Agency Conflicts, Prudential Regulation, and Marking to Market*

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Abstract

We develop a model of a financial institution to show how prudential capital regulation and mark-to-market accounting interact to affect the institution's project choices in the presence of shareholder–debt holder agency conflicts. We demonstrate that, relative to a benchmark "pure historical cost" regime in which assets and liabilities on an institution's balance sheet are measured at their origination values, mark-to-market or fair value accounting could alleviate the inefficiencies arising from asset substitution, but exacerbate those arising from underinvestment due to debt overhang. The inefficiencies due to underinvestment and asset substitution work in opposing directions. An increase in the propensity for asset substitution mitigates underinvestment, and this tradeoff is especially pronounced for highly levered financial institutions. The optimal choices of the accounting measurement regime and prudential solvency constraint balance the conflicts between shareholders and debt holders. Under fair value accounting, the optimal solvency constraint declines with the institution's marginal cost of investment in project quality and the excess cost of equity capital relative to debt capital. The optimal solvency constraint is institution-specific, which suggests that a uniform solvency constraint could be sub-optimal. In fact, if the solvency constraint in the fair value regime is sub-optimally chosen to be tighter than a threshold, historical cost accounting dominates fair value accounting.

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1 Introduction

The role that the prudential regulation of financial institutions based on fair value or "mark–to–market" accounting may have played in the recent financial crisis is the subject of an ongoing debate (e.g., see Laux and Leuz (2009)).[1] Proponents of fair value accounting argue that a balance sheet based on market prices leads to better insights into the current risk profiles of financial institutions. Regulators can intervene in a more timely and effective manner, and tools such as capital requirements can be used to prevent the inefficient choices or continuation of bad projects. Opponents counter that market prices can only provide useful signals to outsiders if the assets and liabilities of institutions trade in frictionless competitive markets, which do not exist for several important claims. Further, prudential regulation based on market values could increase the risks faced by institutions, and induce myopic behavior by preventing the selection of efficient, long-term projects. To the best of our knowledge, however, a trade-off that is central to this debate—regulation based on fair value accounting could mitigate inefficient choices of bad projects, but *simultaneously* hamper the choices of good ones—has not been theoretically formalized.

We develop a theory of how agency conflicts between the shareholders and debt holders of a financial institution, accounting measurement rules, and prudential capital regulation interact to affect the institution's project choices. We show that, relative to the benchmark historical cost regime in which all claims are measured at their origination values, prudential regulation based on market values could mitigate the inefficiency arising from *asset substitution* or risk-shifting (the choice of risky, negative NPV projects), but exacerbate underinvestment due to *debt overhang* (the avoidance of risky, positive NPV projects). The conflicting effects of fair value accounting hold even in the scenario in which the institution's claims are traded in frictionless, competitive markets. Put differently, *even if* prices fully reflect fundamentals, we show that the fair value measurement regime may still be dominated by the historical cost regime. The inefficiencies due to underinvestment and asset substitution work in opposing directions in that an increase in the propensity for asset substitution alleviates underinvestment. The optimal choices of the accounting measurement regime and prudential capital regulation balance the effects of underinvestment and

[1] Throughout the paper, we will use the phrases *mark-to-market accounting* or *fair value accounting* synonymously. While mark-to-market accounting is the use of observable market prices to measure the value of an asset, fair value accounting is a broader term than mark-to-market accounting in the sense that it may use both observable and/or unobservable inputs to measure the value of a claim.

asset substitution. Under fair value accounting, we show that the optimal (value-maximizing) solvency constraint declines with the institution's marginal cost of investment in project quality, and with the excess cost of equity relative to debt financing. Importantly, our results suggest that a uniform solvency constraint across institutions could be sub-optimal. Indeed, we show that, if the solvency constraint in the fair value regime is sub-optimally tighter than a threshold, historical cost accounting actually dominates fair value accounting.

Our theory focuses on financial institutions such as insurance firms and commercial banks that are subject to prudential regulation. Financial institutions differ from non-financial institutions in two important aspects. First, financial institutions are much more highly leveraged than non-financial firms. Second, in contrast with industrial firms, a relatively large proportion of the debt of a financial institution is held by uninformed and widely dispersed debt holders. By the "representation hypothesis" proposed by Dewatripont and Tirole (1994), uninformed debt holders of the institution are represented by a regulator who protects their interests by imposing a solvency constraint to ensure that the institution's leverage ratio is below a threshold.

We capture the aforementioned distinguishing features of financial institutions in a two-period model in which a representative financial institution finances a long-term project through a combination of debt and equity. We follow studies such as Heaton *et al.* (2010) by assuming that there are deadweight costs of equity financing that are represented by equity holders demanding an additional (risk-adjusted) expected return or premium relative to debt holders. The excess cost of equity creates an incentive for the institution to choose a high leverage.[2] The manager behaves in the interests of shareholders. The term "project" could also refer to a "pool" of projects.

The project's cash flows are realized at the end of period 2. The cash flows depend stochastically on the project's *quality* that is only observable by shareholders. The cost incurred by shareholders in choosing the project's quality is a nonnegative random variable. The mean cost increases with the project's quality. At the end of period 1, there is a publicly observable signal about the cash flows of the project. The signal could take either a low or high value indicating poor or good interim performance, respectively, of the project. The probability of receiving the high signal increases

[2] Previous literature proposes a number of reasons for the high leverage levels of financial institutions (e.g. see Allen and Gale (1999) and Santos (2001)). For simplicity, we abstract away from the particular mechanism that leads to a high leverage level, and assume that the various mechanisms manifest in a higher effective cost of equity capital relative to debt capital for the institution.

with the project's quality. Given the signal, if the institution meets the solvency constraint, its shareholders may act opportunistically by engaging in inefficient asset substitution or risk-shifting in period 2. If the institution violates the solvency constraint, control transfers to the regulator who closely monitors its operations and ensures that the ex post efficient continuation strategy—no asset substitution—is chosen in period 2.[3] Consequently, the project's terminal cash flows are affected by its quality that is chosen in the first period, transfer of control to the regulator if the solvency constraint is violated, and potential asset substitution by shareholders in the second period if the solvency constraint is not violated. The institution's capital structure reflects the trade-off between the excess cost of equity relative to debt financing and the agency costs of debt.

We analyze two accounting measurement regimes: a fair value (FV) regime in which the balance sheet of the institution—and, therefore, the solvency constraint—is marked to market every period, and a benchmark "pure" historical cost (HC) regime in which all claims are measured at their origination values. Given a solvency constraint, we first examine the institution's optimal choices of its capital structure, its project quality, and its asset substitution strategy in each regime. We then derive the optimal (value-maximizing) choice of the prudential constraint by the regulator. Finally, we compare the two measurement regimes.

The upshot of our analysis is that, regardless of the accounting measurement regime, there are two key inefficiencies that arise from agency conflicts between shareholders and debt holders. First, because shareholders effectively hold a call option on the terminal payoffs, the higher the debt level, the greater are shareholders' incentives to increase risk by engaging in asset substitution in the second period. Second, the higher the debt level, the lower are shareholders' incentives to make a costly investment to increase project quality in the first period. In other words, the *debt overhang* problem, where a larger proportion of the increased total payoffs from the higher quality project accrues to debt holders, could cause shareholders to *under-invest* in project quality in the first period. More interestingly, our analysis uncovers a subtle, but important tradeoff between asset substitution and underinvestment; an increase in the propensity for asset substitution in the second period *alleviates* underinvestment.

The solvency constraint plays an important role in mediating the inefficiencies arising from

[3] Our analysis does not change if we instead assume that the regulator liquidates the institution's assets where the liquidation payoff equals the institution's value under no asset substitution. Our results, therefore, hold even if the institution's claims are traded in frictionless, competitive markets so that prices fully reflect fundamental values.

asset substitution and debt overhang, but its effectiveness depends on the prevailing accounting measurement regime. Because the balance sheet is not re-measured in the HC regime, the institution automatically meets the solvency constraint at date 1 if it meets it at date 0. Because there is no possibility of a transfer of control at date 1, the solvency constraint has little bite so that the HC regime is plagued with a high incidence of asset substitution. The high incidence of asset substitution, however, *alleviates* underinvestment in the first period. If asset substitution were hypothetically ruled out, the likelihood of choosing high project quality *decreases*. Further, the positive effect of asset substitution on project quality is especially pronounced at relatively high leverage levels that are typical of financial institutions. Stated differently, at high leverage levels, not only do the debt overhang and asset substitution inefficiencies become severe, but, more interestingly, these two inefficiencies move in opposing directions.

The intuition for the trade-off between the asset substitution and underinvestment problems is as follows. At low leverage levels, asset substitution occurs (if at all) only in the low state, i.e., when the value of the signal is low. At low leverage levels, however, the debt overhang problem is also insignificant in that the institution chooses high project quality in the first period. Consequently, at low leverage levels, eliminating the possibility of asset substitution has little impact on the ex ante project quality choice. At higher leverage levels, however, asset substitution is pervasive in that it occurs in both the low and the high intermediate states. The reason is that, as leverage increases, the call option in the low state becomes more out of the money relative to the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Given that the high state is more likely for the high quality project, this, in turn, *increases* the bank's incentives to choose the higher project quality. Consequently, shutting down the possibility of asset substitution eliminates the *differential* rents from asset substitution in the good state relative to the bad state and, therefore, decreases shareholders' ex ante incentives to invest in the higher quality project, that is, the underinvestment problem worsens. Conversely, the high propensity for asset substitution in the HC regime alleviates ex ante underinvestment in project quality.

Consistent with the intuition expressed by proponents of fair value accounting that market prices play a disciplining role, we show that the FV regime does indeed alleviate the asset substitution inefficiency pervasive in the HC regime. Because claims are marked to market in the FV regime, the solvency constraint has bite at the intermediate date 1 so that transfer of control to the regulator occurs if it is violated. Further, such transfer of control occurs when the institution's leverage is above a threshold. However, as discussed above, the incentives for asset substitution are particularly pronounced at high leverage levels, and this is precisely when shutting down the possibility of asset substitution through the transfer of control to the regulator has the biggest negative impact on ex ante investment in project quality. In mitigating asset substitution, therefore, FV accounting exacerbates the debt overhang problem by inducing the shareholders to under-invest in project quality relative to the HC regime.

From a normative perspective, the regulator faces a dilemma in choosing the optimal solvency constraint in the FV regime. A loose solvency constraint aggravates the asset substitution problem because the constraint has less bite. A tight solvency constraint, however, aggravates underinvestment by increasing the likelihood of the transfer of control, thereby curbing potential rents from *ex post* asset substitution. In choosing the solvency constraint, the regulator minimizes the expected inefficiencies arising from asset substitution and underinvestment.

We show that the optimal solvency constraint does not eliminate either inefficiency, that is, both underinvestment and asset substitution are possible at the optimum. Further, the optimal solvency constraint becomes tighter when the marginal cost of investment in project quality or the excess cost of equity capital increase. As the excess cost of equity capital increases, the institution's incentives to use debt financing increase so that asset substitution and underinvestment *both* become more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially increasing underinvestment. If the marginal cost of investment in project quality increases, the debt overhang problem becomes less severe because shareholders have less incentives to raise project quality. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

Our results show that the optimal solvency constraint in the FV regime is *institution-specific* in that it depends on parameters that determine the payoff distribution of the institution's projects. These parameters are likely to vary across institutions even if they belong to a particular category such as commercial banks or insurance firms. A uniform solvency constraint across institutions could, therefore, be suboptimal. In fact, we show that if the solvency constraint is sub-optimally tighter than a threshold, then FV accounting is actually dominated by HC accounting. Our analysis, therefore, highlights the importance of choosing an appropriate accounting measurement regime and tailoring the solvency constraint to the characteristics of the institution.

Our model and results are particularly pertinent to the prudential regulation of highly levered financial institutions that has become a hotly debated issue in the aftermath of the financial crisis. As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in "good" and "bad" states. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as risky asset substitution that occurred in "good" states. Our analysis highlights the fact that, at higher leverage levels that are more typical of financial institutions and where prudential regulation potentially plays a role, the option value of asset substitution is significantly higher in good states. Consequently, shutting down asset substitution through a prudential solvency constraint and the transfer of control to a regulator could have a much bigger negative impact on ex ante investment in project quality. Our study therefore sheds light on the interactions between pervasive asset substitution, underinvestment and the roles that prudential capital regulation and the accounting measurement regime play in balancing the trade-off between asset substitution and underinvestment.

2 Related Literature

We contribute to the growing stream of literature that theoretically analyzes the economic trade–offs of mark–to–market versus historical cost measurement policies. O'Hara (1993) investigates the effect of market value accounting on project maturity and finds that mark-to-market results in a preference for short-term projects over long-term projects. Allen and Carletti (2008) (hereafter, AC) and Plantin, Sapra, and Shin (2008) (hereafter, PSS) are two recent studies that show how fair value accounting may have detrimental consequences for financial stability. In both studies, markets are illiquid and incomplete and therefore a reliance on price signals may lead to inefficiencies. We complement these studies in a number of ways. First, in contrast to the above studies, we analyze the effects of accounting measurement on the capital regulation of financial institutions. Because solvency constraints depend on how the values of assets and liabilities are measured, accounting measurement rules naturally have *real* effects. PSS, instead, assume that managers maximize expected accounting earnings so that accounting has real effects. AC focus solely on fair value accounting. Second, because the issues we examine are different, there are important distinctions in the tensions identified. In our setup, markets are frictionless and competitive so that price signals perfectly impound information about future cash flows. We focus on the effects of agency conflicts between a financial institution's shareholders and its debt holders. We show that, even in the absence of liquidity risk so that prices fully reflect fundamentals, while fair value accounting curbs inefficient risk shifting, it is still inefficient because it exacerbates underinvestment.

Burkhart and Strausz (2009) (hereafter, BS) and Heaton, Lucas, and McDonald (2010) (hereafter, HLM) model the effects of fair value accounting on financial institutions and also assume frictionless and competitive markets so that prices fully reflect fundamentals. BS show that, unlike historical cost accounting, fair value accounting increases the liquidity of a financial institution's assets, which, in turn, increases the institution's asset substitution incentives. Our analysis identifies different frictions, and therefore generates very different conclusions. BS focus on the information asymmetry between the institution's current shareholders and prospective shareholders, while we examine conflicts between debt holders and shareholders. In their environment, fair value accounting reduces information asymmetry that induces asset substitution. In our environment, fair value accounting curbs asset substitution through the intervention of the regulator but unfortunately, the underinvestment problem is exacerbated. HLM build a general equilibrium model of an institution and study how accounting interacts with an institution's capital requirements to affect the social costs of regulation. In their model, financial institutions invest in firms whose technologies are *exogenous* and *fixed*. In contrast, our analysis centers on how the optimal choices of the accounting regime and the solvency constraint anticipate the financial institution's *endogenous* project choices.

Our study is also related to the literature on the capital regulation of banks and, more generally, financial institutions (see Dewatripont and Tirole (1995) and Santos (2001) for surveys). We adopt the perspective in Dewatripont and Tirole (1995) who argue that the main concern of prudential regulation is the solvency of financial institutions that, in turn, is related to their capital structure. Capital structure is relevant because it implies an allocation of control rights (e.g., see Aghion and Bolton (1992)) between shareholders and debt holders. Further, the importance of regulation stems from the fact that small, uninformed debt holders of institutions need a representative to protect their interests. In early studies, Merton (1978) and Bhattacharya (1982) show that capital requirements curb inefficient risk-shifting. However, studies such as Koehn and Santomero (1980), Kim and Santomero (1988), Gennotte and Pyle (1991), and Rochet (1991) argue that capital requirements could alter the equilibrium scale of operations of an institution and, therefore, its optimal asset composition in ambiguous ways. Besanko and Kanatas (1996) show that conflicts of interest between a bank's management and its shareholders could lower, and sometimes even reverse, the beneficial effects of capital regulation in curbing asset substitution. Kahn and Winton (2004) emphasize that risk-shifting incentives are particularly important for financial institutions. We contribute to this literature by showing how solvency constraints optimally balance the inefficiencies arising from asset substitution and underinvestment. More importantly, our study demonstrates how the trade–off between these inefficiencies is affected by the accounting measurement regime.

3 Model

3.1 Environment

A financial institution that finances a long-term project through a combination of debt and equity. The term "project" could refer to a "pool" of projects. Because our theory is broadly applicable to institutions that are subject to prudential regulation such as insurance firms and commercial banks, we deliberately do not model a specific type of institution.[4] Our focus is on agency conflicts between shareholders and debt holders so we assume that the management behaves in the interests of shareholders.

The project's payoff increases stochastically (in the sense of first-order stochastic dominance) in the project's *quality*. The institution chooses the quality of the project through careful analysis and selection. The cost incurred by shareholders in choosing the quality of the project is a nonnegative random variable. The mean cost increases with project quality.

At some interim date before the project's payoffs are realized, there is a publicly observable signal about the performance of the project. At this date,shareholders may act opportunistically

[4] If the institution is an insurance firm, its "creditors" include insurance policyholders. With the pooling of insurance risks, the insurance firm's liabilities arising from insurance claims are similar to a debt obligation. If the institution is a bank, its creditors include depositors and other debt holders.

by engaging in asset substitution or risk-shifting that results in the transfer of wealth from debt holders to shareholders, but lowers the value of the overall project. Asset substitution could be achieved by engaging in off balance sheet derivative transactions, altering the characteristics of the existing project, etc.

The institution operates in a regulated environment. There is a prudential regulator who protects the interests of small and uninformed debt holders by ensuring that, at any point of time, the institution's leverage ratio is not too high. The regulator imposes a prudential or solvency constraint to ensure that the value of the institution's assets are sufficiently high relative to its liabilities (see Dewatripont and Tirole (1994)). If the prudential constraint is violated at the interim date, control transfers to the regulator who closely monitors the institution and ensures that it chooses the efficient continuation strategy—no asset substitution—in the second period.

Our analysis does not change in any way if we, instead, assume that the regulator sells/liquidates the institution's assets where the total payoff is the market value of the assets assuming the efficient continuation strategy—no asset substitution—is chosen in the second period. In other words, our results hold even if the institution's claims are traded in frictionless, competitive markets, that is, there are no deadweight costs arising from the early sale/liquidation of the institution's assets.

We study two accounting measurement regimes: a "pure" historical cost (HC) regime in which the balance sheet of the institution is measured using the original prices of the claims; and a fair value (FV) regime in which the balance sheet of the institution—and therefore the prudential constraint—is marked to market every period using the current market prices of the claims. We view the pure HC regime as a benchmark against which we examine the effects of mark-to-market or fair value accounting. We carry out both positive and normative analyses. For each accounting regime, we first examine the effects of a given solvency constraint on the institution's capital structure, its project quality, and its asset substitution strategy. We then derive the optimal (value-maximizing) choice of the prudential constraint by the regulator. Finally, we analyze the optimal choice of accounting regime. We next describe the ingredients of the model in more detail.

3.2 Long-Term Project and Capital Structure

There are two periods with three dates $0, 1, 2$. All agents are risk-neutral, but, as we discuss below, could have differing discount rates. At $t = 0$, the institution makes a fixed investment A_0

in a long-term project. The institution finances the investment through a combination of debt and equity. Our objective is to study shareholder-debt holder conflicts, especially when the institution's leverage may be high. Similar to studies such as Giammarino *et al.* (1993), Heaton *et al.* (2010), and Mehran and Thakor (2010), there are deadweight costs of equity capital that we model by assuming that equity holders demand a higher (risk-adjusted) expected return on their investment than debt holders. (Other choices of modeling the excess cost of equity capital lead to similar results.) For example, if the institution is an insurance firm, the lower cost of debt capital could arise from the fact that agents have a demand for insurance. The insurance firm's core business is the provision of insurance so that it has a comparative advantage in supplying insurance that it does not possess in raising equity capital. If the institution is a bank, the lower cost of debt could arise from the fact that the bank has a comparative advantage in raising capital from depositors.

Previous literature suggests a number of reasons why financial institutions issue debt and, moreover, have relatively high leverage levels (e.g., see Dewatripont and Tirole (1994), Allen and Gale (1999), Santos (2001)). All these mechanisms have the effect of lowering the "effective" cost of debt relative to equity. Because the economic insights we focus on in this study do not hinge on the particular frictions that give rise to the excess cost of equity, we follow studies such as Giammarino *et al.* (1993), Heaton *et al.* (2010), and Mehran and Thakor (2010) by not modeling them explicitly to simplify the exposition and analysis.[5]

We normalize the cost of debt to 1 and the cost of equity to $1 + \lambda$ over the period between date 0 and date 2, where λ denotes the excess cost of equity. Purely for notational simplicity, we assume that the two periods are of equal length so that the cost of equity over the period between date 1 and date 2 is $\sqrt{1 + \lambda}$. Our implications are qualitatively unaltered if we assume the periods are of unequal length, but the notation becomes more messy. Note that λ represents the additional deadweight loss of equity financing relative to debt financing; λ is *not* a risk premium.

Because financial institutions have substantially higher leverage levels relative to industrial firms, their effective λ is significantly higher in the context of our model. Gropp and Heider (2010) conduct an empirical analysis of the determinants of the capital structures of large U.S and

[5]A fairly straightforward way to endogenize the excess cost of equity capital in our framework is to explicitly incorporate an agency conflict between the shareholders and the manager of the institution. Specifically, we could assume that the manager derives pecuniary private benefits that are proportional to the cash flows to equity. In such a scenario, debt is a "hard" claim that reduces the firm's "free cash flow" that, in turn, provides ex ante incentives to issue debt.

European banks. They document that the median book and market leverage ratios of banks in their sample are 92.6% and 87.3%, respectively, while the corresponding median ratios for non-financial firms are 24% and 23%, respectively. They argue that their findings are consistent with banks facing significantly higher excess costs of equity financing compared with non-financial firms, which could explain their substantially higher leverage levels.

For simplicity, we restrict consideration to debt that pays off at date $t = 2$ with no intermediate interest payments. The amount of debt the institution chooses to issue is determined by its payoff/face value M at maturity. Let the market value of the debt at date $t = 0$ be D_0, which is endogenously determined. The institution therefore finances the remaining amount $E_0 = A_0 - D_0$ through equity. Capital markets are competitive.

If the institution is a bank, its depositors are protected by deposit insurance in practice. We do not incorporate the presence of deposit insurance in our analysis because, as mentioned earlier, we intend our theory to be applicable to a general financial intermediary whose liabilities need not be protected by deposit insurance. Even in the case of banks, a substantial portion of their debt is long-term and uninsured.

It turns out that, even if we restrict ourselves to the specific case in which the institution is a bank and all its debt comprises of insured demand deposits, our implications are unaffected as long as deposit insurance is fairly priced. The reason is that fairly priced deposit insurance—that is, the deposit insurance premium rationally incorporates the institution's optimal choices of capital structure, project quality, and asset substitution—is merely a transfer of funds from shareholders to debt holders. Shareholders pay the deposit insurance premium to the deposit insurer who, in turn, compensates debt holders if the institution defaults. Consequently, although debt is risk-free due to deposit insurance, the deposit insurance premium lowers the value of equity so that the value of the institution—the size of the total pie—is unchanged. Furthermore, the deposit insurance premium is a sunk cost that is incurred *ex ante*. Consequently, the *ex post* value of equity—that is, after deposit insurance and capital structure are in place—is identical to its value in the scenario in which there is no deposit insurance. The upshot of these implications is that none of the institution's decisions—capital structure, project quality, and asset substitution—is affected by the presence of deposit insurance. Because the size of the total pie is unchanged by deposit insurance, the regulator's objective function is also unaltered. The only result that changes is the magnitude of the optimal solvency constraint which increases with deposit insurance because the value of insured debt is higher than that of uninsured debt.[6]

3.3 Project Quality

The terminal cash flows of the project are realized at date $t = 2$. The terminal cash flows, which we describe shortly, are affected by both the *quality* of the project chosen in period 1 and potential *asset substitution* chosen in period 2. We denote the quality of the institution's project by $q \in \{0, q_H\}$ where $0 < q_H \leq 1$. (We normalize the low project quality to zero purely to simplify the notation; our results do not change if the low and high project qualities are both nonzero.) The project quality is only observable by the manager and shareholders. The institution can always invest in a default long-term project, i.e., in a project with a low quality level 0. By carefully analyzing and screening the type of project that it finances, the institution can raise the quality of its project from 0 to q_H. The resources invested by the shareholders in choosing a project of quality $q \in \{0, q_H\}$ is a nonnegative random variable $\widetilde{C}(q)$ with support $(0, \infty)$. The *expected cost* of choosing a project is increasing in its quality. Enhancing the project quality from 0 to q_H requires the institution's shareholders to incur an *additional* expected cost of kq_H. We alternately refer to the additional expected cost kq_H as the *additional investment* to increase project quality. Consequently,

$$E[\widetilde{C}(q_H) - \widetilde{C}(0)] = kq_H. \tag{1}$$

3.4 Intermediate Signal and Prudential Constraint

At the interim date $t = 1$, there is a publicly observable signal of the final payoff of the project. The signal $y \in \{X_L, X_H\}$ where $X_H > X_L > 0$. If the quality of the project is $q \in \{0, q_H\}$, then

$$\Pr(y = X_H) = q \text{ and } \Pr(y = X_L) = 1 - q. \tag{2}$$

By (2), the high quality project first-order stochastically dominates the low quality project, that is, the probability of receiving a high intermediate signal is greater with the higher quality project.

[6] An analysis of the model with deposit insurance is available upon request. Chan et al. (1992), Giammarino et al. (1993) and Freixas and Rochet (1995) examine the feasibility of fairly priced deposit insurance when there is *adverse selection* regarding the bank's projects.

At any date t, the institution faces a solvency constraint imposed by a regulator to protect the interests of the institution's creditors. The solvency constraint requires that the value of the institution's assets be high enough relative to the value of its liabilities. In a fair value accounting regime, where all assets and liabilities are marked to market, the constraint takes the form

$$\frac{D_t}{A_t} \leq c \text{ where } t \in \{0,1\}, \tag{3}$$

where D_t is the *market value* of the institution's debt and $A_t = D_t + E_t$ is the *market value* of the institution's total assets at date t. In (3), the interval $0 \leq c \leq 1$ implies that the institution's leverage ratio must be below a threshold c.[7]

If the prudential constraint is satisfied at date 1—that is, $\frac{D_1}{A_1} \leq c$—the institution's shareholders maintain control for the second period. However, if it is not satisfied—that is, $\frac{D_1}{A_1} > c$—control transfers to the regulator who closely monitors the institution and ensures that it does not engage in asset substitution. We later describe a benchmark accounting regime that we refer to as the *historical cost regime* in which the institution's assets and liabilities are not marked to market.

3.5 Terminal Payoffs

At the beginning of period 2, the shareholders decide whether or not to engage in asset substitution. In particular, given the signal $y = X_i$, where $i \in \{L, H\}$, the shareholders take a *hidden action* that is represented by the ordered pair $(r, z) \in \{(0,0), (r_H, z_H)\}$ that alters the distribution of terminal payoffs of the institution. Given $y = X_i$, the terminal payoff of the institution, $\widetilde{X}$, takes two possible values, either $(1+z)y$ or $(1-z)y$, where

$$\begin{aligned} \Pr(\widetilde{X} &= (1+z)y) = \frac{1}{2} - r \\ \Pr(\widetilde{X} &= (1-z)y) = \frac{1}{2} + r. \end{aligned} \tag{4}$$

We assume that $0 < r_H \leq \frac{1}{2}$ and $0 < z_H \leq 1$. Given the asset substitution strategy (r, z) and public signal y, the expected value of the terminal cash flows of the institution is

$$E(\widetilde{X}|y) = (1 - 2rz)y \ .$$

[7] As we show later, the optimal threshold c may depend on the accounting measurement regime.

From the above, it is clear that the action $(0,0)$ signifies "no asset substitution" because the terminal payoff *conditional* on the intermediate signal is risk-free and equals the value of the signal. On the other hand, the action (r_H, z_H) captures asset substitution because it injects uncertainty in the terminal payoffs, while simultaneously reducing the expected terminal cash flows of the institution from y to $(1-2r_H z_H)y$. We choose the two strategies to be "no asset substitution" and "asset substitution" purely to simplify and sharpen the analysis.

To simplify the algebra, we assume a "recombining" binomial tree when asset substitution is chosen in the high and low states. More precisely, the best possible terminal payoff from asset substitution when the intermediate signal is low (i.e., when $y = X_L$) equals the worst possible terminal payoff from asset substitution when the intermediate signal is high (i.e., when $y = X_H$) so that

$$(1 - z_H)X_H = (1 + z_H)X_L. \tag{5}$$

We also make the following standing assumption on project parameters:

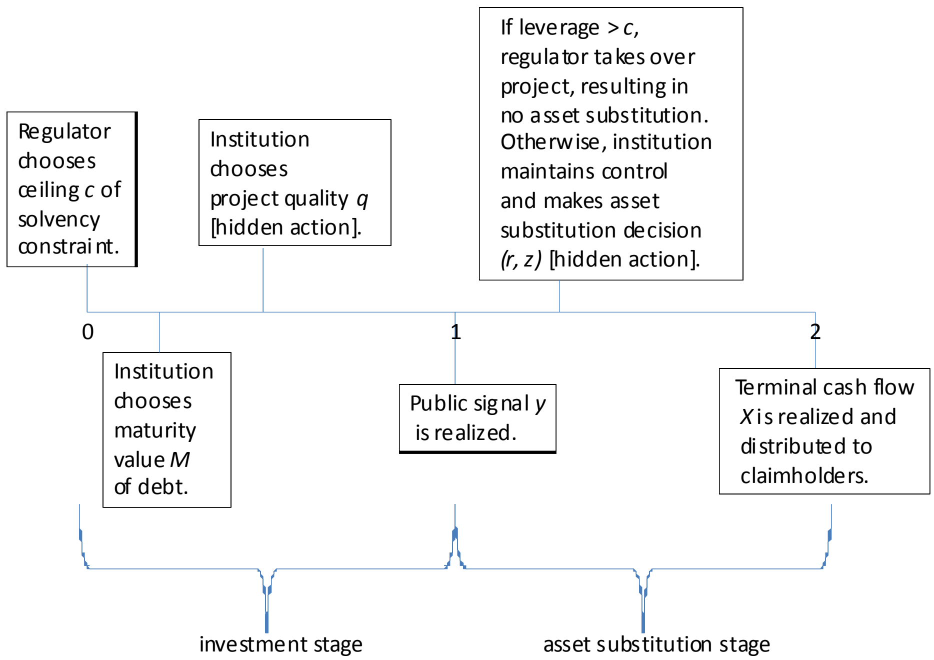
$$\frac{1}{1+\lambda}(1 + z_H)X_L < A_0 < \frac{1}{1+\lambda}X_H - kq_H. \tag{6}$$

The first inequality implies that, conditional on a low intermediate signal at date 1, even the best possible outcome under asset substitution is not sufficient to recover the initial investment A_0. The second inequality ensures that, conditional on a high intermediate signal at date 1, engaging in no asset substitution has a positive net payoff in the sense that the corresponding terminal payoff X_H is greater than the sum of the initial investment A_0 and the expected incremental cost kq_H of choosing high project quality. Assumption (6) ensures that the inefficiencies arising from asset substitution problem are severe enough for prudential regulation to be relevant.

By (2) and (4), the distribution of terminal cash flows $\widetilde{X}$ depends on both the unobservable project quality $q \in \{0, q_H\}$ chosen in period 1 and on the unobservable asset substitution strategy $(r, z) \in \{(0,0), (r_H, z_H)\}$ chosen in period 2. We refer to period 1 as the *investment stage* and to period 2 as the *asset substitution* stage.

Figure 1 summarizes the sequence of events. Figure 2 illustrates how the distribution of terminal cash flows $\widetilde{X}$ depends on the institution's investment q in period 1 and its asset substitution choice

Figure 1: Sequence of Events



r in period 2.

The payoffs of the shareholders and debt holders depend on whether the solvency constraint (3) is violated at the end of period 1 and, therefore, on whether the regulator takes control. If the regulator takes control at $t = 1$, it ensures that the institution chooses the ex post efficient strategy of no asset substitution, that is, it chooses $(r, z) = (0, 0)$ in the second period. The debt holders' payoffs equal the lower of the face value M of the debt or the terminal payoff $\widetilde{X}(q; (r, z))$ of the institution, where we explicitly indicate the dependence of the terminal payoff on the project quality q and the asset substitution strategy (r, z). Shareholders receive the cash flows net of payments to debt holders minus the cost of investment in project quality. Table 1 summarizes the payoffs of the shareholders, the debt holders, and their combined payoffs:

Figure 2: Technology

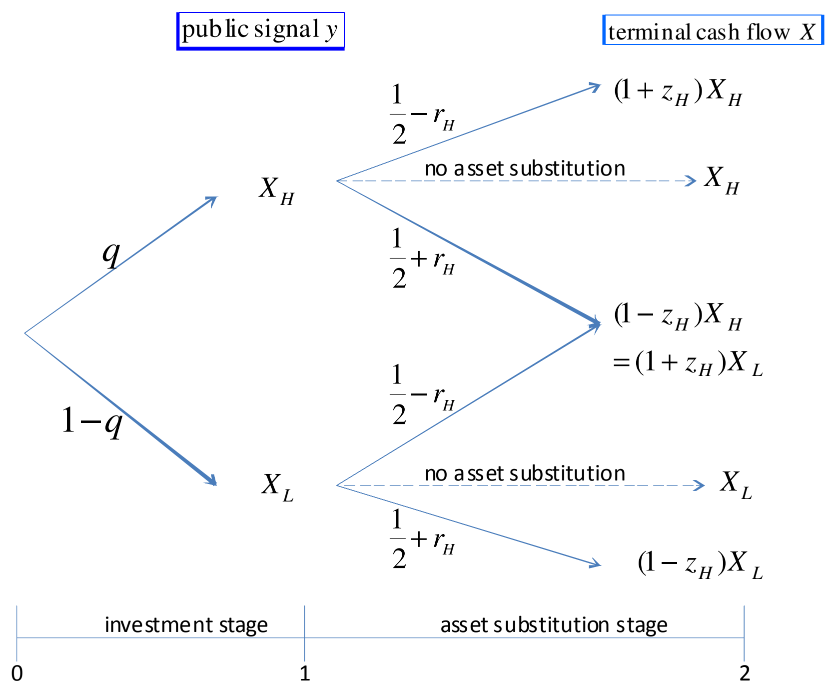


Table 1: Payoffs of Debt Holders and Shareholders

		Institution Maintains Control	Regulator Takes Control
	Date 0	Date 2	Date 2
Debt holders' Payoff		$\min\{M, \widetilde{X}(q;(r,z))\}$	$\min\{M, \widetilde{X}(q;(0,0))\}$
Shareholders' Payoff	$-\widetilde{C}(q)$	$\max\{\widetilde{X}(q;(r,z)) - M, 0\}$	$\max\{\widetilde{X}(q;(0,0)) - M, 0\}$
Total Payoff	$-\widetilde{C}(q)$	$\widetilde{X}(q;(r,z))$	$\widetilde{X}(q;(0,0))$

Note that the payoffs in the scenario where the regulator takes control reflect the fact that the regulator ensures that the institution chooses the ex post efficient continuation strategy—no asset substitution—in the second period, that is, $(r,z) = (0,0)$.

3.6 Measurement Regimes

We study two accounting measurement regimes. The first regime, which should be viewed as a benchmark regime, is a "pure" *historical cost* regime (HC) in which the institution's assets and liabilities are measured at their initial date 0 "origination" values. More precisely, in the context of our model, the prudential constraint is given by

$$\frac{D_0}{A_0} \leq c^{HC} \tag{7}$$

at the initial date $t = 0$ *and* the intermediate date $t = 1$. In the above, D_0 is the initial present value of the institution's debt and A_0 is the acquisition cost of its assets.

The second regime is the *fair value* regime (FV) in which the institution's balance sheet is marked to market every period so that the solvency constraint is given by (3), that is,

$$\frac{D_0}{A_0} \leq c^{FV} \text{ at } t = 0 \text{ and } \frac{D_1}{A_1} \leq c^{FV} \text{ at } t = 1. \tag{8}$$

The superscripts on the solvency constraints in the two regimes reflect the fact that they could differ across the regimes.

In the first best scenario, all decisions are made to maximize the total value of the institution rather than just shareholder value, and the excess cost of equity λ is zero. In this scenario, it is easy to show that the institution always chooses the high quality project and does not engage in asset substitution. Because the incentives of shareholders are aligned with those of creditors, the institution's capital structure and therefore its leverage play no role. Therefore, accounting measurement issues are moot.

In the second-best world, maximizing shareholder value is not necessarily equivalent to maximizing the total value of the institution (that is, the equity value plus the debt value). We analyze each accounting measurement regime using backward induction. We start at the beginning of period 2 when the public signal has been released. For a given solvency constraint, capital structure, and a given public signal, we first derive the transfer of control decision and asset substitution decision in period 2. Next, we derive the project quality decision in period 1, which anticipates the transfer of control and asset substitution decisions in period 2. Next, we determine the capital structure decision at date 0, which is determined by the choice of the face value of debt. Finally, given the institution's optimal capital structure, investment, and asset substitution decisions, we derive the optimal/value-maximizing solvency constraint.

4 Historical Cost Regime

Under the HC regime, because the institution's assets and liabilities are measured at their date 0 origination values until terminal payoffs are realized, the solvency constraint $\frac{D_0}{A_0} \leq c$ is used at *both* $t = 0$ and $t = 1$. In the HC regime, therefore, if the solvency constraint is satisfied at date 0, it is automatically satisfied at date 1. Consequently, there is no transfer of control at date 1.

If we were to extend the model by introducing interim cash flows at date 1, then these would be incorporated in the calculation of the solvency constraint at date 1 in the HC regime. Consequently, the solvency constraint at date 1 would be different from the constraint at date 0 so that transfer of control would, in general, be feasible. Nevertheless, it would still be true that transfer of control is much less likely in the HC regime relative to the FV regime because the solvency constraint is relatively insensitive to interim price signals. Our main implications, which hinge on the significantly different likelihoods of the transfer of control in the two regimes, are unlikely to be significantly altered. To simplify the model and analysis, therefore, we abstract away from interim cash flows.

4.1 Asset Substitution

At date $t = 1$, given the public signal y and the debt face value M, shareholders decide whether to engage in asset substitution by choosing the hidden action $(r, z) \in \{(0, 0); (r_H, z_H)\}$ to maximize

$$\begin{aligned} & \overbrace{E[\max\{\widetilde{X} - M, 0\}|y]}^{\text{shareholder value}} \qquad (9) \\ = \; & (\frac{1}{2} - r) \max\{(1 + z)y - M, 0\} + (\frac{1}{2} + r) \max\{(1 - z)y - M, 0\}. \end{aligned}$$

Proposition 1 (Asset Substitution in HC Regime) *Under the historical cost regime, shareholders choose asset substitution if and only if the maturity value M of debt is sufficiently high, that is, $M > c_0 y$, where $c_0 \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$.*

The intuition for Proposition 1 is well known. Because shareholders effectively hold a call option on the terminal payoff with strike price equal to the face value of debt, it is optimal for them to increase risk by choosing asset substitution when the intermediate signal is sufficiently low relative to the face value of debt.

We also note that, as $\frac{1}{2}-r_H$ (the probability of a good outcome given asset substitution) and/or z_H (the spread of outcomes resulting from asset substitution) increases, asset substitution becomes more attractive to shareholders in period 2. Consequently, the threshold value $c_0 y$ of the face value of debt above which asset substitution takes place *decreases*, that is, asset substitution occurs for a *larger range* of debt face values.

It follows from the proposition that the propensity to choose asset substitution depends on the leverage level of the financial institution which is endogenous. Furthermore, for high leverage levels, asset substitution is likely in both the good state ($y = X_H$) and in the bad state ($y = X_L$); an observation that is important for our subsequent analysis.

4.2 Project Quality

At date 0, given the face value M of debt, shareholders choose the project quality $q \in \{0, q_H\}$ anticipating the asset substitution decision in period 2 given by Proposition 1. In choosing the project quality, shareholders trade off their expected payoff incorporating the period 2 asset substitution decision against the expected investment in project quality. The following proposition describes the optimal choice of project quality by the shareholders.

Proposition 2 (Project Quality in HC Regime) *Under the historical cost regime, shareholders choose low project quality if and only if the maturity value M of debt is sufficiently high: (i) for $k \leq k^*$, q_L is chosen if and only if $M > c_2 X_H$; (ii) for $k > k^*$, q_L is chosen if and only if $M > c_1 X_H$. In the above,*

$$c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H};\ c_2 \equiv (1+z_H) - \frac{k(1+\lambda)}{(\frac{1}{2}-r_H)X_H};\ k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda).$$

Proposition 2 states that shareholders choose the low quality project if the face value of debt is sufficiently high. This is essentially a consequence of the well known "debt overhang" problem. If the amount of debt in the institution's capital structure is sufficiently high, shareholders' incentives to make a costly investment in the higher quality project are curbed because a larger proportion of the increased total payoff from such an investment accrues to debt holders.

A more novel and interesting implication of our analysis is that Propositions 1 and 2 together imply that the debt overhang problem in period 1 is actually *alleviated* by the possibility of asset substitution in period 2. The following corollary makes this precise.

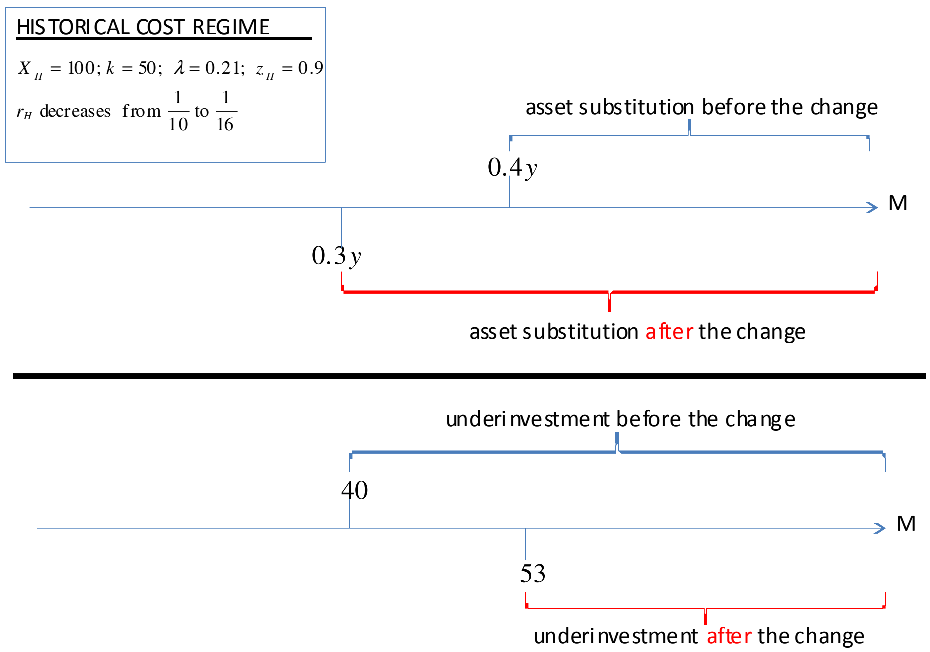
Corollary 1 (Asset Substitution and Underinvestment in the HC Regime) *If r_H decreases and/or z_H increases (i) the threshold level of the debt face value above which asset substitution occurs decreases for any value of the intermediate signal y; (ii) for given k, the threshold level of the debt face value above which the low project quality is chosen increases; and (iii) the threshold level k^* in Proposition 2 increases.*

By the discussion following Proposition 1, a decrease in r_H and/or an increase in z_H increases the incentives for asset substitution in period 2, that is, the range of debt face values for which asset substitution occurs increases for any value of the intermediate signal. The corollary, however, shows that a decrease in r_H and/or an increase in z_H causes the range of debt face values for which the low quality project is chosen to *shrink*. In other words, an increase in the propensity for asset substitution in the second period *increases* the likelihood of choosing high project quality in the first period, that is, it *alleviates* underinvestment in project quality. Furthermore, as r_H decreases and/or z_H increases, the threshold k^* in Proposition 2 increases so that the region $k \leq k^*$ expands while the region $k > k^*$ shrinks. Therefore, not only does the range of debt face values for which the low quality project is chosen shrink in the presence of asset substitution, but as asset substitution becomes more attractive, the latter effect also persists for larger values of k.

Figure 3 illustrates the corollary via a numerical example. In particular, Figure 3 demonstrates how an increase in the propensity of asset substitution via a decrease in the value of r_H from $\frac{1}{10}$ to $\frac{1}{16}$ affects underinvestment in the HC regime. The top half of Figure 3 illustrates that, as r_H declines from $\frac{1}{10}$ to $\frac{1}{16}$, the range of debt face values M for which asset substitution occurs *expands* from $M > 0.4y$ to $M > 0.3y$. The bottom half of Figure 3 shows that, following a decline in r_H, the corresponding range of values of M for which the low quality project is chosen *shrinks* from $M > 40$ to $M > 53$. Hence, an *increase* in the propensity for asset substitution *mitigates* underinvestment in project quality.

The intuition for these results is as follows. At low leverage levels, asset substitution is either non-existent or occurs only in the low state X_L. At low leverage levels, however, the underinvestment problem is also nonexistent in that the high project quality is chosen in the first period as shown by Proposition 2. Since asset substitution occurs (if at all) only in the low state where payoffs are low,

Figure 3: The Trade-Off in the Historical Cost Regime



a change in the incentives for asset substitution triggers little distortion from an ex ante perspective so that the project quality choice in the first period is unaffected. As leverage increases, however, the option to engage in asset substitution becomes more valuable in the high state relative to the low state because the call option in the low state becomes more out of the money relative to the high state. Further, at higher leverage levels, the payoffs from asset substitution are much greater for the high state relative to the low state because the good outcome for the project is realized. Given that the high state is more likely for the high quality project, an increase in the propensity for asset substitution in the second period *increases* the institution's incentives to choose the higher project quality in the first period. Furthermore, as asset substitution becomes more profitable in period 2 (i.e., when r_H decreases and/or z_H increases), the call option in the high state becomes even more valuable so that the incentives to choose the higher quality project persist even for large values of k.

To summarize, at low leverage levels, the debt overhang and asset substitution problems are both minor so that the low ex post rents from asset substitution have little or no impact on ex ante investment in project quality. At high leverage levels, however, asset substitution also occurs in the good state. The potentially large ex post rents from asset substitution in the good state increase the institution's incentives to choose the high quality project in the first period, thereby reducing the underinvestment problem.

4.3 Capital Structure

We now analyze the institution's optimal choice of its capital structure. The bank's *original* shareholders (that is, before capital structure is in place) optimally finance the project by issuing debt and equity rationally anticipating the *ex post* project quality and asset substitution choices. In particular, at $t = 0$, the bank's original shareholders choose the institution's capital structure to maximize their value subject to the date $t = 0$ solvency constraint (7). The value of original shareholders at date zero equals the market value of equity plus the market value of debt. Recall that the cost of debt is normalized to 1 and that of equity to $1 + \lambda$. Consequently, the debt face value, which determines the institution's capital structure, solves

$$M^{HC} = \arg\max_{M} \overbrace{\frac{E(\max\{\widetilde{X} - M, 0\})}{1+\lambda}}^{\text{market value of equity}} + \overbrace{E\left(\min\left\{M, \widetilde{X}\right\}\right)}^{\text{market value of debt}} - \overbrace{E[C(q^{HC})]}^{\text{expected investment in project quality}} - \overbrace{A_0}^{\text{initial investment in project}} \tag{10}$$

subject to the $t = 0$ solvency constraint

$$\frac{D_0}{A_0} \leq c, \tag{11}$$

where D_0 is the market value of debt at date 0. In (10), q^{HC} is the optimal project quality choice and M^{HC} is the optimal debt face value, where the superscripts denote that these are their values in the HC regime.

By (10), M^{HC} balances the trade–off between the excess cost of equity represented by λ and the inefficiencies arising from underinvestment and asset substitution due to the presence of debt in the institution's capital structure. The optimal face value of debt, M^{HC}, depends on the underlying parameters of our environment. In particular, as one would expect, M^{HC} increases in λ, that is, the optimal amount of debt financing increases with the excess cost of equity. The formal result on the institution's optimal capital structure choice is somewhat messy, and is not the central focus of our study. Consequently, we relegate the statement of the result on capital structure and its proof to Lemma 1 in the Appendix. The goal of analyzing the optimal capital structure decision is to emphasize that the central tradeoff between asset substitution and underinvestment that we identified earlier holds when capital structure is endogenized.

4.4 Prudential Constraint

From a normative perspective, the regulator chooses the optimal solvency constraint, c^{HC}, to maximize the total value of the institution. Given that capital markets are competitive, the institution's original shareholders extract all the surplus from its operations. Therefore, in choosing the optimal solvency constraint, c^{HC}, the regulator's problem of maximizing the total value of the institution is equivalent to maximizing the value of original shareholders subject to the solvency constraint (7).

In the HC regime, the solvency constraint does not have any bite at date $t = 1$ and thus transfer of control never occurs. The prudential constraint is only relevant at $t = 0$ because it constrains the shareholders' optimal choice of M. As discussed in Section 4.3, however, the original shareholders optimally choose M to maximize their value that, as we have argued above, is equivalent to maximizing the value of the institution. Constraining the choice of M via the prudential constraint only reduces the institution's date 0 value below the unconstrained maximum. Therefore, c^{HC} should be set as high as possible, that is, it should be set to 1.

Proposition 3 (Optimal Prudential Constraint in HC Regime) *The optimal prudential constraint in the historical cost regime, c^{HC}, is* 1.

To summarize, in the HC regime, because transfer of control does not occur at the end of period 1, there is a prevalence of asset substitution in period 2. Further, as leverage increases, not only does asset substitution become feasible in both the good and the bad state but the *differential* ex post rents from asset substitution for the high state relative to the low state also increase. These differential rents alleviate the underinvestment problem in period 1 by increasing shareholders' incentives to invest in the high quality project. Therefore, the more severe the asset substitution problem is in the second period, the less severe the underinvestment problem is in the first period.

Finally, in the HC regime, because the prudential constraint only restricts the institution's choice of capital structure and plays no role once the capital structure is in place, it should be set as high as possible.

The absence of transfer of control implies that the pure HC regime is essentially equivalent to a regime with no prudential regulation, that is, we could also interpret the benchmark HC regime as a "no regulation" regime. However, as we mentioned earlier, if we modify the model to incorporate the possibility of interim cash flows at date 1, transfer of control becomes feasible in the HC regime at date 1. Nevertheless, it can be shown that, for reasonable parameterizations of the model, transfer of control would still be significantly less likely in the HC regime compared with the FV regime. Our main results hinge on the trade-off between the higher likelihood of transfer of control in the FV regime against the lower incidence of asset substitution. Consequently, the key implications of our study are unlikely to be altered even if we were to complicate the model by incorporating interim cash flows so that transfer of control is possible in the HC regime.

5 Fair Value Regime

In the fair value regime (FV), the institution's balance sheet is marked to market every period so that the prudential constraint is

$$\frac{D_0}{A_0} \leq c \text{ at } t = 0 \text{ and } \frac{D_1}{A_1} \leq c \text{ at } t = 1, \tag{12}$$

where D_t and A_t, respectively, denote the *market values* of the institution's debt and assets at t. If $\frac{D_1}{A_1} > c$, the regulator takes control and closely monitors the institution to ensure that there is no asset substitution in period 2.

5.1 Asset Substitution

The analysis of the fair value regime is significantly more complicated than the historical cost regime. By (12), the prudential constraint at date 1, which determines whether transfer of control occurs, depends on the market values of the institution's debt and assets. These market values are, however, determined *in equilibrium* along with the institution's asset substitution strategy.

More precisely, at $t = 1$, the institution's asset substitution decision (r, z) is unobservable. Consequently, in order to value the institution's debt, the capital market forms a conjecture $(\widehat{r}, \widehat{z})$ about (r, z). Given the date $t = 1$ signal y, the capital market values the institution's debt at

$$\begin{aligned} D_1(y, (\widehat{r}, \widehat{z})) &= E[\min\{M, \widetilde{X}\}|y, (\widehat{r}, \widehat{z})] \\ &= (\frac{1}{2} - \widehat{r}) \min\{M, (1 + \widehat{z})y\} + (\frac{1}{2} + \widehat{r}) \min\{M, (1 - \widehat{z})y\}. \end{aligned} \tag{13}$$

Similarly, at date $t = 1$, the market value of equity is

$$\begin{aligned} E_1(y, (\widehat{r}, \widehat{z})) &= E[\max\{\widetilde{X}(y, (\widehat{r}, \widehat{z})) - M, 0\}] \\ &= \frac{(\frac{1}{2} - \widehat{r}) \max\{(1 + \widehat{z})y - M, 0\} + (\frac{1}{2} + \widehat{r}) \max\{(1 - \widehat{z})y - M, 0\}}{\sqrt{1 + \lambda}}. \end{aligned} \tag{14}$$

These date $t = 1$ market prices along with the prudential constraint determine whether transfer of control occurs. Given the continuation or control transfer outcome, the institution chooses $((r, z) \in \{(0, 0), (r_H, z_H)\}$ in period 2. If transfer of control occurs at date $t = 1$, the regulator ensures that the ex post efficient continuation strategy, i.e., $(r, z) = (0, 0)$ in period 2 is chosen so that the payoff at date 2 is y. If transfer of control does not occur at date $t = 1$, then shareholders could choose whether or not to engage in asset substitution. In a rational expectations equilibrium, the market's conjecture regarding the chosen asset substitution strategy is correct. In other words, given $D_1(y, (\widehat{r}, \widehat{z}))$ and $A_1(y, (\widehat{r}, \widehat{z}))$ and the prudential constraint c, the institution's optimal asset substitution strategy is indeed $(\widehat{r}, \widehat{z})$.

The following proposition characterizes the optimal continuation/transfer of control and asset substitution decisions given the debt face value M and prudential constraint c.

Proposition 4 (Asset Substitution in FV Regime) *Under the fair value regime, shareholders choose asset substitution if and only if the prudential constraint is less than a threshold and the maturity value of debt lies in an intermediate interval. That is, asset substitution is chosen if and only if $c_0 < T(c)$ and $M \in [c_0 y, T(c)y]$, where*

$$c_0 \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H; \; T(c) \equiv \frac{c}{\sqrt{1 + \lambda} - c(\sqrt{1 + \lambda} - 1)}.$$

For $M < c_0 y$, shareholders choose no asset substitution voluntarily. For $M > T(c)y$, no asset substitution is chosen because the prudential constraint is violated and transfer of control occurs.

Note that unlike the HC regime, transfer of control occurs in the FV regime if M is sufficiently large relative to y. In fact, the preceding proposition shows that, regardless of the value of c, as long as $M > T(c)y$, transfer of control occurs. This is a direct consequence of violating the date $t = 1$ solvency constraint. Transfer of control prevents the possibility of asset substitution in period 2.

Furthermore, in the FV regime, for values of c below a threshold, i.e., $T(c) < c_0$ (note that $T(c)$ increases with c), *either* (i) transfer of control occurs *or* (ii) shareholders retain control and *voluntarily* do not choose asset substitution in period 2. Consequently, regulators may eliminate the inefficiencies created by asset substitution by choosing a low value of the constraint c. In fact, as $\frac{1}{2} - r_H$ and/or z_H increases, c_0 shrinks. Therefore as shareholders find asset substitution more enticing in period 2, an even tighter solvency constraint is necessary to eliminate asset substitution.

For relatively high values of c, i.e., $T(c) > c_0$, asset substitution only occurs for intermediate values of M relative to y. For large values of M relative to y, the institution violates the prudential solvency constraint so that transfer of control occurs, and the regulator ensures that no asset substitution is chosen.

To summarize, compared to the HC regime, in which transfer of control does not occur at $t = 1$, the prevalence of asset substitution is lower in the FV regime because transfer of control occurs for large values of M relative to y. A low enough value of c may completely rule out asset substitution. Conversely, a high value of c exacerbates asset substitution. In fact, as c increases so that the ceiling is very high, the FV regime becomes effectively equivalent to the HC regime.

5.2 Project Quality

At date 0, given the face value M of the debt, shareholders choose the project quality q anticipating the transfer of control/continuation and asset substitution decisions described in Proposition 4. The following result describes shareholders' optimal choice of project quality at date $t = 0$.

Proposition 5 (Project Quality in FV Regime) *Under the fair value regime, shareholders choose the low project quality q_L if and only if the maturity value M of debt is sufficiently high.*

Define

$$c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}; \; c_2 \equiv (1+z_H) - \frac{k(1+\lambda)}{(\frac{1}{2}-r_H)X_H}; \; T(c) \equiv \frac{c}{\sqrt{1+\lambda} - c(\sqrt{1+\lambda}-1)}; \quad (15)$$

$$k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda).$$

(i) For $k \leq k^*$:

- *If $T(c) > c_2$, q_L is chosen if and only if $M > c_2 X_H$.*
- *If $T(c) \in [c_1, c_2]$, q_L is chosen if and only if $M > T(c)X_H$.*
- *If $T(c) < c_1$, q_L is chosen if and only if $M > c_1 X_H$.*

(ii) For $k > k^*$: q_L is chosen if and only if $M > c_1 X_H$.

Note that, unlike the HC regime in which the solvency constraint plays no direct role in affecting the choice of project quality, Proposition 5 illustrates the crucial role that it plays in the FV regime in determining the project quality choice $q \in \{0, q_H\}$ when the marginal cost of investment in project quality is below a threshold ($k \leq k^*$).

Recall from Proposition 4 that, the smaller c is, the higher the likelihood of transfer of control. Proposition 5 implies that, the smaller c is, the higher the likelihood of underinvestment, that is, choosing the low quality project $q = 0$. Taken together, these two propositions imply a *positive relationship* between transfer of control and underinvestment.

For low values of c ($T(c) < c_1$), the prudential constraint is relatively tight so that the institution is very likely to exceed it. A high likelihood of transfer of control implies that the incidence of asset substitution is very low. Not surprisingly, the FV regime becomes equivalent to a world of no asset substitution, so that shareholders under-invest in project quality (that is, choose $q = 0$) if and only if $M > c_1 X_H$.

For high values of c ($T(c) > c_2$), the prudential constraint is relatively loose so that transfer of control is highly unlikely and the FV regime becomes equivalent to the HC regime. In fact, for high values of c, we recover the same result obtained in the HC regime: shareholders underinvest in project quality if and only if $M > c_2 X_H$.

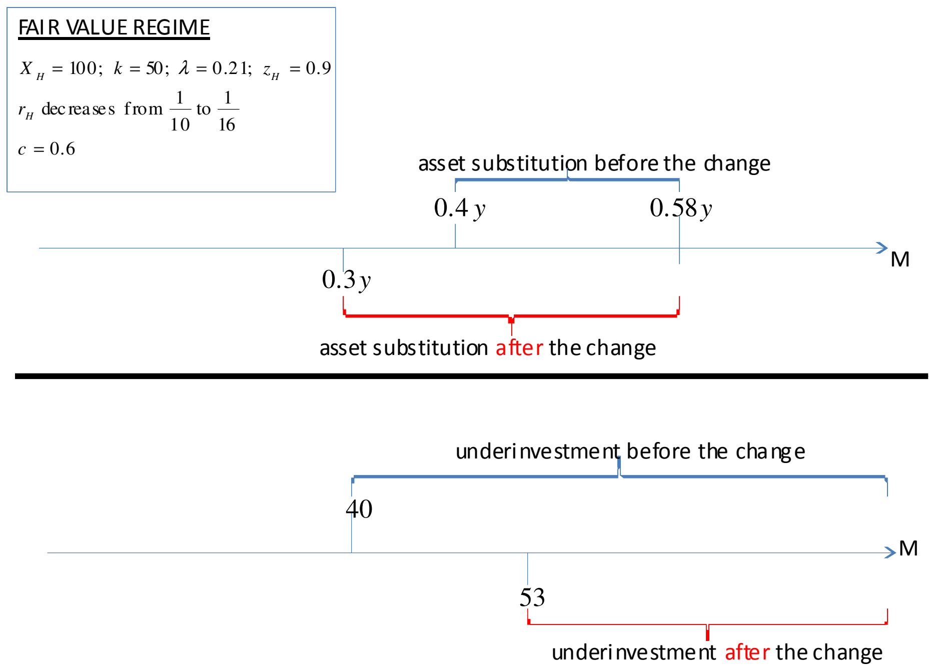
For intermediate values of c ($T(c) \in [c_1, c_2]$), the threshold ($T(c)X_H$) of the face value of debt triggering underinvestment in project quality becomes lower. Thus, as the likelihood of transfer of control increases, the underinvestment problem worsens. To understand this result, note that transfer of control in the FV regime shuts down asset substitution and such transfer of control is more likely the higher the leverage of the bank. But as we discussed earlier, this is precisely when the option value of asset substitution is greater for the high state than for the low state! Consequently, shutting down asset substitution via a change in control in the FV regime has a significant negative impact on the project quality choice in the first period.

Furthermore, as shareholders find asset substitution more attractive in period 2, i.e., as $\frac{1}{2} - r_H$ and/or z_H increases, both c_2 and k^* increase. In other words, as the ex post rents from asset substitution increase, shareholders find asset substitution in the high state even more valuable so that the positive relationship between transfer of control and underinvestment becomes more pervasive as it applies to a larger set of values of the ceiling c and the marginal cost k. The following corollary makes this precise.

Corollary 2 (Asset Substitution and Underinvestment in the FV Regime) *As r_H decreases and/or z_H increases, (i) the range of debt face values for which asset substitution occurs increases for each value of the intermediate signal; (ii) for given k, the range of debt face values for which the low project quality is chosen shrinks; (iii) the threshold k^* in Proposition 5 increases.*

Figure 4 illustrates the corollary using the same numerical example introduced earlier in Figure 3. However, unlike the HC regime, the value of the prudential constraint now matters in the FV regime. We set the value of the prudential constraint, $c = 0.6$. For the chosen parameter values, when r_H declines from $\frac{1}{10}$ to $\frac{1}{16}$, asset substitution incentives increase. For $r_H = \frac{1}{10}$, then asset substitution occurs for $M \in [0.40y, 0.58y]$. For $r_H = \frac{1}{16}$, asset substitution occurs over a larger intermediate range of $M \in [0.30y, 0.58y]$. For $M < 0.30y$, no asset substitution occurs because the debt face value is relatively low so that shareholders have no incentives to asset substitute regardless of the value of y. For $M > 0.58y$, the leverage is so high that the prudential solvency constraint is violated. Control transfers to the regulator and asset substitution is shut down. The bottom half of the figure shows that the decline in r_H from $\frac{1}{10}$ to $\frac{1}{16}$ shrinks the range of debt face values for which underinvestment occurs.

Figure 4: The Trade-Off in the Fair Value Regime



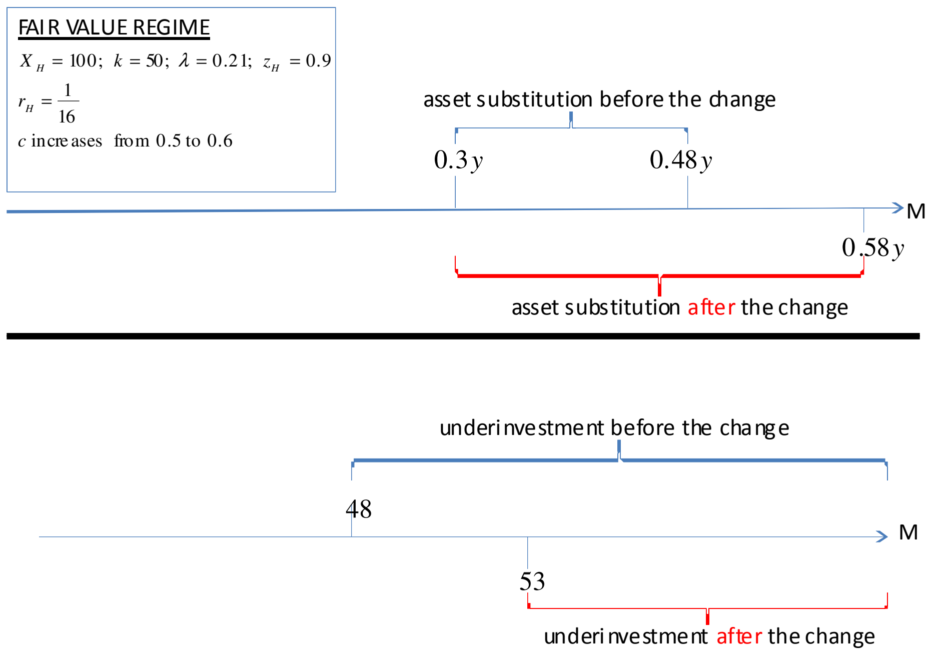
The discussion above along with the intuition for Proposition 4 suggests that, while transfer of control at date $t = 1$ *mitigates* inefficiencies created by asset substitution in period 2, it *exacerbates* inefficiencies created by underinvestment in period 1. As we discuss shortly, the optimal choice of the solvency constraint balances the trade-off between these two sources of inefficiencies.

5.3 Optimal Capital Structure and Prudential Constraint

For a given solvency constraint c, the institution's original shareholders choose its capital structure to maximize their value subject to the prudential constraint (11). Unlike the HC regime, in which there is no transfer of control at $t = 1$, in the FV regime, the shareholders' payoffs are affected by the potential transfer of control at $t = 1$. In Lemma 2 in the Appendix, we analyze the optimization program that determines the institution's optimal capital structure.

We now turn to the derivation of the optimal solvency constraint in the FV regime. In other words, anticipating the institution's optimal capital structure, quality investment, and asset substitution decisions, how should a regulator set the optimal value of the solvency constraint? In

Figure 5: The Effect of Prudential Constraint on the Trade-Off in the Fair Value Regime



choosing the optimal solvency constraint in the FV regime, the regulator faces a dilemma. Choosing a high value of c (a loose constraint) aggravates the asset substitution problem in period 2 while choosing a low value of c (a tight constraint) aggravates underinvestment in project quality in period 1.

In choosing the optimal constraint, the regulator attempts to minimize the expected inefficiencies arising from asset substitution and from underinvestment. To illustrate the trade-off in choosing the optimal prudential constraint in the fair value regime, Figure 5 uses the numerical example illustrated in Figure 4 except that we now fix r_H at $\frac{1}{16}$ and change c from 0.5 to 0.6. The top half of Figure 5 shows that as the the prudential constraint c increases from 0.5 to 0.6 (i.e., the constraint becomes looser), the range of debt face values for which asset substitution occurs expands. The bottom half of Figure 5 shows that such an increase in c shrinks the range of values of M inducing underinvestment.

The next result characterizes the optimal solvency constraint in the FV regime.

Proposition 6 (Optimal Prudential Constraint in FV Regime) *Under the fair value regime,* *the optimal solvency constraint,* c^{FV}, *is* $\frac{1}{1+\frac{k\sqrt{1+\lambda}}{X_H-k(1+\lambda)}}$.

By setting $c^{FV} = \frac{1}{1+\frac{k\sqrt{1+\lambda}}{X_H-k(1+\lambda)}}$, it follows from Proposition 4 and from Proposition 5 that the regulator maximizes the expected surplus by reducing the incidence of asset substitution while tolerating the underinvestment problem arising from excessive transfer of control. With this constraint, however, neither inefficiency is completely eliminated, that is, underinvestment and asset substitution *both* occur at the optimum.

Proposition 6 also shows that the optimal solvency constraint becomes tighter as the excess cost of equity λ or the marginal cost of investment in project quality k increase. As λ increases, the institution's incentives to use debt financing increase so that asset substitution and underinvestment *both* become more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially increasing underinvestment. As k increases, the debt overhang problem becomes less severe because the NPV of the project decreases. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

To summarize, in the FV regime, because the balance sheet of the institution is marked to market every period, the solvency constraint reflects current market values. The institution therefore faces a threat of transfer of control at the end of period 1. If the institution violates the solvency constraint, transfer of control eliminates the possibility of asset substitution in period 2. However such transfer of control takes place precisely when the option value of asset substitution is potentially high. Therefore the regulator faces a dilemma in choosing the prudential constraint. The tighter (looser) the prudential constraint, the higher (lower) the likelihood of transfer of control. Therefore, to reduce the incidence of asset substitution, the solvency constraint must be tightened. Unfortunately, in doing so, the underinvestment problem is aggravated. The regulator trades off the asset substitution problem against the underinvestment problem.

Our results suggest that the key tradeoff between asset substitution and underinvestment, and the role that prudential regulation in mediating the distortions arising from them, are particularly pronounced at high leverage levels where both problems are significant, and prudential regulation plays a role. As we mentioned in Section 3.2, financial institutions are characterized by much higher leverage levels (on average) than non-financial firms. Indeed, Gropp and Heider (2010) document that the average leverage ratio of banks is approximately 90%, while that of non-financial firms is only around 25%. In the context of our model, financial institutions have greater effective costs of equity capital λ that induces them to choose higher leverage levels, which is consistent with the empirical findings and discussion in Gropp and Heider (2010). Consequently, even though asset substitution and debt overhang are also relevant for non-financial firms, our results are especially pertinent to financial institutions. We further discuss the relevance of our results in the context of financial and non-financial firms in Section 7.

6 Fair Value Versus Historical Cost Regimes

Our previous results show that, in both accounting measurement regimes, the institution's total value is reduced by the inefficiencies arising from debt overhang and asset substitution. The solvency constraint mediates these two distortions. In the HC regime, because the balance sheet is not remeasured in the interim date, the solvency constraint has no bite in the interim date. The institution faces no threat of transfer of control so that the incidence of asset substitution is high. In the FV regime, because the balance sheet is marked to market, the solvency constraint serves as a credible threat of transfer of control, thereby alleviating the asset substitution inefficiency pervasive in the HC regime.

Comparing Propositions 1 and 4, we easily see that the incidence of asset substitution is higher in the HC regime than in the FV regime. In fact, for low values of the solvency constraint c, FV eliminates asset substitution. The following proposition compares the under-investment or debt overhang problem in the two regimes (recall the definitions of $k^*, T(c)$ and c_2 in (15).

Proposition 7 (Under-Investment in the Two Regimes) *(i) If the marginal cost of investment is low, i.e., $k \leq k^*$, and the solvency constraint is tight, i.e., $T(c) < c_2$, then (a) the underinvestment problem in the FV regime is worse than in the HC regime and (b) as $\frac{1}{2} - r_H$ and/or z_H increases, the underinvestment problem deteriorates in the FV regime relative to the HC regime.*

(ii) If $k \leq k^*$ and $T(c) > c_2$, or if $k > k^*$, the extent of underinvestment is the same in both regimes.

Note that as the asset substitution problem increases, i.e., $\frac{1}{2} - r_H$ and/or z_H increases, both

k^* and c_2 increase so that the underinvestment region in the FV regime expands relative to the underinvestment region in the HC regime over a larger range of values of parameters. That is, the debt overhang problem in the FV regime worsens.

It follows from the above results that the interplay between asset substitution and underinvestment could cause the HC regime to dominate the FV regime if the solvency constraints in the two regimes do not take their optimal values, c^{HC} and c^{FV}, respectively. However, if the two constraints take their optimal values, it easily follows from our analysis that the FV regime unambiguously dominates the HC regime. This is because the regulator operating in the FV regime can always replicate the HC regime by setting a loose enough solvency constraint so that it will not have bite at the interim date and there is no transfer of control as in the HC regime. Consequently, the FV regime can do at least as well as the HC regime.

Proposition 8 (Comparison Between Measurement Regimes) *Suppose that* $c^{HC} = 1$ *and* $c^{FV} = \frac{1}{1+\frac{k\sqrt{1+\lambda}}{X_H - k(1+\lambda)}}$. *The* FV *regime always dominates the* HC *regime.*

Note that the optimal solvency constraint, c^{FV}, in the FV regime depends on the marginal cost of investment in project quality, k, and the value of the high signal X_H. These parameters are likely to vary across institutions even if they belong to the same category such as commercial banks or insurance firms. The optimal solvency constraint is, therefore, *institution-specific*. Further, the optimal solvency constraint also depends on the excess cost of equity λ that could vary over time and, in particular, with the business cycle.

The above discussion implies that a uniform solvency constraint across institutions may not be optimal. Further, the above result crucially depends on the respective solvency constraints in the HC and FV regimes taking their optimal values. In fact, the following proposition shows that, if the solvency constraint in the FV regime is below a threshold, the HC regime dominates the FV regime.

Proposition 9 (HC Versus FV Regime) *Suppose that* $c^{HC} = 1$. *There exists* $\underline{c} \in (0, c_1)$ *such that for* $c \in [0, \underline{c})$, *the* HC *regime dominates the* FV *regime.*

Proposition 9 shows that, if the solvency constraint in the fair value regime is below a threshold, the historical cost regime would be superior to the fair value regime. Our results, therefore, highlight the importance of not only choosing the appropriate accounting regime, but tailoring the solvency constraint to the characteristics of the institution.

7 Conclusions

In the aftermath of the recent financial crisis, the merits and demerits of prudential regulation based on fair value accounting are being actively debated by academics, practitioners, and regulators. Our study contributes to the debate by showing how prudential regulation and accounting measurement rules interact with the intensity of agency conflicts between a financial institution's shareholders and debt holders. Relative to a benchmark pure "historical cost" regime, fair value accounting could alleviate the inefficiencies arising from asset substitution, but exacerbate those arising from under-investment due to debt overhang. The subtle, but important, opposing effects of fair value accounting on asset substitution and under-investment are especially pronounced at high leverage levels that are typical of financial institutions.

The optimal choices of accounting regime and prudential solvency constraint balance the conflicts between shareholders and debt holders. Under fair value accounting, the optimal solvency constraint declines with the institution's marginal cost of investment in project quality and the excess cost of equity capital relative to debt capital. Our results suggest that a uniform solvency constraint across institutions could be sub-optimal. In fact, we show that, if the solvency constraint in the fair value regime is sub-optimally chosen to be too tight, historical cost accounting actually dominates fair value accounting.

To sharpen the analysis and to highlight the main results in the paper, we developed a two-period binomial model with binary actions. However, we believe that the central trade-off between debt overhang and asset substitution would generalize to a setting with multiple states and multiple actions even though the analysis would be much more complicated. Even in a general setting, the debt overhang and asset substitution problems are either both absent or insignificant at very low leverage levels. At moderate leverage levels, asset substitution is present, but is not severe in that it occurs only in "bad" states. However, at these moderate leverage levels, the debt overhang problem is also not severe so that high quality projects are chosen anyway. Further, shutting down the possibility of asset substitution in bad states has only a minor impact from an ex ante standpoint because payoffs in these states are low to begin with. Consequently, shutting down asset substitution would have only a minor impact on the project quality choice. But, at higher leverage levels, which are more typical of financial institutions and where prudential regulation is relevant, the asset substitution problem is more severe in that asset substitution also occurs in "good" states. Further, at these leverage levels, the expected payoff from asset substitution is much higher in the good states because the corresponding call option is deep out of the money in bad states. Consequently, shutting down asset substitution has a much bigger negative impact on the expected payoffs in the good states than the bad states. Since payoffs are higher in the good states, this, in turn, has a significant negative impact on the ex ante project quality choice.

As discussed above, the key trade-off between asset substitution and debt overhang is particularly pronounced at high leverage levels when both problems are severe, and asset substitution is pervasive in that it occurs in "good" and "bad" states. The tradeoff we identify is especially relevant in the context of the recent financial crisis. Indeed, one of the primary causes of the financial crisis was risky subprime mortgage lending by banks during a period when the economy was booming and credit was cheap. Subprime mortgage lending could be more generally viewed as risky asset substitution that occurred in "good" states. Our study sheds light on the interactions between pervasive asset substitution, underinvestment and the role that prudential capital regulation based on market values plays in balancing the trade-off between asset substitution and underinvestment.

The debt issued by non-financial firms typically has associated covenants that also play the role of effecting a transfer of control if they are violated. As discussed extensively by Dewatripont and Tirole (1994), from a "high level" perspective, prudential regulation of financial firms and debt covenants for non-financial ones are "isomorphic" in that they are both mechanisms that achieve a transfer of control. As such, the main economic rationales for their presence are similar. As they emphasize, however, the debt issued by financial firms differs significantly from that issued by non-financial ones in that the former is held by widely dispersed, uninformed investors who need a "representative" in the form of a regulator. Consequently, the mechanisms through which a transfer of control is achieved differ for financial and non-financial firms. Further, as we discussed earlier, financial institutions differ from non-financial firms in that they have much higher leverage levels, which arise from higher effective costs of equity financing in the context of our model. The tradeoff we identify between asset substitution and debt overhang is particularly pronounced at high leverage levels. Consequently, although debt overhang and asset substitution are also relevant for non-financial firms, the main implications of our analysis are much more pertinent to financial institutions.

Appendix

Proof of Proposition 1

If the shareholders choose (r_H, z_H), it follows from (9) that their value is

$$(\frac{1}{2} - r_H) \max\{(1+z_H)y - M, 0\} + (\frac{1}{2} + r_H) \max\{(1-z_H)y - M, 0\}. \tag{16}$$

However, if they choose $(0, 0)$, their value is

$$\max\{y - M, 0\}. \tag{17}$$

Using expressions (16) and (17), the following table summarizes shareholders' expected payoff from asset substitution ("AS") and that from no asset substitution.

For example, for $M \in [(1-z_H)y, y]$, with probability $\frac{1}{2} - r_H$, AS will produce $(1+z_H)y$, which is larger than M, so shareholders, as a residual claimant, will get $(1+z_H)y - M$, and with probability $\frac{1}{2} + r_H$, AS will produce $(1-z_H)y$, which is smaller than M, so shareholders will get nothing. Therefore, shareholders' expected payoff from AS is $(\frac{1}{2} - r_H)[(1+z_H)y - M]$. In contrast, no AS will always produce y, which is larger than M, so shareholders will get $y - M$. Comparing the expected payoff from AS with that from no AS, $(\frac{1}{2} - r_H)[(1+z_H)y - M]$ versus $y - M$, yields the decision rule: AS if and only if $M > c_0 y$ for $M \in [(1-z_H)y, y]$.

range of M	payoff from AS	payoff from no AS	decision
$M < (1-z_H)y$	$(\frac{1}{2} - r_H)[(1+z_H)y - M]$ $+(\frac{1}{2} + r_H)[(1-z_H)y - M]$	$y - M$	no AS
$M \in [(1-z_H)y, y]$	$(\frac{1}{2} - r_H)[(1+z_H)y - M]$	$y - M$	no AS if $M < c_0 y$ AS if $M > c_0 y$
$M \in [y, (1+z_H)y]$	$(\frac{1}{2} - r_H)[(1+z_H)y - M]$	0	AS
$M > (1+z_H)y$	0	0	AS

The above table implies the following:

Table 2		
range of M	decision	payoff from decision
$M < c_0 y$	no AS	$y - M$
$M \in [c_0 y, (1+z_H)y]$	AS	$(\frac{1}{2} - r_H)[(1+z_H)y - M]$
$M > (1+z_H)y$	AS	0

■

Proof of Proposition 2

The shareholders' expected payoff from q at date $t = 0$ is their expected payoff from asset substitution decision (given in Table 2 in the proof of Proposition 1) minus the cost of investment in q. The shareholders' expected payoffs from choosing q are summarized below for all the feasible values of M.

For example, for $M \in [c_0 X_H, (1+z_H)X_H]$, we know from Table 2 that shareholders will engage in asset substitution both when $y = X_H$ and when $y = X_L$. If $y = X_H$ (which occurs with probability q), Table 2 tells us that the payoff is $(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$; if $y = X_L$ (which occurs with probability $1 - q$), Table 2 tells us that the payoff is 0. Therefore, shareholders' expected payoff from asset substitution is $q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$. Discounting this payoff to its present value at Date 0 by the cost of equity of $1 + \lambda$ and substracting the cost of quality investment of kq yields the shareholders' expected payoff from q for this range of M: $-kq + \frac{1}{1+\lambda} q(\frac{1}{2} - r_H)[(1+z_H)X_H - M]$.

range of M	asset substitution decision	shareholders' expected payoff from q
$M < c_0 X_L$	$(0, 0)$ if $y = X_H$ $(0, 0)$ if $y = X_L$	$-kq + \frac{1}{1+\lambda}\{q[X_H - M]$ $+(1 - q)[X_L - M]\}$
$M \in [c_0 X_L, (1 + z_H)X_L]$	$(0, 0)$ if $y = X_H$ (r_H, z_H) if $y = X_L$	$-kq + \frac{1}{1+\lambda}\{q[X_H - M]$ $+(1 - q)(\frac{1}{2} - r_H)[(1 + z_H)X_L - M]\}$
$M \in [(1 + z_H)X_L, c_0 X_H]$	$(0, 0)$ if $y = X_H$ (r_H, z_H) if $y = X_L$	$-kq + \frac{1}{1+\lambda} q[X_H - M]$
$M \in [c_0 X_H, (1 + z_H)X_H]$	(r_H, z_H) if $y = X_H$ (r_H, z_H) if $y = X_L$	$-kq + \frac{1}{1+\lambda} q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$

Using the above table, we can investigate the shareholders' quality decision by comparing the shareholders' expected payoff from q_H and that from q_L and derive the following decision rules:

Case 1: $M < c_0 X_L$: Shareholders choose q_H if and only if $k < \frac{1}{1+\lambda}(X_H - X_L)$, which is true by assumption in (6). Therefore, shareholders will choose q_H.

Case 2: $M \in [c_0 X_L, (1+z_H)X_L]$: Shareholders choose q_H if and only if $M < \frac{X_H - (\frac{1}{2} - r_H)(1+z_H)X_L - k(1+\lambda)}{\frac{1}{2} + r_H}$.

Case 3: $M \in [(1 + z_H)X_L, c_0 X_H]$: Shareholders choose q_H if and only if $M < c_1 X_H$.

Case 4: $M \in [c_0 X_H, (1 + z_H)X_H]$: Shareholders choose q_H if and only if $M < c_2 X_H$.

Using the above results, we derive the optimal choice of q for different values of k.

(i) $k < k^*$: Shareholders choose q_H if and only if $M < c_2 X_H$.

Proof: In Case 1, shareholders will always choose q_H. In Case 2, shareholders choose q_H if and only if $M < \frac{X_H - (\frac{1}{2} - r_H)(1+z_H)X_L - k(1+\lambda)}{\frac{1}{2} + r_H}$. But even for the highest possible value of M in Case 2, $(1 + z_H)X_L$, that inequality always holds as long as $k < X_H - (1 + z_H)X_L$. Because of the assumption of $(1 + z_H)X_L = (1 - z_H)X_H$, $k < X_H - (1 + z_H)X_L \Leftrightarrow k < z_H X_H$, which is satisfied because $k < k^*$. So q_H will be chosen. In Case 3, shareholders choose q_H if and only if $M < c_1 X_H$. But even for the highest possible value of M in Case 3, $c_0 X_H$, that inequality always holds as long as $k < k^*$, which is true by the assumption for k in this scenario (i). So q_H will be chosen. In Case 4, shareholders choose q_H if and only if $M < c_2 X_H$, which is exactly stated in the statements of this proposition.

(ii) $k \in [k^*, (X_H - (1+z_H)X_L)/(1+\lambda)]$: shareholders choose q_H if and only if $M < c_1 X_H$. (The proof is analogous to that in case (i) and so is omitted.)

(iii) $k > (X_H - (1+z_H)X_L)/(1+\lambda)$: This case is infeasible by the assumption in (6).

We summarize the shareholders' expected payoff from the optimal choice of q in the following:

Table 3 (the case of $k < k^*$)		
range of M	choice of q	shareholders' expected payoff from the optimal choice of q
$M < c_0 X_L$	q_H	$-kq_H + \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$
$M \in [c_0 X_L, (1+z_H)X_L]$	q_H	$-kq_H + \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)(\frac{1}{2} - r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2} - r_H)]M\}$
$M \in [(1+z_H)X_L, c_0 X_H]$	q_H	$-kq_H + \frac{1}{1+\lambda} q_H [X_H - M]$
$M \in [c_0 X_H, c_2 X_H]$	q_H	$-kq_H + \frac{1}{1+\lambda} q_H (\frac{1}{2} - r_H)[(1+z_H)X_H - M]$
$M \in [c_2 X_H, (1+z_H)X_H]$	q_L	0

The payoff for $k > k^*$ is similar and so is omitted. ■

Lemma 1 *Under the historical cost regime, the shareholders' optimal choice of the maturity value M of debt is as follows:*

$\lambda < \lambda_1$:	$\lambda \in [\lambda_1, \lambda_2]$:	$\lambda > \lambda_2$:
$M = c_0 X_L$	$M = c_0 X_H$	$M = c_2 X_H$

,

where

$$c_0 \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H; \; c_2 \equiv (1+z_H) - \frac{k(1+\lambda)}{(\frac{1}{2} - r_H)X_H};$$

$$\lambda_1 \equiv \frac{2r_H z_H}{(1 - 2r_H z_H) - -c_0 \frac{1 - q_H X_H / X_L}{1 - q_H}}; \; \lambda_2 \equiv \frac{2r_H z_H}{(k^* - k)/X_H}.$$

<u>Proof of Lemma 1</u>

We use (10) to derive the shareholders' expected payoff at the time when they make capital structure decision. This payoff is the shareholders' expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus E_0, the shareholders' equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive D_0, the equilibrium debt price at Date 0, and then substitute this price into the shareholders' expected payoff, and finally derive the shareholders' optimal choice of M.

(i) The case where $k < k^*$:

Under the historical cost regime, taking into consideration of the optimal choices of (r, z) and q, we first derive D_0 using (**??**). Table 4 summarizes various values of D_0 for different ranges of M.

For example, for $M \in [(1+z_H)X_L, c_0 X_H]$, we know from Table 3 that shareholders will choose q_H and therefore $D_0 = q_H E[M, X(q_H, (0,0))] + (1-q_H)E[M, X(q_H, (r_H, z_H))]$. Because $X(q_H, (0,0)) = X_H > M$, debtholders expect to receive M; because $X(q_H, (r_H, z_H)) = (1 - 2r_H z_H)X_L < M$, debtholders expect to receive $(1 - 2r_H z_H)X_L$. Therefore, $D_0 = q_H M + (1-q_H)(1 - 2r_H z_H)X_L$.

Table 4	
range of M	D_0
$M \leq c_0 X_L$	M
$M \in [c_0 X_L, (1+z_H)X_L]$	$[q_H + (1-q_H)(\frac{1}{2} - r_H)]M + (1-q_H)(\frac{1}{2} + r_H)(1-z_H)X_L$
$M \in [(1+z_H)X_L, c_0 X_H]$	$q_H M + (1-q_H)(1-2r_H z_H)X_L$
$M \in [c_0 X_H, c_2 X_H]$	$q_H(\frac{1}{2} - r_H)M + q_H(\frac{1}{2} + r_H)(1-z_H)X_H + (1-q_H)(1-2r_H z_H)X_L$
$M \in [c_2 X_H, (1+z_H)X_H]$	$(1-2r_H z_H)X_L$

Substituting D_0 in Table 4 into the shareholders' expected payoff, which is the shareholders' expected payoff from quality decision (given in Table 3 in the proof of Proposition 2) minus $E_0 = A_0 - D_0$, yields the following:

range of M	Shareholders' expected payoff
$M \leq c_0 X_L$	$-A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda}M$
$M \in [c_0 X_L, (1+z_H)X_L]$	$-A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)(\frac{1}{2} - r_H)(1+z_H)X_L]$ $+(1-q_H)(\frac{1}{2} + r_H)(1-z_H)X_L + \frac{\lambda}{1+\lambda}[q_H + (1-q_H)(\frac{1}{2} - r_H)]M$
$M \in [(1+z_H)X_L, c_0 X_H]$	$-A_0 - kq_H + \frac{1}{1+\lambda}q_H X_H + (1-q_H)(1-2r_H z_H)X_L + \frac{\lambda}{1+\lambda}q_H M$
$M \in [c_0 X_H, c_2 X_H]$	$-A_0 - kq_H + \frac{1}{1+\lambda}q_H(\frac{1}{2} - r_H)(1+z_H)X_H + q_H(\frac{1}{2} + r_H)(1-z_H)X_H$ $+(1-q_H)(1-2r_H z_H)X_L + \frac{\lambda}{1+\lambda}q_H(\frac{1}{2} - r_H)M$
$M \in [c_2 X_H, (1+z_H)X_H]$	$-A_0 + (1-2r_H z_H)X_L$

Because regional payoffs are increasing in M, the optimal value of M for a given region is the upper bound of that region. Substituting the regional optimal M into the regional payoff function yields the regional maximal expected payoffs for shareholders at Date 0. For example, for the region of $M \leq c_0 X_L$, the shareholders' expected payoff is increasing in M and so the optimal value of M for this particular region is $c_0 X_L$. Inserting $M = c_0 X_L$ into the payoff function for this region yields $-A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda}c_0 X_L$. Similar analyses apply to other regions. Note that the payoff in the third region is always larger than that in the second region, and therefore we combined those two regions. In addition, the last region is never optimal because the payoff of $-A_0 + (1-2r_H z_H)X_L$ is negative by the assumption that a project with a low quality and asset substitution is a negative NPV project.

Table 5		
range of M	choice of M	Shareholders' *ex ante* payoff
$M \leq c_0 X_L$	$c_0 X_L$	$\pi_I^{HC} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L\}$ $+\frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H)X_L$
$M \in [c_0 X_L, c_0 X_H]$	$c_0 X_H$	$\pi_{II}^{HC} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)(1-2r_H z_H)X_L\}$ $+\frac{\lambda}{1+\lambda}\{q_H(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H)X_H + (1-q_H)(1-2r_H z_H)X_L\}$
$M \in [c_0 X_H, c_2 X_H]$	$c_2 X_H$	$\pi_{III}^{HC} \equiv -A_0 - kq_H$ $+\frac{1}{1+\lambda}\{q_H(1-2r_H z_H)X_H + (1-q_H)(1-2r_H z_H)X_L\}$ $+\frac{\lambda}{1+\lambda}\{q_H[(1-2r_H z_H)X_H - k(1+\lambda)] + (1-q_H)(1-2r_H z_H)X_L\}$

Note that π_I^{HC}, π_{II}^{HC}, and π_{III}^{HC} are all decreasing in λ, and at $\lambda = 0$, $\pi_I^{HC} > \pi_{II}^{HC} > \pi_{III}^{HC}$.

Comparing those three payoffs demonstrates that shareholders prefer π_I^{HC} for low values of λ, π_{III}^{HC} for high values of λ, and π_{II}^{HC} for intermediate values of λ:

$$\pi_I^{HC} > \pi_{II}^{HC} \Leftrightarrow \lambda < \lambda_1;$$

$$\pi_{III}^{HC} > \pi_{II}^{HC} \Leftrightarrow \lambda > \lambda_2.$$

Therefore, the value of λ dictates the choice of M. For example, when $\lambda \in [\lambda_1, \lambda_2]$, shareholders prefer π_{II}^{HC}. To induce it, they set M to be $c_0 X_H$.

(ii) The case where $k > k^*$: The analysis of this case is analogous to that of the preceding case and so is omitted. ■

Proof of Proposition 4

The outcome of control transfer or continuation depends on the prudential constraint (c) and the debt/asset ratio at Date 1 ($\frac{D_1}{A_1}$). For the feasible range of values of M, using equations (**??**) through (**??**), Table 6 shows the corresponding market values D_1 and A_1:

Table 6 Part A

range of M	$(\widehat{r}, \widehat{z}) = (r_H, z_H)$
$M < (1 - z_H)y$	$D_1 = M$ and $E_1 = \frac{(1-2r_H z_H)y - M}{\sqrt{1+\lambda}}$ $\Longrightarrow \frac{D_1}{A_1} = \frac{D_1}{D_1+E_1} < c \Longleftrightarrow M < \frac{c(1-2r_H z_H)}{\sqrt{1+\lambda} - c(\sqrt{1+\lambda}-1)} y$
$M \in [(1 - z_H)y, (1 + z_H)y]$	$D_1 = (\frac{1}{2} - r_H)M + (\frac{1}{2} + r_H)(1 - z_H)y$ and $E_1 = \frac{(\frac{1}{2}-r_H)[(1+z_H)y - M]}{\sqrt{1+\lambda}}$ $\Longrightarrow \frac{D_1}{A_1} = \frac{D_1}{D_1+E_1} < c \Longleftrightarrow M < \frac{c(\frac{1}{2}-r_H)(1+z_H) - (1-c)\sqrt{1+\lambda}(\frac{1}{2}+r_H)(1-z_H)}{[\sqrt{1+\lambda} - c(\sqrt{1+\lambda}-1)](\frac{1}{2}-r_H)} y$
$M > (1 + z_H)y$	$D_1 = (1 - 2r_H z_H)y$ and $E_1 = 0$ $\Longrightarrow \frac{D_1}{A_1} = \frac{D_1}{D_1+E_1} = 1 \geq c$

Table 6 Part B

range of M	$(\widehat{r}, \widehat{z}) = (0, 0)$
$M < y$	$D_1 = M$ and $E_1 = \frac{y-M}{\sqrt{1+\lambda}}$ $\Longrightarrow \frac{D_1}{A_1} = \frac{D_1}{D_1+E_1} < c \Longleftrightarrow M < T(c)y$
$M > y$	$D_1 = y$ and $E_1 = 0$ $\Longrightarrow \frac{D_1}{A_1} = \frac{D_1}{D_1+E_1} = 1 \geq c$

Note that Table 6 implies the following three general facts:

(i) Given $(\widehat{r}, \widehat{z}) = (r_H, z_H)$, for $M > (1 + z_H)y$, transfer of control will occur for sure, for $M \in [(1 - z_H)y, (1+z_H)y]$, transfer of control will occur if and only if $M > \frac{c(\frac{1}{2}-r_H)(1+z_H) - (1-c)\sqrt{1+\lambda}(\frac{1}{2}+r_H)(1-z_H)}{[\sqrt{1+\lambda} - c(\sqrt{1+\lambda}-1)](\frac{1}{2}-r_H)} y$, and for $M < (1 - z_H)y$, transfer of control will occur if and only if $M > \frac{c(1-2r_H z_H)}{\sqrt{1+\lambda} - c(\sqrt{1+\lambda}-1)} y$.

(ii) Given $(\widehat{r}, \widehat{z}) = (0, 0)$, for $M > y$, transfer of control will occur for sure, and for $M < y$, transfer of control will occur if and only if $M > T(c)y$.

Claim: When $T(c)y \leq c_0 y$, shareholders will choose $(r, z) = (0, 0)$.

Proof. Suppose the conjecture is that $(\widehat{r}, \widehat{z}) = (0, 0)$.

For $M > y$, transfer of control will occur and so $(r, z) = (0, 0)$.

For $M < y$, transfer of control will occur and so $(r, z) = (0, 0)$ if $M > T(c)y$ and continuation will occur if $M < T(c)y$. In the latter case, because $T(c)y \leq c_0 y$, by Proposition 1, shareholders will choose $(r, z) = (0, 0)$.

Altogether, shareholders will choose $(r, z) = (0, 0)$ in all cases, thereby confirming the conjecture.

■

Claim: When $T(c)y > c_0 y$, shareholders will choose $(r, z) = (r_H, z_H)$ if $M \in [c_0 y, T(c)y]$ and choose $(r, z) = (0, 0)$ in all other cases.

Proof. (i) Suppose the conjecture is that $(\widehat{r}, \widehat{z}) = (0, 0)$ when $M > T(c)y$.

For $M > y$, transfer of control will occur and so $(r, z) = (0, 0)$. For $M \in [T(c)y, y]$, transfer of control will occur and so $(r, z) = (0, 0)$. Altogether, shareholders will choose $(r, z) = (0, 0)$ in both cases, thereby confirming the conjecture.

(ii) Suppose the conjecture is that $(\widehat{r}, \widehat{z}) = (0, 0)$ when $M < c_0 y$.

Continuation will occur because $M < c_0 y < T(c)y$. By Proposition 1, shareholders will choose $(r, z) = (0, 0)$, thereby confirming the conjecture.

(iii) Suppose the conjecture is that $(\widehat{r}, \widehat{z}) = (r_H, z_H)$ when $M \in [c_0 y, T(c)y]$.

Note that $[c_0 y, T(c)y] \in [(1 - z_H)y, \frac{c(\frac{1}{2} - r_H)(1+z_H) - (1-c)\sqrt{1+\lambda}(\frac{1}{2} + r_H)(1-z_H)}{[\sqrt{1+\lambda} - c(\sqrt{1+\lambda} - 1)](\frac{1}{2} - r_H)} y]$. Therefore, continuation will occur. Because $M > c_0 y$, by Proposition 1, shareholders will choose $(r, z) = (r_H, z_H)$, thereby confirming the conjecture. ■

The above two claims imply the following:

Table 7		
When $T(c)y > c_0 y$		
range of M	*decision*	*payoff from decision*
$M < c_0 y$	no AS	$y - M$
$M \in [c_0 y, T(c)y]$	AS	$(\frac{1}{2} - r_H)[(1 + z_H)y - M]$
$M \in [T(c)y, y]$	no AS	$y - M$
$M \in [y, (1 + z_H)y]$	no AS	0
When $T(c)y \leq c_0 y$		
range of M	*decision*	*payoff from decision*
$M < y$	no AS	$y - M$
$M \in [y, (1 + z_H)y]$	no AS	0

■

Proof of Proposition 5

The shareholders' expected payoff from choosing q at date $t = 0$ is their expected payoff from asset substitution decision (given in Table 7 in the proof of Proposition 4) minus the cost of quality investment.

For low values of $T(c) \leq c_0$, the payoffs are summarized in Table 8 Part A, and for high values of $T(c) > c_0$, the payoffs are summarized in Table 8 Part B.

Table 8 Part A: $T(c) \le c_0$	
range of M	shareholders' expected payoff from choosing q
$M < X_L$ region 1	no AS regardless of y $-kq + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [X_L, X_H]$ region 2	no AS regardless of y $-kq + \frac{1}{1+\lambda}q[X_H - M]$
$M \in [X_H, (1+z_H)X_H]$ region 3	no AS regardless of y $-kq$

Table 8 Part B: $T(c) > c_0$	
range of M	shareholders' expected payoff from choosing q
$M < c_0 X_L$ region 1	no AS regardless of y $-kq + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [c_0 X_L, T(c)X_L]$ region 2	no AS if $y = X_H$ and AS if $y = X_L$ $-kq + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)(\frac{1}{2} - r_H)[(1+z_H)X_L - M]\}$
$M \in [T(c)X_L, X_L]$ region 3	no AS regardless of y $-kq + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [X_L, c_0 X_H]$ region 4	no AS regardless of y $-kq + \frac{1}{1+\lambda}q[X_H - M]$
$M \in [c_0 X_H, T(c)X_H]$ region 5	AS if $y = X_H$ and no AS if $y = X_L$ $-kq + \frac{1}{1+\lambda}q(\frac{1}{2} - r_H)[(1+z_H)X_H - M]\}$
$M \in [T(c)X_H, X_H]$ region 6	no AS regardless of y $-kq + \frac{1}{1+\lambda}q[X_H - M]$
$M \in [X_H, (1+z_H)X_H]$ region 7	no AS regardless of y $-kq$

Using Table 8, we derive the following optimal quality choices in the FV regime.

We first analyze the case where $T(c) \le c_0$. It is obvious that shareholders always choose q_H in region 1 and q_L in region 3. In region 2, the shareholders choose q_H if and only if $M < c_1 X_H$.

For the case where $T(c) > c_0$, using Table 8, a similar analysis can be done. It is obvious that shareholders always choose q_H in regions 1, 2, and 3 and q_L in region 7. In regions 4 and 6, the shareholders choose q_H if and only if $M < c_1 X_H$. In region 5, the shareholders choose q_H if and only if $M < c_2 X_H$.

(i) $k < k^*$: In regions 1, 2, and 3, shareholders always choose q_H. Furthermore, $k < k^*$ implies that $c_2 X_H > c_1 X_H$, which implies that shareholders choose q_H in region 4. The choices of q in regions 5 and 6 depend on the values of c. When $T(c) < c_1$, shareholders always choose q_H in region 5 but chooses q_H in region 6 if and only if $M < c_1 X_H$. When $T(c) > c_2$, shareholders always choose q_L in region 6 but chooses q_H in region 5 if and only if $M < c_2 X_H$. When $T(c) \in [c_1, c_2]$, shareholders always choose q_H in region 5 and q_L in region 6; therefore, the cutoff value of M dividing the q_H and q_L regions is the boundary of regions 5 and 6, that is, $T(c)X_H$. Finally, shareholders always choose q_L in region 7.

We summarize the shareholders' expected payoff from the optimal choice of q in the following, where payoffs in Scenarios A, C, and D are expressed in terms of payoffs in Scenario B:

Table 9 Scenario A: $T(c) \leq c_0$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < X_L$	q_H	π_3
$M \in [X_L, c_1 X_H]$	q_H	π_6
$M \in [c_1 X_H, (1+z_H)X_H]$	q_L	π_7

Table 9 Scenario B: $T(c) \in [c_0, c_1]$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < c_0 X_L$	q_H	$\pi_1 \equiv \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\} - kq_H$
$M \in [c_0 X_L, T(c)X_L]$	q_H	$\pi_2 \equiv \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)(\frac{1}{2} - r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2} - r_H)]M\} - kq_H$
$M \in [T(c)X_L, X_L]$	q_H	$\pi_3 \equiv \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\} - kq_H$
$M \in [X_L, c_0 X_H]$	q_H	$\pi_4 \equiv \frac{1}{1+\lambda} q_H [X_H - M] - kq_H$
$M \in [c_0 X_H, T(c)X_H]$	q_H	$\pi_5 \equiv \frac{1}{1+\lambda} q_H (\frac{1}{2} - r_H)[(1+z_H)X_H - M] - kq_H$
$M \in [T(c)X_H, c_1 X_H]$	q_H	$\pi_6 \equiv \frac{1}{1+\lambda} q_H [X_H - M] - kq_H$
$M \in [c_1 X_H, (1+z_H)X_H]$	q_L	$\pi_7 \equiv 0$

Table 9 Scenario C: $T(c) \in [c_1, c_2]$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < c_0 X_L$	q_H	π_1
$M \in [c_0 X_L, T(c)X_L]$	q_H	π_2
$M \in [T(c)X_L, X_L]$	q_H	π_3
$M \in [X_L, c_0 X_H]$	q_H	π_4
$M \in [c_0 X_H, T(c)X_H]$	q_H	π_5
$M \in [T(c)X_H, (1+z_H)X_H]$	q_L	π_7

Table 9 Scenario D: $T(c) > c_2$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < c_0 X_L$	q_H	π_1
$M \in [c_0 X_L, T(c)X_L]$	q_H	π_2
$M \in [T(c)X_L, X_L]$	q_H	π_3
$M \in [X_L, c_0 X_H]$	q_H	π_4
$M \in [c_0 X_H, c_2 X_H]$	q_H	π_5
$M \in [c_2 X_H, (1+z_H)X_H]$	q_L	π_7

(ii) $k > k^*$: The analysis is analogous to that in (i) and thus is omitted. ∎

Lemma 2 *In the fair value regime, the shareholders' optimal choice of the maturity value M of debt is as follows:*

$T(c) > c_2$:	$\lambda < \lambda_2$: $M = c_0 X_H$	$\lambda > \lambda_2$: $M = c_2 X_H$	
$T(c) \in [c_1, c_2]$:	$\lambda < \lambda_3$: $M = c_0 X_H$	$\lambda > \lambda_3$: $M = T(c) X_H$	,
$T(c) < c_1$:	$M = c_1 X_H$		

where

$$c_0 \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H; \; c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}; \; c_2 \equiv (1 + z_H) - \frac{k(1+\lambda)}{(\frac{1}{2} - r_H) X_H}; \; T(c) \equiv \frac{c}{\sqrt{1+\lambda} - c(\sqrt{1+\lambda} - 1)};$$

$$\lambda_2 \equiv \frac{2 r_H z_H}{(k^* - k)/X_H}; \; \lambda_3 \equiv \frac{2 r_H z_H}{(\frac{1}{2} - r_H) T(c) + (\frac{1}{2} + r_H)(1 - z_H) - c_0}.$$

Proof of Lemma 2

We derive the shareholders' expected payoff at the time when they make capital structure decision. This payoff is the shareholders' expected payoff from quality decision (given in Table 9 in the proof of Proposition 5) minus E_0, the shareholders' equity investment, which equals $A_0 - D_0$. Therefore, in the following, we first derive D_0, the equilibrium debt price at Date 0, and then substitute this price into the shareholders' expected payoff, and finally derive the shareholders' optimal choice of M.

Table 9 identifies four scenarios, A, B, C, and D. We first analyze Scenario B.

Scenario B: Taking into consideration of the optimal choices of (r, z) and q, we first derive $D_0 = E[\min\{M, \widetilde{X}\}]$:

Table 10: Scenario B: $T(c) \in [c_0, c_1]$

range of M	D_0
$M < c_0 X_L$	M
$M \in [c_0 X_L, T(c) X_L]$	$[q_H + (1 - q_H)(\frac{1}{2} - r_H)]M + (1 - q_H)(\frac{1}{2} + r_H)(1 - z_H) X_L$
$M \in [T(c) X_L, X_L]$	M
$M \in [X_L, c_0 X_H]$	$q_H M + (1 - q_H) X_L$
$M \in [c_0 X_H, T(c) X_H]$	$q_H(\frac{1}{2} - r_H)M + q_H(\frac{1}{2} + r_H)(1 - z_H) X_H + (1 - q_H) X_L$
$M \in [T(c) X_H, c_1 X_H]$	$q_H M + (1 - q_H) X_L$
$M \in [c_1 X_H, (1 + z_H) X_H]$	X_L

We now substitute the equilibrium values of D_0 in Table 10 into the expected payoff at the time when shareholders make capital structure decision, which is the shareholders' expected payoff from quality decision (given in Table 9) minus $E_0 = A_0 - D_0$. We do it region by region. As it turns out, the regional payoff increases in M, and so the regional optimal M is the upper bound of the region. Evaluating the regional payoff at the regional optimal M yields the regional maximal payoffs in the following:

Table 11: Scenario B: $T(c) \in [c_0, c_1]$		
range of M	choice of M	shareholders' *ex ante* payoff
$M < c_0 X_L$	$c_0 X_L$	$\pi_{B1} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$
$M \in [c_0 X_L, T(c)X_L]$	$T(c)X_L$	$\pi_{B2} \equiv -A_0 - kq_H$ $+\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)(1-2r_H z_H)X_L\}$ $+\frac{\lambda}{1+\lambda}\{[q_H + (1-q_H)(\frac{1}{2} - r_H)]\frac{c}{\sqrt{1+\lambda}-c(\sqrt{1+\lambda}-1)}X_L$ $+(1-q_H)(\frac{1}{2}+r_H)(1-z_H)X_L\}$
$M \in [T(c)X_L, X_L]$	X_L	$\pi_{B3} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda}X_L$
$M \in [X_L, c_0 X_H]$	$c_0 X_H$	$\pi_{B4} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L\}$ $+\frac{\lambda}{1+\lambda}\{q_H(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H + (1-q_H)X_L\}$
$M \in [c_0 X_H, T(c)X_H]$	$T(c)X_H$	$\pi_{B5} \equiv -A_0 - kq_H$ $+\frac{1}{1+\lambda}\{q_H(1-2r_H z_H)X_H + (1-q_H)X_L\}$ $+\frac{\lambda}{1+\lambda}\{q_H[(\frac{1}{2} - r_H)T(c) + (\frac{1}{2}+r_H)(1-z_H)]X_H$ $+(1-q_H)X_L\}$
$M \in [T(c)X_H, c_1 X_H]$	$c_1 X_H$	$\pi_{B6} \equiv -A_0 - kq_H + q_H X_H + (1-q_H)X_L - \lambda k q_H$
$M \in [c_1 X_H, (1+z_H)X_H]$	$[c_1 X_H,$ $(1+z_H)X_H]$	$\pi_{B7} \equiv -A_0 + X_L$

It is straightforward to show the following results:

$$\pi_{B3} > \pi_{B1};\ \pi_{B3} > \pi_{B2};\ \pi_{B4} > \pi_{B3};\ \pi_{B6} > \pi_{B4};\ \pi_{B6} > \pi_{B5};\ \pi_{B6} > \pi_{B7}.$$

Therefore, π_{B6} is the highest payoff in Scenario B. To induce π_{B6}, by Table 11, shareholders must choose $M = c_1 X_H$.

Scenario A: By similar reasoning process demonstrated in Scenario B, we can derive π_{A1} to π_{A3} for Scenario A, using Table 9 Scenario A, where π_{A1} to π_{A3} are the regional maximal payoffs for the three regions in Scenario A. It is straightforward to show the following results:

$$\pi_{A1} = \pi_{B3};\ \pi_{A2} = \pi_{B6};\ \pi_{A3} = \pi_{B7}.$$

Therefore, $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenario A. To induce π_{A2}, shareholders must choose $M = c_1 X_H$.

Scenario C: By similar reasoning process demonstrated in Scenario B, we can derive π_{C1} to π_{C6} for Scenario C, using Table 9 Scenario C, where π_{C1} to π_{C6} are the regional maximal payoffs for the six regions in Scenario C. It is straightforward to show the following results:

$$\pi_{C1} = \pi_{B1};\ \pi_{C2} = \pi_{B2};\ \pi_{C3} = \pi_{B3};\ \pi_{C4} = \pi_{B4};\ \pi_{C5} = \pi_{B5};\ \pi_{C6} = \pi_{B7}.$$

Therefore, π_{C4} and π_{C5} dominate the other payoffs. $\pi_{C4} > \pi_{C5}$ if and only if $\lambda < \lambda_3$. To induce

π_{C4}, shareholders must choose $M = c_0 X_H$; to induce π_{C5}, shareholders must choose $M = T(c)X_H$.

Scenario D: By similar reasoning process demonstrated in Scenario B, we can derive π_{D1} to π_{D6} for Scenario D, using Table 9 Scenario D, where π_{D1} to π_{D6} are the regional maximal payoffs for the six regions in Scenario D. It is straightforward to show the following results:

$$\begin{aligned}
\pi_{D1} &= \pi_{B1};\ \pi_{D2} = \pi_{B2};\ \pi_{D3} = \pi_{B3};\ \pi_{D4} = \pi_{B4}; \\
\pi_{D5} &= -A_0 - kq_H + \frac{1}{1+\lambda}\{q_H(1-2r_H z_H)X_H + (1-q_H)X_L\} \\
&\quad + \frac{\lambda}{1+\lambda}\{q_H[(1-2r_H z_H)X_H - k(1+\lambda)] + (1-q_H)X_L\}; \\
\pi_{D6} &= \pi_{B7}.
\end{aligned}$$

Therefore, π_{D4} and π_{D5} dominate the other payoffs. $\pi_{D4} > \pi_{D5}$ if and only if $\lambda < \lambda_2$. To induce π_{D4}, shareholders must choose $M = c_0 X_H$; to induce π_{D5}, shareholders must choose $M = c_2 X_H$.■

Proof of Proposition 6

The proof of Lemma 2 shows that π_{B6} is the highest payoff in Scenario B and that $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenario A.

It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario C. Specifically, region 6 in Scenario B does not exist in Scenario C.

It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario D. Specifically, not only region 6 in Scenario B does not exist in Scenario D, but also the payoff in region 5 in B is larger than the payoff in region 5 in D.

Therefore, the highest payoff among all the four scenarios is $\pi_{A2} = \pi_{B6}$, and so the regulator will choose c to induce Scenarios A and B.

Recall that Scenario A will be viable when $T(c) \leq c_0$. Recall also that Scenario B will be viable when $T(c) \in [c_0, c_1]$. Therefore, to induce Scenario A and/or B, it suffice for the regulator to set $c \leq c_1$. However, any further reduction of c below c_1 will constrain shareholders' choice of M at date 0 and therefore damage their *ex ante* welfare. This tension gives rise to the socially optimal constraint, $T(c) = c_1 \Leftrightarrow c = \frac{1}{1+\frac{k\sqrt{1+\lambda}}{X_H - k(1+\lambda)}}$. ■

Proof of Proposition 7

If $k > k^*$, from Propositions 2 and 5, the threshold value of M above which triggers underinvestment in both regimes is $c_1 X_H$.

If $k \leq k^*$, from Proposition 2, the threshold value of M above which triggers underinvestment in the *HC* regime is $c_2 X_H$. From Proposition 5 it is $c_1 X_H$, $T(c)X_H$, or $c_2 X_H$ under the *FV* regime. The underinvestment region is larger under *FV* regime than that under *HC* regime if and only if $c_1 X_H < c_2 X_H \Leftrightarrow k \leq k^*$, which is true by assumption. Furthermore, $T(c)X_H < c_2 X_H \Leftrightarrow T(c) < c_2$,which is true when $T(c)X_H$ is the threshold value. When $\frac{1}{2} - r_H$ and/or z_H increases, both c_2 and k^* increases, thereby expanding the underinvestment (q_L) region. ■

Proof of Proposition 9

We already know from the proof of Proposition 8 that at $T(c) = c_1$, the fair value regime dominates the historical cost regime. We show in the following that at $c = 0 \Rightarrow T(c) = 0$, the historical cost regime dominates the fair value regime. Taken together these two facts, by continuity, there must exist $\underline{c}$ such that for $c \in [0, \underline{c})$ the historical cost regime dominates the fair value regime.

Under the fair value regime, when $c = 0$, it must be the case that the debt/asset ratio at date 0, $\frac{D_0}{A_0}$, exceeds or equals $c = 0$. For the business to continue beyond date 0, the shareholders must choose the minimal face value of debt in order to satisfy the solvency constraint. Therefore, $M(c = 0) = 0$. This face value of debt implies that the shareholders' *ex ante* payoff, given in Table 9 Scenario A in the proof of Proposition 5, is

$$\begin{aligned} & -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L - M] \\ = & -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L], \end{aligned}$$

which is less than its counterpart in the historical cost regime, given in Table 5 in the proof of Lemma 1,

$$\pi_I^{HC} \equiv -A_0 - kq_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H)X_L.$$

Therefore, at $c = 0$, the historical cost regime dominates the fair value regime. ■

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