Comprehensive Correlation and Capital Estimates for a Canadian SME Portfolio: Implications for Basel

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ABSTRACT

We present a unique and rich source of Canadian SME credit data, and use it to estimate asset correlations among risk- and size-based segments of this crucial segment of the financial sector. The availability of such a rich dataset, concentrated within a single portfolio, presents a significant contrast with the existing literature on asset correlation, which has typically relied on aggregated data sets of SME borrowers with, in many cases, limited historical span. Our estimates of SME asset correlations from this high risk SME portfolio point to a lack of patterns across SME risk and size segments. These results run counter to the assumptions found in Basel III while also providing for implementation suggestions for regulators considering SME portfolios. We use internally-calibrated asset correlations to generate internally-calibrated capital charges and compare them with those obtained under Basel III. We show that the application of dual classification creates capital misallocation within the SME portfolio – allowing for two varying levels of prudence – as compared to the internally calibrated capital charges. The Basel treatment's strong and controversial assumptions on the behaviour of Small and Medium Enterprise (SME) borrowers are challenged in this paper and we present clear policy recommendations allowing for the coherent allocation of required regulatory capital charges and internally calibrated capital charges.

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1. Introduction

The Basel III treatment of portfolio credit risks places strong and controversial assumptions on the behaviour of Small and Medium Enterprise (SME) borrowers within a credit portfolio. These assumptions include decreasing asset correlations with decreasing borrower size, for SME borrowers treated under the Corporate asset class, while SME borrowers treated under the Retail asset class are, by default, assigned generally lower asset correlations. Under both treatments, Basel III proposes a negative relationship between asset correlation and probability of default (PD).

Empirical evidence on these strong assumptions has been mixed. Duellmann and Scheule (2003) and Lopez (2004) find evidence of increasing asset correlation with borrower size, while Dietsch and Petey (2004) find evidence to the contrary, rejecting that assumption. On the relationship between asset correlations and PD the results appear weaker but nonetheless contradictory, with Lopez (2004) showing signs of a negative relationship and Gordy (2000) and Dietsch and Petey (2004) finding a positive relationship. This work, and especially that focused on SME borrowers, has been marked by aggregated data sets and generally limited historical span.

We present a unique and rich source of Canadian SME credit data, and use it to estimate the portfolio credit risk characteristics of this crucial segment of the financial sector. The availability of such a rich dataset, made up of over 25,000 SME borrowers spanning a time period from 1997 to 2010, concentrated within a single portfolio presents a significant contrast with the existing literature which has typically relied on aggregated data sets of SME borrowers with, in many cases, limited historical span. For example, Dietsch and Petey (2002) estimate portfolio credit risk over a database of 220,000 French SME borrowers, accounting for more than two thirds of all French SMEs, Dietsch and Petey (2002, p. 305), spanning the period 1995 to 1999; Dietsch and Petey (2004) estimate SME portfolio credit risk characteristics over an aggregated database of 440,000 French and 280,000 German borrowers spanning the periods 1995 to 2001, and 1997 to 2001, respectively; Duellmann and Scheule (2003) use another aggregated database of over 53,000 predominantly small and private-owned German borrowers spanning the period 1991 to 2000, while Jacobson, Linde, and Roszbach (2005) study the riskiness of SME borrowers as compared to larger borrowers over an aggregated database of approximately 60,000 Swedish borrowers spanning the period 1997 to 2000.

In and of itself, the use of aggregated data presents a potential for a dilution of risk characteristics and, for single institutions looking to benefit from SME portfolio credit risk analysis, a potential for a misrepresentation of the risks as they may relate to a single lending entity; see, e.g., Basurto and Padilla (2006) and Dietsch and Petey (2004).

In this paper, assumptions on the level and relationships of asset correlations in Basel III across SME segments are tested. We present a careful and robust partition of our data into homogenous segments of borrowers, benefiting from significant data depth to present dual segmentations of our borrowers by Risk and Size Groups (RG and SG, respectively). We use methods presented in Gordy (2000) to estimate asset corrleations from default rates using a single factor implementation of the asset value model (*AVM*) within these segments and evaluate patterns across them.

Our results show that SME portfolios typically exhibit low asset correlation values, and we find no empirical evidence of either a positive relationship with size or a negative relationship with PD for our set high risk SME borrowers. This result runs counter to Basel III specifications of a negative relationship between asset correlations and PD, and counters the Corporate asset class assumption of a positive relationship between asset correlation and Size. This finding of low asset correlation values and no relationship between asset correlations and Size appears to provide some support to specifications under the Retail-Other treatment. In addition, our results on the lack of relationships between asset correlations and Size and PD, contrast with the literature wherein such relationships have been deduced from generally weak empirical evidence.

Having generated internally-calibrated asset correlation values for our portfolio of SME exposures, we implement a simulation-based Merton-type single factor asset value model for the estimation of portfolio capital charges. In this exercise, asset correlations are subjected to a prudential boost, allowing for internally-calibrated capital charges of the same scale as those found in Basel III. This exercise benefits from a partial implementation analysis of the Basel III Internal Ratings Based Approach (IRBA) portfolio credit risk framework in which various assumptions are sequentially toggled on and off. Using results from this exercise we are able to obtain estimates of the IRBA capital charges under various cases. A comparison of boosted internally calibrated capital charges and Basel III capital charges reveals capital misallocation among Size segments within an SME portfolio.

In view of these findings, we suggest a remedy for capital misallocation within SME portfolios through the removal of size-based adjustments within SME segments. Such a case exists in the Retail-Other treatment of SME borrowers, but is limited in its applicability to all SME segments due to exposure limits and other restrictions on its use. The dropping or easing of these limits may provide an avenue through which SME capital allocation within the framework may be corrected.

Our paper is divided into the following sections: Section 2 presents a brief review of the Basel treatment of SMEs, as well as the empirical and theoretical literature on SME correlations and credit behaviours; Section 3 presents our unique dataset; Section 4 presents our internally calibrated asset correlations and PDs; Section 5 presents internally calibrated capital charges for our SME portfolio and compares them to those obtained under various partial implementations of the Basel framework, and finally; Section 6 presents our conclusions and directions for further research.

Our approach in presenting our work will be to focus on results, relegating technical discussions and descriptions to the appendices. In particular, Appendix A will present a detailed review of the Basel III frameworks for portfolio credit risk, including a review of the Internal Ratings Based Approach risk weighting function; Appendix B will provide a description of the single factor asset value model framework used to estimated asset correlations and generate an SME portfolio loss distribution, and finally; Appendix C will present a technical discussion of the prudential boost methodology applied in this paper and the parameters used to generate internally calibrated capital charges.

2. The Basel Treatment of SME Borrowers and a Review of Literature on SME Correlations

The Basel III recognition of SMEs as borrowers with divergent credit risk characteristics from their Corporate counterparts is reflected in both the Standardized Approach (SA) and the Internal Ratings Based Approach (IRBA); for more on the divergent treatment of SMEs and Corporate borrowers in the Basel Accords, see, for example, Hennek and Truck (2006), Altman and Sabato (2005), Dietsch and Petey (2004) and Jacobson, Linde, and Roszbach (2005). Within the IRBA framework, the risk-weighting function incorporates pre-calibrated risk components and adjustments which play a crucial role in determining final capital requirements and allocations. For SME exposures, these pre-calibrated components are set to values that effectively reduce the capital required for SMEs with respect to their Corporate counterparts; for the Canadian implementation of Basel III, see OSFI (2012), for the revised Basel III capital framework document release by the Basel Committee for the Supervision of Banks (BCBS), see BCBS (2011), for the revised Basel II capital framework released by the BCBS, see BCBS (2006) – note, the IRBA and SA treatments in the Basel frameworks, as they relate to SMEs have remained unchanged in the transition from Basel II to Basel III.

In particular, the recognition of SMEs in the Basel Accords can be categorized through the following:

Standardized Approach (SA)

i. Almost exclusively unrated by external agencies, SME borrower loans classified as Corporate exposures generally warrant a 100% risk weight. These borrowers will also, in general, exhibit elevated risk profiles, so that for an externally rated Corporate borrower loan with a comparably elevated risk profile, a higher risk weight of 150% could be applied; see BCBS (2006, p. 23).

ii. Basel II allows for the classification of SME borrower loans under the Retail-Other asset class, with a 75% risk weight, given some restrictions; see BCBS (2006, p. 23).

Internal Ratings Based Approach (IRBA)

i. For SMEs in the Corporate asset class, a size-based adjustment can be applied within the risk-weighting function to lower capital charges through lower asset correlations; see BCBS (2006, p. 64) and Equation (3) in Appendix A.

ii. Within the Corporate asset class, SME properties such as shorter terms (maturities) and higher PDs generally lend themselves to lower capital charges within the Internal Ratings Based Approach framework; see BCBS (2006, p. 64) and Equations (1) and (2) in Appendix A.

iii. For SMEs in the Retail asset class, Size and Term-to-Maturity have no impact on capital charges, however, lower overall capital charges are obtained through lower overall asset correlations; see BCBS (2006, p. 77) see Equations (4), in particular, in Appendix A.

A major focus of our work in this paper is the testing of these Basel specifications on the level and patterns of asset correlation across Risk and Size segments of an SME portfolio. In Appendix A we take a closer look at the Basel treatment of SMEs, examining the internal risk-weighting function, the assumptions on SMEs as they are presented in that function, and the capital calculation methodology used in the SA and the IRBA. Our implementation of the IRBA will focus on the Advanced IRB (AIRB) implementation wherein internal estimates of loss given default (LGD) will be generated for secured and unsecured SME loans.

Within the SME asset correlation literature, three divergent theoretical arguments for the presence of increasing/decreasing patterns in asset correlation across borrower size segments can be found. Below we present a brief review of these arguments, closely following the review presented in Duellman and Scheule (2003).

The first argument, referred to as the "business sector argument", presents size-based patterns in asset correlation as proxies for asset correlation patterns across industries. This argument is bolstered by variances in predominant borrower sizes across industries, with highly cyclical industries dominated by large borrowers and less cyclical industries dominated by small borrowers. Duellman and Scheule (2003) provide an empirical test for this argument by examining three sectors considered to be highly cyclical (manufacturing; construction; and automotive), along with three sectors considered less cyclical (transport & communication services; health & financial services; and other public & personal services). Their results provide support for the business sector argument by finding that in highly cyclical industries SME borrowers account for a small proportion (approximately 15%) of total borrowers, while in the less cyclical industries they account for a significant proportion of borrowers (between 30% and 40%). As such, the evidence presents support for the theoretical arguments for increasing asset correlations with increasing Size.

The second argument presents large borrowers as better diversified firms as compared to their smaller counterparts. This better diversification reduces idiosyncratic risks among large borrowers, thereby increasing their exposure to systematic risks relative to smaller borrowers. This hypothesis is contested by Roll (1988) which presented empirical work suggesting that small firms displayed higher diversification than larger borrowers.

Contrary to the first two arguments, the third argument, referred to as the "financial accelerator" hypothesis, suggests that asset correlations are in fact inversely related to borrower size. The hypothesis, put forward in Bernanke and Gertler (1995) and Bernanke, Gertler, and Gilchrist (1996) holds that smaller borrowers' reliance on bank loans for financing, as compared to larger borrowers who can access capital markets, renders them more vulnerable to macroeconomic shocks and their effects on credit-market conditions. In particular, empirical work in Bernanke, Gertler, and Gilchrist (1996) suggests this negative relationship between borrower size and asset correlation holds even when controlling for industry.

On asset correlation patterns across risk segments two theoretical arguments; again, see, Duellman and Scheule (2003) for a broader discussion of these arguments. In the first argument, it is proposed that borrowers with elevated sensitivities to macroeconomic developments may choose more conservative capital structures, thereby reducing overall riskiness. This theory then indicates that borrowers with higher asset correlations may display lower probabilities of default. In the second argument, it is proposed that if an increase in a borrower's credit risk is initiated by idiosyncratic events, then the relative importance of idiosyncratic risks to systematic risks increases.

In the following sections we will estimate SME asset correlations based on our single institution portfolio. Our results will be compared to other empirical work in the literature dealing with SME asset correlations.

3. A Canadian High Risk SME Credit Risk Dataset

Our study of Small and Medium Enterprise (SME) portfolio credit risk is centred on the unique characteristics of our Canadian portfolio and the *Financing Company* in which it resides. In particular, we note that the *Financing Company* (the name of which cannot be disclosed due to confidentiality) that is the source of our data is a specialized SME financer that specifically targets high risk niches within the SME loans market, both in terms of borrowers of diminished size (e.g., assets, sales, etc.) and industries which have historically faced some level of under-servicing from Canadian financial institutions. Our SME loans portfolio is composed of over 35,000 loans to over 25,000 borrowers, totalling over $10 billion in Canadian dollars Outstanding ($OS). Impaired loans, as well as loans and borrowers classified as Performing but subject to "watch list" monitoring are also excluded from our analysis.

Historical defaults are compiled from January 1997 to December 2010, covering a period of 13 years. Risk Groups are classified in increasing order of riskiness as 1-3 (least risky), 4-5, 6, 7 and 8-9 (most risky). Size Groups are classified from smallest to largest as *≤$100,000* (smallest borrowers), *$100,000-$250,000*, *$250,000-$1,000,000*, and *>$1,000,000* (largest borrowers). Here, the Size measurement is based on the borrower's maximal total commitment to the *Financing Company* at last origination, including $OS to other borrowers with common ownership on the *Financing Company* books. This size classification differs from the sales-based classification generally used in the Basel framework. To that end, we present Table 1 wherein a mapping between the two is presented using *Financing Company* internal data. Note that a more granular Size Bucket classification is used; where applicable, certain Size Buckets are amalgamated to create the Size Groups described above.

In order to properly identify the discussion as pertaining to either the single segmentation or the dual segmentation of the data, we will generally use the "overall" adjective when referring to the single segmentation, and the "segment" adjective when referring to the dual segmentation. For each segment, the PD is calculated as the weighted-mean of default rates over our sample period of 12 years. This method represents industry best practice and is widely applied in the literature; see, e.g., Standard & Poor's (2011, p. 2) and Dietsch and Petey (2002, p. 311).

Table 2 maps the *Financing Company* Risk Groups (RG) against the S&P credit ratings – as represented by their estimated one year default rates; PD estimates will be discussed in greater detail in Section 4, with particular emphasis on RG-SG segments, our discussion in this segment will relate focus on portfolio-descriptive aspects. As can be seen in Table 2, the Financing Company risk grades occupy the high risk spectrum of these ratings. In particular, we observe that the lowest risk RG has an overall PD equivalent to that of a BB- S&P rating, with the highest risk RG being roughly equivalent to B- S&P rating. This minimum overall PD of 1.3% for the lowest risk clients emphasises the high risk nature of *Financing Company* portfolio, and compares to such SME low risk grade PD values as 0.19% and 0.49% in Dietsch and Petey (2004, p. 779) for their portfolios of French and German borrowers, respectively. In Section 4 we return to Table 2 in greater detail with regard to our estimates of asset correlations their primary drivers.

Loss Given Default is set at 73% and 41% for unsecured and secured loans, respectively. These values reflect downturn values specific to our high risk SME portfolio. No assumptions are made on the type of collateral used.

Table 3 presents distributions of borrowers and borrower dollars outstanding ($OS) as of March 2009 in our portfolio. The Table indicates decreasing proportions of extremely high risk borrowers (8-9 RG) with increasing Size. Put another way, our portfolio reveals a generally decreasing tolerance for high risk clients with increasing Size, such that smaller borrowers are more likely to have a high risk rating and larger borrower a lower risk rating. Perhaps strikingly, a distinguishing characteristic of this portfolio may be found in the high proportion of borrowers in the 8-9 RG (29.1%). Comparing again to Dietsch and Petey (2004, p. 777), we observe that this percentage is comparable to that found in proportion of borrowers occupying the riskier half of their risk classes for both their French and German portfolios.

In terms of Size, we observe in Table 1 average sales of approximately $3.6 million for our two smallest borrower segments. In Table 3 these two segments are observed to account for approximately 50% of borrowers in our portfolio, and approximately 8% of $OS. Unsurprisingly, Table 3 shows that the proportion of $OS allocated in the portfolio grows dramatically with increasing borrower Size. Our analysis of the portfolio also reveals interesting statistics with respect to exposure amount, with a median borrower exposure amount ($OS) equal to approximately $150,000, and less than 1% of borrowers having exposures greater than $5 million. These statistics reveal a portfolio in which low exposures dominate to an extent greater than that observed at other institutions. For instance, Allen and Saunders (2002, p. 144) approximate a middle-market portfolio with a test portfolio with 2,500 obligors with average exposures of £894,000. In addition, these statistics present interesting avenues for research on the potential for granularity effect with a changing SME portfolio composition; see, for example, Gordy and Lutkebohmert (2007) for more on the granulairty effect and its presence in the application of the Basel IRBA framework.

4. Internally Calibrated SME Probabilities of Default and Asset Correlations

In this section dual segmentations are applied to the data in the calculation of probabilities of default as well as asset correlations. The application of dual segments to our data provides us with a useful convention in that it allows for the formation of homogenous segments of SME borrowers on which research on credit relationships and characteristics can be undertaken. The significant depth of our portfolio differentiates this data from other studies in which this dual segmentation has been applied to aggregated data sets; see, for example Dietsch and Petey (2004) and Duellman and Scheule (2003).

Table 4 provides probability of default estimates, along with PD variance and normalized standard deviations by Risk and Size Groups. We observe an overall portfolio PD equal to 4.6%. Overall, PDs are shown to increase monotonically with Risk Group such that PDs for the 1-3 RG and the 8-9 RG are equal to 1.30% and 8.75%. PDs by overall Size Group are shown to decrease monotonically as Size increases. We therefore observe PDs for the smallest borrowers equal to 8.32% and PDs for the largest borrowers equal to 2.37%. This pattern in default rates by Size is not surprising given the distributions of borrowers by Risk Group in each Size Group; see Table 3.

Observing our data by RG-SG segment, we observe that overall RG patterns are observed in all SGs. We similarly observe a repeat of overall Size patterns in all RGs except the 8-9 RG where a U-shape pattern is observed, such that, unlike other RGs, we observe an increase in the relative riskiness of borrowers in the *>$1,000,000* Size Group as compared to those in the *$250,000 - $1,000,000* SG. This U-shape for the largest riskiest borrowers may be reflective of a willingness to tolerate elevated risk characteristics among smaller borrowers while acknowledging the severe circumstances under which a larger borrower would find himself in the elevated risk grouping. Put another way, high PDs among larger borrowers in the 8-9

RG may be reflective of deteriorating financial conditions among those borrowers while decreasing PD patterns with increasing Size among other borrowers in the 8-9 RG may be reflective of the risk appetite at or near authorization for smaller borrowers.

Table 5 depicts the internally calibrated SME asset correlations using the data presented in Table 4. Our results, reviewed below, indicate no evidence in support of strong relationships between asset correlation and either RG or SG. Results here were generated through the use of the methodology presented in Appendix B. In particular, joint probability estimates for borrowers in the same segment were generated using segment PDs and PD volatilities, as presented in Table 4, and the application of Equation (10).

Specifically, we observe an overall portfolio asset correlation of 0.34%, the lowest observed value among all segments in our portfolio. For the overall RGs, we do not observe evidence of either monotonically increasing or decreasing patterns of asset correlation with PD. Specifically, we observe the lowest asset correlation values at the 8-9 RG (0.93%) and the 1-3 RG (0.98%), while the highest asset correlation values are observed at the 4-5 RG (1.49%) and the 7 RG (1.30%). For the overall SGs, we do not observe a pattern in asset correlation by Size over the four fixed SGs. In particular, we observe the highest asset correlation for the *\$100,000 - \$250,000* SG (0.77%) and the lowest asset correlation in the *≤\$100,000* SG (0.34%). This lack of pattern at the overall level is repeated at the dual-segment level when controlling for Size and for Risk.

Our results fail to confirm the presence of strict relationships between asset correlations and either PD or Size over a dataset of high risk SME borrowers. In addition, internally calibrated asset correlations are observed to be much lower than those found in Basel III, such that for all segments internally calibrated asset correlations are lower than the Basel III minimum of three percent. This is not uncommon in the literature, with possible explanations including reduced default rate means and volatilities over the time period of measurement, the use of default data versus other sources such as loss data or market-based data, and prudential considerations in the Basel Accords; see, for example, Frey (2008).

Our results suggest that asset correlations are closer in value and behaviour to those found in Basel III IRB Retail-Other specification as opposed to those found in the Corporate asset class specifications for SMEs. Specifically, we find that there is no fixed relationship between asset correlations and Size, nor is there a fixed relationship between PD and asset correlation.

These results run counter to the Size-asset correlation relationship programmed into the Basel III IRB Corporate asset class risk-weighting function, and the PD-asset correlation relationship found in both the Retail-Other and Corporate asset class risk-weighting function. Our results also contribute to the debate on the specification of these relationships by providing direct evidence on the absence of such relationships.

In particular, empirical work in Gordy (2000, p. 134), using the same framework used here, estimates asset correlations for various S&P risk grades and shows an increasing relationship with increasing PD. In Dietsch and Petey (2004, p. 780), SME asset correlations are evaluated over aggregated data sets of borrowers in France and Germany using three SME Size groupings and eight risk grades. Results indicate a generally increasing pattern with increasing PD, overall, but no strict relationship - within Size groups the results show even less homogeneity in pattern. Examining results by overall Size, the authors observe decreasing asset correlations with increasing Size over the three SME Size groups. Examining results by Size and PD segments, the authors observe a mixture of patterns; see Table 4, Dietsch and Petey (2004, p. 780).

Lopez (2004, p. 273) finds evidence of decreasing asset correlations with increasing PD at the overall level for datasets of borrowers worldwide, in the US, Japan, and Europe. These results find some support when controlling for Size, however, a universally monotonic relationship is not clear; see Table 4, Lopez (2004, p. 275). Examining overall results by Size, the author finds evidence of strictly increasing relationship across all geographically-defined portfolios; see Table 3, Lopez (2004, p. 274). This result is upheld when controlling for PD; see Table 4, Lopez (2004, p. 275).

Finally, we observe that the Basel framework provides for asset correlations ranging from 3% to 30% (applied to the High Volatility Commercial Real Estate (HVCRE) asset class; see BCBS (2006, p. 66) for details). For SMEs, the range maximum is reduced to 24% - or 20% for those SMEs benefitting from a maximal size-adjustment in the Corporate asset class; see Appendix A. Compared to these prudential regulatory levels, internally calibrated asset correlations derived in this paper within a single factor framework appear to be of a significantly lower level. This discrepancy in the overall level of internally calibrated asset correlations and those found in the Basel regulatory framework retains a sharp focus both in the academic literature and practical implementations of portfolio credit risk frameworks.

In particular, Chernih, Henrard, and Vanduffel (2010) review asset correlation results found in the literature and segregate them by source data type. Their survey – replicated in Table 6 – suggest that the type of source data (i.e., default data vs. market-based equity data) may play a significant role in the setting of overall asset correlation levels; see Table 1 and Table 2 in Chernih, Henrard, and Vanduffel (2010, p. 53). Citing this work, Frye (2008) observes that the maximum asset correlation obtained with observed defaults as the source data, is approximately 10%, with that figure dropping to 2.3% for some studies; see, for example, Hamerle, Liebig, and Roesch (2003b). This maximum figure of 10% asset correlations, when estimated over default data, compares to a minimum of 10% asset correlations when estimated over equity data, and is attributed to observed and conceptual differences in the underlying data; see Frye (2008).

Commenting on low asset correlation levels obtained in their respective studies, Dietsch and Petey (2004) and Duellman and Scheule (2003) suggest that the use of aggregated data may engender some over-diversification within their data sets and therefore be a possible source of low correlation values. Dietsch and Petey (2004) also suggest their shortened time series as a potential source of reduced asset correlations due to the lack of a full economic cycle over the time period considered.

In contrast, our research benefits from the use of non-aggreagted data, specific to one institution targeting high-risk SME borrowers. In addition, we benefit from a time series with 13 years of data. Despite our longer time series, however, we observe that the period covered is comprised of a prolonged period of economic growth along with low volatilities in our observed default rates, pointing to another potential source of low overall asset correlation levels.

Finally, we reiterate the high risk nature of our dataset. Returning to Table 2, we compare normalized default rate volatilise obtained in our study with those observed in Standard & Poor's (2011) over the period 1981 to 2010. As can be seen in Table 2, our normalized default rate volatilities are considerably lower than those observed over the Corporate defaults studied in Standard & Poor's (2011). As the primary empirical driving factor of our asset correlation estimation, these low levels for the normalized volatilities can be classified as a significant contributor to our low asset correlation values.

5. Internally Calibrated Capital Charges as Compared to those under Basel III

Having estimated PDs and asset correlations by Risk and Size Group segments, we move to the estimation of capital charges for our SME portfolio. In particular, we generate capital charges using a Monte Carlo simulation-based single factor asset value model using internally calibrated PDs and asset correlations from Section 4. In Appendix B a description of the simulation process is given, along with the methodology used to allocate capital charges back to individual borrowers and borrower segments. This allocation methodology focuses on 99.9% realizations within our multiple loss distribution generations and the capital contributions of our SME borrowers within each realization; see Equations (13 and (14).

In order to generate capital levels comparable to those found in the Basel framework, we perform an ad-hoc conservatism factor adjustment, or boost, to the low level of asset corrlations obtained in our internal estimation; see Table 7. This exercise, described in Appendix C, is similar to that in Dietsch and Petey (2002) wherein an SME portfolio credit risk model is designed and estimated from SME default data. In that paper, the authors find low overall asset correlation values, averaging approximately 2%, and choose to input Basel II IRB asset correlations equal to 20% for Corporate borrowers and 8% for Retail borrowers to generate capital charges comparable to the regulatory framework; see Dietsch and Petey (2002, pp. 307-308). In this paper, we use a bounded log odds methodology to calibrate our overall asset correlation of 0.34% to the average asset correlation of 8.5% obtained under the implementation of Basel IRBA (Case 2, see below), safeguarding throughout the asset correlation patterns obtained in Section 4 and presented in Table 5.

Table 8 presents our internally calibrated SME capital charges, as well as those obtained under various partial implementation cases. Our boosted internally calibrated capital charges reveal strictly decreasing capital charges with increasing Size, both at the overall level and when controlling for risk. As expected, this result lies in contradiction with the results obtained under the full Basel implementation (Case 2) wherein borrowers are divided along two asset classes, Corporate or Retail-Other, and all other applicable adjustments are implemented (e.g., size-adjustment, maturity-adjustment, etc). In particular, under Case 2, we observe two broad levels of capital, the generally lower capital charges – that decrease with increasing Size – obtained for the segments of smaller borrowers, classified as Retail-Other; and the generally higher capital charges obtained for the segment of the largest borrowers, classified as Corporate.

More specifically, under Case 2 we observe overall capital charges of 7.8% for the *≤$100,000* SG, decreasing to 4.9% for the *$250,000 - $1,000,000* the segment with the largest sized borrowers eligible for Retail-Other classification. For the *>$1,000,000* SG, we observe an overall capital charge of 9.8%. In contrast, internally calibrated capital charges range between 20.2% for the *≤$100,000* SG, and 5.5% for the *>$1,000,000* SG. These results are repeated when controlling for Risk. (note, for the 7 RG, a slight decrease in capital charges when compared to the 6 RG for some SGs is attributed to lower LGD values in that category).

In addition, we generate internally calibrated capital charges using PDs and boosted asset correlations by RG, irrespective of borrower Size. These PDs are identical to those used in Case 2. Our results confirm those observed under the RG-SG calibration, and indicate that the inclusion of size segments in the calibration of PDs and asset correlations results in an amplification of trends observed in the RG calibration.

Our results are indicative of a significant misallocation of capital under the Basel dual-asset class capital regime. In order to better pinpoint the sources of this misallocation, we present several other cases of the partial implementation of the Basel framework.

In particular, we present Case 3, referred to as the “naïve” implementation, in which we classify all of our exposures as Corporate exposures and withhold any maturity or size adjustments; Case 4 applies a size adjustment to the “naïve” implementation; Case 5 applies Retail – Other classifications, where applicable, to the “naïve” implementation, and; Case 8 in which the Retail-Other asset class specification is applied to all borrowres. Results for this exercise are generated using partial implementations of Appendix A Equation (1), as well as Equations (2) and (3), for loans classified as Corporate, and Equation (4) for loans classified as Retail-Other.

Our results under these cases, also presented in Table 8, reveal no misallocation of capital under Cases 3 and 8. That is, we observe strictly decreasing capital charges with increasing Size under both those implementations. However, in the presence of size adjustments for the Corporate asset class (Case 4) or the application of a dual-regime framework (Case 5), in which SMEs with exposures greater than \$1.25m are ineligible for Retail-Other treatment, misallocation is observed. As such, our results lead us to conclude that, when applied to an exclusively SME portfolio, the Basel III specifications on size can lead to severe overcharging of larger borrowers with respect to their smaller counterparts.

This capital misallocation has significant impacts on such institutional-significant activities as the pricing of loans, the evaluation of performance for lending units associated with different borrower size segments, and an inconsistency between an SME bank’s regulatory capital charges and its internal capital charges.

6. Conclusions

In this paper we benefit from a unique database of over 25,000 SME borrowers from a single institution, with 13 years of heavily populated default history, to generate estimates of probabilities of default and asset correlations for homogenous segments of SME borrowers defined by risk grade and size.

These estimates are used to test the Basel III specifications for loans to SME borrowers in banks' portfolios. In particular, our estimates of SME asset correlations reveal no empirical evidence of either a positive relationship with size or a negative relationship with PD. This result runs counter to Basel III specifications of a negative relationship between asset correlations and PD, and counters the Corporate asset class assumption of a positive relationship between asset correlation and size. Our finding of low asset correlation values and no relationship between asset correlations and size our results appear to provide some support to credit risk specifications under the Retail-Other treatment.

The estimation of low correlation values in the empirical literature is not uncommon, and recent research has highlighted several potential sources for the discrepancy between correlations estimated from default or loss data, and correlations estimated from market-based sources. Using a prudential boost to internally estimated asset correlations across Risk and Size Group segments, we generate internally calibrated capital charges for our portfolio according to a simulation-based single factor asset value model. Our results reveal that Basel III leads to misallocation of capital charges, such that in some cases, smaller and riskier SME borrowers are charged less than larger and safer SME borrowers. These Basel III capital charges can represent cases of under- or over-charging of capital to borrowers as compared to the capital charges they would incur under internally-calibrated models of portfolio credit risk.

Engaging in a Partial Implementation exercise of the Basel IRBA framework, we are able to pinpoint the sources of this misallocation and present potential remedies for its alleviation within an SME portfolio. In particular, our results reveal that the inclusion of size-based adjustments in the Corporate asset class, and significantly, the application of a dual Corporate-Retail asset class framework over an SME portfolio, can lead to breaks in Basel capital charges as compared to internally-calibrated capital charges. This condition may be alleviated through the removal of size-based adjustments within SME segments. Such a case exists in the Retail-Other treatment of SME borrowers, but is limited in its applicability to all SME segments due to exposure limits and other restrictions on its use. The dropping or easing of these limits may provide an avenue through which SME capital allocation within the framework may be corrected.

Our results present an interesting incision into the debate surrounding the presence of patterns and relationships between SME asset correlations and other characteristics, most markedly size and credit quality. The methods used to obtain these results have relied on traditional techniques, with the application of alternative techniques presenting interesting avenues for further research. Another avenue for further research on SME correlations presents itself in the study of these correlations within (and across) industry segments – such a study would allow for more incisive commentary on the "business sector argument" presented in Section 2. Finally, our results on capital allocations within an SME portfolio present an interesting area for further research. In particular, it is interesting, and perhaps imperative, that the $1.25m exposure threshold presented in the Basel framework be examined in order to establish when, or with what segment growth, if any, a portfolio of high risk SMEs such as ours would cease to behave according to the precepts of the Retail-Other treatment, and behave according to those of the Corporate asset class treatment. Similarly, a study on the approximation error generated through the application to an SME portfolio of the IRBA framework's infinite granularity assumption could prove interesting.

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Appendix A - A Closer Look at the Basel Treatment of SMEs

Standardized Approach (SA)

Given the overwhelming predominance of unrated borrowers among SMEs, most SME borrowers classified under the Corporate asset class obtain SA risk weights of 100%. Alternatively, SME borrower exposures classified as Retail – Other are accorded a risk weight of 75%, see BCBS (2006, p. 23).

In order to calculate capital charges for a given portfolio under the SA approach, exposures are multiplied first by the corresponding risk weight and second by the minimum regulatory capital requirement. Under Basel III that minimum total regulatory capital requirement is set at 10.5%. In this paper, the calculation of portfolio capital requirements under the SA method will use the Basel II minimum capital ratio of 8% so that for each loan we multiply the corresponding dollar Exposure and SA risk weights by 8% to generate capital charges for that loan.

Internal Ratings Based Approach (IRBA)

Our discussion of the IRBA will center on the Advanced IRB (AIRB) implementation, wherein banks are required to supply their own estimates of PD, LGD and Maturity. We present a general form of the IRBA risk weighting function below:

$$K = \left[LGD \times N\left[\frac{1}{\sqrt{1-R}} \times N^{-1}(PD) + \sqrt{\frac{R}{1-R}} \times N^{-1}(0.999)\right] - PD \times LGD\right] \times \left[\frac{1+(M-2.5)\times b(PD)}{1-1.5\times b(PD)}\right]. \quad (1)$$

where, (K) represents the percent capital charge in excess of EL; (PD) is the average Probability of Default defined over a given segment; (LGD) is the downturn Loss Given Default; see OSFI (2011, pp. 152-153) for details; (R) is the asset correlation; ($N[\cdot]$) is the cumulative distribution function (CDF) for a standard normal variable, and ($N^{-1}[\cdot]$) is the inverse CDF of a standard normal variable; (M) is the loan Term-to-Maturity; and $b(PD)$ is a smoothed regression maturity function, such that the slope of the adjustment function with respect to (M) decreases as the (PD) increases – specifically:

$$b(PD) = \left(0.11852 - 0.05478 \times ln(PD)\right)^2. \quad (2)$$

For each loans under Basel III, the risk weight is obtained by multiplying *K* by 12.5. Once the risk weight has been determined, risk weighted assets under the IRBA are summed, boosted by 6%, and then multiplied by minimum required capital ratio, say 10.5%. In our implementations of the IRBA in Section 5, we will use capital charges as proxied by *K* for each borrower segment. For the Corporate asset class, Equation (3) provides the asset correlation function, as found in BCBS (2006, p. 64):

$$R = 0.12 \times \frac{(1-exp(-50 \times PD))}{(1-exp(-50))} + 0.24 \times \left[1 - \frac{(1-exp(-50 \times PD))}{(1-exp(-50))}\right] - 0.04 \times \left[1 - \frac{S-6.25}{56.25}\right], \quad (3)$$

where (*S*) is the borrower size as measured by sales (applicable to Corporate SME exposures). Specifically, borrowers with annual sales ranging between \$6.25m and \$56.25m, as specified in OSFI (2011, p. 149), receive a negative adjustment ranging between 4% and 0%. Borrowers with sales of \$6.25m or less obtain a size adjustment of 4% while those with \$56.25m or more receive no adjustment. While the BCBS puts forward both intuition and empirical evidence as justification for this relationship, there is a concession that the empirical evidence supporting it is not conclusive, BCBS (2005, p. 12) – in the subsequent section of this paper we will review the theoretical and empirical literature on the relationship between asset correlation and borrower size.

$$R = 0.03 \times \frac{(1-exp(-35 \times PD))}{(1-exp(-35))} + 0.16 \times \left[1 - \frac{(1-exp(-35 \times PD))}{(1-exp(-35))}\right]. \quad (4)$$

Equation (4) presents the asset correlation equation under the Retail-Other asset class, to which borrowers classified as SMEs may be subjected; see BCBS (2006, p. 77). For a borrower to be eligible for Retail-Other specification, a financial institution's exposure to that borrower may not be greater than \$1.25m, OSFI (2011, p. 40). A quick comparison of Equations (3) and (4) reveals the absence of a size adjustment for SME borrowers, as well as lower asset correlation limits as compared to the Corporate asset class. We also observe a distinction in the calibration of the relationship between asset correlation and PD. Specifically, we note that the Corporate asset class correlation and the Retail-Other asset class correlation functions are based on an exponential weighting function with a "k-factor" set to 50 for Corporate exposures and 35 for Retail-Other exposures. This k-factor determines the pace of decrease of the asset correlation with respect to the PD such that given the above calibrations, the asset correlation decreases quicker for Corporate borrowers as opposed to Retail borrowers.

Finally, the asset correlations derived for the Retail asset class are applied to the IRB risk-weighting function given in Equation (3) with the exclusion of the Maturity adjustment (last bracket on the right-hand side of the equation).

The Basel II IRB framework therefore presents two specifications for SME asset correlations and capital charge allocation, and thus two conceptual frameworks for SME credit behaviour. Under the Corporate asset class, SME asset correlations are highly sensitive to PD values, and can range between 8% and 24%, depending on Size (as well as PD). Capital charges for SMEs under this specification increase with longer loan maturities. Under the Retail asset class, SME asset correlations are measured independently of Size; asset correlations are still inversely related to PDs but compared to Corporate exposures this relationship is significantly dampened. Asset correlations are lower than those for SMEs classified as Corporate exposures and range between 3% and 16% (depending on PD), and capital charges are not determined by maturity.

By segmenting our portfolio into homogenous subportfolios by Size and Risk Groups, and explicitly measuring the asset correlations in our SME portfolio, our work will directly test these assumptions and conceptual frameworks on the behaviour of asset correlations.

Capital charges generated under partial implementation cases are calculated on the loan level and then aggregated by segment (e.g., RG-SG segments). For all partial implementation cases, results are presented as dollar-weighted percentages of aggregated exposures ($OS) for each segment, as well as for the overall portfolio.

Appendix B - Asset Value Model Methodology for Estimating Asset Correlations and Capital

For a given segment (ζ_A) of borrowers sharing some common characteristic, e.g., the same credit rating, we define for each borrower (i) the standard normal latent factor $y_{\zeta_A,i}$, such that:

$$y_{\zeta_A,i} = w_{\zeta_A} x + \sqrt{\left(1 - w_{\zeta_A}^2\right)} \cdot e_i, \tag{5}$$

where x is the systematic factor e_i, is the idiosyncratic factor and each is an independent standard normal variate. Characterising the systematic factor as being representative of the state of the economy, borrowers' dependence on the business cycle can be measured by the weighting w_{ζ_A} on x. Given two borrowers from two different segments, (ζ_A) and (ζ_B), the covariance between their latent factors is then defined as:

$$Cov\left[y_{\zeta_A,i}; y_{\zeta_B,J}\right] = w_{\zeta_A} \cdot w_{\zeta_B} \tag{6}$$

Borrower, (i)'s status at the end of a given time horizon is set to default if:

$$w_{\zeta_A} x + \sqrt{\left(1 - w_{\zeta_A}^2\right)} e_i < N^{-1}\left(\bar{p}_{\zeta_A}\right), \tag{7}$$

where $N^{-1}(\cdot)$ denotes the inverse cumulative standard normal distribution function and $\bar{p}_{\zeta_A}$ is the unconditional, or long-term, probability of default for segment (ζ_A).

Following Gordy (2000), and using Equation (7), we define the joint probability of default for two borrowers (i) and (k) in the same segment (ζ_A) is then given by:

$$\begin{aligned} &\Pr\left[y_{\zeta_A,i} < N^{-1}\left(\bar{p}_{\zeta_A}\right) \, and \; y_{\zeta_A,k} < N^{-1}\left(\bar{p}_{\zeta_A}\right) \middle| x\right] \\ &= \Pr\left[y_{\zeta_A,i} < N^{-1}\left(\bar{p}_{\zeta_A}\right) \middle| x\right] \cdot \Pr\left[y_{\zeta_A,k} < N^{-1}\left(\bar{p}_{\zeta_A}\right) \middle| x\right] = \bar{p}_{\zeta_A}^2. \end{aligned} \tag{8}$$

Next, we use Equation (8) to define the variance of the conditional probability of default as a function of the asset correlation and the unconditional probability of default. Then, using the method proposed in Gordy (2000), and our empirically calibrated unconditional probability of defualt and conditional probability variance we will estimate the representative asset correlation for segment (ζ_A), $w_{\zeta_A}^2$.

Specifically, we write the variance for the conditional probability of default $p_{(\zeta_A)}(x)$ as the following:

$$\begin{aligned} Var\left[p_{(\zeta_A)}(x)\right] &= E\left[p_{(\zeta_A)}(x)^2\right] - \left(E\left[p_{(\zeta_A)}(x)\right]\right)^2 \\ &= E\left[Pr\left[y_{\zeta_A,i} < N^{-1}\left(\bar{p}_{\zeta_A}\right) \; and \; y_{\zeta_A,k} < N^{-1}\left(\bar{p}_{\zeta_A}\right)\middle|x\right]\right] - \bar{p}_{\zeta_A}^2. \end{aligned} \tag{9}$$

Given the assumptions of standard normality for the latent variables $y_{\zeta_A,i}$ and $y_{\zeta_A,k}$, and the correlation $w_{\zeta_A}^2$, based on Equation (6), we can now write:

$$Var\left[p_{(\zeta_A)}(x)\right] = Bivnorm\left(N^{-1}\left(\bar{p}_{\zeta_A}\right), N^{-1}\left(\bar{p}_{\zeta_A}\right), w_{\zeta_A}^2\right) - \bar{p}_{\zeta_A}^2. \tag{10}$$

Here we assume serial independence for the systematic factor realizations and conditional independence between borrower defaults; Dietsch and Petey (2004). To calculate $\left(w_{\zeta_A}^2\right)$, we first calculate the conditional variance $Var\left[p_{(\zeta_A)}(x)\right]$ as a function of the data-derived unconditional variance $Var\left[p_{(\zeta_A)}\right]$, the average number of healthy borrowers in a given segment $H_{(\zeta_A)}$ across the beginning of one year periods, and the unconditional probability of default for that segment, $\left(\bar{p}_{\zeta_A}\right)$; see, for example, Dietsch and Petey (2002, p. 313). The resultant equation is given below:

$$Var\left[p_{(\zeta_A)}(x)\right] = \frac{Var\left[p_{(\zeta_A)}\right] - E\left[1/H_{(\zeta_A)}\right] \cdot \bar{p}_{\zeta_A} \cdot \left(1 - \bar{p}_{\zeta_A}\right)}{1 - E\left[1/H_{(\zeta_A)}\right]}. \tag{11}$$

Given the joint probability of default found in Equation (8), we can calculate the default correlation (DC) between two borrowers in the same segment (ζ_A) as:

$$DC_{\zeta_A} = \frac{Bivnorm\left(N^{-1}\left(\bar{p}_{\zeta_A}\right), N^{-1}\left(\bar{p}_{\zeta_A}\right), w_{\zeta_A}^2\right) - \bar{p}_{\zeta_A}^2}{\left(\bar{p}_{\zeta_A} \cdot \left(1 - \bar{p}_{\zeta_A}\right)\right)}. \tag{12}$$

Applying Equation (8) the representative asset correlation is then estimated for each segment. To generate a loss distribution, we use a Monte Carlo simulation method to draw realizations of the single systematic factor *x*, as well as realizations of the idiosyncratic factor e_i, for each borrower (i). For each previously defined segment (e.g., Risk and Size) the corresponding internally estimated asset correlation is used to define the movement of the latent factor of a borrower in that segment according to the systematic factor and that borrower's randomly generated idiosyncratic factor. Default is assigned to borrower (*i*) if Equation (7) is found to hold. Taking the exposure as given and multiplying by a given LGD a loss is calculated for a given loan. Aggregating across borrowers we obtain a portfolio loss for a given draw of the systematic factor x. Portfolio EC values are derived over 150,000 simulations, the 99.9% VaR and the portfolio EL. For each loan, capital is allocated according to average VaR contributions as measured across 300 realizations of a 99.9% VaR. For each segment, EC values are given as the dollar-weighted average across all loans for borrowers in that segment. EC values are presented along with the percentage change in EC in going from an asymptotic implementation to a simulation-based implementation. Taking the 99.9% VaR of the portfolio loss distribution and subtracting the portfolio EL yields an Economic Capital or risk capital charge for the portfolio. The portfolio EL is calculated as the sum of the individual obligor ELs.

Specifically, we define the VaR contribution of an obligor j as:

$$C_j^{VaR_\alpha} = E\left[L_j \middle| L = VaR_\alpha\right]; \tag{13}$$

see, for example, Mausser and Rosen (2008. Pg 691). Taking Equation (15), we then proceed by simply repeating our simulation procedure; for each execution, we save the VaR_α run, maintaining our realized obligor loss under an $(L = VaR_\alpha)$ simulation. For each obligor, we then average over our realizations and obtain a set of $\left(C_j^{VaR_\alpha}\right)$ such that:

$$L = \sum_{j=1}^{N} C_j^{VaR_\alpha} \cdot L_j. \tag{14}$$

Appendix C - Asset Correlation Boost Methodology

We perform an ad-hoc conservatism factor adjustment to the low level of asset corrlations obtained in our estimation. This exercise is similar to that in Dietsch and Petey (2002) wherein an SME portfolio credit risk model is designed and estimated from SME default data. In that paper, the authors find low overall asset correlation values, averaging approximately 2%, and choose to input Basel II IRB asset correlations equal to 20% for Corporate borrowers and 8% for Retail borrowers to generate capital charges comparable to the regulatory framework; see Dietsch and Petey (2002, pp. 307-308).

A bounded log odds ratio adjustment is applied to all segments such that the overall estimated asset correlation of 0.34%, see Table 5, is equal to pre-specified value. For example, suppose that we want to adjust our estimated segment asset correlations $\{a_1, a_2, \ldots\}$ such that the overall asset correlation *A* is equal to some value *B,* subject to the condition that no segment asset correlation $\{b_1, b_2, \ldots\}$ is less than some lower boundary value *L* or greater than some upper boundary value *U*. The applied adjustment to each segment asset correlation would then be given by the following:

$$b_i = \frac{L + U \times exp(ln(a_i/(1 - a_i)) + X)}{1 + exp(ln(a_i/(1 - a_i)) + X)}, \tag{15}$$

where,

$$X = ln\left(\frac{B - L}{U - B}\right) - ln\left(\frac{A}{1 - A}\right). \tag{16}$$

The idea of these boosts, ultimately, is to provide internally measured asset correlations that can be practically applied within a prudentially concordant portfolio credit risk framework. An important aspect of this practicality is the level at which asset correlations are set with respect to the international regulatory requirements presented in Basel III. To that end, we use 3% and 24% as the lower and upper bounds, respectively, in our adjustment.

Table 7 presents the results of our boost by Risk and Size Group segmentations. The overall portfolio asset correlation is adjusted to the average asset correlation obtained in the full AIRB implementation (Case 2), equal to 8.5%. Capital charge results using these boosted values, and compared to charges obtained in the Partial Implementation exercise are given in Table 8.

Tables

Table 1

Borrower Size Buckets and Annual Sales

Size Bucket ('000)	Loans (%)	$OS (%)	MtM	Sales $ (m)
≤ $100	20%	2%	42	$3.6
$100 - $250	23%	6%	56	
$250 - $1000	31%	21%	74	$4.3 - $5.4
$1000 - $3000	5%	31%	97	$12.7
$3000 - $5000	17%	16%	116	
> $5000	4%	24%	132	$46.1
Overall	100%	100%	61	$5.2

Table 1 presents summary information on the *Financing Company* portfolio by Size Bucket. In particular, we note in the fifth column average annual sales (in $ millions) by Size Bucket, as well as the Months-to-Maturity in the fourth column for each Size Bucket, and in the fourth column. In addition, columns two and three show the percentage of the overall portfolio accounted for by each Size Bucket, both in terms of number of loans (column two) and $ Outstanding (column three).

Table 2

***Financing Company* SME PDs and Ratings as Compared to S&P PDs and Ratings**

One Year Default Rates, Average, Standard Deviation, Normalized SD							
S&P	**Mean**	**Std Dev**	**Norm SD**	**FC RG**	**Mean**	**Std Dev**	**Norm SD**
AAA	0.00%	0.00%					
AA+	0.00%	0.00%					
AA	0.01%	0.08%	8.0				
AA-	0.03%	0.10%	3.3				
A+	0.05%	0.15%	3.0				
A	0.07%	0.14%	2.0				
A-	0.07%	0.02%	0.3				
BBB+	0.16%	0.32%	2.0				
BBB	0.26%	0.35%	1.3				
BBB-	0.31%	0.47%	1.5				
BB+	0.67%	0.96%	1.4				
BB	0.88%	0.83%	0.9				
BB-	1.47%	1.79%	1.2	1-3	1.30%	0.34%	0.3
B+	2.47%	2.12%	0.9	4-5	2.29%	0.67%	0.3
				6	3.24%	0.79%	0.2
				7	4.63%	1.12%	0.2
B	7.17%	4.62%	0.6				
				8-9	8.75%	1.54%	0.2
B-	9.99%	7.95%	0.8				
CCC/C	23.56%	12.69%	0.5				

Table 2 provides descriptive statistics for the *Financing Company* SME loans and Standard & Poor's (S&P) rated corporate debt. Statistics are given by rating for the Mean, Standard Deviation (Std Dev) and Normalized Standard Deviation (Norm SD). S&P statistics were measured over the 1921 – 2010 observation period while *Financing Company* (FC) statistics were measured over the 1997 – 2010 period. Results show significantly higher normalized standard deviations for the FC Risk Groups (RGs). Source: Default, Transition, and Recovery: 2010 Annual Global Corporate Default Study and Rating Transitions. Standard & Poor's RatingsDirect on the Global Credit Portal: March 30, 2011

Table 3

Portfolio Borrower and Dollar Distributions

Distribution of Borrowers across Risk & Size Groups (%)					
Risk Group	**Size Group ('000)**				
	≤ $100	**$100 - $250**	**$250 - $1000**	**> $1000**	**Overall**
1-3	3.9	5.8	7.9	8.0	25.6
4-5	4.0	5.3	7.5	5.4	22.2
6	1.3	2.0	3.7	2.6	9.6
7	2.8	3.2	4.9	2.8	13.7
8-9	12.7	7.7	6.3	2.4	29.1
Overall	24.7	24.0	30.3	21.2	100.0

Distribution of Borrower $OS across Risk & Size Groups (%)					
Risk Group	**Size Group ('000)**				
	≤ $100	**$100 - $250**	**$250 - $1000**	**> $1000**	**Overall**
1-3	0.4	1.5	5.3	28.5	35.7
4-5	0.4	1.5	5.3	19.0	26.2
6	0.1	0.6	2.8	8.9	12.4
7	0.2	0.8	3.5	8.3	12.8
8-9	1.0	1.8	3.8	6.4	13.0
Overall	2.1	6.2	20.7	71.1	100.0

Table 3 describes the distribution of borrowers and dollars across our high risk Canadian SME portfolio. The portfolio consists of borrowers and $OS in Risk Groups 1 (least risky) to 8-9 (riskiest). Size Groups range from ≤*$100,000* (smallest borrowers) to >*$1,000,000* (largest borrowers) and are based on the total commitment to a borrower at last authorization.

Table 4

Probability of Default by Risk and Size Group

Size Group ('000)	Risk Group 1-3	4-5	6	7	8-9	Overall
Probability of Default (%)						
≤$100	2.81	4.34	5.59	8.15	11.39	8.32
$100 - $250	1.71	2.85	3.63	4.01	7.71	4.58
$250 - $1000	1.00	1.72	2.78	3.50	6.68	3.25
> $1000	0.68	1.49	1.92	3.02	7.43	2.37
Overall	1.30	2.29	3.24	4.63	8.75	4.56
Variance (%)						
≤$100	0.0056	0.0079	0.0087	0.0306	0.0369	0.0080
$100 - $250	0.0012	0.0084	0.0123	0.0147	0.0418	0.0071
$250 - $1000	0.0010	0.0039	0.0024	0.0105	0.0403	0.0032
> $1000	0.0005	0.0021	0.0017	0.0023	0.0186	0.0021
Overall	0.0011	0.0045	0.0063	0.0125	0.0237	0.0031
Normalized Standard Deviation						
≤$100	0.27	0.20	0.17	0.21	0.17	0.11
$100 - $250	0.20	0.32	0.31	0.30	0.27	0.18
$250 - $1000	0.31	0.36	0.18	0.29	0.30	0.17
> $1000	0.35	0.31	0.22	0.16	0.18	0.20
Overall	0.26	0.29	0.24	0.24	0.18	0.12

Table 4 presents the Probabilities of Default as calculated over a period spanning 13 years starting in January 1997 and ending in December 2010. Defaults were aggregated over a given calendar year and then divided over the count of healthy customers at the start of the year. A healthy-weighted average of default rates over the time period then gives the PDs presented above. Normalized standard deviations in the third panel are calculated as the ratio of the square root of the variance (second panel) over the PD (first panel).

Table 5

Internally Calibrated SME Asset Correlations by Size and Risk Group

Size Group ('000)	Asset Correlation (%)					
	Risk Group					Overall
	1-3	4-5	6	7	8-9	
≤ \$100	1.32	0.92	0.68	1.33	0.99	0.34
\$100 - \$250	0.67	1.92	1.88	1.92	1.96	0.77
\$250 - \$1000	1.31	2.06	0.58	1.70	2.34	0.60
> \$1000	1.46	1.45	0.77	0.49	0.93	0.68
Overall	0.98	1.49	1.17	1.30	0.93	0.34

Table 5 presents asset correlations calculated by Risk and Size Group segments. Asset correlations correspond to PD and PD variance values given in Table 4.

Table 6

Asset Correlations Derived from Various Data Sources

Source Study	Default data source	Results (%)
Gordy (2000, Table 2)	S&P	1.5 - 12.5
Cespedes (2002)	Moody's	10
Hamerle, Liebig, and Roesch (2003a)	Unknown	Max 2.3
Hamerle, Liebig, and Roesch (2003b)	S&P 1982-1999	0.4-6.04
Frey and McNeil (2003, Table 1)	S&P 1981-2000	6.5-6.9-9.1
Dietsch and Petey (2004)	Coface 1994-2001	0.12-10.72
Jobst and De Servigny (2005)	S&P 1981-2003	4.7-14.6
Duellman and Scheule (2003)	DB 1987-2000	0.5-6.4
Jakubik (2006)	BF 1988-2003	5.7
Source Study	**Asset data source**	**Results (%)**
Duellmann, Scheicher, and Schmieder (2008)	MKMV Credit Monitor	10.2
Zeng and Zhang (2001)	MKMV source	9.46 - 19.98
Akhavein, Kocagil, and Neugebauer (2005)	Equity	20.92 - 24.09
Lopez (2002)	MKMV Portfolio Manager	11.25
de Servigny and Renault (2002)	Equity	6

Table 6 replicates asset correlation results presented in Chernih, Henrard, and Vanduffel (2010, p. 53) Tables 1 and 2. Results show a large discrepancy between asset correlation results generated from market equity data and those generated from default data sources. S&P: Standard and Poor's; DB: Deutsche Bundesbank; BF: Bank of Finland; MKMV: Moody's KMV.

Table 7

Boosted Asset Correlations by Risk and Size Group

Size Group ('000)	Asset Correlation (%)					
	Risk Group					Overall
	1-3	4-5	6	7	8-9	
≤$100	15.2	13.3	11.7	15.3	13.7	8.5
$100 - $250	11.7	17.1	17.0	17.1	17.2	12.4
$250 - $1000	15.2	17.4	10.9	16.5	18.0	11.1
> $1000	15.7	15.7	12.4	10.1	13.4	11.7
Overall	13.7	15.8	14.6	15.1	13.4	8.5

Table 7 presents boosted asset correlation values. A bounded log odds adjustment method is applied to the original internally-calibrated correlations such that the overall (Overall) portfolio asset correlation is boosted from 0.34% to 8.5%, while maintaining existing patterns and relationships by Risk and Size Group. The 8.5% value corresponds to the average correlation across all loans under Case 2.

Table 8

Internally Calibrated Simulation Based Capital Charges vs. Basel II

Size Group ('000)	Risk Group 1-3	4-5	6	7	8-9	Overall
Capital Charges under CASE 2: AIRB						
≤$100	6.0%	7.2%	7.2%	7.9%	8.8%	7.8%
$100 - $250	5.2%	6.2%	6.3%	6.2%	7.3%	6.3%
$250 - $1000	4.1%	4.9%	5.2%	5.1%	5.7%	4.9%
> $1000	8.6%	9.7%	10.1%	10.8%	13.2%	9.8%
Overall	7.8%	8.5%	8.8%	9.0%	9.9%	8.5%
Internally Calibrated (RG-SG) Capital Charges						
≤ $100	12.9%	14.7%	14.1%	23.2%	24.8%	20.2%
$100 - $250	6.0%	13.1%	14.5%	14.5%	21.5%	14.2%
$250 - $1000	4.5%	7.9%	6.3%	10.7%	16.4%	8.8%
> $1000	3.6%	5.9%	5.3%	5.8%	12.7%	5.5%
Overall	3.9%	6.8%	6.0%	8.0%	16.0%	7.0%
Internally Calibrated (RG) Capital Charges						
≤ $100	6.9%	12.0%	12.7%	16.9%	21.1%	16.1%
$100 - $250	5.9%	10.4%	11.1%	13.4%	17.6%	11.9%
$250 - $1000	4.6%	8.2%	9.1%	11.2%	13.6%	8.9%
> $1000	4.6%	7.8%	8.8%	11.0%	13.4%	7.5%
Overall	4.7%	8.1%	9.0%	11.3%	14.6%	8.5%
Capital Charges under CASE 3: Naïve						
≤$100	9.6%	12.1%	12.8%	15.3%	20.1%	15.9%
$100 - $250	8.3%	10.4%	11.3%	12.1%	16.8%	12.1%
$250 - $1000	6.5%	8.2%	9.2%	10.0%	13.0%	9.1%
> $1000	6.3%	7.7%	8.7%	9.8%	12.7%	8.0%
Overall	6.5%	8.0%	9.0%	10.1%	13.9%	8.6%
Capital Charges under CASE 4: S-AIRB						
≤$100	7.5%	9.3%	9.7%	11.5%	15.2%	12.1%
$100 - $250	6.5%	8.0%	8.5%	9.1%	12.7%	9.2%
$250 - $1000	5.1%	6.3%	7.0%	7.5%	9.8%	6.9%
> $1000	5.4%	6.5%	7.2%	8.0%	10.4%	6.7%
Overall	5.4%	6.6%	7.2%	8.0%	10.9%	7.0%
Capital Charges under CASE 5: R-AIRB						
≤$100	6.0%	7.2%	7.2%	7.9%	8.8%	7.8%
$100 - $250	5.2%	6.2%	6.3%	6.2%	7.3%	6.3%
$250 - $1000	4.1%	4.9%	5.2%	5.1%	5.7%	4.9%
> $1000	6.3%	7.7%	8.7%	9.8%	12.7%	8.0%
Overall	5.9%	7.0%	7.8%	8.3%	9.6%	7.2%
Capital Charges under CASE 8: All Retail						
≤$100	6.0%	7.2%	7.2%	7.9%	8.8%	7.8%
$100 - $250	5.2%	6.2%	6.3%	6.2%	7.3%	6.3%
$250 - $1000	4.1%	4.9%	5.2%	5.1%	5.7%	4.9%
> $1000	3.9%	4.6%	4.9%	5.1%	5.6%	4.5%
Overall	4.0%	4.8%	5.0%	5.2%	6.1%	4.8%

Table 8 presents Partial Implementation capital charges (underlined) as compared to internally calibrated ones (not underlined); see Section 5 for further details.