > 0$, then bank 1 pays in full and the game degenerates to a two player bargaining game in which renegotiations never break down. If bank 1 is insolvent and bank 2 is solvent then $x_{23} = p_{23} = d_{23}$ and $p_{13} = 0$ because of $\Phi > \Phi^*$. Equation (32) then simplifies to $x_{13} \geq 0$ which holds because of Equation (29). If both banks are insolvent under clearing then condition (32) simplifies to $x_{13} + x_{23} \geq 0$ which again holds because of Equation (29).

The proof works similarly when other banks propose first. Intuitively the bankruptcy costs above Φ^* ensure that all banks get zero payments from counterparties under the bankruptcy mechanism and thus prefer any feasible negotiated solution under which all banks survive. This completes the proof. ■

Proof of Proposition 3

To facilitate the exposition of the proof, we reverse the order of the choice of the contract terms and effort in (23), and write the profit of bank i as

$$\pi_i^{CV} = \text{Max}_{h_i} \left[\text{Max}_{a(h_i)} \left[E \left[v_i - \omega_i^{CV} - \gamma \cdot h_i^2 \right] \right] \right],$$

that is we first fix the effort level of the bank, and then choose the contract terms as a function of this effort level. Take any effort level h^*, fix $b = 0$ as presumed and consider the optimal choice a. With the effort choice fixed, the distributions of $\tilde{A}_i$, $i = 1, 2, 3$ are fixed. Now with a choice of any $a > 0$, the ex-post profit conditional on any realization of the asset values from the clearing vector for i is $\max(\tilde{A}_i - L_i + r_i - a, 0)$, while with $a = 0$, the profit would be $\max(\tilde{A}_i - L_i, 0)$. However, by the definition of the clearing vector in (2) we have that $r_i \leq a$, which implies that the profit with $a = 0$ is greater than or equal to the profit with $a > 0$. This completes the proof. ■

Proof of Proposition 4

Without loss of generality assume that bank 3 is the first proposer. The case where all banks have positive outside equity e is trivial as all banks pay in full and all banks survive. For the remainder of the proof we consider only cases in which at least one bank has negative e.

The high face value of $a \geq 2L$ guarantees feasibility since the amount owed to the weaker bank is greater or equal its shortfall with probability 1. In addition, the circular structure of large interbank debt implies that there are no "sink banks" that block the flow of resources and hence the first proposer optimizes the liquidation costs of the entire system (see Example 4 for an example of inefficient liquidation policy when debt is not circular).

We look at the following cases:

(i) Assume that $e_1 < 0$, $e_2 < 0$, and $e_3 > a$ and write $a = 2L + a'$, with $a' \geq 0$. Consider the payoff under the bankruptcy mechanism first. Bank 3 pays in full and banks 1 and 2 get liquidated. Under the rules of the bankruptcy mechanism banks 1 and 2 will keep L to cover senior depositors and pass on any remaining assets after liquidation costs, so that bank 3 receives $p_{23} = 2L + a' + A_1(1 - \Phi) - L + A_2(1 - \Phi) - L = (A_1 + A_2)(1 - \Phi) + a'$.

Bank 3 is then left with its equity after payment to and from the interbank market for a total of $w_3 = \max(e_3 - (2L + a') + p_{23}, 0) = \max(\sum_i e_i - (A_1 + A_2)\Phi, 0)$.

The high face value of the circular interbank loan thus effectively forces all the liquidation costs of banks 1 and 2 on bank 3 and makes it optimal for bank 3 to seek a bailout through renegotiations to avoid these liquidation costs. Since banks 1 and 2 get liquidated under the bankruptcy mechanism they will agree to any offer that allows them to survive. To implement the bailout bank 3 offers $x^*_{31} = -e_1 - e_2$ and $x^*_{23} = 0$. In the subgame bank 1 will keep $-e_1$ so that it can pay its depositors off in full and pass on $-e_2$ to bank 2, which is the smallest offer that bank 2 is willing to accept. Banks 1 and 2 both survive and bank 3 gets an overall payoff of $v_3 = e_3 - x_{31} + x_{23} = \sum_i e_i \geq w_3$. The face value of $a \geq 2L$ ensures that the bailout is feasible since the lower bound for e_i is $-L$ which implies that $x^*_{31} \leq 2L \leq p_{31}$.

(ii) Some banks have positive outside equity value but receive zero under the bankruptcy mechanism, i.e. $w_1 = w_2 = 0$. Banks 1 and 2 will accept any offer that lets them survive and it is easy to show that bank 3's optimal offer is $x^*_{31} = \max(-e_1, -e_1 - e_2, 0)$ and $x^*_{23} = \min(\max(e_2, e_1 + e_2, 0), a)$. It can be easily verified that bank 3 is able to extract all the equity and can achieve a payoff of $v_3 = \sum_i e_i$.

(iii) In the case where $w_1 > 0$ or $w_2 > 0$, bank 3 as the proposer must ensure that banks are at least as well off with renegotiations as under the bankruptcy mechanism.

Start with the case in which $w_1 = e_1 + p_{31} - a > 0$. Bank 3 then pays $x_{31} = p_{31}$ to bank 1 and bank 1 has to pay bank 2 in full because it is solvent. Bank 1 is then left with a payoff $v_1 = w_1$ and will accept bank 3's offer. Bank 3 will then extract all of bank 2's resources by setting $x_{23} = \min(e_2 + a, a) > L$. It is straightforward to check that all banks have enough to survive.

If $w_1 < 0$ and $w_2 > 0$ bank 2 is solvent and must pay in full, $x_{23} = a \geq 2L$. Bank 2's payment is enough to ensure survival of both, banks 3 and 1, even if they realize an asset value of zero. Bank 3 has to offer bank 1 enough so that it will survive, i.e. $x_{31} = \max(-e_1, 0)$.

This completes the proof. ∎

Proof of Proposition 5

Without loss of generality suppose that $A_1 > A_2 > A_3$. Because of the constraints on b and c, with netting bank i always has an obligation to pay to bank $j, j > i$, an amount d_{ij}, which is either $d_{ij} = b(A_i - A_j) > 0$ for the asset swap or $d_{ij} = c\max(L - A_j, 0) - c\max(L - A_i, 0) > 0$ for the CDS contract. For the proof we go through every possible combination of banks being in fundamental default, i.e. they are insolvent even if they get paid in full. We then examine for each case whether renegotiations are necessary.

(i) All banks are solvent. In this case no bargaining is necessary and all banks pay in full.

(ii) Bank 1 is in fundamental default, bank 2 is not in fundamental default, which yields:

$$e_1 - d_{12} - d_{13} < 0 \quad (33)$$

$$e_2 + d_{12} - d_{23} > 0 \quad (34)$$

To show that this case is not possible with asset swaps subtract equation (34) from equation (33) to get $e_1 - e_2 - 2d_{12} - d_{13} + d_{23} < 0$. Substitute the payoffs for the asset swap contracts to get $A_1 - A_2 - 2b(A_1 - A_2) - b(A_1 - A_3) + b(A_2 - A_3) < 0$ or $A_1 - A_2 - 2b(A_1 - A_2) - b(A_1 - A_2) < 0$ which simplifies to $(A_1 - A_2)(1 - 3b) < 0$. The last inequality cannot hold based in the assumptions that $A_1 > A_2$ and $b < 1/3$. Therefore, case (ii) cannot arise with asset swaps.

To show that case (ii) cannot occur with CDS we have to walk through all possible combinations of CDS payoffs. The CDS' payoffs are determined by the sign of the e_i. For all banks to survive it must be that $e_1 + e_2 + e_3 > 0$. Hence at least one bank must have a positive e_i and since bank 1 has the highest asset value by assumption it must be that $e_1 > 0$. We have to consider three subcases:

(a) Assume that all $e_i > 0$. Then no CDS pays out and all $d_{ij} = 0$. Substituting the zero CDS payoffs into equation (33) yields $e_1 < 0$, which contradicts our assumption.

(b) Assume that $e_1 > 0, e_2 > 0$ and $e_3 < 0$. In this case $d_{12} = 0$ and $d_{13} = d_{23} = c(L - A_3)$. Substituting the CDS payoffs into equations (33) and (34) yields $e_1 < c(L - A_3)$ and $e_2 > c(L - A_3)$, respectively. This is inconsistent with the assumption that $e_1 > e_2$ and thus case (b) cannot occur.

(c) Assume that $e_1 > 0$ and $e_2 < 0, e_3 < 0$. In this case $d_{12} = c(L - A_2)$ and $d_{23} = c(L - A_3) - c(L - A_2)$. Substituting the CDS payoffs into equation (34) yields $e_2 + 2c(L - A_2) - c(L - A_3) > 0$ or $e_2(1 - 2c) + ce_3 > 0$. This is inconsistent with the assumption of $c < 1/2$ and case (c) which assumes that $e_2 < 0$ and $e_3 < 0$. Thus case (c) cannot occur.

Therefore, case (ii) cannot arise with CDS.

(iii) Bank 2 is in fundamental default and bank 3 is not in fundamental default which yields

$$e_2 + d_{12} - d_{23} < 0 \tag{35}$$

$$e_3 + d_{13} + d_{23} > 0 \tag{36}$$

To show that this case is not possible in the case of asset swaps subtract equation (36) from equation (35) to get $A_2 - A_3 - 2d_{23} + d_{12} - d_{13} < 0$. Substituting the swap payoffs we get $A_2 - A_3 - 2b(A_2 - A_3) + b(A_1 - A_2) - b(A_1 - A_3)$ which simplifies to $(A_2 - A_3)(1 - 3b) < 0$. The last inequality cannot hold based in the assumptions that $A_2 > A_3$ and $b < 1/3$. Therefore, case (iii) cannot arise with asset swaps.

To show that case (iii) cannot occur with CDS we again walk through all possible combinations of CDS payoffs. As in case (ii) we know that $e_1 > 0$ and we have to consider three subcases:

(a) Assume that all $e_i > 0$. Then no CDS pays out and all $d_{ij} = 0$. Substituting the zero CDS payoffs into equation (35) yields $e_2 < 0$ which is inconsistent with case (a).

(b) Assume that $e_1 > 0, e_2 > 0$ and $e_3 < 0$. In this case $d_{13} = d_{23} = c(L - A_3)$. Substituting the CDS payoffs into equation (36) yields $e_3 + 2c(L - A_3) > 0$ or $e_3(1 - 2c) > 0$. This is inconsistent with the assumption that $e_3 < 0$ and $c < 1/2$. Thus case (b) cannot occur.

(c) Assume that $e_1 > 0$ and $e_2 < 0, e_3 < 0$. In this case $d_{13} = c(L - A_3)$, and $d_{23} = c(L - A_3) - c(L - A_2)$. Substituting the CDS payoffs into equation (36) yields $e_3 + 2c(L - A_3) - c(L - A_2) > 0$ or $e_3(1 - 2c) + ce_2 > 0$. This is inconsistent with the assumption of $c < 1/2$ and case (c) which assumes that $e_2 < 0$ and $e_3 < 0$. Thus case (c) cannot occur.

Therefore, case (iii) cannot arise with CDS.

(iv) Bank 3 is in fundamental default, i.e. $e_3 + d_{13} - d_{23} < 0$. Banks 1 and 2 cannot pay more to bank 3 than the full face value of their obligations and thus there exists no feasible allocation that would allow bank 3 to survive.

This completes the proof. ■

Proof of Proposition 6

We know from Proposition 5 that there is no need to renegotiate the swap contract. Profits thus equal those under the bankruptcy mechanism and either all banks survive whenever $\sum_i e_i > 0$ or else all banks are liquidated. The liquidation pattern is identical to that with the interbank debt contract as stated in Proposition 4. Rewriting equation (24), banks maximize the expected profit in the non-liquidation states minus deposit insurance and effort costs.

$$\pi_i = \max_{h_i} \int_{\pi_i > 0} \pi_i df(A) - \omega_i - \gamma h_i^2, \tag{37}$$

where $f(A)$ denotes the density of the asset values. The first order condition with respect to the effort choice is

$$\frac{\partial \pi}{\partial h_i} = \frac{\partial}{\partial h_i} \int_{\pi_i > 0} \pi_i df(A) - \frac{\partial \omega_i}{\partial h_i} - 2\gamma h \tag{38}$$

We now show that for a fixed effort level, the partial derivative (38), is greater for the interbank debt contract when $a > 2L$ than for the perfectly hedged asset swap contract ($b = 1/3$).

For a fixed level of effort under both contracts banks banks default exactly in the same states. The deposit insurance system equally distributes the aggregate liquidation costs across banks, and since holding effort constant, the liquidation costs are identical for the two hedging strategies, the second term on the right hand side of equation (38) must be the same. For a fixed level of effort the third them on the right hand side is also identical under the two hedging contracts.

Now fix the asset values for all banks except bank i. Then there has to exist an $\underline{A}_i(h)$ such that the aggregate outside equity, $\sum_j e_j$, is positive for all $A_i > \underline{A}_i(h)$. We can then write the first term on the right hand side in equation (38) as

$$\frac{\partial}{\partial h_i} \int_{\underline{A}_i(h)}^{\infty} \pi_i(A_i) df(A_i) = -\pi_i(\underline{A}_i(h)) \frac{\partial \underline{A}_i(h)}{\partial h_i} + \int_{\underline{A}_i(h)}^{\infty} \frac{\partial \pi_i(A_i)}{\partial h_i} df(A_i). \tag{39}$$

By definition the aggregate equity in the banking system at the point $\underline{A}_i$ is zero, so must each bank's profit. The first term on the right hand side of equation (39) is therefore zero. The only term that differs in the first order condition across contracts is thus $\frac{\partial \pi_i(A_i)}{\partial h_i}$.

With the $b = 1/3$ asset swap contract, no renegotiations are necessary and two thirds of an incremental increase in bank i's assets are passed on to other banks and thus $\partial \pi_i / \partial h_i = 1/3 \cdot \partial A_i / \partial h_i$. With an interbank debt contract two outcomes are possible: if no renegotiations are necessary (because for all banks $i : A_i > L$), then the bank can keep any incremental value in its assets for itself and $\partial \pi_i / \partial h_i = \partial A_i / \partial h_i$. If renegotiations are necessary the bank becomes the proposer with probability 1/3 in which case it can extract all the surplus and thus can keep any increase in the asset value for itself. Several outcomes are possible when other banks propose depending on the distribution of asset values but in the worst case the bank has to pass on all its asset value to another bank and increases in effort have zero value for the bank. With an interbank debt contract we have therefore $\partial \pi_i / \partial h_i \geq 1/3 \cdot \partial A_i / \partial h_i$ Because asset values A increase in effort, the difference in the first order condition (38) between the interbank loan contract is thus always non-negative:

$$\frac{\partial \pi}{\partial h_i} \mid_{a \geq 2L} - \frac{\partial \pi}{\partial h_i} \mid_{b=1/3} \quad \geq \quad 0. \tag{40}$$

The bank will thus choose a higher effort level under the debt contract than under the swap contract and thus make a higher profit. ■

Proof of Proposition 7

The promised payment on interbank debt obligations is fixed and hence does not depend on the source of variation (hedgeable or unhedgeable) in asset values. By Proposition 4, bank liquidations are perfectly correlated and occur whenever $\sum_i e_i < 0$, so that the liquidation probability is independent of the bankruptcy regime. For a given realization of asset values, renegotiations will cause banks to split the aggregate surplus $\sum_i e_i$ in a way that might depend on the bankruptcy regime. But since ex-ante all banks are identical, each bank's ex-ante profit will not change with the bankruptcy regime. ■

Table 1: Optimal OTCDs and Asset Quality

	Optimal Contract a	b	c	Effort	Profit (x 100)	Inefficient Liquidations Ex-post (%)	Ex-ante (%)	Probability Liq.(%)	Def.(%)	Deposit Ins.(%)
Panel A: 70 Percent Hedgeable Risk in Strong Bankruptcy Regime (Base Case)										
With Renegotiations										
Optimal contract	0.25	0.00	0.20	0.082	18.324	0.00	0.17	1.25	16.43	0.52
Interbank loans	0.25	0.00	0.00	0.082	18.322	0.00	2.44	1.26	16.85	0.52
Asset Swaps	0.00	0.25	0.00	0.045	16.886	0.00	80.32	4.15	5.25	1.69
CDS	0.00	0.00	0.90	0.083	15.952	0.00	93.50	7.54	8.18	3.18
Without Renegotiation										
Optimal contract	0.00	0.20	0.00	0.053	16.570	1.48	86.56	5.33	5.33	2.16
Interbank loans	0.00	0.00	0.00	0.088	15.545	0.00	94.93	8.68	8.68	3.77
Asset Swaps	0.00	0.20	0.00	0.053	16.570	1.48	86.56	5.33	5.33	2.16
CDS	0.00	0.00	0.90	0.083	15.698	1.21	93.88	8.18	8.18	3.41
No Hedging	0.00	0.00	0.00	0.088	15.545	0.00	94.93	8.68	8.68	3.77
Panel B: 100 Percent Unhedgeable Risk in Strong Bankruptcy Regime										
With Renegotiations										
Optimal contract	0.25	0.00	0.00	0.082	18.528	0.00	5.74	0.74	16.50	0.30
Without Renegotiation										
Optimal contract	0.00	0.00	0.00	0.088	15.550	0.00	97.14	8.67	8.67	3.77
No Hedging	0.00	0.00	0.00	0.088	15.550	0.00	97.14	8.67	8.67	3.77
Panel C: 100 Percent Hedgeable Risk in Strong Bankruptcy Regime										
With Renegotiations										
Optimal contract	0.25	0.00	0.50	0.084	18.233	0.00	0.42	1.49	5.16	0.62
Interbank loans	0.25	0.00	0.00	0.082	18.221	0.00	1.96	1.52	17.28	0.63
Asset Swaps	0.00	0.25	0.00	0.044	17.494	0.00	49.74	2.48	2.48	1.00
CDS	0.00	0.00	0.80	0.081	17.626	0.00	61.11	3.07	4.91	1.24
Without Renegotiation										
Optimal contract	0.00	0.15	0.20	0.060	17.642	0.05	54.01	2.67	2.67	1.07
Interbank loans	0.00	0.00	0.00	0.088	15.538	0.00	93.82	8.70	8.70	3.78
Asset Swaps	0.00	0.25	0.00	0.044	17.494	0.00	49.74	2.48	2.48	1.00
CDS	0.00	0.00	0.50	0.084	17.308	0.01	74.74	3.93	3.93	1.60
No Hedging	0.00	0.00	0.00	0.088	15.538	0.00	93.82	8.70	8.70	3.78
Panel D: 70 Percent Hedgeable Risk in Weak Bankruptcy Regime										
With Renegotiations										
Optimal contract	0.20	0.00	0.40	0.079	18.282	0.00	7.83	1.35	23.57	0.56
Interbank loans	0.20	0.00	0.00	0.080	18.268	0.00	20.25	1.41	23.78	0.58
Asset Swaps	0.00	0.25	0.00	0.045	16.886	0.00	80.32	4.15	5.25	1.69
CDS	0.00	0.00	0.80	0.083	15.893	0.00	93.69	7.71	8.19	3.24
Panel E: Base Case with Reserve Requirement										
With Renegotiations										
Optimal contract	0.20	0.00	0.20	0.080	18.543	0.00	1.84	0.67	11.37	0.28
Interbank loans	0.20	0.00	0.00	0.080	18.542	0.00	9.30	0.68	11.86	0.28
Asset Swaps	0.00	0.20	0.00	0.048	17.326	0.00	87.15	3.10	3.68	1.25
CDS	0.00	0.00	0.90	0.078	16.784	0.00	95.04	5.25	6.02	2.19
Without Renegotiation										
Optimal contract	0.00	0.15	0.20	0.056	17.131	1.23	90.44	3.87	3.87	1.55
Interbank loans	0.00	0.00	0.00	0.083	16.248	0.00	96.55	6.74	6.74	2.90
Asset Swaps	0.00	0.15	0.00	0.057	17.098	0.82	91.43	4.04	4.04	1.64
CDS	0.00	0.00	0.70	0.079	16.487	1.03	95.68	6.04	6.04	2.50
No Hedging	0.00	0.00	0.00	0.083	16.248	0.00	96.55	6.74	6.74	2.90

Results are obtained obtained from Monte-Carlo simulation of banks' asset value processes as in Assumption 1 and the renegotiation process in Section 2. Ex-post and ex-ante liquidations are in definitions 2 and 3, respectively. Probabilities of inefficient liquidations are conditional on the bank being liquidated. Banks not making full payments on its interbank commitments are said to be in default. Banks are liquidated if they are insolvent after renegotiations. Parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Table 2: Optimal Contract and Effort Choice in Base Case For Alternative Pricing Kernels

	Optimal Contract a	b	c	Effort	Profit (x 100)	Inefficient Liquidations Ex-post (%)	Ex-ante (%)	Probability Liq.(%)	Def.(%)	Deposit Ins.(%)
Panel A: Pricing Kernel=(1,4,9)										
With Renegotiations										
Optimal contract	0.15	0.00	0.00	0.089	14.462	0.00	50.85	1.62	12.45	4.42
Interbank loans	0.15	0.00	0.00	0.089	14.462	0.00	50.85	1.62	12.45	4.42
Asset Swaps	0.00	0.15	0.00	0.064	13.786	0.00	88.49	5.32	5.68	5.13
CDS	0.00	0.00	0.00	0.090	12.961	0.00	94.99	8.59	8.59	6.35
Without Renegotiation										
Optimal contract	0.00	0.10	0.00	0.073	13.534	0.49	91.59	6.42	6.42	5.52
Interbank loans	0.00	0.00	0.00	0.090	12.961	0.00	94.99	8.59	8.59	6.35
Asset Swaps	0.00	0.10	0.00	0.073	13.534	0.49	91.59	6.42	6.42	5.52
CDS	0.00	0.00	0.00	0.090	12.961	0.00	94.99	8.59	8.59	6.35
No Hedging	0.00	0.00	0.00	0.090	12.961	0.00	94.99	8.59	8.59	6.35
Panel B: Pricing Kernel=(1,4,25)										
With Renegotiations										
Optimal contract	0.00	0.05	0.00	0.083	12.228	0.00	93.51	7.27	7.35	6.96
Interbank loans	0.00	0.00	0.00	0.091	12.125	0.00	95.06	8.54	8.54	7.18
Asset Swaps	0.00	0.05	0.00	0.083	12.228	0.00	93.51	7.27	7.35	6.96
CDS	0.00	0.00	0.00	0.091	12.125	0.00	95.06	8.54	8.54	7.18
Without Renegotiation										
Optimal contract	0.00	0.05	0.00	0.083	12.177	0.21	93.58	7.35	7.35	7.01
Interbank loans	0.00	0.00	0.00	0.091	12.125	0.00	95.06	8.54	8.54	7.18
Asset Swaps	0.00	0.05	0.00	0.083	12.177	0.21	93.58	7.35	7.35	7.01
CDS	0.00	0.00	0.00	0.091	12.125	0.00	95.06	8.54	8.54	7.18
No Hedging	0.00	0.00	0.00	0.091	12.125	0.00	95.06	8.54	8.54	7.18

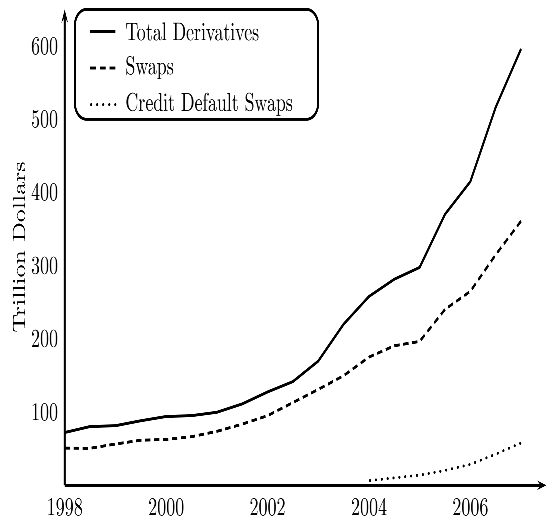
Results are obtained obtained from Monte-Carlo simulation of banks' asset value processes as in Assumption 1 and the renegotiation process in Section 2. Ex-post and ex-ante liquidations are in definitions 2 and 3, respectively. Probabilities of inefficient liquidations are conditional on the bank being liquidated. Banks not making full payments on its interbank commitments are said to be in default. Banks are liquidated if they are insolvent after renegotiations. Parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Table 3: Optimal OTCDs and Asset Quality Under a Regulatory Constraint $a \leq 0.05$ with Renegotiations.

	Optimal Contract			Effort	Profit	Inefficient Liquidations		Probability		Deposit
	a	b	c		(x 100)	Ex-post (%)	Ex-ante (%)	Liq.(%)	Def.(%)	Ins.(%)
Base case	0.05	0.10	0.90	0.065	17.423	0.00	79.10	3.46	7.10	1.40
Only Unhedgeable Risk	0.05	0.00	0.00	0.086	17.111	0.00	94.24	4.69	9.40	1.93
Only Hedgeable Risk	0.05	0.07	0.80	0.068	17.123	4.74	73.37	4.08	4.08	1.59
Weak Regime	0.05	0.10	0.80	0.066	17.407	0.00	79.75	3.52	7.06	1.42
Pricing Kernel=(1,4,9)	0.05	0.00	0.00	0.090	13.944	0.00	90.01	4.78	9.46	5.10
Pricing Kernel=(1,4,25)	0.00	0.05	0.00	0.083	12.228	0.00	93.51	7.27	7.35	6.96

Results are obtained obtained from Monte-Carlo simulation of banks' asset value processes as in Assumption 1 and the renegotiation process in Section 2. Ex-post and ex-ante liquidations are in definitions 2 and 3, respectively. Probabilities of inefficient liquidations are conditional on the bank being liquidated. Banks not making full payments on its interbank commitments are said to be in default. Banks are liquidated if they are insolvent after renegotiations. Parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$. The base case, only unhedgeable risk, only hedgeable risk, and the weak regime correspond to Panels A,B,C, and D, respectively in Table 1. The cases of the two pricing kernels correspond to Panels A and B in Table 2

Figure 1: Ratio of Outstanding OTC Derivatives to Asset Values at Banks



The figure shows the notional amounts outstanding of over-the-counter derivatives (OTCDs) held at banks and dealers in the G10 countries Source: BIS Quarterly Review 2008.

Figure 2: Structure of Banking System with Interbank OTCD Hedges Prior to Netting

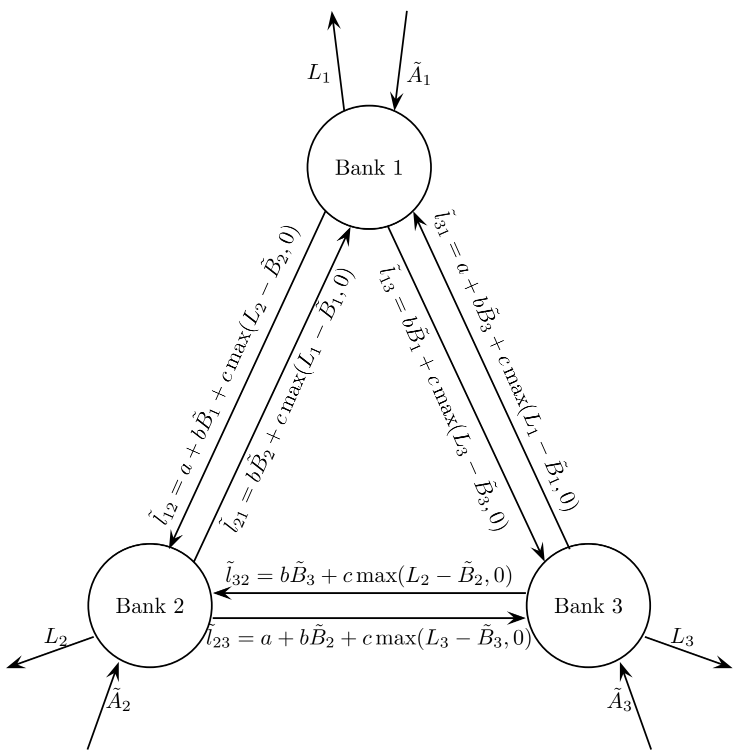
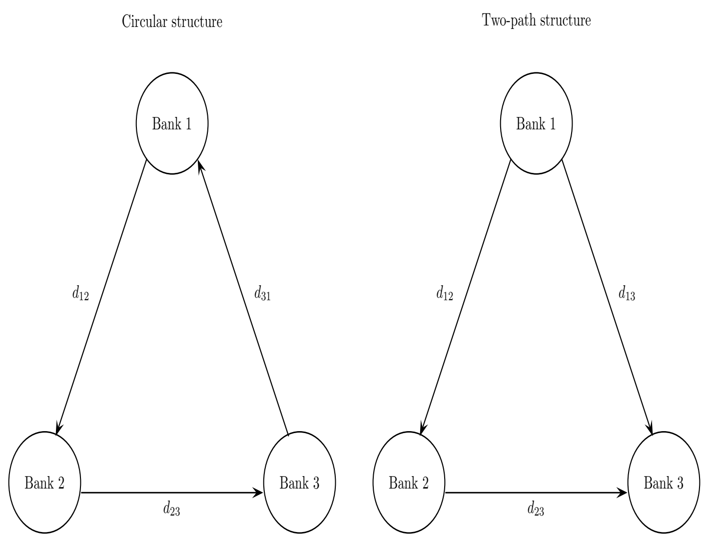
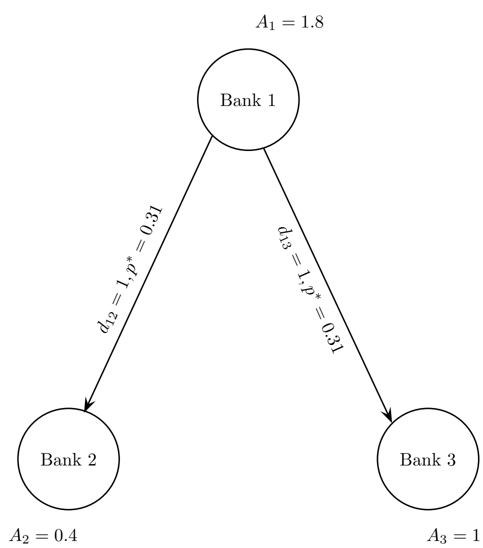


Figure 3: Possible Network Structures



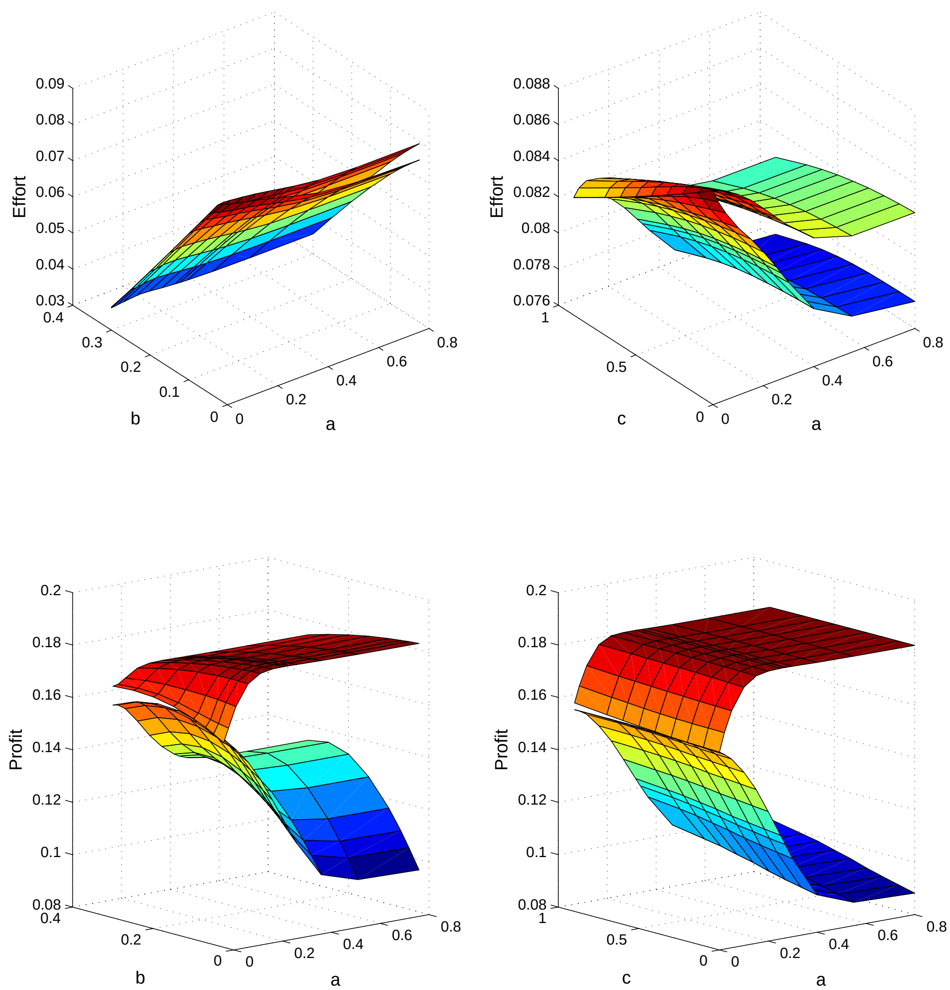
After netting, only two network structures are possible. One in which payments flow in a circle, the other one has two paths of payments flowing from bank 1 to bank 3.

Figure 4: Example of Inefficient Renegotiation



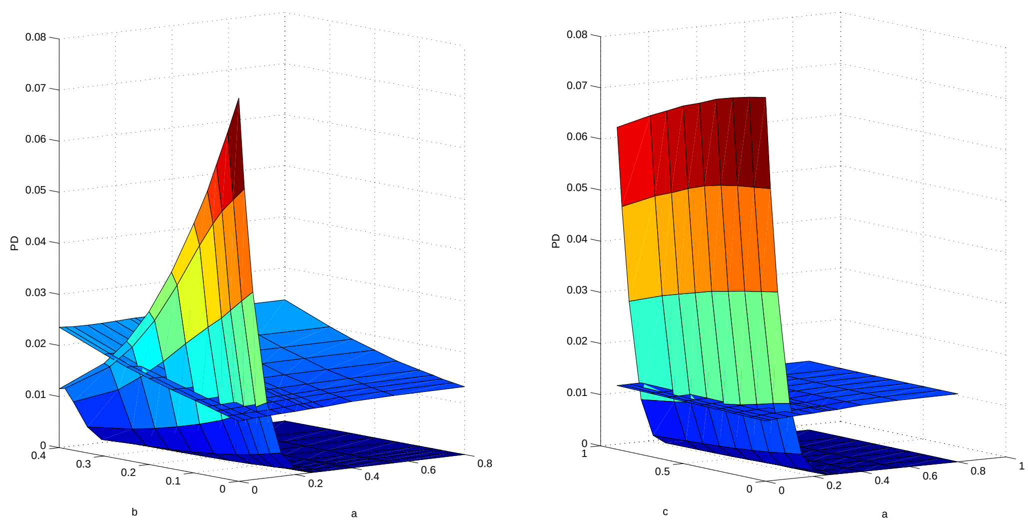
Network structure and asset values for example 3.

Figure 5: Bank Profits and Effort Choices for Alternative OTCD Contracts in the Base Case



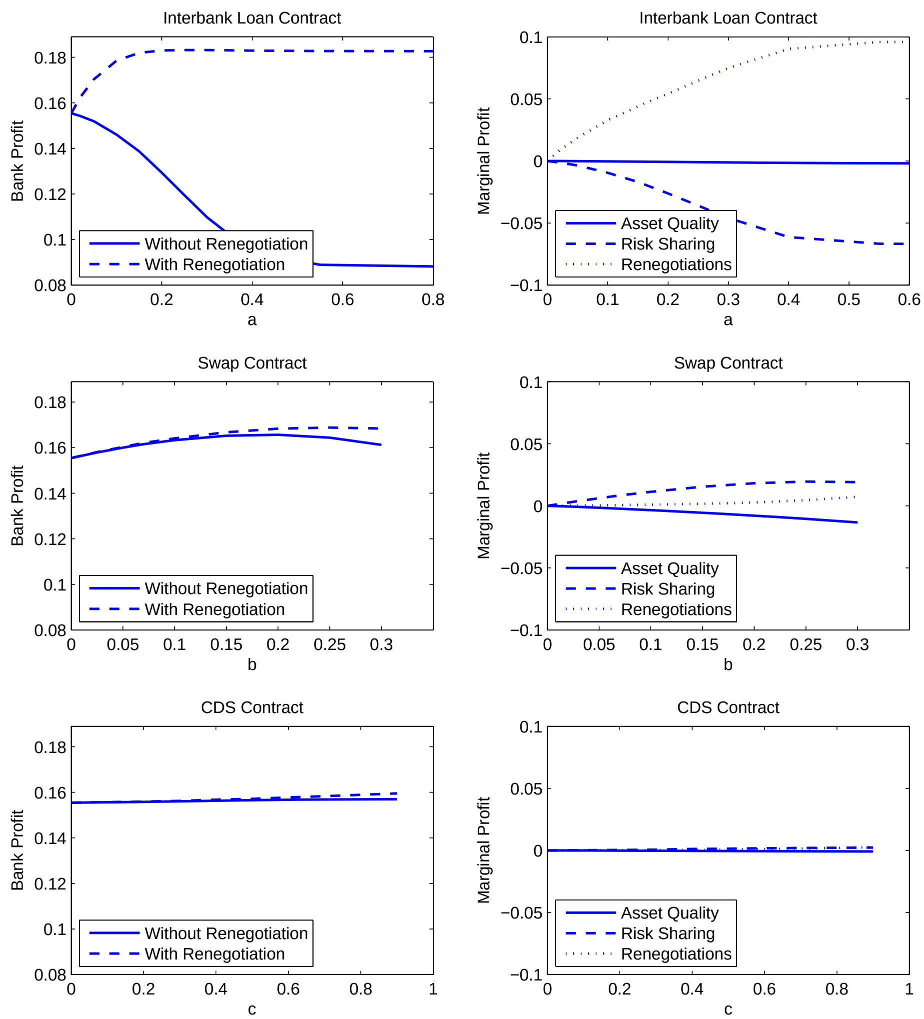
We display the optimal effort (top panels) and bank profit (bottom panels) of the individual bank for different exposures of straight debt a, asset swaps b, and CDS c. The upper and lower surfaces are for the cases with and without renegotiations, respectively. Results are for the "Base Case" in Section 3.2 in which the parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Figure 6: Decomposition of Banks' Liquidation Probability (No Unhedgeable Risk)



We decompose the liquidation probability of a bank for different levels of interbank debt (a), asset swaps (b), and CDS (c) contracts. The relatively flat surface represents liquidations when $\sum_i e_i < 0$, while the spiked surface represents liquidations where the $\sum_i e_i > 0$ but payments required for renegotiations cannot be spanned by the outstanding OTCDs. Results are for the "Base Case" but with no unhedgeable risk ($\zeta = 1$) in Section 3.2 in which the parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Figure 7: Overall profit as well as Marginal Profits from Asset Quality, Diversification, and Renegotiation for OTCD Contracts - Base Case

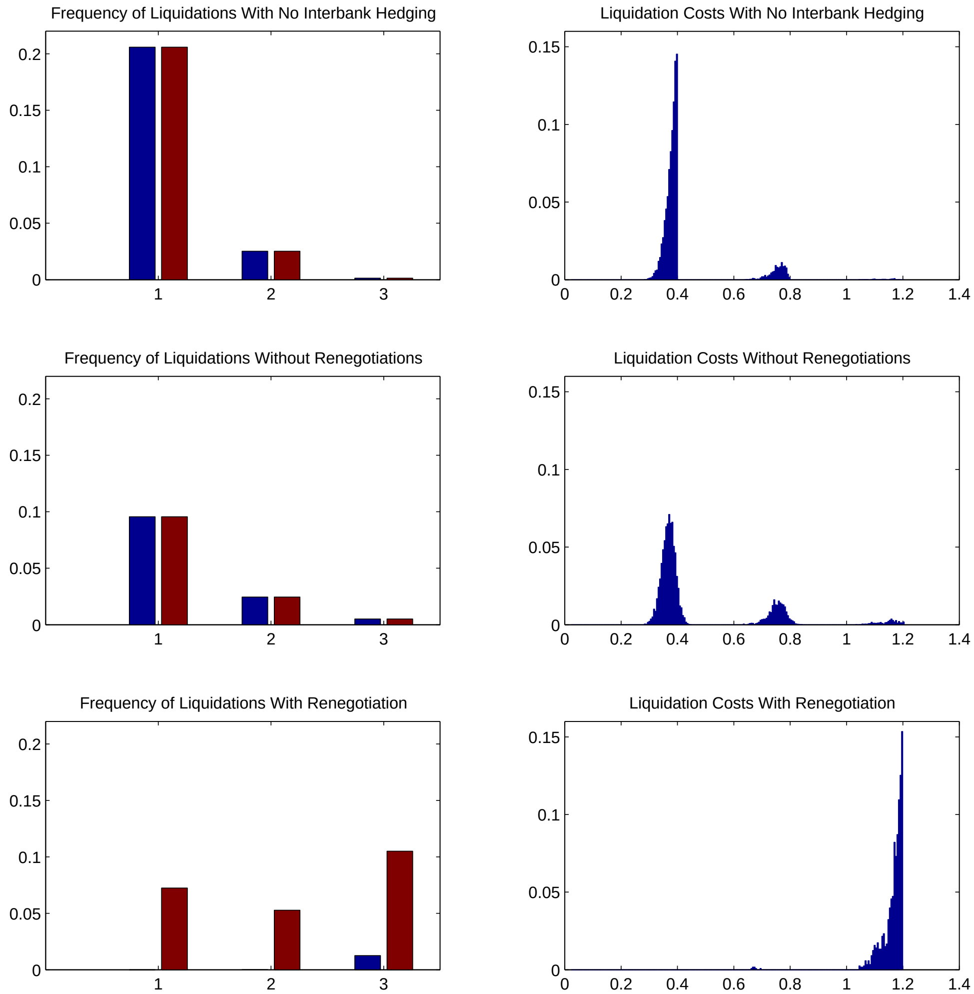


We display the overall profit (left panels) as well as the marginal profits from three different sources for each type of contract (right panels). Overall profit is computed assuming that banks only enter interbank loans, asset swaps, and CDS, respectively. Marginals are computed as follows:

$$\begin{aligned}\text{Marginal renegotiation recovery impact} &= \pi^R(f, h(f)) - \pi^{BM}(f, h(f)) \\ \text{Marginal risk sharing impact} &= \pi^{BM}(f, h(f)) - \pi^{BM}(0, h(f)) \\ \text{Marginal asset quality impact} &= \pi^{BM}(0, h(f)) - \pi^{BM}(0, h(0)),\end{aligned}$$

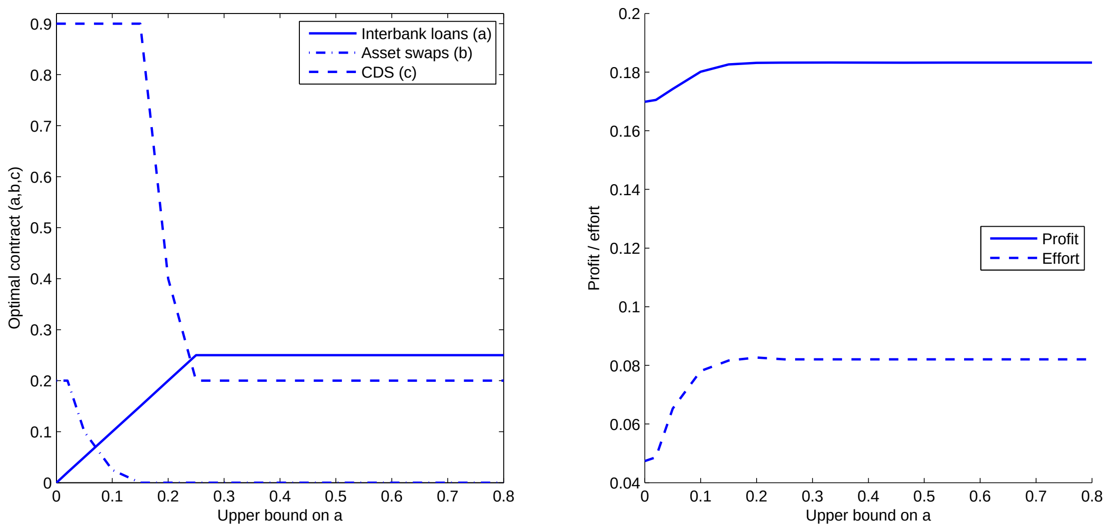
where π^R and π^{BM} denote the profits with payoffs from interbank contracts settled by renegotiation and the bankruptcy mechanism respectively. f denotes the optimal choice of the hedging contract, and $h(f)$ is the optimal effort the bank uses to maintain asset quality given this contract choice. Results are for the "Base Case" in Section 3.2 in which the parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Figure 8: Credit and Systemic Risk Distributions for the Base Case



Using Monte-Carlo simulations, we study the distributions of credit and systemic risk for cases: no interbank hedging (**top panels**); with interbank hedging but no renegotiations (**middle panels**); and with interbank hedging and renegotiations (**bottom panels**). In the **left panels**, we plot two bars at each integer 1,2,3, the frequency of liquidations (left bar) or defaults (right bar) that will be observed in a three bank system. The **right panels** shows histograms of dead weight losses given at conditional on at least one default for each case. Results are for the "Base Case" in Section 3.2 in which the parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1$.

Figure 9: Optimal Contract Choice In the Base Case When Interbank Loans are Constrained



In the left panel, we plot the optimal choice of interbank loans, assets swaps, and CDS for given limits on the interbank debt contracts. The right panel shows optimal effort (dashed line, right axis) and bank profit (solid line, left axis) given these optimally chosen contracts. Results are for the "Base Case" in Section 3.2 in which the parameter values are: $\mu_0 = 0.1579, \mu_1 = 0.3, \gamma = 2, \rho = 0.1, \sigma = 0.13, \phi = 0.1, \zeta = 0.7, M_n = 1$ for all $n, \Phi = 0.1.$

References

Allen, F., and A. Babus, 2009, “Networks in Finance,” in Paul R. Kleindorfer, Yoram Wind, and Robert E. Gunther (ed.), *The network challenge: strategy, profit, and risk in an interlinked world*, Wharton School Publishing.

Allen, F., and D. Gale, 2000, “Financial Contagion,” *Journal of Political Economy*, 108, 1–34.

Angelini, P., G. Maresca, and D. Russo, 1996, “Systemic Risk in the Netting System,” *Journal of Banking and Finance*, 20, 853–886.

Berger, A. N., R. DeYoung, M. J. Flannery, D. Lee, and Özde Öztekin, 2008, “How Do Large Banking Organizations Manage Their Capital Ratios?,” Working Paper.

Billio, M., M. Getmansky, A. Lo, and L. Pelizon, 2010, “Measuring Systemic Risk in the Finance and Insurance Sectors,” Working Paper, Massachussetts Institute of Technology.

Bolton, P., and D. Scharfstein, 1996, “Optimal Debt Structure and the Number of Creditors,” *Journal of Political Economy*, 104, 1–25.

Brusco, S., and F. Castiglionesi, 2007, “Liquidity Coinsurance, Moral Hazard and Financial Contagion,” *Journal of Finance*, 62, 2275–2302.

Cont, R., A. Moussa, and A. Minca, 2010, “Too Interconnected to Fail: Contagion and Systemic Risk in Finanical Networks,” Working Paper, Columbia University.

David, A., 2001, “Pricing the Strategic Value of Putable Securities in Liquidity Crises,” *Journal of Financial Economics*, pp. 63–99.

Degryse, H., and G. Nguyen, 2004, “Interbank Exposures: An Empirical Examination of Systemic Risk in the Belgian Banking System,” National Bank of Belgium, Working Paper No.43.

Diamond, D. W., and P. Dybvig, 1983, “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, pp. 401–419.

Dubey, P., J. Geanakopolos, and M. Shubik, 2005, “Default and Punishment in General Equilibrium,” *Econometrica*, 73, 1–37.

Duffie, D., N. Garleanu, and L. H. Pederson, 2005, “Over-the-Counter Markets,” *Econometrica*, 73, 1815.

Duffie, D., N. Garleanu, and L. H. Pederson, 2007, “Valuation in Over-the-Counter Markets,” *Review of Financial Studies*, 20, 1865–1900.

Eisenberg, L., and T. Noe, 2001, “Systemic Risk in Financial Systems,” *Management Science*, 47, 236–249.

Elsinger, H., A. Lehar, and M. Summer, 2006, “Risk Assessment for Banking Systems,” *Management Science*, 52, 1301–1314.

Furfine, C., 2003, “Interbank Exposures: Quantifying the Risk of Contagion,” *Journal of Money, Credit, and Banking*, 35, 111–128.

Garleanu, N., and J. Zwiebel, 2009, “Design and Renegotiation of Debt Covenants,” *Review of Financial Studies*, 22, 749–791.

Gauthier, C., A. Lehar, and M. Souissi, 2010, “Macroprudential Capital Requirements and Systemic Risk,” Working Paper Bank of Canada and Haskayne School of Business, University of Calgary.

Hackbarth, D., C. A. Hennessy, and H. Leland, 2007, “Can the Tradeoff Theory Explain Debt Structure?,” *Review of Financial Studies (Forthcoming)*, 20.

Hart, O., and J. Moore, 1998, "Default and Renegotiation: A Dynamic Model of Debt," *Quarterly Journal of Economics*, 109, 1–42.

Humphrey, D. B., 1986, "Payments Finality and Risk of Settlement Failure," in Anthony S. Saunders, and Lawrence J. White (ed.), *Technology and Regulation of Financial Markets: Securities, Futures and Banking* . pp. 97–120, Lexington Books, MA.

Leitner, Y., 2005, "Financial Networks: Contagion, Commitment, and Private Sector Bailouts," *Journal of Finance*, 60, 2925–2953.

Macho-Stadler, I., D. Perez-Castrillo, and D. Wettstein, 2007, "Sharing the Surplus: An Extension of the Shapley Value for Enviroments with Externalities," *Journal of Economic Theory*, 135, 339–356.

Maskin, E., 2003, "Bargaining, Coalitions, and Externalities," Presidential Address to Econometric Society.

Rajan, R. G., and L. Zingales, 1998, "Power in the Theory of the Firm," *The Quarterly Journal of Economics*, 113, 387–432.

Ray, D., and R. Vohra, 2001, "Coalitional Power and Public Goods," *Journal of Political Economy*, 109, 1355–1384.

Roberts, M. R., and A. Sufi, 2009, "Renegotiation of Financial Contracts: Evidence from Private Credit Agreements," *Journal of Financial Economics*, 93, 159–184.

Sheldon, G., and M. Maurer, 1998, "Interbank Lending and Systemic Risk," *Swiss Journal of Economics and Statistics*, 134, 685–704.

Singh, M., and J. Aitken, 2009, "Counterparty Risk, Impact on Collateral Flows and Role for Central Counterparties," IMF Working Paper 09/173, International Monetary Fund.

Stulz, R., 2009, "Credit Default Swaps and the Credit Crisis," Working Paper Series 15384, NBER.

Thompson, J. R., 2010, "Counterparty Risk in Financial Contracts: Should the Insured Worry About the Insurer?," *Quarterly Journal of Economics*, 125, 1195–1252.

Tirole, J., 1988, *The Theory of Industrial Organization*, The MIT Press, Cambridge, Massachusetts.

Upper, C., and A. Worms, 2004, "Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion?," *European Economic Review*, 48, 827–849.

Wells, S., 2002, "UK interbank exposures: Systemic Risk Implications," Bank of England Financial Stability Review.