Credit and Liquidity in Interbank Rates: a Quadratic Approach

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Abstract

In this paper, we propose a quadratic term-structure model of the EURIBOR-OIS spreads. As opposed to OIS, EURIBOR rates incorporate credit and liquidity risks. Indeed, a bank that lends on the unsecured market requires compensations for facing (a) the risk of default of the borrowing bank and (b) the risk of its own possible future funding needs. Our approach allows us to decompose the whole term structure of spreads into credit and liquidity components. Our no-arbitrage econometric framework makes it possible to identify risk premia associated with each of these two risks. Our results shed a new light on the effects of unconventional monetary policy carried out in the Eurosystem. In particular, our findings suggest that most of the recent easing in the euro interbank market comes from a decrease in liquidity-related risk premia.

JEL Codes: E43, E44, G12, G21

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1 Introduction

Since the beginning of the financial crisis, the interbank market has been carefully scrutinized by commentators and policy-makers, both in Europe and in the US. This paper focuses on the spreads between the *Euro Interbank Offered Rates* (EURIBORs) and their risk-free counterparts, proxied by the *Overnight Indexed Swap* rates (OIS). This spread is considered as a crucial indicator at the very core of the financial crisis: it reveals not only banks' concerns regarding the credit risk of their counterparts, but also their own liquidity needs.

Disentangling those credit and liquidity effects has essential policy implications. If a rise in spreads reflects poor liquidity, policy measures should aim at improving funding facilities. On the other hand credit concerns should be treated by enhancing debtors' solvency (see Codogno, Favero, and Missale (2003)). This question is very important in the euro area, where most of the unconventional monetary operations conducted by the European Central Bank focused on the curbing of interbank risk (see Gonzales-Paramo (2011)). Many attempts have been made to provide a credit/liquidity decomposition of the interbank risk (see next Section), but whereas most studies reckon that liquidity risk has been an important driver of interbank yields during the last 5 years, there is no consensus on the precise size of these effects: Schwarz (2009) estimates that one third of the EURIBOR-OIS 1-month spread is linked to liquidity in January 2008, whereas Filipovic and Trolle (2013) find that nearly all the spread is liquidity-related at that date.

In this paper, we present a new technique to investigate credit and liquidity risks in interbank markets. Our method is based on a reduced-form no-arbitrage term-structure model of the EURIBOR-OIS spreads. Considering the whole term structure of spreads to perform such an exercise is important for at least two reasons. By including several maturities in our sample, we first improve the quality and precision of our model estimation. Second, the term structure dimension of our analysis can be exploited to identify the part of the spreads that corresponds to risk premia, thereby extending the existing literature on interbank risks. Risk premia are the components of yields or spreads that would not exist if (a) economic agents or investors – in our case, banks – were risk-neutral or (b) the risks involved in the considered asset were not systematic, i.e. if they could be diversified away (see e.g. Longstaff, Pan, Pedersen, and Singleton (2011)). Since (a) and (b) are not likely to hold in the case of euro-area interbank risks, risk premia are expected to be present in the EURIBOR-OIS spreads. At the same time, these risk premia cannot account for the whole spreads. Indeed, even if agents were risk-neutral, the EURIBOR-OIS spread would not be zero. In that case, the spreads would just equal expected losses stemming from the total amount of risk – credit- and liquidity-related – that a bank faces when lending to another bank.

Our methodology aims at decomposing the spreads along two dimensions: credit vs. liquidity and expected vs. risk-premia parts. This is achieved with the use of a no-arbitrage framework involving credit and liquidity intensities. In order to clarify the interpretation of the latter, we propose a stylized interbank-market model where these intensities appear naturally. In this model, when a bank lends to another bank, it is exposed to two kinds of risks: the first corresponds to the default of the borrowing bank and the second pertains to the difficulty to meet potential liquidity needs the lending bank may face over the loan period. In order to respect the non-negativity of the intensities, we take them equal to non-negative quadratic functions of two Gaussian latent factors. Hence, our model belongs to the class of quadratic term-structure models (QTSM). Our identification scheme and the interpretation of the factors rely on credit and liquidity proxies. The estimation is performed using a recently-introduced quadratic Kalman filter (see Monfort, Renne, and Roussellet (2014)).

The model is estimated over a 6-year period, between August 2007 and September 2013. Both credit and liquidity components account for the fluctuations of the spreads over that period, with a higher average contribution of liquidity risks. Our results suggest further that both kinds of risk command substantial risk premia, pointing towards the systematic nature of credit and liquidity interbank risks. We illustrate how the existence of credit-risk premia translates into substantial differences between model-implied physical and risk-neutral probabilities of default.

The spreads' decomposition allows us to explore the consequences of unconventional monetary policies conducted by the ECB during this period. Our findings support the claim that the recent 3-year ECB loans to euro commercial banks (i.e. the *Very Long-Term Refinancing Operations*, or VLTROs) and the announcement of the still-unused ECB sovereign-bond purchase program (i.e. the *Outright Monetary Transactions*, or OMTs) have helped to reduce the perception of liquidity risk and its related risk premium. However, we find little evidence that the ECB large-scale asset-purchase programs of 2010 and 2011 (i.e. the *Securities Market Programs*, or SMP 1 & 2) have had any significant impact on the interbank risk. Eventually, we find that unconventional monetary policies have had very modest, if any, effects on the credit part of the spreads.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 details the construction of interbank rates. Section 4 develops the quadratic term-structure model. Section 5 describes the identification strategy and shows the estimation results. Section 6 performs the decomposition of EURIBOR-OIS spreads and discusses the impact of the ECB unconventional monetary policies; it also derives risk-premia-corrected default probabilities of banks. The last section concludes. Proofs are gathered in the Appendices.

2 Literature Review

In most term structure models, the authors assume that the default intensity and/or the short-term rate are affine functions of the underlying factors. A quadratic specification however possesses several advantages over the standard affine case. Constantinides (1992) shows that a standard QTSM with a specific quadratic short-term interest rate can generate positive yields for all maturities and more flexibility in the term structure to fit bond data. Leippold and Wu (2002) generalize the quadratic term structure models showing that this specification provides closed-form or semi closed-form formulas for bond pricing of most fixed-income derivatives. Ahn, Dittmar, and Gallant (2002) provide further empirical evidence that QTSM often outperforms the standard affine term structure specification (ATSM). Leippold and Wu (2007) study the joint behavior of exchanges rates and bond yields using QTSM models for Japan and the US. More recently, Andreasen and Meldrum (2011) and Kim and Singleton (2012) exploit the QTSM framework to model the term structure of interest rates in a context of extremely low monetary-policy rates. Turning to the credit literature, Hordahl and Tristani (2012) use a quadratic specification to model euro-area sovereign spreads, and Doshi, Jacobs, Ericsson, and Turnbull (2014) consider a quadratic intensity to price corporate credit default swaps. Our paper also adopts a quadratic approach in order to impose positivity of the risk intensities and spreads, and it takes advantage of a new well-suited technique to estimate the model, namely the quadratic Kalman filter (see Monfort, Renne, and Roussellet (2014)).

Our identification scheme follows several studies that rely on reduced-form no-arbitrage models to identify credit and liquidity components in the term structures of yields or spreads (e.g. Liu, Longstaff, and Mandell (2006), Feldhutter and Lando (2008), Longstaff, Mithal, and Neis (2005)). At the heart of these studies are credit/liquidity intensities whose fluctuations affect the whole term structure of spreads. As in Monfort and Renne (2014a), the present paper allows for some dynamic interactions between credit and liquidity risks, consistently with the theoretical predictions of, among others, Goldstein and Pauzner (2005) or He and Xiong (2012).

Our paper also relates to the interbank spreads literature. A wide range of studies deals with the determinants of interbank spreads: Taylor and Williams (2009) claim that counterparty risk was the main driver of the LIBOR-OIS spread, Michaud and Upper (2008) and Gyntelberg and Wooldridge (2008) find that credit and liquidity factors both played a role, while the results by Schwarz (2009) and Filipovic and Trolle (2013) suggest that liquidity risk has accounted for most of the LIBOR-OIS and EURIBOR-OIS spread variations over the period 2007-2009. In comparison, Smith (2010) emphasizes that most of the variation in the risk premia of interbank spreads is explained by credit risk. Finally, Angelini, Nobili, and Picillo (2011) highlight the main role of macro-factors to account for the dynamics of unsecured/secured money-market spreads. The measured impact of unconventional monetary policies is ambiguous: Taylor and Williams (2009) find no effects of the Fed's intervention in 2008, contrary to Christensen, Lopez, and Rudebusch (2014a). According to the latter, Fed's liquidity injections (TAF, for *Term Auction Facility*) reduced significantly the 3-month maturity interbank spread by about 70 basis points. Carpenter, Demiralp, and Eisenschmidt (2014) find that non-standard monetary-policy measures contributed to sustained lending activity by lowering funding volatility. Angelini, Nobili, and Picillo (2011) measure a modest impact of ECB exceptional 3-month refinancing operations, in contradiction with Abbassi and Linzert (2012). Cecioni, Ferrero, and Secchi (2011) provide a comprehensive review of the quantitative assessment regarding the relative importance of the interbank spreads' drivers, as well as of the effects of unconventional monetary policies in the U.S. and euro-area interbank markets.

3 Interbank market rates and risks

3.1 The unsecured interbank rates

The interbank money market is at the heart of bank funding issues. It is an over-the-counter market (OTC) where interbank loans are negotiated with maturities ranging from one day to to twelve months. As banks do not possess the same characteristics and underlying risks, there is no uniqueness of interbank rates. Only the disaggregated rates are really representative of the funding issues of each institution. However, such data are not publicly available and in order to conduct an analysis on interbank risks, a more aggregated measure must be considered.

The Euro Interbank Offered Rate (EURIBOR) provides a daily measure of the interest rates at which banks can raise unsecured funds from other financial institutions in the euro wholesale money market, for maturities ranging from one week to twelve months. A daily survey is sent to a panel of 30 to 50 creditworthy banks in the Euro area; the question of the survey is: *what are the rates at which euro interbank term deposits are being offered within the Eurozone by one prime bank to another?* The EURIBORs are then trimmed means of the contributed rates, the 15% of each tail being erased.

The loans that underlie the EURIBOR are unsecured, that is the lending bank does not receive collateral as protection against default by the borrowing one. Therefore, these rates carry some compensation for solvency issue that we refer to as credit risk. Furthermore, through an interbank loan, a lending bank exposes its funds during the time-to-maturity of the loan although those funds might be needed to cover the bank's own shortfalls. Since an unsecured interbank loan is highly specific to the identity of both counterparties, its unwinding is a costly task. This is taken into consideration by the lending bank at the inception of the loan, which gives rise to an extra compensation in the loan rate.

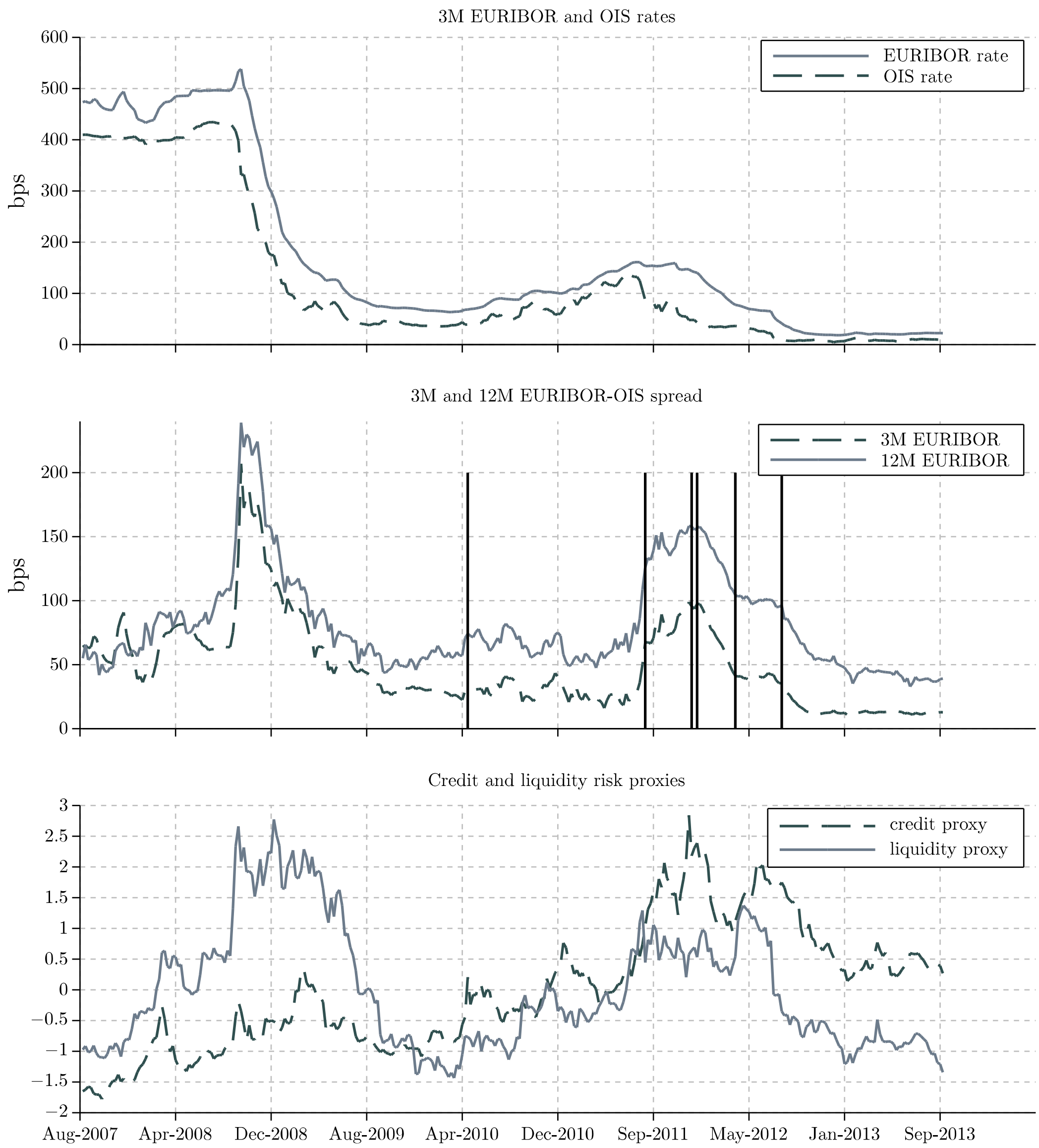
While there are no reliable data on volumes in term money markets, anecdotal evidence suggests that the financial crisis has resulted in a sharp decline in unsecured term money market volumes (see Eisenschmidt and Tapking (2009)). In spite of this, there is evidence that EURIBOR rates remain reliable proxies for bank funding costs. Typically, data collected from the ECB Short Term European Papers (STEP) database suggest that EURIBOR are very close to quotations of certificates of deposits issued by banks. For instance, it appears that the average of the spreads between (a) the issuance yields for certificates of deposits with an initial maturity comprised between 101 and 200 days and (b) the 5-month EURIBOR rate was lower than 3 basis points over the 2008-2012 period. Moreover, using U.S. data, Kuo, Skeie, and Vickery (2012) find that public interbank yield data beyond Libor are moderately informative about bank funding costs.

Figure 1 presents the evolution of the 3-month EURIBOR from August 2007 to September 2013. During the first year, the rate is stable around 500 basis points. The Lehman bankruptcy of September 2008 is followed by a sharp decline in EURIBOR of about 400 basis points, to 80 basis points. From mid-2010 onwards, the EURIBOR rises slowly to 150 basis points in September 2011 and decays to nearly 20 basis points during the recent period. Table 1 presents the descriptive statistics for 3, 6, 9, and 12-month EURIBORs.

3.2 The interbank *risk-free* rate

In this paper, the risk-free rates are proxied by the Overnight Indexed Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg indexed on overnight interbank rates, the EONIA in the euro-area case. OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew

Figure 1: Level of 3M rates and spreads



Notes: Top panel: plot of the 3M EURIBOR (light grey) and 3M OIS (dashed dark grey). Middle panel: plot of the 3M (dashed dark grey) and 12M (lighter grey) EURIBOR-OIS spreads. Units are in basis points. Bottom panel: credit (dashed dark grey) and liquidity (light grey) proxies; these proxies are demeaned and standardized. Time ranges from August 31, 2007 to September 13, 2013.

significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants as a proxy of the risk-free interbank yield curve (see e.g. Joyce, Lasaosa, Stevens, and Tong (2011) or BIS (2005)). As no principal is exchanged, the OIS requires nearly no immobilization of capital. Further, due to netting and credit-enhancement mechanisms (including call margins), the counterparty risk is limited in the case of a swap contract (Bomfim (2003)).

The upper panel of Figure 1 displays the 3-month OIS rate from August 2007 to September 2013. While this chart shows that EURIBOR and OIS rates present strong common fluctuations, the middle panel also highlights that the spread between the two rates has undergone substantial variations over the last five years. In the next subsection, we discuss the term structure of the EURIBOR-OIS spreads.

3.3 Preliminary analysis of the EURIBOR-OIS spreads

Being mostly stable before August 2008, the spread abruptly increased during Lehman crisis until December 2008, the 3-month spread peaking at 200 basis points, where a slow decay begins (see Figure 1, bottom). For sake of comparison, before summer 2007, the EURIBOR-OIS spread was around ten basis points; part of this deviation was accounted for by the fact that the EURIBOR is an offer rate while the OIS is a mid rate. Then, following a long stabilization period between August 2009 and 2010, a sharp rise stroke again in mid-2011. Since the beginning of 2012, the EURIBOR-OIS spreads have decreased, alternating between a linear decreasing trend and stable phases.

Standard descriptive statistics of spreads are provided in Table 1. The means of spreads increase with respect to maturity, from 48 basis points (3-month maturity) to 82 basis points (12-month maturity). This indicates a positive slope in the term structure of spreads, which is graphically illustrated by the bottom panel in Figure 1: except at the very beginning of the sample, the 12-month spread is always larger than the 3-month spread, up to around 50 basis points in late 2011.

Whereas the standard deviations are respectively stable and decreasing with maturity for OIS and EURIBOR rates, the standard deviations of spreads slightly increase

Table 1: Descriptive statistics of EURIBOR and OIS rates

	min	max	amplitude *bps*	mean	std	skewness	excess kurtosis
EURIBOR 3M	18.4	538.1	519.7	172.0	165.1	1.12	−0.35
EURIBOR 6M	29.4	543.1	513.7	190.8	158.9	1.10	−0.32
EURIBOR 9M	38.8	546.3	507.5	202.3	155.1	1.07	−0.31
EURIBOR 12M	47.4	549.3	501.9	213.0	152.1	1.06	−0.29
OIS 3M	4.5	434.6	430.1	123.6	145.7	1.29	−0.03
OIS 6M	2.35	442.85	440.5	125.1	144.9	1.30	0.03
OIS 9M	−0.5	453.5	454	127.9	143.7	1.29	0.05
OIS 12M	−1.1	465.3	466.4	131.2	142.2	1.27	0.07
Spread 3M	9.9	206.9	197	48.4	34.9	1.61	3.37
Spread 6M	19.6	222.5	202.9	65.7	36.5	1.62	3.44
Spread 9M	26.8	227.9	201.1	74.4	38.1	1.63	3.05
Spread 12M	32.9	239	206.1	81.8	40.0	1.54	2.38

Notes: Those figures are computed with weekly data ranging from August, 31 2007 to September, 13 2013.

with maturity. Regarding higher-order moments, Table 1 indicates that spreads are more positively skewed than the rates in level; also, contrary to the latter, spreads are heavy-tailed (positive excess kurtosis). The heavy-tail behavior is typically illustrated during the Lehman crisis on Figure 1, where both 3-month and 12-month spreads peak to 207 and 239 basis points, respectively. These levels are about 4 standard-deviation far from their respective sample means.

A principal component analysis performed on the four EURIBOR-OIS spreads proves that the first two principal components captures most of spread fluctuations, explaining 99.7% of the whole variance of the spreads (96.4% and 3.3% for the first and second principal components respectively).

3.4 Credit and liquidity proxies

In this subsection, we introduce credit and liquidity proxies on which we will base our identification of credit and liquidity parts of EURIBOR-OIS spreads. In the next sections, we relate these proxies to the factors driving our term structure model. This subsection ends by providing regression-based evidence of the presence of credit/liquidity effects in EURIBOR-OIS spreads.

The liquidity proxy we will use in our term-structure model is the first principal component of a set of three liquidity-related variables. These variables are chosen in order to capture different aspects of liquidity pricing, namely market and funding liquidity. While market liquidity is reflected by the difference between market and fundamental value of an asset, funding liquidity relates to the scarcity of capital (see Brunnermeier and Pedersen (2009)). Our first two proxies are mostly related to market liquidity whereas the last one is mostly related to funding liquidity. Nearly 60% of the total variance is explained by the first principal component.

- A first liquidity-pricing factor is the *KfW-Bund spread* (5-year maturity). KfW is a public German agency. KfW bonds are guaranteed by the Federal Republic of Germany. Hence, they embed the same credit quality as their sovereign counterparts, the so-called Bunds. KfW bonds being less liquid than their sovereign counterpart, the KfW-Bund spread essentially reflect liquidity-pricing effects (see Schwarz (2009), Monfort and Renne (2014b) or Schuster and Uhrig-Homburg (2012)). In the same spirit, Longstaff (2004) computes liquidity premia based on the spread between U.S. Treasuries and government-guaranteed bonds issued by Refcorp.

- A second liquidity factor is the Tbill-repo spread, computed as the yield differential between the 3-month German T-bill and the 3-month general-collateral repurchase agreement rate (repo). From an investor point of view, the credit qualities of the two instruments are comparable (as argued by Liu, Longstaff, and Mandell (2006)). The differential between the two rates corresponds to the convenience yield, that can be seen as a premium that one is willing to pay when holding highly-liquid Treasury securities (see e.g. Feldhutter and Lando (2008)).

- A third factor is based on the Bank Lending Survey conducted by the ECB on a quarterly basis. This survey is addressed to senior loan officers of a representative sample of around 90 euro-area banks; it addresses issues such as credit standards for approving loans as well as credit terms and conditions applied to enterprises and households. Our indicator is based on the following specific question of the survey: *Over the past three months, how has your bank's liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?* The respondents can answer $++$, $+$, 0, $-$ or $--$ to that question. We compute the proportion of $-$ and $--$ as a ratio of total answers. To obtain weekly series, we assign the same value to all weeks in a quarter (step function).

The credit proxy is the first principal component of a set of 36 Euro-zone bank CDS. We use 5-year CDS denominated in USD since these are the most traded – and therefore the most liquid – ones. Eight are German, six Italian, five Spanish, four French, four Dutch, three Irish, three Portuguese, two Austrian, and one Belgian. Nearly 72% of the total variance is explained by the first principal component.

Table 2 presents the results of preliminary regressions of the spreads on these credit- and liquidity-related proxies. The explanatory variables of regressions (1) to (4) are the two proxies presented above. In regressions (5) to (8), we have added the squares of these proxies and in the last four regressions, we replace the liquidity proxy by its three constituents and the credit proxy by the first three principal components of the

36 CDS. Liquidity factors enter all regressions with a positive sign. More surprisingly, this is not the case for credit variables: the first credit principal component is negative in half of the regressions, especially for shorter-term spreads. Regressions (5) to (8) show that coefficients of squared principal components are statistically significative and that these additional variables result in a substantial increase in the R squared, which points toward the existence of non-linear relationships between the proxies and the spreads. The R squared lie between 58% and 72% across specifications; while the quality of the fit is therefore relatively high, it will be significantly improved in the model we propose in the sequel of this paper.

4 The model

4.1 Notations and preliminary remarks

We consider the pool of the N banks of the EURIBOR panel. At date t, market participants get the new information $w_t = \{r_t, X_t, d_t, \ell_t\}$, where r_t is the short-term risk-free rate between dates t and $t+1$, $X_t = (x_{c,t}, x_{l,t})'$ is a (2×1) vector whose components are respectively credit- and liquidity-related, and where d_t and ℓ_t are two N-dimensional vectors of binary variables $d_t^{(i)}$ and $\ell_t^{(i)}$, with $i \in \{1, \dots, N\}$. While $d_t^{(i)}$ defines the credit state of bank i at date t, $\ell_t^{(i)}$ defines its liquidity status. In the following, we will make more precise the implications of defaults ($d_t^{(i)} = 1$) or liquidity shocks ($\ell_t^{(i)} = 1$) for interbank-loan payments. The next subsection notably shows how these shocks translate into credit and liquidity intensities. Note that the mechanisms proposed here are highly stylized and our approach eventually is reduced-form in nature; this simple framework is nevertheless aimed at helping better apprehend these intensities.

Finally, we denote by $\underline{w_t}$ the cumulative information up to date t, that is $\underline{w_t} =$

Table 2: Regressions of EURIBOR-OIS spreads on credit and liquidity proxies

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Spreads	3M	6M	9M	12M	3M	6M	9M	12M	3M	6M	9M	12M
Liquidity	KfW−Bund									0.186	0.184	0.197	0.214
										(0.012)	(0.012)	(0.013)	(0.013)
	Tbill−repo									0.047	0.027	0.038	0.051
										(0.026)	(0.027)	(0.027)	(0.028)
	BLS									0.107	0.098	0.102	0.111
										(0.014)	(0.015)	(0.015)	(0.015)
	PC1	0.268	0.286	0.294	0.294	0.215	0.228	0.243	0.254				
		(0.013)	(0.013)	(0.014)	(0.014)	(0.015)	(0.016)	(0.017)	(0.018)				
Credit	PC1	−0.089	−0.035	0.013	0.059	−0.133	−0.074	−0.024	0.021	−0.049	0.018	0.063	0.104
		(0.013)	(0.013)	(0.014)	(0.014)	(0.012)	(0.013)	(0.014)	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)
	PC2									0.046	0.098	0.084	0.05
										(0.023)	(0.024)	(0.024)	(0.025)
	PC3									−0.006	−0.022	−0.02	−0.02
										(0.012)	(0.012)	(0.013)	(0.013)
Squared	Liq. PC1					0.055	0.06	0.053	0.042				
						(0.011)	(0.012)	(0.013)	(0.014)				
	Credit PC1					0.112	0.105	0.098	0.093				
						(0.01)	(0.01)	(0.011)	(0.012)				
	Adj. R^2	0.58	0.592	0.601	0.602	0.708	0.698	0.684	0.667	0.684	0.701	0.712	0.72

Notes: This table reports results of regressions of the EURIBOR-OIS spreads of different maturities (3, 6, 9 and 12 months) on credit- and liquidity-related variables. Three liquidity-related variables are used: the KfW-Bund spread, the Tbill-repo spread and a variable based on the ECB Bank Lending Survey (BLS); we also compute the first principal component of these three variables (PC1 in the liquidity part of the table). The credit variables are the first three principal components of a set of 36 euro-area bank CDS. Some regressions also involve the squared values of the first PC of credit and liquidity variables. Intercepts are not shown. Robust standard errors are reported in parentheses.

$(w_t', w_{t-1}', ...)'$.

4.2 Intensities and EURIBOR rates

We assume that the panel of banks is homogeneous, in the sense that, conditional on $\underline{w_t}$, the default probabilities and the probabilities of being affected by a liquidity shock are the same for all the banks of the EURIBOR panel. This assumption notably implies that, at each date t, there is a single rate prevailing for interbank unsecured loans between t and a future date $t+h$. This interest rate is denoted by $R_{t,h}^{EUR}$. By definition of this rate, an interbank loan between dates t and $t+h$ of unit face value provides the borrower with the amount $B(t,h) = \exp(-hR_{t,h}^{EUR})$ at date t. Note that the pricing formulas derived in this paper feature continuously-compounded interest rates: denoting by z a market-quoted interest rate and applying the money-market day-count convention (ACT/360), the corresponding continuously-compounded rate is given by $\ln(1 + d \times z/360) \times 365/d$ where d is the residual maturity of the considered instrument, expressed in days.

Suppose that, at date t, bank i lends $B(t,h)$ to bank j for a period of length h. The maturity date is $t+h$ and, assuming no premature termination of the loan, the repayment is 1. Now, consider an intermediary date t^* (i.e. $t < t^* \leq t+h$). At date t^*, if bank j defaults or if bank i is hit by a liquidity shock, this terminates the interbank loan and the resulting payoffs are as follows:

- If bank j defaults at date t^* ($d_{t^*}^{(j)} = 1$), then bank i will not obtain full repayment at $t+h$. Instead, at date t^*, it recovers a fraction $\theta_c < 1$ of the "market value" of the loan that would have prevailed at date t^* in the absence of default. This market value corresponds to the face value of the loan discounted by the EURIBOR $R_{t^*,t+h}^{EUR}$. This set up builds on the "recovery at market value" assumption of Duffie and Singleton (1999).

- When bank i is hit by a liquidity shock at date t^* (i.e. $\ell_{t^*}^{(i)} = 1$), bank i has to find some cash in a limited period of time to meet an unexpected liquidity need. It may do so by negotiating a premature termination of the loan with bank j. The latter agrees, but at a discount: the repayment at date t^* is expressed as a fraction $\theta_l < 1$ of the aforementioned "market value" of the loan. Such a mechanism of costly liquidation is in the spirit of Ericsson and Renault (2006) or He and Xiong (2012).

In that context, the value of the loan at date $t+1$ writes:

$$B(t+1, h-1)\left\{(1 - d_{t+1}^{(j)} + \theta_c d_{t+1}^{(j)})(1 - \ell_{t+1}^{(i)} + \theta_l \ell_{t+1}^{(i)})\right\}.$$

Now, instead of lending the amount $B(t,h)$ to bank j at date t, bank i can roll-over this amount between dates t and $t+h$. In the latter case, the rate that applies is the risk-free short-term rate r_t (and $r_{t+1}, \dots, r_{t+h-1}$ for subsequent periods). In particular, this strategy yields the payoff $B(t,h)\exp(r_t)$ at date $t+1$ (this payoff is deterministic and known at date t). At equilibrium, bank i is indifferent between these two alternative investment strategies, which implies that:

$$\begin{aligned}&B(t,h) = \exp(-r_t) \times \\ &\mathbb{E}^{\mathbb{Q}}\left[B(t+1,h-1)\left\{(1 - d_{t+1}^{(j)} + \theta_c d_{t+1}^{(j)})(1 - \ell_{t+1}^{(i)} + \theta_l \ell_{t+1}^{(i)})\right\} \Big| \underline{w_t}, d_t^{(j)} = 0\right], \quad (1)\end{aligned}$$

where $\mathbb{E}^{\mathbb{Q}}$ denotes the expectation under the risk-neutral (pricing) measure.

Let us introduce the default and liquidity intensities $\lambda_{c,t}$ and $\lambda_{\ell,t}$. These intensities are assumed to depend on the factors $x_{c,t}$ and $x_{\ell,t}$, respectively, and are defined through (for any banks i and j):

$$\left\{\begin{array}{rcl}\mathbb{E}^{\mathbb{Q}}(d^{(j)}_{t+1}|\underline{w_t}, X_{t+1}, d^{(j)}_t = 0) & = & 1 - \exp(-\lambda_{c,t+1}) \\ \\ \mathbb{E}^{\mathbb{Q}}(\ell^{(i)}_{t+1}|\underline{w_t}, X_{t+1}, \ell^{(i)}_t = 0) & = & 1 - \exp(-\lambda_{\ell,t+1}).\end{array}\right. \tag{2}$$

The previous system of Equations implies that $\exp(-\lambda_{c,t+1})$ and $\exp(-\lambda_{\ell,t+1})$ are probabilities, and that $\lambda_{c,t+1}$ and $\lambda_{\ell,t+1}$ must be positive at all times. When these intensities are small, they are close to the default probabilities and to the probabilities of being hit by the liquidity shock, respectively. Besides, a first order approximation yields:

$$\left\{\begin{array}{rcl}\mathbb{E}^{\mathbb{Q}}((1-\theta_c)d^{(j)}_{t+1})|\underline{w_t}, X_{t+1}) & = & 1 - \exp(-(1-\theta_c)\lambda_{c,t+1}) \\ \\ \mathbb{E}^{\mathbb{Q}}((1-\theta_\ell)\ell^{(i)}_{t+1})|\underline{w_t}, X_{t+1}) & = & 1 - \exp(-(1-\theta_\ell)\lambda_{\ell,t+1}).\end{array}\right.$$

Under the assumption that, conditional on $(\underline{w_t}, X_{t+1})$, liquidity shocks ℓ_{t+1} and defaults d_{t+1} are independent (under $\mathbb{Q}$), and introducing the total intensity $\lambda_t = (1-\theta_c)\lambda_{c,t} + (1-\theta_\ell)\lambda_{\ell,t}$, the law of iterated expectations in Equation (1) yields:

$$B(t,h) \quad = \quad \mathbb{E}^{\mathbb{Q}}_t\left[B(t+1,h-1)\exp(-r_t - \lambda_{t+1})\right], \tag{3}$$

where $\mathbb{E}^{\mathbb{Q}}_t(\bullet)$ denotes the $\mathbb{Q}$-expectation conditional to $\underline{w_t}$. After additional recursive uses of the law of iterated expectations (starting from Equation (3) for $B(t+h-1,1)$ and iterating backwards), and assuming that d_t and ℓ_t do not $\mathbb{Q}$-Granger-cause X_t, we get (see Proposition 3 in Monfort and Renne (2014b)):

$$B(t,h) \quad = \quad \mathbb{E}^{\mathbb{Q}}_t\left[\exp(-r_t - \lambda_{t+1} - \cdots - r_{t+h-1} - \lambda_{t+h})\right]. \tag{4}$$

Since $B(t,h) = \exp(-hR^{EUR}_{t,h})$, we have:

$$R^{EUR}_{t,h} = -\frac{1}{h}\ln\left\{\mathbb{E}^{\mathbb{Q}}_t\left[\exp(-r_t - \lambda_{t+1} - \cdots - r_{t+h-1} - \lambda_{t+h})\right]\right\}. \tag{5}$$

4.3 OIS swap rates and the EURIBOR-OIS spreads

An OIS is an interest-rate derivative that allows for exchanges between a fixed-interest-rate cash flow and a variable-rate cash flow. At maturity, the payoff received by the fixed-rate payer is the difference between (a) the notional (W, say) inflated with the date-t OIS (fixed) rate (i.e. $W\exp\left\{hR^{OIS}_{t,h}\right\}$) and (b) the same notional capitalized with the realized short-term rates (i.e. $W\exp\{r_t + \ldots + r_{t+h-1}\}$). Note that the latter expression implicitly reckons that the OIS reference rate –that is the EONIA rate– corresponds to the risk-free rate r_t, thereby assuming that lending on the overnight interbank market preserves the lending bank from (i) liquidity and (ii) credit risk. The rationale behind (i) and (ii) are the following:

(i) By rolling its cash on the overnight market (at the EONIA rate), a bank is not exposed to the risk of having to liquidate longer-term investments upon the realization of the liquidity shock.

(ii) While the EONIA is an unsecured-transaction rate, the extremely-short maturity of these transactions substantially reduces the credit-risk exposure of the lending bank. This point is corroborated by a comparison of EURIBOR-OIS spreads with spreads between Repo rates – where credit-risk effects are kept at a minimum through collateralization schemes – and OIS rates: over 2007-2013, the mean absolute value of the 3-month Repo-OIS spread is about 10 times smaller than the one of the EURIBOR-OIS spread of the same maturity (the former being of a few basis points).

At the inception date of the swap, there is no cash-flow exchange between the two counterparties, that is, the discounted values of the two legs are initially the same:

$$W\mathbb{E}_t^{\mathbb{Q}}\left[\exp(hR_{t,h}^{OIS})\exp\left\{-r_t-\ldots-r_{t+h-1}\right\}\right]=W,$$

or:

$$R_{t,h}^{OIS}=-\frac{1}{h}\log\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-r_t-\ldots-r_{t+h-1}\right\}\right]. \tag{6}$$

As in, e.g., Berndt, Douglas, Duffie, Ferguson, and Schranz (2005), Pan and Singleton (2008) or Longstaff, Pan, Pedersen, and Singleton (2011), we assume that the short-term risk-free interest rate and the intensity processes are independent under $\mathbb{Q}$. Denoting by $S(t,h)$ the EURIBOR-OIS spread of maturity h, it follows that:

$$S(t,h)=R_{t,h}^{EUR}-R_{t,h}^{OIS}=-\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\sum_{i=1}^{h}-\lambda_{t+i}\right)\right]\right). \tag{7}$$

Equation (7) shows that, under these assumptions, the study of EURIBOR-OIS spreads does not require the modeling of the short-term risk-free interest rate r_t. In the following, we impose a factor structure and a specification for the modeling of both the credit and liquidity intensities to obtain pricing formulas for the interbank spreads.

4.4 Intensity specification and factor dynamics

Each of the two intensities depends on a single respective factor. These credit and liquidity factors are respectively denoted by $x_{c,t}$ and $x_{\ell,t}$. We formulate their dynamics to allow for interactions, authorizing lagged Granger causality. The two factors are contemporaneously influenced by independent Gaussian idiosyncratic shocks $\varepsilon_{c,t}$ and $\varepsilon_{\ell,t}$. Summing up, the joint dynamics of the risk factors $X_t=(x_{c,t},x_{\ell,t})'$ can be described by the following $VAR(1)$ representation:

$$X_t = \mu + \Phi X_{t-1} + \varepsilon_{t+1}, \tag{8}$$

where $\varepsilon_{t+1} = (\varepsilon_{c,t+1}, \varepsilon_{\ell,t+1})' \sim \mathcal{IIN}^{\mathbb{P}}(0, I_2)$.

Now, it remains to specify the relationship between the intensities $(\lambda_{c,t}, \lambda_{\ell,t})$ and the factors $(x_{c,t}, x_{\ell,t})'$. In a preliminary analysis, whose results are not reported here for sake of brevity, we postulated a linear relationship between the intensities and the factors, within a standard Gaussian affine term-structure model. However, the results were not satisfying, the model clearly violating the non-negativity of spreads. The model-implied frequencies of generating negative spreads (i.e. considering their marginal densities) was huge and close to 50% for all maturities. This comes from the facts that (a), in such a model, the distribution of model-implied spreads is Gaussian and that (b) consistently with the high persistence of observed spreads, the resulting model-implied variance of the spreads is large. This failure illustrates the inappropriateness of Gaussian ATSM to model such spreads. Therefore, following Doshi, Jacobs, Ericsson, and Turnbull (2014) or Gouriéroux and Monfort (2008), we set a quadratic relationship between the intensities and the associated factors:

$$\lambda_{c,t} = \Lambda_c x_{c,t}^2 \quad \text{and} \quad \lambda_{\ell,t} = \Lambda_\ell x_{\ell,t}^2 \,. \tag{9}$$

This ensures that the underlying probabilities of liquidity and default events are constrained between 0 and 1, both $\lambda_{c,t}$ and $\lambda_{\ell,t}$ being positive (see Equation 2). In turn, this implies that the spreads at any maturity are positive, which can be seen from Equation (7). Besides, an additional advantage of this modeling is that it allows to accommodate heteroskedasticity in the spreads (see Ahn, Dittmar, and Gallant (2002)).

The risk-neutral dynamics of the intensities results from the specification of a stochastic discount factor (SDF). We denote by $M_{t,t+1}$ the SDF between t and $t+1$. It is assumed to be exponential-affine in $(\varepsilon_{c,t+1}, \varepsilon_{\ell,t+1})'$:

$$M_{t,t+1} = \exp\left[\Gamma_t' \left(\varepsilon_{c,t+1}, \varepsilon_{l,t+1}\right)' - \frac{1}{2}\Gamma_t'\Gamma_t - r_t\right], \tag{10}$$

where Γ_t is the vector of market prices of risks. As is standard in the term structure literature (see Piazzesi (2010)), we make those market prices of risk time-varying by making them affine in the current level of the risk factors:

$$\Gamma_t = \Gamma_0 + \Gamma \left(x_{c,t}, x_{\ell,t}\right)', \tag{11}$$

where Γ_0 and Γ are respectively a (2×1)-dimensional vector and a (2×2)-dimensional matrix. Note that though our SDF specification includes the short-term interest rate r_t, we do not have to assign it to any specific dynamics or distribution and let it unspecified; it is just assumed here that it is independent from ε_t under the physical measure, which implies that the same holds true under the risk-neutral measure for our SDF specification (see Monfort and Renne (2014b)). With this SDF and the physical dynamics given in Equation (8), it can be easily shown that the risk-neutral dynamics of $(x_{c,t}, x_{\ell,t})'$ is given by a Gaussian $VAR(1)$ with shifted parameters. More specifically, we have:

$$X_t = \mu^* + \Phi^* X_{t-1} + \varepsilon^*_{t+1}, \tag{12}$$

where $\varepsilon^*_{t+1} = (\varepsilon^*_{c,t+1}, \varepsilon^*_{\ell,t+1})' \sim \mathcal{IIN}^{\mathbb{Q}}(0, I_2)$. The mapping between the parameters defining the historical and the risk-neutral dynamics depends on these prices of risk:

$$\mu^* = \mu + \Gamma_0 \quad \text{and} \quad \Phi^* = \Phi + \Gamma. \tag{13}$$

4.5 Recursive pricing formulas

Putting together the intensity specifications of Equation (9) and the risk-neutral dynamics given by Equation (12), it can be shown that our model belongs to the class of Quadratic Term Structure Models (QTSM). We show in Appendix A.1 that the spreads $S(t,h)$ of Equation (7) can be expressed as a quadratic combinations $x_{c,t}$ and $x_{\ell,t}$. This results from the fact that the conditional Laplace transform of the vector $(X'_{t+1}, Vec(X_{t+1}X'_{t+1})')'$ given $\underline{X_t}$ is exponential affine in $(x'_t, Vec(x_t x'_t))'$ (see Gouriéroux and Sufana (2011) or Cheng and Scaillet (2007)) and, therefore, the process $(X'_{t+1}, Vec(X_{t+1}X'_{t+1})')'$ is affine. We have:

$$\begin{aligned} S\left(t,h\right) &= -\frac{1}{h}\left(\Theta_{0,h} + \Theta'_{1,h}X_t + X'_t\Theta_{2,h}X_t\right) \\ &\triangleq \theta_{0,h} + \theta'_{1,h}X_t + X'_t\theta_{2,h}X_t. \end{aligned} \tag{14}$$

The factor loadings $\theta_{0,h}$, $\theta_{1,h}$ and $\theta_{2,h}$ are maturity-dependent and are functions of risk-neutral dynamics parameters and of Λ, which is the (2×2)-dimensional diagonal matrix containing $(1-\theta_c)\Lambda_c$ and $(1-\theta_\ell)\Lambda_\ell$ on its diagonal. The loadings $\Theta_{0,h}$, $\Theta_{1,h}$

and $\Theta_{2,h}$ can be computed recursively as (see Appendix A.1):

$$\begin{cases} \Theta_{0,h} &= \Theta_{0,h-1} + \Theta'_{1,h-1}\left[I_n - 2\left(\Theta_{2,h-1} - \Lambda\right)\right]^{-1}\left(\mu^* + \frac{1}{2}\Theta_{1,h-1}\right) \\ &+ \quad \mu^{*\prime}\left(\Theta_{2,h-1} - \Lambda\right)\left[I_n - 2\left(\Theta_{2,h-1} - \Lambda\right)\right]^{-1}\mu^* - \frac{1}{2}\log\left|I_n - 2\left(\Theta_{2,h-1} - \Lambda\right)\right| \\ \Theta_{1,h} &= \Phi^{*\prime}\left\{\left[I_n - 2\left(\Theta_{2,h-1} - \Lambda\right)\right]^{-1}\left[\Theta_{1,h-1} + 2\left(\Theta_{2,h-1} - \Lambda\right)\mu^*\right]\right\} \\ \Theta_{2,h} &= \Phi^{*\prime}\left(\Theta_{2,h-1} - \Lambda\right)\left[I_n - 2\left(\Theta_{2,h-1} - \Lambda\right)\right]^{-1}\Phi^*, \end{cases} \tag{15}$$

where initial conditions are given by $\Theta_{0,0} = 0$, $\Theta_{1,0} = (0,0)'$, and $\Theta_{2,0} = [0]_{i,j\in\{1,2\}}$. One of our main objectives is to decompose spreads into a credit and a liquidity component. A necessary condition to obtain such twofold decomposition is that $\Theta_{2,h}$ is diagonal for all maturities h. This condition constrains Φ^* to be diagonal.

5 Estimation procedure

5.1 Identification strategy: linking proxies and latent factors

In the following, we relate latent factors $x_{c,t}$ and $x_{\ell,t}$ to our credit and liquidity proxies, that we respectively denote by $P_{c,t}$ and $P_{\ell,t}$. Recall that these proxies are first principal components of sets of credit- and liquidity-related variables.

We assume that – up to a measurement error term – the proxies are quadratic functions of the corresponding latent factors. This relationship, of the same kind of the one relating the latent factors to modeled spreads, is consistent with the fact that several variables used in the computation of proxies are also homogeneous to interest rates. Formally:

$$\begin{cases} P_{c,t} &= \pi_{c,0} + \pi_{c,1} x_{c,t} + \pi_{c,2} x_{c,t}^2 + \sigma_{\nu_c} \nu_{c,t} \\ P_{\ell,t} &= \pi_{\ell,0} + \pi_{\ell,1} x_{\ell,t} + \pi_{\ell,2} x_{\ell,t}^2 + \sigma_{\nu_l} \nu_{l,t}, \end{cases} \tag{16}$$

where $\nu_{c,t}$ and $\nu_{l,t}$ are Gaussian standardized and uncorrelated noises. These measurement errors authorize the proxies to imperfectly represent the underlying corresponding risk, addressing potential concerns regarding the fact that our proxies are not pure measures of credit and liquidity risks. For instance, CDS contracts may be affected by liquidity issues. It is also worth stressing that, even though risk factors $x_{c,t}$ and $x_{\ell,t}$ are contemporaneously uncorrelated, their VAR(1) dynamics authorizes the presence of lagged Granger causality between them. Equations (16) therefore imply that the credit (resp. liquidity) proxy is a combination of past (resp. past and current) liquidity shocks, of past and current (resp. past) credit shocks and of an error $\nu_{c,t}$ (resp. $\nu_{l,t}$).

It can be remarked that our proxies are mostly built on market rates and a no-arbitrage consistency argument would impose some parametric restrictions on the loadings of our proxies equations. The reasons for not imposing those constraints in our estimation technique are twofold. First, the pricing of CDS contracts imply non-linearities – notably because CDS are not homogeneous to zero-coupon rates – and rely mostly on approximation techniques (see for instance Filipovic and Trolle (2013)). Those non-linearities would greatly increase the computational complexity of the estimation. Second, our liquidity proxy is a blend of variables of different nature. We more likely see it as a broad liquidity indicator rather than a tradable instrument. Building a comprehensive model that allows for the pricing of a variety of fixed-income instruments is beyond the scope of this paper.

5.2 State-space model and estimation strategy

The state-space representation of the model is obtained by gathering: (a) the $\mathbb{P}$-dynamics of the factors $x_{c,t}$ and $x_{\ell,t}$ (Equation (8)), (b) the spread formulas (Equation (14)) and (c) the proxies measurement equations (Equation (16)). More specifically, the measurement equations are:

$$\begin{aligned} S(t,h) &= \theta_{0,h} + \theta_{1,h}' X_t + X_t' \theta_{2,h} X_t + \sigma_\eta \eta_{t,h} \qquad \forall h \in \{13, 26, 39, 52 \text{ weeks}\} \\ P_{i,t} &= \pi_{i,0} + \pi_{i,1} x_{i,t} + \pi_{i,2} x_{i,t}^2 + \sigma_{\nu,i} \nu_{i,t} \qquad \forall i = \{c, \ell\}, \end{aligned} \tag{17}$$

where the components of the vector of pricing errors η_t and $\nu_{i,t}$ are independent Gaussian white noises with unit variance. Parameters $\pi_{i,0}$, $\pi_{i,1}$, and $\pi_{i,2}$ are not constrained by model-implied restrictions, contrary to the loadings $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ that derive from Equations (14) and (15). Appendix A.3 presents additional restrictions that we impose on model parameters in order to ensure a positive correlation between the proxies and the corresponding intensities.

The estimation data cover the period from August 31, 2007 to September 13, 2013 at the weekly frequency (end of week data). Interest rates and CDS data are extracted from Bloomberg. The EURIBOR-OIS spreads of the following maturities enter the measurement equations: 3, 6, 9, and 12 months.

The model parameters are estimated by maximizing the likelihood function, which is approximated by means of a Kalman-type algorithm. Whereas recent articles use extensively the so-called Unscented Kalman Filter (UKF, see for instance Filipovic and Trolle (2013) or Christoffersen, Dorion, Jacobs, and Karoui (ming)), we rely on the Quadratic Kalman filter (QKF) of Monfort, Renne, and Roussellet (2014), which is specifically fitted to quadratic measurement equations and which shows nice performances in this context. The filtering algorithm is detailed in Appendix A.2. Once the model parameters are estimated, a final call of the algorithm provides us with estimates of the latent factors.

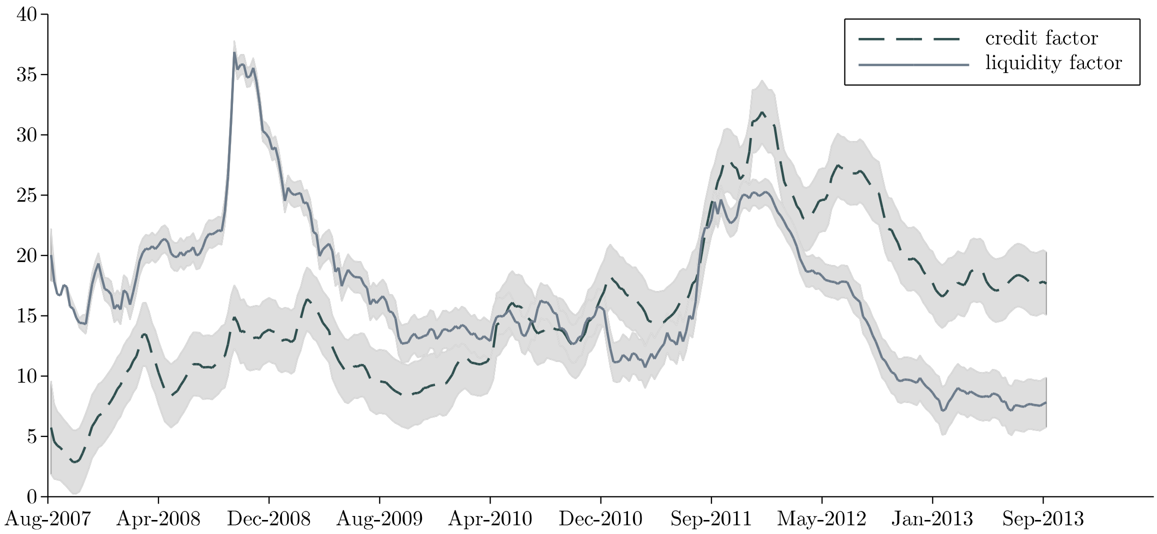
5.3 Estimation results

Table 3 reports the estimates of the physical and risk-neutral dynamics parameters of $x_{c,t}$ and $x_{\ell,t}$. Both processes are highly persistent, especially under the risk-neutral measure (with eigenvalues of 1 and 0.998). The fact that risk factors are more persistent under the pricing measure than under the physical measure is common in the literature (see e.g. Pan and Singleton (2008)). Intuitively, this feature implies that bad times tend to last longer under $\mathbb{Q}$ than under $\mathbb{P}$, which translates into risk premia. In a preliminary estimation, we found that the Granger causality from credit to liquidity was insignificantly different from zero. Hence it has been imposed exactly to zero in a second pass of maximization. On the other hand, the liquidity factor significantly Granger causes the credit factor, which implies some liquidity feedback in the credit risk. Table 3 also reports the market prices of risk parameters, which can be directly backed out from physical and risk-neutral parameters (see Equation (13)).

Figure 2 presents the filtered time-series of the factors. Whereas they possess roughly the same patterns as the credit and liquidity proxies, the quadratic specification and the measurement errors allows for a greater flexibility in the factor's behavior. In particular, the liquidity factor peaks are shorter in duration than those of the corresponding proxy.

The remaining parameter estimates are gathered in Table 4. Both intensities loadings are significantly different from zero, and we observe that $(1-\theta_\ell)\Lambda_\ell > (1-\theta_c)\Lambda_c$ (last row of Table 4). This means that liquidity shocks are the main drivers of the short-term fluctuations in the total intensity since the innovations $(\varepsilon_{c,t}, \varepsilon_{\ell,t})$ of factors $(x_{c,t}, x_{\ell,t})$ are of unit variance and given that the total intensity λ_t is given by

Figure 2: Estimated credit and liquidity factors



Notes: Time ranges from August 31, 2007 to September 13, 2013. The grey shaded areas are the 95% confidence intervals of the latent factors (this uncertainty is the one associated with the filtering technique).

$(1-\theta_c)\Lambda_c x_{c,t}^2 + (1-\theta_\ell)\Lambda_\ell x_{\ell,t}^2$. However, the credit factor is more persistent than the liquidity one under $\mathbb{Q}$ (see the diagonal elements of Φ^* in Table 3); the relative importance of credit in the spread therefore increases with maturity. This will be illustrated below.

The variance estimate $\widehat{\sigma}_\eta^2$ associated with the error terms in the spread equation is 0.007, which translates into an average pricing error of 8 basis points for all maturities. This implies that the model captures 95% of the variation of the spreads, which is much higher than the R-squared obtained in our preliminary regressions (Table 2).

Besides, the estimated model proves to be able to capture part of the heteroskedasticity in spreads. Indeed, unreported results suggest that the model-implied conditional volatility of spreads exhibits a 60% correlation with realized volatility (measured using daily data on a 2-month rolling window). Note that this is due to our quadratic framework, a standard Gaussian model being unable to generate time-variation in conditional yields' variance.

Table 3: Factor parameter estimates

	$\mathbb{P}-$dynamics			$\mathbb{Q}-$dynamics			Market prices of risk		
	μ	$x_{c,t-1}$	$x_{\ell,t-1}$	μ^*	$x_{c,t-1}$	$x_{\ell,t-1}$	Γ_0	$x_{c,t-1}$	$x_{\ell,t-1}$
$x_{c,t}$	0.107	0.960	0.023	1.097	1	0	0.990	0.040	-0.023
	(0.020)	(0.009)	(0.005)	(0.311)	–	–	(0.318)	(0.010)	(0.005)
$x_{\ell,t}$	0.210	0	0.962	0.168	0	0.998	-0.042	0	0.036
	(0.011)	–	(0.004)	(0.040)	–	(0.001)	(0.043)	–	(0.004)

Notes: Standard errors are in parentheses. The '—' sign indicates either that the constraint is binding or that the value is calibrated, thus the parameter is not estimated and its estimator has therefore no standard deviation.

Table 4: Parameter estimates of measurement equations

Equation	Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
$P_{c,t}$	$\pi_{c,0}$	-1.977	$\pi_{c,1}$	0.132	$\pi_{c,2}$	0.00001
		(0.217)		(0.024)		(0.0007)
$P_{\ell,t}$	$\pi_{\ell,0}$	-1.370	$\pi_{\ell,1}$	0.039	$\pi_{\ell,2}$	0.002
		(0.092)		(0.008)		(0.0002)
noise	$\sigma^2_{\nu_c}$	0.1	$\sigma^2_{\nu_\ell}$	0.1	σ^2_{η}	0.007
		–		–		(0.0004)
λ_t	$(1-\theta_c)\Lambda_c$	0.00009	$(1-\theta_\ell)\Lambda_\ell$	0.00134		
		(0.00002)		(0.00009)		

Notes: Standard errors are in parentheses. The '—' sign indicates that the value is calibrated.

6 Decomposing EURIBOR-OIS spreads

In this section, we present the model-implied decomposition of EURIBOR-OIS spreads for all maturities. We can perform our spread decomposition along two dimensions: credit vs. liquidity on the one hand (as in e.g. Filipovic and Trolle (2013)) and risk premia vs. expected components on the other hand (as in e.g. Pan and Singleton (2008)).

6.1 The decomposition method

First, we decompose observed spreads into credit and liquidity components. Remember from Equation (7) that the spread of maturity h involves the conditional $\mathbb{Q}$-expectations of both credit and liquidity intensities up to maturity. To obtain the effects on credit only (say), we simply put $\Lambda_\ell = 0$ and recompute the counterfactual spread implied by this restriction. More formally, if we denote by $S_c(t,h)$ and $S_\ell(t,h)$ the respective credit and liquidity components of the observed spread, we have:

$$\begin{aligned} S_c(t,h) &= -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{\sum_{i=1}^{h} -(1-\theta_c)\lambda_{c,t+i}\right\}\right]\right) \\ &\triangleq \theta_{0,h}^{(c)} + \theta_{1,h}^{(c)} x_{c,t} + \theta_{2,h}^{(c)} x_{c,t}^2 \end{aligned} \tag{18}$$

$$\begin{aligned} S_\ell(t,h) &= -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{\sum_{i=1}^{h} -(1-\theta_\ell)\lambda_{\ell,t+i}\right\}\right]\right) \\ &\triangleq \theta_{0,h}^{(\ell)} + \theta_{1,h}^{(\ell)} x_{\ell,t} + \theta_{2,h}^{(\ell)} x_{\ell,t}^2, \end{aligned} \tag{19}$$

where $\theta_{0,h}^{(c)}$, $\theta_{1,h}^{(c)}$, $\theta_{2,h}^{(c)}$; and $\theta_{0,h}^{(\ell)}$, $\theta_{1,h}^{(\ell)}$, and $\theta_{2,h}^{(\ell)}$ can be computed recursively using the formulas presented in System (15), imposing respectively $\Lambda_\ell = 0$ and $\Lambda_c = 0$. Since the factors $x_{c,t}$ and $x_{\ell,t}$ are independent under the risk-neutral measure, we obtain an exact decomposition of the modeled spread and, for the observed spread we get:

$$S(t,h) = S_c(t,h) + S_\ell(t,h) + \sigma_\eta \eta_{t,h} \,, \tag{20}$$

where $\sigma_\eta \eta_{t,h}$ exactly matches the measurement errors included in the measurement equations (Equation (17)). Given their relative small size and following the usual approach, we neglect those measurement errors in the analysis and consider only the decomposition of the modeled spread $S(t,h) - \sigma_\eta \eta_{t,h}$.

Spreads can be split in an other dimension. Indeed, our estimation strategy provides us with both the physical and the risk-neutral dynamics of the factors. This knowledge enables us to extract risk premia from observed spreads. Risk premia are defined as the differentials between observed (or model-implied) spreads and the ones that would prevail if investors were risk-neutral. In the latter case, which corresponds to the expectation hypothesis, spreads would be those obtained by using the physical dynamics to compute the expectation term in Equation (7). Using the estimated $\mathbb{P}$-dynamics parameters and the fact that λ_t is the same function of X_t under $\mathbb{P}$ and $\mathbb{Q}$, we calculate a new set of factor loadings under the expectation hypothesis. To perform the credit/liquidity decomposition of this expected component, we use exactly the same method as previously (Equations (18) and (19), replacing the $\mathbb{Q}$ dynamics by the $\mathbb{P}$ dynamics), setting respectively $\Lambda_\ell = 0$ and $\Lambda_\ell = 0$ to obtain the credit and liquidity parts of the expected components of the spreads. We denote these components by $S_c^{\mathbb{P}}(t,h)$ and $S_\ell^{\mathbb{P}}(t,h)$.

6.2 Decomposition results

The decomposition of the 6- and 12-month maturity spreads are represented in Figure 3. On average, the liquidity component accounts for most of the spread averages over the sample period, representing more than 75% of the spreads for all maturities (see Table 5). This average share, which is comprised on average between 10% and 25%, increases with respect to maturity. The first row of charts in Figure 3 illustrates that the liquidity factor accounts for much of the high-frequency variations in the spreads, in particular during the distress period of late 2008 (after the Lehman collapse) and in end 2011 (in a period of particular strain in the European sovereign markets) no matter the maturity.

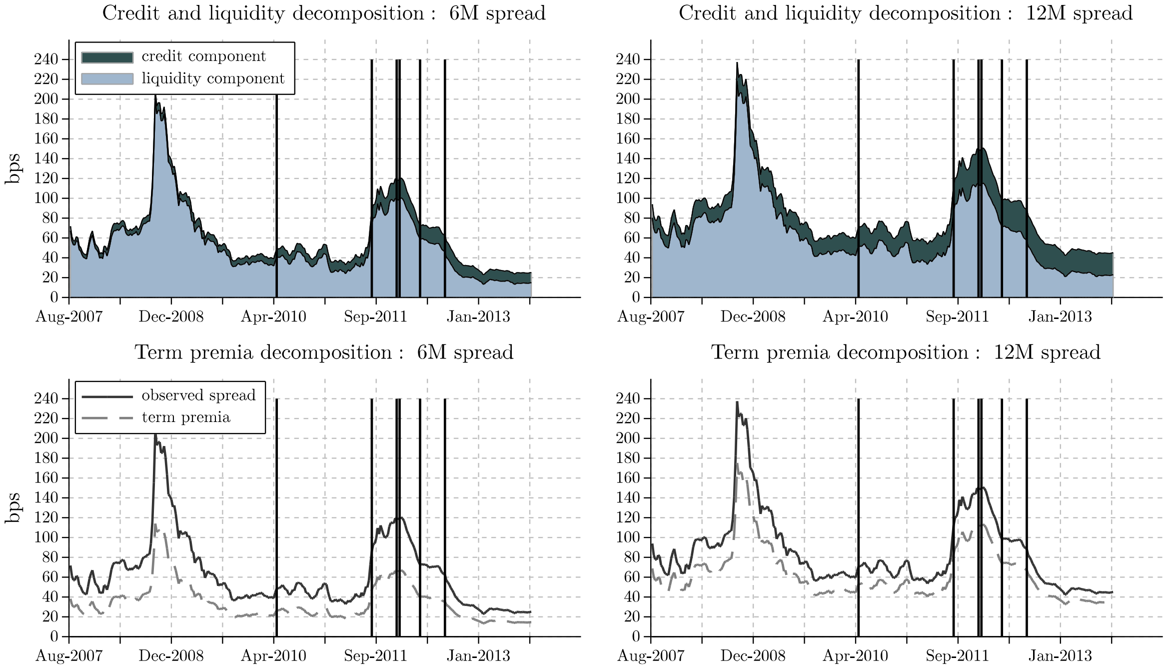
Table 5: Descriptive statistics of EURIBOR-OIS components

		Total spread		Risk premium	
		Credit	Liquidity	Credit	Liquidity
average level (in bps)	Spread 3M	5.43	48.22	4.22	15.50
	Spread 6M	9.35	53.70	8.28	28.21
	Spread 9M	14.49	59.42	13.53	38.47
	Spread 12M	20.82	65.36	19.97	47.44
average (% of spread avg)	Spread 3M	10.21	90.73	7.86	28.89
	Spread 6M	14.97	85.94	13.14	44.74
	Spread 9M	19.77	81.09	18.31	52.05
	Spread 12M	24.36	76.46	23.17	55.04

Notes: The modeled spreads are decomposed into four components, along two dimensions: credit vs. liquidity and expected part vs. risk premium. The risk premia are the parts of the spreads that would not exist is investors were risk-neutral. The table shows for instance that for the 9-month maturity, 70% of the EURIBOR-OIS spread correspond to risk premia, a quarter of which ($\simeq 18/(18+52)$) being accounted for by aversion to credit risk.

The second row of Figure 3 displays the decomposition of the observed spread into the risk premium and the expected component: the risk premium component and the observed spread have very similar features, and are positively and highly correlated. Together with Table 5, we see that the share of the spreads explained by risk premia is increasing with the maturity: for the 3-month spread, credit and liquidity risk premia account respectively for 8% and 29% of the total spread average ; and for the 12-

Figure 3: 6M EURIBOR-OIS spreads decomposition



Notes: Date ranges from August 31, 2007 to September 13, 2013. Units are in basis points. Top panel represents the stacked components of the spread: light grey component is the liquidity component and the dark grey corresponds to the credit component. Bottom panel represents the modeled spread and its term premia. The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi's London speech (last axis).

month spread, respectively 23% and 55% (see third and fourth columns of Table 5). In times of distress (Lehman collapse or the European debt crisis), the level of risk premia, which are the compensations for exposures to non-diversifiable systematic risk, increases for all maturities.

Figure 4 confirms the previous statements by presenting decomposition of the term structure of EURIBOR-OIS spreads at different dates. In particular, the second and third rows show respectively the decomposition of the expected component of spreads and of risk premia. Under the expectation hypothesis (i.e. in the absence of risk premia), the liquidity risk term structure is downward sloping whereas the credit component is smaller and almost constant with respect to maturity. Conversely, looking at the last row of Figure 4, both credit and liquidity risk premia are upward sloping with respect to maturity. Note that these features are not specific to the four chosen dates.

In the next section, we exploit the time series and the term structure of the spreads components to analyze the effectiveness of unconventional monetary policies in the Eurozone.

6.3 The impact of unconventional monetary policy on interbank risk

The main programs of unconventional monetary policies in the Eurozone can be broadly separated into three periods. The *Securities Market Program* (SMP) consisted in sterilized bond-buying on the secondary market. It was designed to "ensure depth and liquidity in [...] market segments that are dysfunctional" and was implemented in May 2010 and August 2011. Later, on the December 8, 2011, the ECB disclosed the design of *Very Long Term Refinancing Operations* (VLTRO), whereby 3-year maturity open market operations were proposed in the form of reverse repo. Two allotments were granted on December 21, 2011 and on the February 29, 2012, of respectively EUR489bln and EUR530bln Euros to 523 and 800 banks. More recently, during August 2012, Mario Draghi announced the setting of *Outright Monetary Transactions* (OMT) in his London speech. Conditionally on fiscal adjustments or precautionary programs enforcement by candidate countries, the ECB is ready to trade in secondary sovereign bond markets with "no ex ante quantitative limits". Whereas this framework has been announced it has not been applied in practice yet.

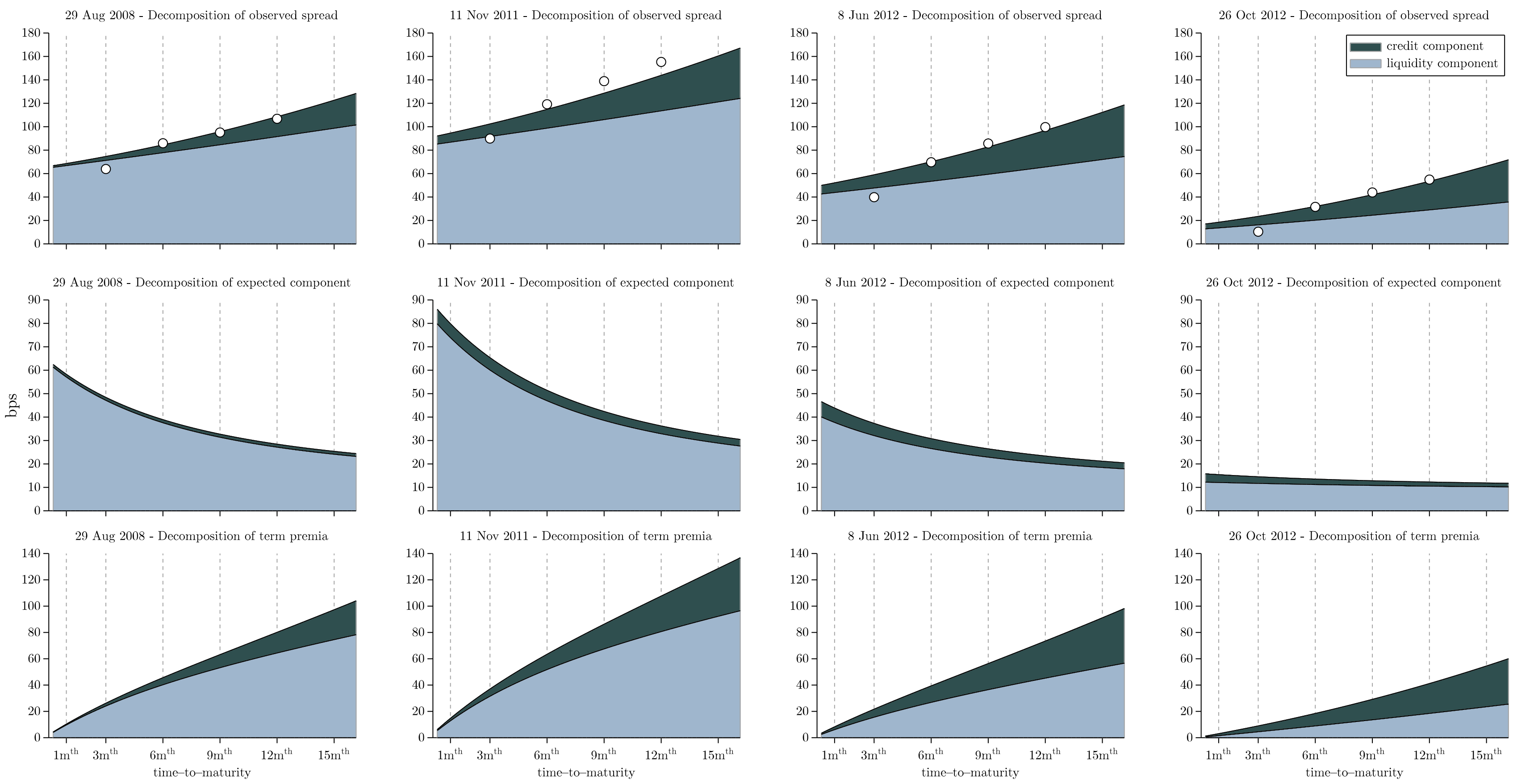
Interestingly, the EURIBOR-OIS spreads have decreased continuously since the VLTRO announcement in December 2011. This drop has led many commentators (and central bankers) to claim that the ECB unconventional refinancing operations were successful in alleviating interbank market tensions. In particular, according to ECB officials, the non-standard VLTRO operations addressed "only the liquidity side of the [interbank market] problem" (see Draghi (2012)'s interview with the Wall Street Journal, published on February 24, 2012). Our results support this view as the liquidity component of the spreads has slowly faded away since the VLTRO announcement date (see Figure 3, first row). A further positive effect can also be attributed to the OMT announcement through liquidity (see the last vertical bar in the charts).

The same pattern can be observed in Figure 4. After the SMP and before the VLTRO announcement (second column of charts), liquidity risk still accounts for most of the term structure of interbank spreads with between 90 to 120 basis points depending on the maturity. However, after the VLTRO allotments, liquidity risk represents only 40 to 60 basis points across maturities (see third column of Figure 4) and further drops to around 20 basis points for all maturities after the OMT announcement (fourth column). In comparison, looking at both Figures 3 and 4, those policy measures had only a small impact on the credit components of the spreads: between November 2011 and October 2012, its range goes from [10 bps, 50 bps] to [5 bps, 40 bps]. Even though there is a small drop in the credit component, the evidence of the effectiveness of unconventional monetary policies on credit risk is far thinner than on liquidity risk.

Turning to the second and third rows of Figure 4, it appears that unconventional monetary policies were followed by decreases in both the expected components and the risk premia. Furthermore, we observe that these drops mainly come from the liquidity parts of the spreads, showing that the VLTROs and OMT have had an effect on both decreasing the expectations of credit and liquidity risks, and were successful in alleviating the effects of aversion to this source of risk.

All in all, even if the EURIBOR-OIS spreads have not really reacted to the 2010 SMP program, our results suggest that the more recent unconventional monetary

Figure 4: Decomposition of EURIBOR - OIS term structure



Notes: First to fourth columns correspond respectively to the following dates: August 29, 2008; November 11, 2011; January 8, 2012; and October 26, 2012. First row represents the stacked components of the term structure: the light grey component is the liquidity component and the dark one is the credit component. The white dots are the observed spreads. Second row presents the same components under the expectation hypothesis. The graph at the bottom represents decomposition of the risk premia, that are the differences between the total modeled spreads and the expected components of the spreads (the sum of the last two rows of chart results in the first row.).

policy measures undertaken within the Eurosystem have contributed to improve bank liquidity positions and to stabilize the credit risk in the Eurozone. The next subsection focuses on this latter aspect by showing how these measures have affected the bank probabilities of default.

6.4 Model-implied probabilities of default

Following Doshi, Jacobs, Ericsson, and Turnbull (2014), we present an additional by-product of our framework, which is the computation of model-implied probabilities of default (PDs). In our model, the panel of banks is homogeneous and the probabilities of default are not bank-dependent. Formally, for any bank i, we have:

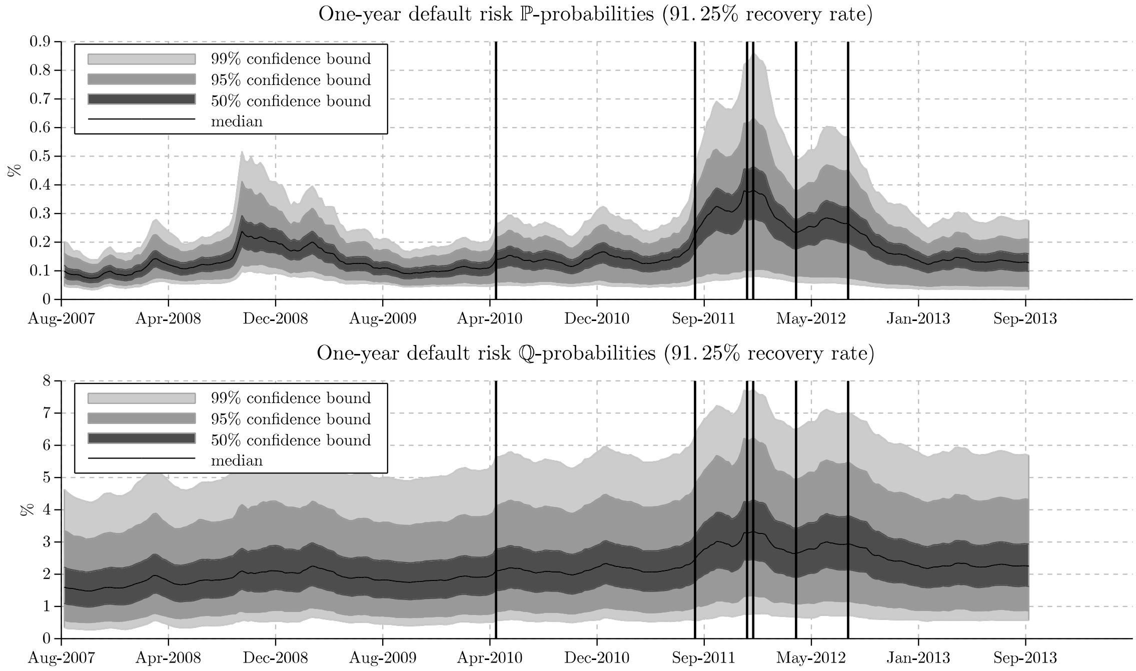
$$\begin{aligned}
\mathbb{P}(d_{t+h}^{(i)} = 1 | d_t^{(i)} = 0, \underline{w_t}) &= 1 - \mathbb{P}(d_{t+1}^{(i)} = 0, \dots, d_{t+h}^{(i)} = 0 | d_t^{(i)} = 0, \underline{w_t}) \\
&= 1 - \mathbb{E}_t^{\mathbb{P}}(\exp(-\lambda_{c,t} - \dots - \lambda_{c,t+h})) \\
&= 1 - \mathbb{E}_t^{\mathbb{P}}\left(\exp\left(\Lambda_c[x_{c,t+1}^2 + \dots + x_{c,t+h}^2]\right)\right). \qquad (21)
\end{aligned}$$

The last term of the previous equation is a multi-horizon Laplace transform of $x_{c,t}^2$, which can be computed analytically by means of recursive formulas of the same kind as those presented in System (15) (replacing μ^* and Φ^* by μ and Φ, and redefining Λ as the matrix with $(\Lambda_c, 0)$ on its diagonal.). The computation requires an estimate of the default recovery rate θ_c. To the best of our knowledge, the existing literature presents no euro-area figure that can serve as a basis for the calibration of such a parameter. Hence, we set it to 91.25%, which is the recovery rate on unsecured deposits on U.S. banks with at least \$5bn assets (see Kuritzkes, Schuermann, and Weiner (2005)). Christensen, Lopez, and Rudebusch (2014b) note that such a recovery rate is high – compared to usual corporate-bond recovery rates – because an unsecured deposit is more senior in the liability structure of a bank than senior unsecured debt.

Figure 5 displays the physical (upper plot) and risk-neutral (lower plot) one-year PDs resulting from this computation. Confidence bands are added on the plots; these bands reflect the uncertainty regarding the model parameterization. These confidence bands are obtained by drawing 1000 sets of model parameters from their asymptotic joint distribution. For each set of parameters, we use the quadratic Kalman filter to estimate time series of $(x_{c,t}, x_{\ell,t})$ and compute the implied (time series of) PDs. For each date, the confidence intervals are based on the percentiles of the 1000 simulated PDs. It appears that risk neutral probabilities are far higher than their physical counterparts, the deviations being accounted for by sizable credit-risk premia. These findings are in line with those of a large body of empirical studies highlighting the substantial deviations existing between physical and risk-neutral PDs (see e.g. Monfort and Renne (2014a) in the case of sovereign issuers and Elton, Gruber, Agrawal, and Mann (2004) in the case of corporate issuers). The existence of credit-risk premia constitutes one of the main explanations for the so-called credit-spread puzzle (see e.g. Amato and Remolona (2003)). This puzzle corresponds to the observation that observed credit spreads tend to be higher than average credit-losses (while they should be equal under some conditions, that notably include the risk-neutrality of investors).

Our estimated physical probabilities of default are roughly comprised between 0.1% and 0.4%. While small, this order of magnitude is however consistent with historical default data of investment-grade issuers. For instance, Moody's (2011) reports that, on average over the period 1983-2010, the one-year default rate of a A-rated financial institutions is of 0.1%. (The median rating of EURIBOR-panel banks is A (across the three main rating agencies.) On a longer time-scale, Moody's (2013) indicates that the default rate of A-rated corporates has been of 0.10% (respectively 0.06%) over the period 1920-2013 (respectively 1970-2013). For lower-rated investment-grade issuers (Baa using the Moody's rating system, which is equivalent to the BBB rating

Figure 5: Default probabilities of banks under the physical and pricing measures



Notes: Time ranges from August 31, 2007 to September 13, 2013. The upper plot show the model-implied one-year probability of default of a bank of the panel (banks are assumed to share the same characteristics). This probability is derived using Equation (21). The lower chart shows the risk-neutral probability of default, which is obtained by using the same formula replacing the physical dynamics parameters by the risk-neutral ones. Shaded areas are the 50% to 99% confidence bounds of these probabilities. The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi's London speech (last axis).

of S&P), the default rates for these two periods are respectively of 0.27% and of 0.17%. Figure 5 illustrates that VLTROs and the OMT announcement (represented by the last four vertical bars on the chart) were effective in reducing bank probabilities of default whether corrected from risk premia or not. However, at the end of the sample, these probabilities remain higher than their mid-2007 value.

Conclusion

We develop a no-arbitrage two-factor quadratic term structure model for the EURIBOR-OIS spreads across several maturities, from August 2007 to September 2013. To identify credit and liquidity components in the spreads, we exploit credit and liquidity proxies based on CDS prices, market liquidity and funding liquidity measures. Our decomposition handles potential interdependence between credit and liquidity risks and is consistent across maturities. We find that the liquidity risk generates most of the variance of the spread over the estimation period. The credit risk is less volatile, but represents more than half of the spread level in late 2012. Our decomposition allows us to shed new light on the effects of unconventional monetary policy of the ECB on the interbank risk. We show that whereas the bond-purchase programs of 2010 and 2011 were not followed by decreases in any of the EURIBOR-OIS spread components, the VLTROs and the OMT announcements have had a substantial impact, mainly on the liquidity risk. At the end of the sample, the liquidity risk is at its lowest since the beginning of the financial crisis.

A Appendix

A.1 Solving for yield/spread loadings in a QTSM

A.1.1 Computing the Laplace transform of $Z_t = \left[X_t', Vec(X_t X_t')\right]'$

Lemma A.1 *If* $\varepsilon_{t+1}^* \sim \mathcal{N}(0, I)$, *we have*

$$\mathbb{E}_t\left[\exp(\theta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*)\right] = \frac{1}{|I-2V|^{1/2}} \exp\left[\frac{1}{2}\theta'(I-2V)^{-1}\theta\right]. \tag{22}$$

Proof 1 *It can be shown that*

$$\forall u \in \mathbb{R}^n, \quad \int_{\mathbb{R}^n} \exp(-u'Qu + \nu'u)\,\mathrm{d}u = \frac{\pi^{n/2}}{|Q|^{1/2}} \exp\left(\frac{1}{4}\nu' Q^{-1}\nu\right). \tag{23}$$

Therefore, we have:

$$\begin{aligned}
\mathbb{E}_t\left[\exp(\theta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*)\right] &= \int_{\mathbb{R}^n} \exp(\theta'\varepsilon + \varepsilon' V \varepsilon) \frac{1}{(2\pi)^{n/2}} \exp\left(-\frac{1}{2}\varepsilon' I \varepsilon\right) d\varepsilon' \\
&= \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\varepsilon'\left(\frac{1}{2}I - V\right)\varepsilon + \theta'\varepsilon\right] \mathrm{d}u \\
&= \frac{1}{|I-2V|^{1/2}} \exp\left[\frac{1}{2}\theta'(I-2V)^{-1}\theta\right] \blacksquare
\end{aligned}$$

Let X_t be a random vector of size n following Gaussian VAR(1) dynamics: $X_t = \mu + \Phi X_{t-1} + \Omega\varepsilon_t$, where ε_t are i.i.d. normalized Gaussian vectors, and $\Sigma = \Omega\Omega'$ is the conditional variance-covariance matrix of X_t. We define Z_t as the augmented vector of factors composed of X_t and of its vectorized outer-product, that is: $Z_t = \left[X_t', Vec(X_t X_t')\right]'$.

Let us consider $u \in \mathbb{R}^n$ and V a square symmetric matrix of size n. The conditional Laplace transform of Z_{t+1} is denoted by φ_t and defined by:

$$\varphi_t(u,V) = \mathbb{E}_t\Big\{ \exp\big[(u', Vec(V)') \times Z_{t+1}\big]\Big\} = \mathbb{E}_t\Big\{ \exp\big[u'X_{t+1} + X'_{t+1}VX_{t+1}\big]\Big\}$$

In the following, we compute the explicit affine form of the conditional Laplace transform of Z_{t+1}. Let us first consider the term in the expectation; substituting $\mu + \Phi X_t + \Omega\varepsilon_{t+1}$ for X_{t+1} leads to:

$$\begin{aligned} \exp\{u'X_{t+1} + X'_{t+1}VX_{t+1}\} \quad = \quad & \exp\{u'(\mu + \Phi X_t) + \mu' V \mu + 2\mu' V \Phi X_t + X_t'\Phi' V \Phi X_t\} \\ \times \quad & \exp\Big\{[u'\Omega + 2(\mu + \Phi X_t)'V\Omega]\,\varepsilon_{t+1} + \varepsilon'_{t+1}\,[\Omega' V \Omega]\,\varepsilon_{t+1}\Big\} \end{aligned}$$

Taking the conditional expectation leaves the first part of the previous expression unchanged as everything is known in t. For the second part of the previous expression, we apply Lemma A.1 and algebraic computation leads to:

$$\begin{aligned} & \mathbb{E}_t\left[\exp\left\{[u'\Omega + 2(\mu + \Phi X_t)'V\Omega]\,\varepsilon_{t+1} + \varepsilon'_{t+1}\,[\Omega' V \Omega]\,\varepsilon_{t+1}\right\}\right] \\ = \; & \exp\Bigg\{ -\frac{1}{2}\log\left|I_n - 2\Omega' V \Omega\right| + \frac{1}{2}u'\Omega(I_n - 2\Omega' V\Omega)^{-1}\Omega' u + 2u'\Omega(I_n - 2\Omega' V \Omega)^{-1}\Omega' V \mu \\ + \; & 2\mu' V\Omega(I_n - 2\Omega' V \Omega)^{-1}\Omega' V \mu \\ + \; & \Big[2u'\Omega(I_n - 2\Omega' V\Omega)^{-1}\Omega' V \Phi + 4\mu' V \Omega(I_n - 2\Omega' V\Omega)^{-1}\Omega' V \Phi\Big] X_t \\ + \; & X_t'\Big[2\Phi' V \Omega(I_n - 2\Omega' V \Omega)^{-1}\Omega' V \Phi\Big] X_t\Bigg\}. \end{aligned}$$

Putting together the first and the second part in the expectation, we obtain: $\varphi_t(u,V) = \exp\{a_1(u,V)'X_t + X_t'a_2(u,V)X_t + b(u,V)\}$, where:

$$
\begin{aligned}
a_1(u,V) &= \Phi'\left[u + 2V\mu + 2V\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'u + 4V\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'V\mu\right] \\
a_2(u,V) &= \Phi'\left[V + 2V\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'V\right]\Phi \\
b(u,V) &= u'\mu + \mu'V\mu - \frac{1}{2}\log\left|I_n - 2\Omega'V\Omega\right| + \frac{1}{2}u'\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'u \\
&+ \quad 2u'\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'V\mu + \quad 2\mu'V\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega'V\mu.
\end{aligned}
$$

Then, noticing that:

$$
\Omega(I_n - 2\Omega'V\Omega)^{-1}\Omega' = \left[\Omega^{-1'}(I_n - 2\Omega'V\Omega)^{-1}\Omega^{-1}\right]^{-1} = \left[\Sigma^{-1} - 2V\right]^{-1},
$$

we can simplify the previous expressions and obtain:

$$
\begin{aligned}
a_2(u,V) &= \Phi'V(I_n - 2\Sigma V)^{-1}\Phi \\
a_1(u,V) &= \Phi'\left[(I_n - 2V\Sigma)^{-1}(u + 2V\mu)\right] \\
b(u,V) &= u'(I_n - 2\Sigma V)^{-1}\left(\mu + \frac{1}{2}\Sigma u\right) + \mu'V(I_n - 2\Sigma V)^{-1}\mu - \frac{1}{2}\log\left|I_n - 2\Sigma V\right|.
\end{aligned}
$$

A.1.2 Calculation of our model's loadings

Let us denote by λ_t the total intensity, that is: $\lambda_t = (1-\theta_c)\lambda_{c,t} + (1-\theta_\ell)\lambda_{\ell,t}$. We have: $\lambda_t = X_t'\Lambda X_t$ where $\Lambda = diag[(1-\theta_c)\Lambda_c, (1-\theta_\ell)\Lambda_\ell]$. We can then re-express the pricing formula (7) as:

$$
S(t,h) = -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\sum_{i=1}^{h}X_{t+i}'\Lambda X_{t+i}\right\}\right]\right),
$$

which is the log of the multihorizon Laplace transform of a quadratic combination of Gaussian variables. Let us postulate that: $S(t,h) = \theta_{0,h} + \theta'_{1,h}X_t + X'_t\theta_{2,h}X_t$. (We know that the model belongs to the class of quadratic term structure models, and that the spreads at all maturities can be expressed as a quadratic combination of X_t.) Using the law of iterated expectation, we obtain the following recursion:

$$
\begin{aligned}
S(t,h) &= -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\mathbb{E}_t^{\mathbb{Q}}\left(\exp\left\{-\sum_{i=1}^{h}X'_{t+i}\Lambda X_{t+i}\right\}\bigg|\underline{X_{t+h-1}}\right)\right]\right)\\
&= -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\sum_{i=1}^{h-1}X'_{t+i}\Lambda X_{t+i}\right\}\varphi_{t+h-1}^{\mathbb{Q}}(0,-\Lambda)\right]\right)\\
&= -\frac{1}{h}\log\left(\mathbb{E}_t^{\mathbb{Q}}\Bigg[\exp\left\{-\sum_{i=1}^{h-1}X'_{t+i}\Lambda X_{t+i}\right\}\Big[b^{\mathbb{Q}}(0,-\Lambda)+\right.\\
&\qquad\left. a_1^{\mathbb{Q}}(0,-\Lambda)'X_{t+h-1} + X'_{t+h-1}a_2^{\mathbb{Q}}(0,-\Lambda)X_{t+h-1}\Big]\Bigg]\right)\\
&\triangleq -\frac{1}{h}\left(\Theta_{0,h} + \Theta'_{1,h}X_t + X'_t\Theta_{2,h}X_t\right).
\end{aligned}
$$

Eventually, the recursive equations of system (15) are obtained by using the closed-form coefficients of the conditional Laplace transform of the previous section, plugging the risk-neutral parameters μ^* and Φ^* (and recalling that we have we have $\Omega = I_2$).

A.2 The Quadratic Kalman Filter

The QKF is based on the fact that the measurement equations are quadratic in the latent factor $X_t = (x_{c,t}, x_{l,t})'$ but affine in the augmented vector $Z_t = (X'_t, Vec(X_tX'_t))'$. This stacked vector Z_t defines a new state-space representation, and new factor dynamics. Recall from Equation (8) that the physical dynamics of X_t writes:

$$X_{t+1} = \mu + \Phi X_t + \Omega\varepsilon_{t+1}$$

where $\Omega\Omega' = \Sigma = I_2$ and $\varepsilon_{t+1} \sim \mathcal{IIN}(0, I_2)$. Relying on Monfort, Renne, and Roussellet (2014), we express the augmented state vector (physical) dynamics as:

$$Z_t = \widetilde{\mu} + \widetilde{\Phi} Z_{t-1} + \widetilde{\Sigma}_{t-1}^{1/2} \xi_t$$

such that:

$$\widetilde{\mu} = \begin{pmatrix} \mu \\ Vec(\mu\mu' + \Sigma) \end{pmatrix} , \quad \widetilde{\Phi} = \left(\begin{array}{c|c} \Phi & 0 \\ \hline \mu \otimes \Phi + \Phi \otimes \mu & \Phi \otimes \Phi \end{array}\right)$$

$$\widetilde{\Sigma}_{t-1} \equiv \widetilde{\Sigma}(Z_{t-1}) = \left(\begin{array}{c|c} \Sigma & \Sigma\Gamma'_{t-1} \\ \hline \Gamma_{t-1}\Sigma & \Gamma_{t-1}\Sigma\Gamma'_{t-1} + \big(I_{n^2} + \Lambda_n\big)(\Sigma \otimes \Sigma) \end{array}\right)$$

$$\Gamma_{t-1} = I_n \otimes (\mu + \Phi X_{t-1}) + (\mu + \Phi X_{t-1}) \otimes I_n$$

Λ_n being the $n^2 \times n^2$ matrix, partitioned in $(n \times n)$ blocks, such that the (i, j) block is $e_j e_i'$, and the distribution of ξ_t is unknown. Let Y_t be the set of measured variables, thus $Y_t = \big[S(t, 13),\ S(t, 26),\ S(t, 39),\ S(t, 52),\ P_{c,t},\ P_{\ell,t}\big]'$. The measurement equations can be transformed in affine functions of Z_t:

$$\begin{pmatrix} S(t,h) \\ P_{c,t} \\ P_{\ell,t} \end{pmatrix} = \begin{pmatrix} \theta_{0,h} \\ \pi_{c,0} \\ \pi_{l,0} \end{pmatrix} + \begin{pmatrix} \theta^{(c)}_{1,h} & \theta^{(\ell)}_{1,h} & \theta^{(c)}_{2,h} & 0 & 0 & \theta^{(\ell)}_{2,h} \\ \pi_{c,1} & 0 & \pi_{c,2} & 0 & 0 & 0 \\ 0 & \pi_{\ell,1} & 0 & 0 & 0 & \pi_{\ell,2} \end{pmatrix} Z_t + \begin{pmatrix} \sigma_\eta \eta_{t,h} \\ \sigma_{\nu_c} \nu_{c,t} \\ \sigma_{\nu_l} \nu_{l,t} \end{pmatrix},$$

$$\Longrightarrow Y_t \triangleq A + \widetilde{B} Z_t + D\zeta_t$$

Approximating the conditional distribution of Z_{t+1} given Z_t by a Gaussian distribution and considering the augmented state-space model based on Z_t, a standard linear Kalman filter can be used for filtering and estimation purposes. In order to get the global likelihood maximum, the estimation is achieved in two steps. The *Artificial Bee Colony* stochastic algorithm (see Karaboga and Basturk (2007)) is used to find the potential maxima areas of parameters. The results are then used as starting values for a usual simplex maximization algorithm and the best estimate is selected. The full algorithm is presented in Table 6 taken from Monfort, Renne, and Roussellet (2014):

Table 6: Quadratic Kalman Filter (QKF) algorithm

Initialization:		$Z_{0\|0} = \widetilde{\mu}^u$ and $P^Z_{0\|0} = \widetilde{\Sigma}^u$.
State prediction:	$Z_{t\|t-1}$	$\widetilde{\mu} + \widetilde{\Phi} Z_{t-1\|t-1}$
	$P^Z_{t\|t-1}$	$\widetilde{\Phi} P^Z_{t-1\|t-1} \widetilde{\Phi}' + \widetilde{\Sigma}(Z_{t-1\|t-1})$
Measurement prediction:	$Y_{t\|t-1}$	$A + \widetilde{B} Z_{t\|t-1}$
	$M_{t\|t-1}$	$\widetilde{B} P^Z_{t\|t-1} \widetilde{B}' + V$
Gain:	K_t	$P^Z_{t\|t-1} \widetilde{B}' M^{-1}_{t\|t-1}$
State updating:	$Z_{t\|t}$	$Z_{t\|t-1} + K_t(Y_t - Y_{t\|t-1})$
	$P^Z_{t\|t}$	$P^Z_{t\|t-1} - K_t M_{t\|t-1} K_t'$

Note: $\widetilde{\mu}^u$ and $\widetilde{\Sigma}^u$ are respectively the unconditional mean and variance of process Z_t. In the filtering method, we impose consistency between the linear and the quadratic part of Z_t by constraining the filtered X_t to be equal to the square root of the filtered quadratic components of Z_t.

A.3 Identifiability and estimation constraints

A.3.1 Parameter contraints

For interpretation purposes, the fluctuations of credit and liquidity proxies are required to correlate positively with the associated intensities. Formally, this is obtained by imposing that, for most values of the factors $x_{c,t}$ and $x_{\ell,t}$, the intensities and the proxies are monotonously increasing with respect to the corresponding factor ($x_{c,t}$ or $x_{\ell,t}$). Recall that the intensity functions are purely quadratic, i.e. of the form $\Lambda_i x_{i,t}^2$, with Λ_c and Λ_ℓ strictly positive to ensure that the intensities are always non-negative. We therefore impose that each $x_{i,t}$ is positive most of the time.

$$\forall i = \{c, \ell\}, \qquad \mathbb{P}\left(x_{i,t} < 0\right) = \alpha \iff \mathbb{E}(x_{i,t}) = -q_{\mathcal{N}(0,1)}(\alpha)\sqrt{\mathbb{V}(x_{i,t})},$$

where $\mathbb{E}(\bullet)$ and $\mathbb{V}(\bullet)$ are the unconditional expectation and variance operators (under the physical measure), and $q_{\mathcal{N}(0,1)}(\alpha)$ is the level-α quantile of the normalized Gaussian distribution and α is typically a small number. We impose the same thing for the proxies, namely:

$$\forall i = \{c, \ell\}, \qquad \mathbb{P}\left(x_{i,t} < -\frac{\pi_{i,1}}{2\pi_{i,2}}\right) = \alpha \iff \mathbb{E}(x_{i,t}) = -\frac{\pi_{i,1}}{2\pi_{i,2}} - q_{\mathcal{N}(0,1)}(\alpha)\sqrt{\mathbb{V}(x_{i,t})}.$$

We impose $\pi_{i,1} > 0$ and $\pi_{i,2} > 0$, which implies that the constraints on the proxies are over-verified. Then, since our factors are jointly Gaussian, the first two unconditional moments are easily computed:

$$\begin{aligned} \mathbb{E}(X_t) &= (I_2 - \Phi)^{-1}(\mu_c,\, \mu_\ell)' \\ Vec[\mathbb{V}(X_t)] &= (I_4 - \Phi \otimes \Phi)^{-1} Vec(I_2) =: (\upsilon_c, \upsilon_{c\ell}, \upsilon_{c\ell}, \upsilon_\ell)', \end{aligned}$$

Eventually, we get the condition:

$$(\mu_c, \mu_\ell)' = (I_2 - \Phi)\left[-q_{\mathcal{N}(0,1)}(\alpha)\left(\sqrt{\upsilon_c}, \sqrt{\upsilon_\ell}\right)'\right]. \tag{24}$$

In the estimation, we set $\alpha = 0.025$. We also control the accuracy of the fit of the proxies, and impose that both $\sigma^2_{\nu_c}$ and $\sigma^2_{\nu_l}$ equal 0.1 (a tenth of the proxies' variance).

A.3.2 Identifiability

In order to see if our model parameters are identifiable, we consider an affine transformation $\widetilde{X}_t$ of X_t, i.e. $\widetilde{X}_t = m + MX_t$, and we check that, if we have an observationally equivalent model when X_t is replaced by $\widetilde{X}_t$, then we necessarily have $m = 0$ and $M = I_2$ (i.e. $\widetilde{X}_t \equiv X_t$).

As the proxies are respectively functions of only one component of X_t, M has to be diagonal. Hence the alternative factors can be written: $\widetilde{x}_{i,t} = M_i x_{i,t} + m_i$ for $i = \{c, \ell\}$. The conditional variance of $\widetilde{X}_t$ must be equal to I_2, thus $M = I_2$. At that stage, we have that $\widetilde{X}_t = m + X_t$. $\widetilde{X}_t$ therefore follows a $VAR(1)$ with the same auto-regressive matrix than X_t. Since Equation (24) also has to apply for $\widetilde{X}_t$, the latter necessarily features the same dynamics as X_t; therefore $m = 0$ and $\widetilde{X}_t \equiv X_t$.

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