Do LTV and DSTI caps make banks more resilient?

Michel DIETSCH*
University of Strasbourg and ACPR – Banque de France

Cécile WELTER-NICOL
ACPR – Banque de France

August 2014

Abstract

This study provides responses to the question of the effectiveness of Loan-To-Value (LTV) and Debt Service-To-Income (DSTI) caps to contribute to financial stability. Using a lender's risk management perspective, the paper provides a new methodology extending the standard asymptotic single risk factor to a multifactor framework, the additional factors being linked to LTV or DSTI tranches. On the basis of a unique database containing 850 896 individual housing loans, the results demonstrate the efficiency of credit standards which constrain the households' indebtedness. On average, credit risk tends to grow in line with the increase of LTV and DSTI tranches. But our findings show also that the relationship between the risk's growth and the ratios' growth is not monotonic. Portfolio credit risk culminates in tranches close to the 100% LTV and the 35% DSTI thresholds. It is more the combination of LTV and DSTI ratios than the use of each ratio separately that help to maintain the total portfolio credit risk at manageable levels.

Keywords: Housing finance, Loan-to-Value, Debt service to Income, credit risk, economic capital.
JEL Codes: R31, R38, G21, G32

*Corresponding author: michel.dietsch@acpr.banque-france.fr

1. Introduction

Loan-to-value (LTV) and Debt service-To-Income (DSTI) caps are frequently considered as desirable tools of macro prudential regulation to dampen credit growth and avoid housing boom and bust. But they are also useful as micro-prudential tools to control mortgage defaults. In France, banks are using such caps in their lending decision since a long time. They condition credit to a minimum down payment and a so-called 33% minimum rule under which debt burden cannot exceed one third of the household's comprehensive disposable income. Despite lending terms loosening, France financial and property markets were resilient during the 2000s, as emphasized recently by the IMF (IMF, 2013). Therefore, France supplies a good experimental field to assess the effectiveness of LTV and DSTI caps to restrain the growth of unexpected losses in housing loans markets.

At the theoretical level, Campbell and Cocco (2012) have documented the channels through which LTV and mortgage affordability affect mortgage default. They show that lower down payment decrease the level of negative equity that triggers default. Moreover, higher borrowing constraints, that may be associated with higher DSTI, accelerate the default by decreasing the trigger level of negative home equity. In brief, levels of LTV and DSTI ratios are correlated to defaults, and a loosening in these credit standards leads to a concentration of defaults. Using other theoretical frameworks, Laufer (2013) and Hatchondo and ali. (2013) confirm the Campbell and Cocco findings. Empirical evidence verifies these theoretical predictions. Sherlund (2008) and Mayer, Pence and Sherlund (2009) find that higher LTV ratios lead to more defaults in US mortgage markets and have been important contributors to the subprime crisis. The drop in house prices caused many borrowers' outstanding mortgage liability to exceed their home value, and this negative home equity level triggered their decision to default on their mortgages. Guiso and al. (2013) have shown that the most likely cause of the increasing proportion of strategic defaults in mid-2009 was the decline in house prices.

However, the mortgage market characteristics vary significantly across countries (Campbell, 2012, Campbell and al., 2012). Homeownership and households' indebtedness rates as well as mortgage forms and lending terms are different and housing loans are funded by a wide variety of mechanisms. Different countries experienced also different histories with inflation

and interest rates. Therefore, different regulations and different government involvement in mortgage markets could produce different delinquency rates.
In France, mortgages are not a dominant form of loans. Loans are standard amortizing loans and to secure loans, French banks do not necessarily ask for a mortgage but more often rely on the guarantee provided by a residential property loan guarantor. In case of default, the guarantor pays back the loan value (including accrued interests) to the bank and manages the resolution of the failure on its own. This form of guarantee has increased significantly since the beginning of the 1990s. More than 51% of outstanding housing loans were secured by a guarantee at end 2013, while nearly 36% were guaranteed by a mortgage (ACPR, 2014). Moreover, under the French rule of "common pledge", every lender is entitled to seek reimbursement of the debt by taking control of all assets or income sources of the borrower who defaults, what allows maintaining losses to quite low levels. This combination of loans with recourse and guarantee scheme provide a good protection to the lender against housing price risk, what helps to understand that defaults occur mainly for cash-flow reasons. In this context, LTV and DSTI caps aim to restrict the borrower's probability of default by adjusting the loan contract terms to the borrower's indebtedness capacity more than to the value of its house.

In this paper, considering the French case, we explore, in a first step, the contribution of LTV and DSTI to the default rate, at the borrower's level. However, what matters is also to know if too high values of these ratios could put the lender's solvency in danger, Therefore, in a second step, we consider the bank portfolio's level and we adopt the lender's risk management perspective. In this perspective, the lender's unexpected losses depend mainly upon the common sensitivity of the borrowers to systematic risk factors, such as macroeconomic or geographical factors, which determine income and real interest shocks and house prices changes. The realization of adverse values of these factors may produce a wave of simultaneous defaults as it was the case in the U.S. subprime market, when real interest rates rose and housing prices dropped sharply. In mortgage markets, theoretical and empirical research has demonstrated that certain categories of borrowers, such as low income borrowers (Ergungor, 2011) or borrowers choosing adjustable rate mortgages (Campbell and Cocco, 2003, 2012, Koijen, Van Hemert and Van Niewerburg, 2009, Van Hemert, 2009), may exhibit high sensitivities to external shocks which may greatly increase their propensity to default on their loans. Here, we assume that higher level of LTV or DSTI ratios could also reveal higher sensitivity to shocks and create a potential concentration of defaults, in accordance with the theoretical predictions of models of mortgage defaults (Campbell and Cocco, 2011, Laufer, 2013, Hatchondo and al., 2013).

Capturing this specific feature calls for no longer considering credit risk at the borrower's level but instead modeling risk at the level of the complete portfolio of loans. Therefore, in this paper, considering the credit-risk issue as a problem of risk management for the lender, we use economic capital measures. Economic capital is defined as an estimate of the worst possible decline in the bank's amount of capital at a specified level of confidence within a chosen time horizon. Thus, economic capital can be viewed as the amount of capital that should be retained by the bank as a direct function of the risks to which the bank is exposed on its credit portfolio. This implies the computation of potential unexpected losses over the chosen time horizon. At the bank level, this refers to some assessment of its global solvency. This approach can also be applied to specific sub-portfolios in order to assess the potential losses they expose the bank to. However, the determination of their capital amount should still be done at the bank level in order to take into account the diversification effects.

More precisely, we compute quantile-based measures of potential unexpected losses at the portfolio and sub-portfolio levels, i.e. in sub-portfolio grouping borrowers who are assumed to shared common risk characteristics because they are in the same LTV and DSTI tranches. In fact, borrowers are heterogeneous, and this heterogeneity could potentially create portfolio credit-risk concentration or, on the contrary, be a source of credit portfolio diversification. From the lender's perspective, the credit risk is sustainable if holding exposures on groups of borrowers in different of LTV or DSTI tranches do not generate excessive portfolio losses as a consequence of risk concentration. Therefore, in this study, we propose an extension of the standard asymptotic single-risk-factor model (Gordy, 2000), which also underlies Pillar 1 of the Basel 2 framework for credit risk, to compute the marginal contributions of different sub-portfolios – which are identified in terms of LTV or DSTI tranches - to the total portfolio's unexpected future losses.

This paper uses a unique database of housing loans provided by a major French banking group. The database covers the period of the 2000s. One of its particularities is that it refers to a large variety of clienteles, from households borrowing on the regular housing loans market to low-income borrowers using regulated loans providing public financial assistance. The database gives information not only about loans characteristics (amount, maturity, type of interest rate, type of loans, regulated or not, loan-to-value and loan-to-income ratios) but also on borrowers characteristics (such as the age of the borrower, its marital status, its profession and personal savings). Information is also available about the borrowers' ratings, including default. All in all, the database represents around one fifth of the French housing loans market.

In this paper, we restrict the analysis to loans financing household's own residence ownership[1]. Our results demonstrate that credit risk is much lower in portfolios' segments characterized by low level of LTV and DSTI ratios and that credit risk tends to grow in line with the increase of such ratios. That allows inferring that maintaining strict DSTI and LTV caps helps to restrict portfolios' credit risk and that any relaxation of credit standards taking the form of higher LTV and DSTI ratios tends to feed an increase of the potential unexpected loan losses. But our results also show that the relationship between the risk's growth and the ratios' growth is not monotonic. In particular, the contribution to total portfolio risk of portfolio's segments which regroup borrowers choosing the highest level of the two ratios is actually not higher than the contribution of portfolio's segments in sub-portfolios where these ratios are lower. In other words, the level of credit risk to which the lender is exposed is not higher in the latter sub-portfolio. In fact, the reason is that banks' solvency benefit from a strict monitoring of these ratios. This paper demonstrates that banks can play with the borrower's downside payment and debt service to income ratio to avoid excessive growth of credit risk. Using these tools, banks can adjust the lending terms to the borrowers' own financial situation or constraints, allowing these borrowers to access to home ownership. Therefore, it is more the combination LTV and DSTI ratios than the use of each ratio separately that help to maintain the contribution of higher LTV or higher DSTI groups of borrowers to the total portfolio credit risk at manageable levels, what is in line with the Campbell and Cocco (2011) predictions. Moreover, our results demonstrate that by efficiently allocating capital to borrowers who differentiate from each other in terms of their own

[1] The decision to borrow for rental investment financing obeys other factors which are specific to these investors. In particular, because people choosing rental investment are less financially constrained, the LTV ratio – or the down-payment ratio - is largely determined by an objective of optimal allocation of household's savings between investments in real estate and financial assets. For this reason, banks could allow borrowers to reduce their personal contribution if the result is to maintain an optimal proportion between real estate assets and financial assets in the household's total portfolio. Moreover, decisions to invest obey also exogenous factors such as fiscal incentives.

characteristics and of the characteristics of their loans, banks can raise significant diversification benefits in housing loans portfolios.

Our findings also demonstrate that the heterogeneity captured by credit ratings, or PDs, the only source of heterogeneity in the asymptotic one factor framework, fails to describe the effective heterogeneity in default rates within large portfolios. Adding new risk factors allows controlling for potential risk concentration or diversification effects. Indeed, risk factors associated to credit standards such as the LTV or DSTI ratios appear to have significant effects on the heterogeneity of credit risk in housing loans portfolios.

The paper is organized as follows. Section 2 is devoted to the literature review. Section 3 presents the data and explores the relationship between LTV and DSTRI ratios and the rate of default on housing loans. In section 4, we set up the multifactor model of portfolio credit risk. In section 5, we presents the economic capital implication of holding portfolios with high LTV and DSTI ratios borrowers. Section 6 concludes and shows policy implications of our findings.

2. Relation to the literature

This paper relies on the strand of the literature trying to understand the channels through which LTV and housing loan affordability determine defaults. Davis and Van Nieuwerburgh (2014) have supplied a rather extensive review of major theoretical findings on how changes in mortgage choice and house prices can explain the boom and bust of the 2000s. And numerous empirical studies verify these findings.

The main channel through which lower down payments could have been important contributors to the rise of default rate during the subprime crisis is through the negative equity threshold, as demonstrated theoretically by Campbell and Cocco (2011). The LTV represents the equity stake that households have in their houses. A higher LTV increases the probability of negative home equity, what favors the rational decision to default. Thus, any decrease in house prices cause borrowers' outstanding mortgage liability to exceed their home value, and for these borrowers default can increase their wealth. In the US, the default rate pattern line up with the patterns in the LTV and DSTI ratios. Sherlund (2008) and Mayer, Pence and Sherlund (2009) document that negative equity and a higher LTV ratio lead to more defaults in US mortgage markets. They find substantial evidence that decline in house prices is a key factor in the rise of defaults in US mortgage markets at the end of the 2000s. Using also US data, Bajari, Chu and Oark (2008), and Furlong and Takhtamanova (2012) bring similar evidence that one main driver of default in the recent subprime crisis is the decrease in home prices. Evidence shows that problem mortgages in the US surge beyond what job growth alone suggested (Duca and al., 2010, 2011). A sizable relaxation of mortgage standards tends to raise the effective demand for housing and to feed the price bubble. On the contrary, in countries with more stable credit standards, any overshooting of house prices owed more to traditional housing supply and demand factors than to credit conditions (Duca and al., 2010). Considering the Korean market, Igan and Kang (2011) found that LTV and DSTI limits are associated with a decline in house price appreciation and in number of transactions.

However, default is costly for the borrower. As emphasized by Campbell and Cocco (2011), borrowing constraints are also relevant for the default decision. Negative home equity is a necessary but not sufficient condition for default. The negative equity default threshold depends on the degree to which households are credit constrained. At low level of negative equity, financially distressed borrowers would prefer to avoid costly default. In fact, according to Campbell and Cocco (2003), and Campbell (2006), households base their decision on risk-management considerations. The choice of loan characteristics reflects household’s adjustment to borrowing constraints to manage interest rate risk, house price risk and inflation and income risks, while maintaining the utility of the non-durable goods consumption. Thus, any decrease of down payment, and any relaxing of financial effort help to loosen the borrowing constraints that are encountered by borrowers. Therefore, in case of house prices shocks, at low level of negative equity, more constrained borrowers would default more frequently on their mortgages (without recourse) than less constrained ones, while at high level of negative equity, all borrowers would choose to default whatever the degree of the credit constraints. Bajari and al. (2008), Bhutta and al. (2010) and Foote and al. (2008) show results consistent with these predictions for the US mortgage market. Other found that higher DSTIs at origination contribute to a higher probability of default, although these effects appear to be less strong than those of LTV, and seems to be inconsistent over time (Ding et al., 2011, Foote et al., 2009). There is also some evidence that the default rates of lower income borrowers are more sensitive to other factors than LTV, in addition to being generally higher (Quercia et al., 2012). High levels of LTV and DSTI ratios are correlated to defaults, and a loosening in LTV and DSTI standards leads to a concentration of delinquencies (Chan, Gedal, Been and Haughwout, 2013). Finally, grounding of this empirical evidence, Laufer (2013) and Hatchondo and al. (2013) have built models of households decision in which they demonstrate that policies aiming to impose tighter borrowing constraints - under the form of LTV and DSTI caps - and better lenders' legal protection (stronger recourse) might reduce the default rate. Their results confirm the findings of Campbell and Cocco (2011).

Lenders' risk management practices can also explain differences in delinquency rates, particularly for higher risk borrowers (Moulton (2010), Ergungor (2010), Moulton and Ergungor (2011), Rosen (2011)). These findings suggest that the better performance of local banks come from relationship banking. While transaction banking may be sufficient for higher income borrowers with good credit quality, relationship banking may play a role for low income borrowers, when soft information and monitoring are needed. The authors verify that relationship banking is critical for the sustainability of credit to borrowers with lower income and higher risk. Research findings demonstrate also the impact of predatory lending. Agarwal and al. (2014) demonstrated that these practices contributed to around one third of the rise of mortgage defaults rates among subprime borrowers. Other studies show that less informed and less sophisticated subprime borrowers are more prone to choose risky loans (Bucks and Pence (2008), Mayer, Pence and Sherlund (2009)). Moreover, credit standards relaxation may come from mortgage regulation itself. Using Indian data, Campbell and al., 2012, demonstrate that government intervention and State regulation aiming to promote homeownership, in particular leverage restrictions, may have contributed to a surge in delinquency during the 2000s in India. So, this evidence shows that this cause of relaxation of lending terms could also favor defaults. On the contrary, using French data and considering the availability of regulated loans for low income borrowers, Dietsch and Petey (2013) show that supplying loans with financial assistance under the form of downside subsidies helps financially constrained borrowers to absorb income shocks and thereby allows these borrowers to present on average the same level of credit risk as borrowers using market loans without any assistance.

However, this literature has two drawbacks. First, most of the studies devoted to the impact of LTV and DSTI caps consider loans markets where mortgages are the dominant type of loans.

Moreover, theoretical and empirical findings apply to mortgages without recourse. They show that the attractiveness of default to a borrower varies with the lender's ability to recover house value in case of default. In most European countries, housing loans are mainly with recourse, what means that lenders benefit from a higher protection. Under the French law, all housing loans are with recourse: any lender is able to pursue a defaulted borrower and take control of all its assets to recover the entire balance of the defaulted loan. In addition, in France, a majority of residential loans are secured with a guarantee instead of a mortgage, what means that borrowers share directly loan losses by putting their money in a mutual fund. These features help to understand why default occurs quite exclusively for cash-flows reasons and not for home equity extraction reasons. Default may be more likely attributed to negative income shocks rather than to negative house prices shocks. Accordingly, the loan supply is more conditioned by the solvency of the borrower than by the value of its house. A second drawback is that main findings are focusing on the borrower solvency, more than on the lender's solvency. In this paper, we will change this perspective and put the emphasis on issues related to portfolio credit risk management.

3. The data and the relationships between credit standards and the default rate

This paper uses a unique database provided by a major French banking group. As mentioned previously, a particularity of this database is that it covers a large variety of clienteles from households borrowing on the regular housing loans market to low-income borrowers using regulated loans providing financial assistance. The database provides information about loans characteristics (amount, maturity, type of interest rate, type of loans, regulated or not, loan-to-value and loan-to-income ratios) and also on borrowers characteristics (such as the age of the borrower, its marital status, its profession and personal savings). The database provides also borrowers' ratings at the loan's origination. The dataset retains housing loans which destination is to finance home ownership and considers the borrower level. Our dataset contains 850 896 loan records[2] and represents around one fifth of the French housing loans market (home ownership financing). Complete individual information is provided over the 2002 to 2010 period.

[2] Each file comprises one or several loans contracted in the 2000s or before and still living at least one year during the 2000s period of this study.

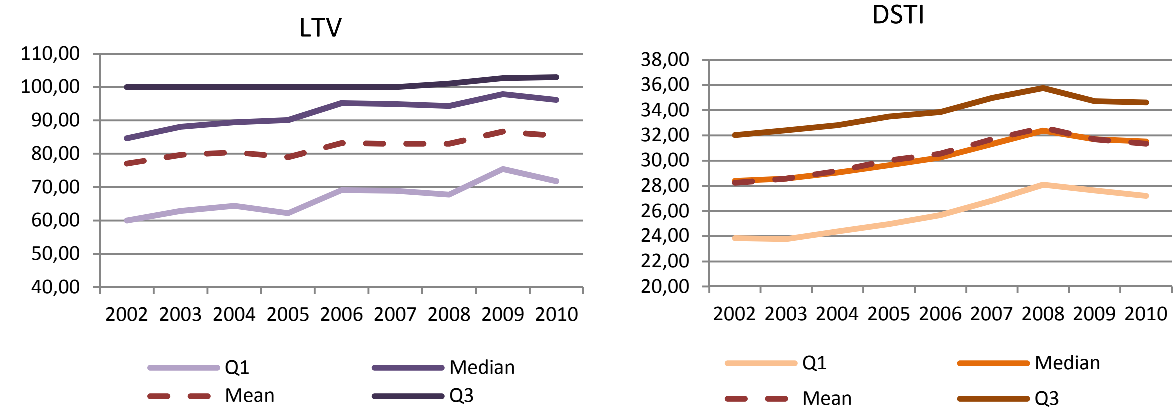
Here, the analysis is focused on the two the down payments in proportion of the investment at the origination, which is a proxy for the LTV ratio, and the DSTI ratio, which measures the loan's burden relatively to the borrower's income at the origination of the loan. In France, the default rate did not explode as it did in other European or non European mortgage markets, despite the rapid growth of the French housing loans market and the change in credit standards in the first half of the 2000s, to comply with the increase of house prices, The main purpose of this section of the paper is to document the relationship between the two credit standards and the rate of default.

3.1. The changes in LTV and DSTI credit standards during the 2000s

We have segmented the portfolio in four segments depending on the borrowers' LTV and on the borrowers' DSTI tranches. These segmentations were chosen by testing different possible segmentations by using a logit model of the rate of default. We retained the segmentation that produced the most significant differences in the coefficients associated to the fixed affects by segment. This procedure allows isolating the upper segment grouping borrowers with the highest level of each ratio. The upper class regroups borrowers whose ratios are far over the thresholds value of 100% for the LTV ratio or 33% for the DSTI ratio. The borrowers belonging to these upper segments share personal characteristics which show that they are not necessarily riskier than the other borrowers, as we will document it in what follows. Notice that this methodology also allows isolating the segment just below the upper one which regroups borrowers who are close to the current LTV threshold and DSTI thresholds ((around 95% to 100% for the LTV ratio and equal to the so called French 33% rule for the DSTI ratio). And, lastly, the two first segments correspond to the smallest values of the ratios (LTV lower than 70% and 95% successively, DSTI lower than 25% and 33%). All in all, this segmentation allows distinguishing borrowers who take very different choices of these ratios as a solution of their financial constraints at the origination and who suffer different borrowing constraints as a consequence of their choices.

Figure 1 presents the distribution and the change of LTV and DSTI ratios for new loans through the 2000s.

Figure 1 – Changes in LTV and DSTI ratios through the 2002-2010 period



Source : Bank data and authors' computations

Figure 1 documents the rise of DSTI and LTV ratios at the origination through the 2000s in a context of a spectacular growth of the number of loan contracts, their average amount and maturity (ACPR, 2014). All the distribution of DSTI ratio shifts from 2002 to 2008, and then tends to move back, stabilizing at a higher level. Whereas the so called French 33% rule corresponds to the upper quartile at the beginning of the period, it finally matches with median and mean in 2010. Significant growth can be observed concerning LTV ratio, except that upper quartile stays almost steady through the period. But the median grows up to 2010, reaching almost 100 %. Observation shows that the values of LTV and DSTI ratios are not independent and that lender combines the two criterions in their lending decisions.

Thus, in the upper class of LTV, that has known the more rapid growth during the period, the share of borrowers with higher level of DSTI ratios represents less than the majority of loans even if this share has decreased from the beginning of the period to its end. Moreover, in this upper LTV class, the relative share of borrowers with DSTI ratios over the 33% threshold is not very different from the proportion we observe in the lower LTV ratio classes. In the upper class of DSTI, the proportion of borrowers with LTV ratio over 100% tends to be lower than in the penultimate class in the second period. All these observations demonstrate that banks manage simultaneously the values of the two ratios. Indeed, the DSTI ratio, the amount, the maturity and the share of adjustable rate tend first to increase with the level of LTV, when the LTV ratio is lower than 100%, but then in the upper class of LTV (over 100%) the value of the DSTI ratio decreases significantly (see Appendix A for detail). In other words, lenders tend to allow highest values of LTV only if they have the possibility to secure lower values of DSTI. The two ratios seem to be managed simultaneously.

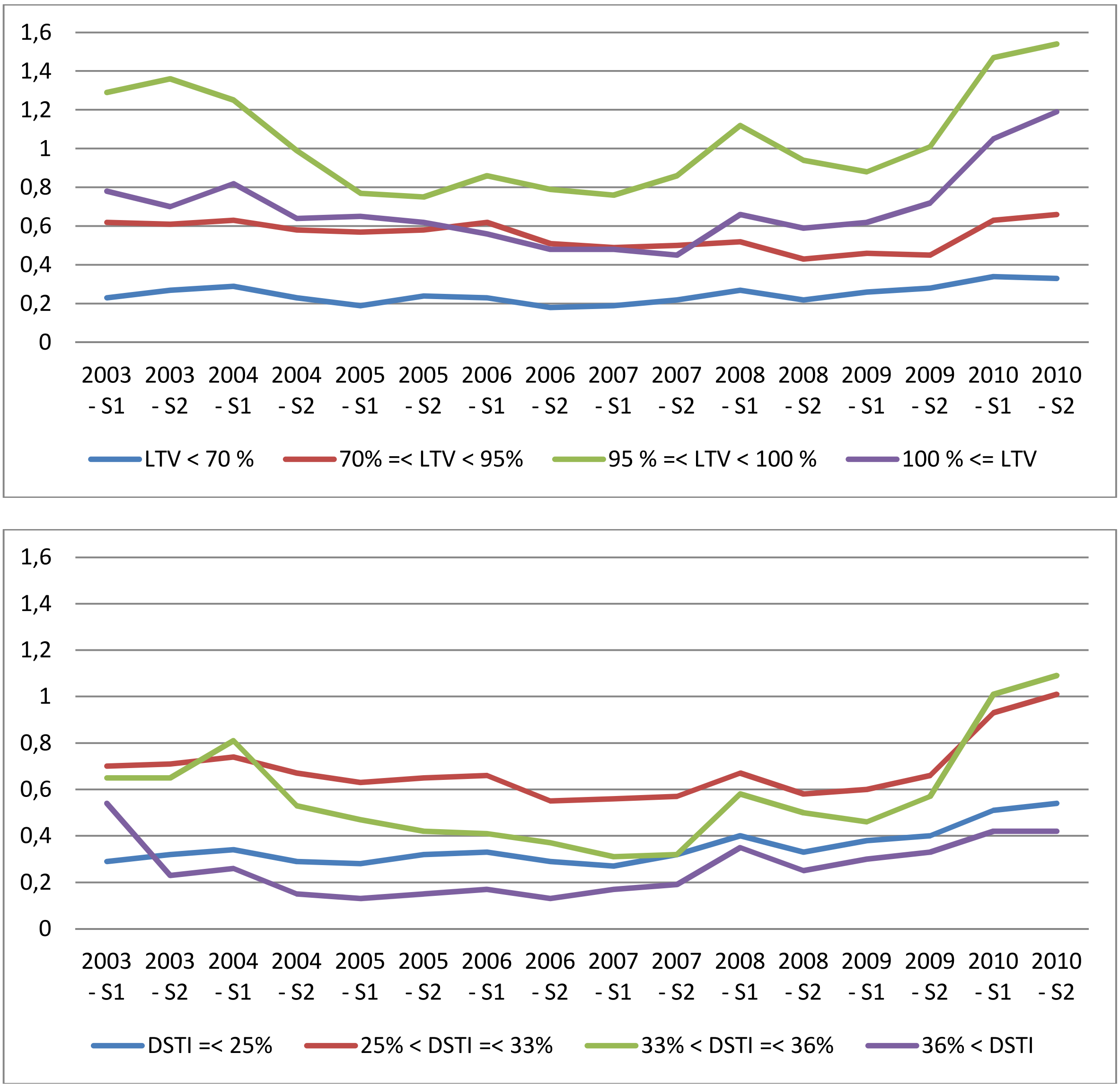
The same kind of observation can be made when considering DSTI classes. In the upper class of the DSTI ratio, the average level of the LTV ratio is lower than in the lower DSTI classes. It also appears that the borrowers located in the class of LTV or DSTI close to the common threshold of 100% and 33% are suffering on average more severe borrowing constraints as demonstrated by the high level of the other ratio, the high levels of the borrowed amount of duration, and finally the higher proportion of borrowers choosing adjustable rates.

3.2. The impact of LTV and DSTI on the default rate

The following figures 2 and 3 document the rise of default rate during the 2000s by classes of LTV or DSTI. Figure 2 shows that after a period of stability during the first part of the 2000s, the default rates begin to growth in 2007. The rise of default rate characterize more particularly the two classes of high LTV ratios (LTV < 95%). However, despite the growth of default rates over time, it’s important to note that default rates associated to LTV over 100% stay smaller than ones associated to LTV between 95% and 100%, all over the period under study. Therefore lending terms seems to be more managed when banks consider the borrowers crossing the 100% LTV ratio threshold.

Figure 2 also shows how default rates increased in the 2000s in each DSTI class. First, we observe an upward trend of default rates after 2007 in all DSTI classes, then we observe that the rate of default begins to increase with the DSTI ratio in the three first classes. But, the rate is lower in the upper class of borrowers whose ratio is over 36%: these borrowers are characterized on average by the smallest default rate over the period.

Figure 2 - The annual default rate (in %) by class of LTV and DSTI in the 2000s

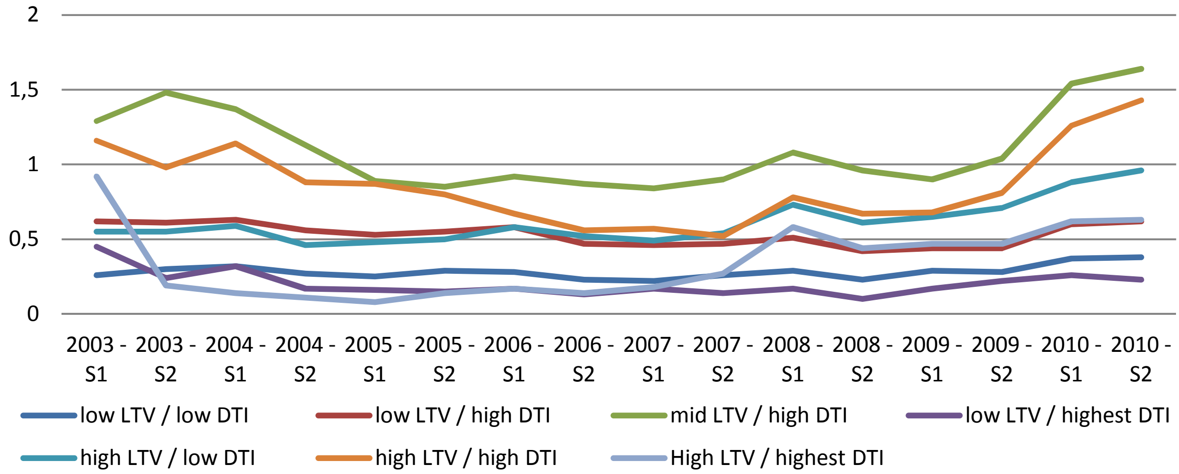


Source : Bank data and authors' computations

To complete the analysis on the two distinct macroprudential tools just shown, we now look at a segmentation crossing LTV ratio and DSTI ratio. From the sixteen feasible classes, we retain only seven[3], trying to group together classes with similar borrowers' characteristics. Figure 3 presents the default rates through these classes.

[3] The seven classes are built as followis : low LTV and low DSTI (LTV < 95 % and DSTI < 25 %), low LTV and high DSTI (LTV < 95 % and 25% < DSTI < 36%), low LTV and highest DSTI (LTV < 95% and 25% < DSTI < 36%), high LTV and low DSTI (LTV > 95% and DSTI < 25%), mid LTV and high DSTI (95% < LTV < 100% and 25% < DSTI < 36%), high LTV and high DSTI (LTV > 100% and 25% < DSTI < 36%), and high LTV and highest DSTI (LTV > 95% and DSTI > 36%).

Figure 3 - The annual default rate (in %) by class mixing LTV and DSTI ratios in the 2000s



Source : Bank data and authors' computations

As observed before, the highest default rates are not associated to the highest LTV and highest DSTI tranches. On the contrary, we observe that the highest LTV and DSTI ratios are not associated with higher default rates. In fact, the highest default rates are found in tranches where borrowers are close to the threshold values of the LTV and DSTI ratios. We will show latter that these borrowers are the most financially constrained.

3.3. How borrowers' characteristics impact LTV and DSTI choices

By looking at the borrowers characteristics, we can determine which characteristics are the more frequently associated with defaulted borrowers. From the database, we have selected the following borrower's characteristics: its saving rate, its marital status, its profession or socio-economic status and its age. We also observe how these characteristics vary in two periods of successive vintages.

Table 1 shows the distribution of borrowers' characteristics according to the LTV classes, and by period. In the two periods, small LTVs are associated to almost older borrowers – who likely have accumulated more savings - and people who have good professions and earnings. On the other hand, workers and young borrowers have on average higher LTVs ratios. So, borrowers with small LTV ratio seem less financial constrained. In the class of borrowers with high LTV ratio, this high level can be explained by the fact that they are frequently single or living alone, so that their savings rate is lower. However, borrowers in this class tend to be older and they benefit from higher income jobs, what means that their ability to support higher debt burden is better over their life cycle than in the penultimate class where borrowers seem to be more financially constrained.

Table 1 - Distribution of borrowers' characteristics according to the LTV classes

		Whole portfolio		*LTV < 70%*		*70% =< LTV < 95%*		*95% =< LTV < 100%*		*100% =< LTV*	
		2002 to 2005 vintages	***2006 to 2010 vintages***	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*
Saving rate	*No savings*	**22,0%**	**22,1%**	16,5%	16,3%	16,4%	16,3%	19,3%	17,8%	34,0%	32,0%
	]0%,10%[	**17,7%**	**23,1%**	13,6%	18,0%	13,9%	17,5%	27,1%	30,5%	23,3%	30,0%
	[10%, and more]	**60,4%**	**54,8%**	70,0%	65,7%	69,7%	66,3%	53,6%	51,7%	42,8%	38,1%
Bank account quality	*Others qualities*	**3,9%**	**6,0%**	3,3%	5,6%	3,7%	5,6%	4,2%	5,9%	4,8%	6,6%
	Average	**18,1%**	**30,9%**	13,4%	24,4%	17,3%	29,5%	21,0%	33,8%	23,6%	37,0%
	Good or very good	**77,9%**	**63,1%**	83,3%	70,0%	79,0%	64,9%	74,8%	60,3%	71,6%	56,5%
Marital status	*Single*	**16,0%**	**19,7%**	17,0%	25,8%	16,5%	24,9%	10,4%	12,9%	17,9%	19,7%
	Married or cohabiting	**73,0%**	**71,4%**	62,6%	52,0%	75,2%	65,5%	85,7%	82,4%	71,2%	72,2%
	Others	**11,0%**	**34,4%**	20,4%	32,2%	8,3%	31,9%	4,0%	39,0%	11,0%	33,2%
socio-economic status	*Workers*	**31,5%**	**31,0%**	20,8%	22,3%	33,7%	29,3%	42,5%	40,0%	30,0%	25,6%
	Bureaucrates	**26,4%**	**28,0%**	27,2%	24,7%	26,6%	34,2%	23,3%	25,0%	27,6%	28,2%
	Middle managers	**21,9%**	**24,1%**	20,0%	21,7%	22,7%	22,4%	23,4%	25,8%	21,4%	24,8%
	Top wealthy	**20,2%**	**16,8%**	31,9%	31,3%	17,1%	14,1%	10,8%	9,2%	21,1%	21,5%
Age of borrower	*age under 35*	**44,0%**	**49,9%**	25,1%	25,6%	49,5%	48,6%	57,1%	58,4%	44,5%	51,2%
	age between 35 and 45	**35,0%**	**31,4%**	37,0%	32,0%	34,7%	33,1%	32,5%	30,7%	35,1%	30,7%
	age between 45 and 55	**14,8%**	**12,3%**	23,4%	22,9%	12,2%	12,7%	8,4%	7,9%	14,9%	12,3%
	age over 55	**6,3%**	**6,4%**	14,5%	19,6%	3,7%	5,6%	2,1%	3,0%	5,5%	5,9%

Source : Bank data and authors' computations

Table 2 shows the distribution of borrowers' characteristics according to the DSTI classes. It shows first that borrowers with small DSTI ratio are more frequently married or cohabiting – what means that the households benefit from two sources of income - and they are also older. In addition, borrowers in the upper DSTI class tend to be less financially constrained. Indeed, even if borrowers in the two highest DSTI ratio classes are more frequently single and have a lower saving rate, what may explain their high DSTI level, they tend to be older and have better jobs.

Table 2 - Distribution of borrowers' characteristics according to the DSTI classes

4.	5.	***Whole portfolio***		***DSTI =< 25%***		***25% < DSTI =< 33%***		***33% < DSTI =< 36%***		***36% < DSTI***	
		2002 to 2005 vintages	***2006 to 2010 vintages***	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*	*2002 to 2005 vintages*	*2006 to 2010 vintages*
Saving rate	*No savings*	**22,0%**	**22,1%**	19,1%	17,7%	22,2%	22,3%	24,4%	24,6%	24,3%	23,4%
	]0%,10%[	**17,7%**	**23,1%**	11,3%	14,5%	19,0%	23,4%	21,2%	27,6%	22,5%	26,5%
	[10%, and more]	**60,4%**	**54,8%**	69,6%	67,8%	58,8%	54,3%	54,4%	47,8%	53,2%	50,1%
Bank account quality	*Others qualities*	**3,9%**	**6,0%**	3,2%	5,3%	3,7%	5,6%	4,5%	5,9%	5,3%	7,1%
	Average	**18,1%**	**30,9%**	14,4%	26,3%	18,5%	31,3%	20,8%	32,8%	21,5%	32,7%
	Good or very good	**77,9%**	**63,1%**	82,4%	68,4%	77,9%	63,1%	74,7%	61,3%	73,2%	60,3%
Marital status	*Single*	**16,0%**	**19,7%**	11,0%	13,0%	17,2%	19,3%	22,2%	25,9%	23,3%	21,4%
	Married or cohabiting	**73,0%**	**71,4%**	75,7%	76,0%	73,2%	72,5%	64,2%	65,1%	60,2%	67,2%
	Others	**11,0%**	**8,9%**	13,3%	11,0%	9,6%	8,2%	13,6%	9,0%	16,5%	11,5%
socio-economic status	*Workers*	**31,5%**	**31,0%**	30,4%	32,1%	32,7%	31,3%	25,2%	29,1%	28,3%	26,0%
	Bureaucrates	**26,4%**	**28,0%**	23,8%	22,4%	27,2%	29,7%	30,1%	23,5%	26,7%	36,8%
	Middle managers	**21,9%**	**24,1%**	20,5%	22,8%	22,7%	25,7%	20,8%	23,2%	19,2%	15,2%
	Top wealthy	**20,2%**	**16,8%**	25,3%	22,8%	17,4%	13,4%	23,9%	24,2%	25,9%	22,0%
Age of borrower	*age under 35*	**44,0%**	**49,9%**	31,7%	32,0%	49,3%	53,2%	44,5%	55,1%	36,0%	43,7%
	age between 35 and 45	**35,0%**	**31,4%**	37,7%	32,8%	33,9%	30,9%	32,8%	30,7%	39,1%	35,1%
	age between 45 and 55	**14,8%**	**12,3%**	19,7%	18,9%	12,5%	11,1%	16,4%	10,2%	18,2%	13,8%
	age over 55	**6,3%**	**6,4%**	10,9%	16,2%	4,4%	4,8%	6,3%	4,0%	6,8%	7,4%

Source : Bank data and authors' computations

To summarize, while previous features confirm that borrowers in the lowest LTV or DSTI tranches are likely the less financial constrained, they also show that borrowers with the highest ratios are not necessarily more constrained. In fact, the more financially constrained borrowers are those in the LTV and DSTI classes which are the closest to the thresholds (i.e. approaching the 100% LTV ratio and the 33% DSTI ratio).

4. The measurement of portfolio credit risk

To be reliable, any measure of portfolio credit risk should, first, properly quantify portfolio-wide credit risk, second, correctly assess dependency across obligors and the risk of credit concentration, and third, permit risk to be allocated at the segment level to establish the cartography of risk within the portfolio. Thus, in this section, we present a multi-factor extension of the structural single factor model (Gordy, 2000, 2003) to take into account borrowers' heterogeneity and multiple sources of credit risk (Dietsch and Petey, 2014). Then, we specify this model as a generalized linear mixed model (GLMM) to produce estimates of the credit risk parameters we need for the calibration of the model. Finally, we use these risk parameters as inputs in the computation of the potential losses that may occur at the total portfolio and sub-portfolios levels. By computing the contributions of specific sub-portfolios to total potential losses, this procedure allows to detect situations where sub-portfolios generate potentially large number of correlated defaults or, on the contrary, portfolio diversification benefits.

4.1. The common structure of the single factor and multifactor models

The multifactor model belongs to the class of structural credit risk models devised by Merton (1974). Thus, losses at the portfolio level can be defined as the sum of individual losses on defaulting loans in the portfolio, adjusted for the severity of these losses. Thus, if u_i is defined as the loss given default (LGD) of an obligor i and if Y_i is defined as the default indicator variable of obligor i (Y_i takes the value of 1 if there is a default and 0 otherwise), then the total portfolio losses L may be computed as follows:

$$L = \sum_{i=1}^{n} u_i Y_i$$

In structural credit-risk models, default occurs if the situation of a borrower crosses an default threshold that is calibrated in accordance with the stationary (long-term) default probability $\overline{p_i}$ of obligor *i.,*:

$$Y_i = 1 \Leftrightarrow U_i = w'_i s + \sqrt{1 - w'_i R w_i \varepsilon_i} < \Phi^{-1}(\bar{p}_i) \quad (1)$$

Here, the financial health of obligor i is represented by a latent (unobservable) variable U_i, and the level of U_i is determined by the realizations s of a set of S, w_i is the vector of sensitivities (or factor loadings) of the i-th borrower to the systematic factors and ε_i is a specific risk factor for borrower i. In the above equation, R is the correlation matrix of the risk factors, assuming that the risk factors are multivariate Gaussian. Φ is the standard normal cumulative distribution function. U_i is standard normal. Specific risk factors are assumed to be uncorrelated among obligors and independent from systematic factors.

Thus, given a realization s of the systematic risk factor, equation (1) can be rewritten such as a default occurs when:

$$\varepsilon_i < \frac{\Phi^{-1}(\bar{p}_i) - w'_i s}{\sqrt{1 - w'_i R w_i}}$$

As the borrower's specific risk factor is normally distributed, the default probability conditional to s follows the standard normal cumulative distribution function. Moreover, assuming that specific risk can be entirely diversified away, then losses can be approximated by their expected value conditional to s (Gordy, 2000). Conditional portfolio losses are then defined as follows:

$$L(s) \approx \sum_{i=1}^{n} u_i \, \Phi\left[\frac{\Phi^{-1}(\bar{p}_i) - w'_i s}{\sqrt{1 - w'_i R w_i}}\right] \qquad (2)$$

This framework is known as the asymptotic multi-factor framework of credit risk (e.g., Lucas et al., 2001). Equation (2) assumes that each obligor can be characterized by his individual default threshold and factor sensitivities. However, in retail loan portfolios, default rates are generally computed based on rating grades, and sensitivities to risk factors cannot be computed on an individual basis. Thus, assumptions are required to reduce the number of parameters of the loss variable. A common assumption is that obligors who belong to the same rating notch j will share the same default threshold. Moreover, one could further assume that the vector of risk factor sensitivities is the same for obligors who share the same characteristic. Hence, assuming the existence of a portfolio that is composed of K segments, losses can be rewritten as follows:

$$L(s) \approx \sum_{k=1}^{K} \sum_{i=1}^{n_k} u_i \, \Phi \left[\frac{\Phi^{-1}(\bar{p}_j) - w'_k s}{\sqrt{1 - w'_k R w_k}} \right] \qquad (3)$$

The implementation of the multifactor model requires the specification of the dependence structure of risk factors and the estimation of the default thresholds and sensitivities to systematic risk factors. When using a random effect specification of the risk factors, there is a correspondence between the conditional default probability of equation 3 and econometric approach grounding on generalized linear mixed models (GLMMs).

4.2. Econometric estimation of the portfolio's credit-risk parameters

The implementation of the multifactor model requires determining the risk factors. In mortgages portfolios, as shown before, sources of heterogeneity can be linked to the loan characteristics, which allow distinguishing different portfolio's segments. Here, the problem is to identify the risk factors leading to borrowers' default. A natural way in searching for explicit risk factors would be to make explicit the latent factor in the Merton framework by introducing a set of macroeconomic or sector variables. However, retail banking markets are local by nature and lack of time series data on potential risk factors at the local level may limit the implementation of such an approach, which is commonly used to compute dependency structure in corporate assets portfolios. That is the reason why we choose to add latent factors that can be linked to observable loan characteristics. Therefore, to implement a multifactor approach, portfolio segmentation has to be built by identifying groups of borrowers with the same observable characteristics – here, LTV and DSTI levels - which expose them to the same risk factors. This approach invites to use a random effects specification by segmenting borrowers' defaults histories according to a combination of risk factors.

Using a random effect specification of the risk factors, we can estimate the default thresholds and factor sensitivities by implementing an econometric model that belongs to the class of generalized linear mixed models (GLMMs) and combines fixed and random effects for observable and (latent) unobservable factors, respectively[4]. Here, the fixed effect which corresponds to the default threshold is defined by the rating class, and the random effects are defined by a segmentation of the portfolio by one or several loan characteristics. The central variable in equation (3) is the conditional default probability. Within the framework of GLMM models, this conditional default probability is defined as follows. Let Y_t be an (N × 1) vector of observed default data at time *t* and γ_t be the (K × 1) vector of random effects. The conditional expected default probability of obligor *i* at time *t* is then:

$$P(Y_{ti} = 1|\gamma_t) = \Phi(x'_{ti}\beta + z_i\gamma_t)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function [5], β denotes the vector of parameters associated with the fixed effect (the borrower's rating class) and z_i is the design matrix of the random effects, here an identity matrix with size the number of random effects. If the rating scale is properly built, we expect the β parameters which correspond to the default thresholds associated to the ratings to be ordered and increasing as credit quality decreases. In the above equation, $x'_{ti} = [0, \ldots, 1, \ldots, 0]$ is a (1 × J) vector of dummies defining the rating of borrower *i* at time *t*. The random effects are assumed to follow a multivariate standard normal distribution with covariance matrix Σ and correlation matrix *R*. Because we assume that borrowers within segments are interchangeable, the estimations of Σ and β do not involve individual borrowers but instead use the quarterly default rates within segments. Assuming that defaults are independent conditional on random effects, the number of defaults in the portfolio is binomially distributed. The conditional probability of $Y_t = (Y_{t1} = 1, \ldots, Y_{tn} = 1)$ is then

$$P(Y_t = y_t|\gamma_t) = \prod_{i=1}^{n_t} P(Y_{ti} = 1|\gamma_t)^{y_i}\big(1 - P(Y_{ti} = 1|\gamma_t)\big)^{1-y_i}, \forall\, y_i \in \{1, 0\}^{n_t}$$

Further assuming that the random effects are serially independent (but possibly cross-sectionally correlated in the case of multiple random effects), the unconditional probability of Y_t is as follows, defining θ as the parameter vector that comprises all unknowns in Σ, *R* and *g* as the multivariate Gaussian distribution:

[4] Detailed presentations of the implementation of GLMM models in credit-risk modeling can be found in McNeil and Wendin (2007).

[5] We focus on the probit link function because the normal distribution is the underlying link function that is assumed by the Basel 2 framework of credit risk.

$$f(y_t|\beta,\theta) = \int P(Y_t = y_t|\gamma_t)\, g(\gamma_t|\theta)d\gamma_t$$

The likelihood function with serially independent random effects is finally:

$$L(\beta,\theta|data) = \prod_{t=1}^{T} f(y_t|\beta,\theta)$$

4.3. Capital allocation within a multi-factor credit-risk model.

Once the credit-risk parameters are estimated, we can build the distribution of losses at the portfolio level by a Monte Carlo simulation of the risk factors, with each realization of risk factors being converted into a conditional default probability at the fixed/random effects sub-portfolio level as defined by equation (3), and lastly, into conditional expected losses at the portfolio level. However, to assess the credit risk of a given type of borrower within the portfolio, we need to compute its contribution to economic capital. This calculation requires the portfolio-wide economic capital to be allocated to sub-portfolios or individual assets. From the findings of Tasche (1999) and Gouriéroux et al. (2000), the marginal contributions to a portfolio value-at-risk (VaR) can be expressed as the expected loss on a given exposure, conditional on losses reaching this VaR:

$$RCVAR_i = E[L_i|L = VaR_\alpha(L)] = \frac{E\left[L_i \mathbf{1}_{VaR_\alpha(L)=L}\right]}{P[L = VaR_\alpha(L)]} \qquad (4)$$

Equation (4) indicates that if there is a positive probability for losses to reach a portfolio's VaR, then the computation of marginal contributions will rely heavily on the ability to estimate individual losses as aggregate losses approach this VaR. Thus, in the context of a Monte Carlo simulation, the conditional mean may be based only on a limited number of simulations, producing unreliable estimates. To improve the estimation procedures, some authors (Tasche, 2009, Glasserman and Li, 2005, Egloff and Leippold, 2010) have used importance sampling. Importance sampling consists of shifting the parameters of a distribution in ways that increase the likelihood of observing certain desired realizations of the variables. The main difficulty with respect to this approach relates to the choice of the alternative distribution $F*$. In this study, we follow the methodology of Tasche (2009) and shift only the risk factor (S) means in the following manner:

$$S_i^* = S_i - E_F[S_i] + \mu_i \; with \; \mu_i = E[S_i | L = VaR_\alpha(L)]$$

The next step is the computation of the conditional expectation as defined by equation (4). Because the computation of VaR is accomplished through Monte Carlo simulations, both the realizations of the risk factors and the resulting credit losses are known. This information permits the utilization of the non-parametric Naradaya-Watson estimator for conditional expectations. If the standard normal density is used as the kernel and h is used to denote the bandwidth of the kernel, then the estimator of the conditional expectation for risk factor k may be defined as follows:

$$\hat{E}[S_k | L = VaR_\alpha(L)] = \frac{\sum_{t=1}^{T} S_k \Phi\left(\frac{VaR_\alpha(L) - L_t}{h}\right)}{\sum_{t=1}^{T} \Phi\left(\frac{VaR_\alpha(L) - L_t}{h}\right)} \qquad with \; h = 1.06\sigma_L T^{-1/5}$$

Assuming perfect granularity of the portfolio, it is possible to compute a single marginal contribution based on the rating/segmentation variable combination rather than by proceeding at the loan level. For borrowers with rating j with characteristic k, losses are then approximated by the following expression:

$$L(s_k) \approx \sum_{j=1}^{n_k} u_j \, \Phi\left[\frac{\Phi^{-1}(\bar{p}_j) - w'_k s_k}{\sqrt{1 - w'_k R w_k}}\right]$$

Once the shifts in the means are computed for all of the risk factors, the next step in the analysis is to obtain realizations of the risk factors under the new distribution to once again compute the aggregate losses for the portfolio and the individual losses within each sub-segment and rating grade. Tasche (2009, proposition 4.2) establishes that conditional on VaR, the expected losses under the natural distribution can be defined as follows, with δ as the likelihood ratio between distributions F and F*:

$$E_F[L_i | L = VaR_\alpha(L)] = \frac{E_{F^*}[L_i \delta | L = VaR_\alpha(L)]}{E_{F^*}[\delta | L = VaR_\alpha(L)]}$$

As discussed above, these conditional expectations can be computed with the Naradaya-Watson estimator, and simulations of risk factors and losses can be obtained under the shifted distribution. Lastly, these expected losses can be aggregated across ratings for each modality of the segmentation variable to compute segment-wide economic capital requirements.

5. How LTV and DSTI ratios management can preserve banks' solvency

Here, we estimate the efficiency of LTV and DSTI caps as tools to control lender's exposure to credit risk. Applying the multifactor model, first, we will consider the two segmentations which rely on these ratios. Then, we will build a segmentation combining the LTV and DSTI criterions. In what follows, we will use successively two types of results of the multifactor model when measuring the impact of each ratio.

Firstly, the matrixes of variance-covariance among factors allow assessing the existence of concentration or diversification effects. Concentration would be high either if the variance within each portfolio segment is high or if the covariance between this segment and other are high. On the contrary, diversification benefits exist if the covariances between LTV or DSTI segments are weak or negative. Therefore, one issue is to know if the lender could exploit the heterogeneity across borrowers located in different segments where these borrowers are exposed to different risk factors. For instance, borrowers with low down payment or highly leveraged, who suffer higher borrowing constraints, could be sensitive to real interest or income shocks, while borrowers with high down payment or weakly leveraged would not be exposed to these risks but instead would be more exposed than the latter to house prices changes that could affect their home equity.

Secondly, we will use results related to the computation of economic capital requirements. Here, results will be expressed under the form of a capital ratio which relates capital requirements needed to cover potential unexpected losses to total exposures of each segment. In fact, we will compare three capital ratios: first, the economic multifactor ratio computed by using a multifactor model which takes into account additional risk sources, secondly, the economic single factor ratio, which uses the standard ASRF model to compute asset correlations and, lastly, the regulatory capital ratio built by using the Basel 2 regulatory formula in the IRB approach (we have assumed a conservative 15% LGD rate). The comparison of the multifactor capital ratio with the single factor capital ratio also allows detecting the existence of portfolio diversification benefits, if the capital ratio provided by a multifactor model is lower than the capital provided by the single factor model. The comparison of the economic capital ratios with the regulatory ones allow to detect potential situation where the regulatory capital requirements might be insufficient to cover extreme losses in segments characterized by high levels of the LTV or DSTI ratios. To compute capital requirements, whatever the model, we took a quite conservative 15% LGD value and a (Basel 2) 99.9% quantile of the probability distribution function.

5.1. The impact of LTV on capital requirements

The loans portfolios are segmented using the four tranches of the LTV ratio presented above. Table 3 shows the covariance matrix provided by the multifactor GLMM model and table 4 presents the value of the capital ratios by segment of LTV (see Appendix B.1. for details on estimation results).

Table 3 - Variance – Covariance matrix among groups of borrowers in different LTV tranches

	LTV < 70 %	70% =< LTV < 95%	95 % =< LTV < 100 %	100 % <= LTV
LTV < 70 %	0.0093	-0.0025	-0.0034	-0.0050
70% =< LTV < 95%	-0.0025	0.0103	0.0145	0.0163
95 % =< LTV < 100 %	-0.0034	0.0145	0.0228	0.0226
100 % <= LTV	-0.0050	0.0163	0.0226	0.0267

Source: bank data and authors' computation. Most covariances' values are significant: see appendix B.

Table 3 shows that the covariance among borrowers (diagonal of the matrix) is higher in the two segments characterized by the highest value of the LTV ratio. Observation shows also that there are strong covariances between these two segments. Correlated defaults are more frequent there than in segments with lower LTV ratios. Such results tend to demonstrate that the same latent systematic risk factors affect simultaneously borrowers with higher LTV ratios. On the contrary, the level of the covariance is quite low in segments which group borrowers with low LTV ratios, what means that borrowers in these segments are more immune to common latent systematic risk factors. Moreover, negative covariance show that diversification benefits occur when including borrowers located in the latter LTV tranches in the total portfolio.

Higher covariance associated to higher LTV ratios level means that higher LTV values may produce more correlated defaults – i.e. more credit risk concentration - in the portfolio's corresponding segments. That is in line with the observed growth of the rate of defaults after 2007 in the population of borrowers choosing higher LTV ratio. Indeed, capital ratios results show that the marginal contribution to total portfolio credit risk is higher in the segments with higher value of the LTV ratios. This is likely the direct consequence of the higher correlation between borrowers' situations but also of the higher PDs of these borrowers, on average. Therefore, our results seem to validate the usefulness of strict LTV standards to manage portfolio credit risk and control its growth. However, results also show that the capital ratio is not increasing monotonically with the level of the LTV ratio. In fact, the marginal contribution to total risk of the group of borrowers with LTV ratio over 100% is lower than the contribution to risk of borrowers located in the LTV tranche close to the 100% threshold.

So, one issue is to know why borrowers in the upper LTV tranche contribute less to the total unexpected losses than borrowers in the closest 95% to 100% LTV tranche. As we have shown above, the upper LTV tranche is composed of borrowers who are less financially constrained than borrowers belonging to the closest tranche. Descriptive analysis of borrowers' characteristics has also shown that the borrowers' average PD is lower in the upper class of LTV ratio than in the closest one. As mentioned before, this result might be explained by the fact that borrowers in this segment are older and wealthier. However, less risky borrowers are not necessary less prone to default simultaneously. Our results (table 3) show on the contrary that covariances are reaching very similar levels in the two segments. In fact, beside the lower level of the PDs, the lower capital ratio in the segment of higher LTV ratio could also come from the control the banks exert on the other characteristics of the loan, such as the loan amount and maturity. Indeed, previous descriptive statistics have shown that average loan amount and maturity are lower in the higher LTV tranche than in the 95-100% tranche. Moreover, loan contracts terms include lower debt service to income and a majority of fixed interest rate loans. These borrowers likely suffer less financial constraints than the borrowers in the closest class. Thus, banks tend to limit the amount of their exposures in this segment by managing the entire set of credit standards.

The comparison of capital ratios computed by using a multifactor model and a single factor model (table 4) shows the capacity of one bank that includes borrowers with different LTV levels in its portfolio to manage its portfolio's credit risk. If the weighted average of multifactor capital ratios (first column of the table 4), that represents the weighted average of the marginal contributions of each LTV tranche to the total risk of the portfolio, is lower than the weighted average of single factor capital ratios (second column of table 4), that means that adding borrowers exposed to different sources of risk in the same portfolio contribute to reduce total risk. Thus, the comparison confirms the existence of diversification benefits when including heterogeneous sub-populations of borrowers exposed to different factors of risk. Moreover, our findings demonstrate that the heterogeneity captured by credit ratings, or PDs, the only source of heterogeneity in the single factor framework, fails to describe the effective heterogeneity in default rates within large portfolios. Systematic risk factors associated to loan standards such as the LTV ratio appear to have significant effects on the heterogeneity of credit risk. Here, additional risk factors linked to loan characteristics tend to lower the capital requirements due to risk diversification effects.

And, finally, the comparison of capital ratios computed by using a multifactor model and the Basel 2 regulatory formulas show that the regulatory capital requirements cover widely the amount of capital that is needed to absorb the loan losses, whatever the segment. Therefore, building additional capital buffers to cover potentially procyclical additional credit risk related to LTV changes seems to be not necessary.

Table 4 - Comparison of capital ratios using LTV segmentation – loans for homeownership

	Economic capital ratio in % Multifactor model	Economic capital ratio in % Single factor model	Regulatory capital ratio in % Basel 2 IRB approach	share of borrowers in the whole portfolio
LTV < 70 %	0.06	0.72	1.33	37.4%
70% =< LTV < 95%	0.21	0.74	1.81	35.0%
95 % =< LTV < 100 %	0.50	1.38	2.48	8.7%
100 % <= LTV	0.36	1.23	2.02	18.9%
Total	0.21	0.88		100%

Source: bank data and authors' computation.
Note: The value of economic capital for the total portfolio in the multifactor approach is the weighted average value of the four segments

5.2. The impact of DSTI on capital requirements

Here, the loans portfolios are segmented using four tranches of the DSTI ratio.

Table 5 - Variance – Covariance matrix among groups of borrowers in **different DSTI** tranches

	DSTI =< 25%	25%<DSTI=<33%	33% <DSTI =<36%	36% < DSTI
DSTI =< 25%	0.008481	-0.00073	0.000479	0.000308
25% < DSTI =< 33%	-0.00073	0.01446	0.01349	0.01242
33% < DSTI =< 36%	0.000479	0.01349	0.01444	0.01116
36% < DSTI	0.000308	0.01242	0.01116	0.01117

Source: bank data and authors' computation. Most covariances' values are significant: see appendix B.

Table 5 shows that the covariance among borrowers (diagonal of the matrix) is higher in the intermediate segments characterized by medium (25% to 33%) or high (33% to 36%) values of the DSTI ratio (See Appendix B.1. for more estimation results). In addition, comparison shows also that the covariances between these two tranches are stronger than between the other tranches. Correlated defaults are more frequent in these tranches than in tranches with highest or lowest DSTI ratios. That means that the borrowers in the two intermediate tranches and more particularly in the 33% to 36% ones are more sensitive to common systematic risk factors which are specific to these tranches (and could be related to stronger borrowing constraints) than the borrowers in other tranches.

Higher covariance associated to these DSTI tranches means that these tranches might generate credit risk concentration. Here, capital ratios results show that the portfolio credit risk is higher in the two intermediate tranches where the covariance is stronger. That reflects the higher correlation between borrowers' situation in these segments. However, as in the case of the LTV ratio segmentation, results also show that the capital ratio is not increasing monotonically with the level of the DSTI ratio. In fact, the capital ratio is lower in the group of borrowers with DSTI ratio over 36% than in group of borrowers with lower DSTI ratio. In other terms, highest DSTI ratios do not systematically produce more correlated defaults. Again, it is useful to understand why borrowers in the upper tranche of DSTI ratio contribute less to the total portfolio's credit risk than borrowers in the closest tranches. The results presented above show that borrowers with the highest DSTI ratio are on average older and wealthier than the borrowers with lower DSTI ratio. Moreover, in the tranche with highest

DSTI ratio, the average characteristics of loan contracts show that a majority of loans have characteristics that generate lower default rates, such as higher downside payments, shorter maturity and a majority of fixed rate loans. Thus, all in all, these borrowers seem suffer less stringent financial constraints than the borrowers in the closest class. These characteristics illustrate the existence of a kind of trade-off between higher constraints associated to high levels of DSTI ratio and lower constraints coming from the other credit standards.

As in the case of the LTV ratio, the comparison of capital ratios computed by using a multifactor model and a single factor model (table 6) shows that diversification benefits dominate concentration effects in the portfolios, due to the relatively low level of covariances between risk factors associated to the different segments of the portfolio. This comparison shows again the capacity of the bank that includes borrowers with different DSTI levels in its portfolio to manage its portfolio's credit risk. The lower value of the weighted average of multifactor capital ratios than of the value computed for the single factor capital ratios shows that adding borrowers with different level of their debt ratio who expose them to different sources of risk contribute in this case to reduce the total risk of the portfolio. Thus, the comparison confirms the existence of diversification benefits when including heterogeneous sub-populations of borrowers exposed to different factors of risk.

Table 6 - Comparison of capital ratios using DSTI segmentation

	Economic capital ratio in % Multifactor model	Economic capital ratio in % Single factor model	Regulatory capital ratio in % Basel 2 IRB approach	share of borrowers in the whole portfolio
DSTI =< 25%	0.09	0.89	1.91	41.9%
25% < DSTI =< 33%	0.28	1.04	2.42	44.5%
33% < DSTI =< 36%	0.29	1.13	2.32	7.6%
36% < DSTI	0.25	0.95	1.47	6.0%
Total	0.20	0.97		100%

Source: bank data and authors' computation
Note: The value of economic capital for the total portfolio in the multifactor approach is the weighted average value of the four segments

And, finally, the comparison of capital ratios computed by using a multifactor model and the Basel 2 regulatory formulas show again that the regulatory capital requirements cover widely the amount of capital that is needed to absorb the loan losses, whatever the segment. Here, in all DSTI segments, the (multifactor) economic capital requirements are at least three times covered by the regulatory capital requirements. Again, building additional capital buffers to cover potentially procyclical additional credit risk related to DSTI changes seems to be not necessary.

5.3. The impact of the combination of LTV and DSTI

Finally, we consider a segmentation crossing the LTV and DSTI ratios. Table 7 shows the covariance matrix given by the multifactor GLMM model and table 8 provides the value of the capital ratios by segment crossing DSTI and LTV. Eight segments are considered here (See Appendix B.1. for estimation results).

Table 7 - Variance – Covariance matrix among groups of borrowers distinguished by crossing DSTI and LTV tranches

	low LTV / low DSTI	low LTV / high DSTI	mid LTV / high DSTI	low LTV / highest DSTI	high LTV / low DSTI	high LTV / high DSTI	High LTV / highest DSTI
low LTV / low DSTI	0.01744	-0.00112	-0.00024	-0.00527	-0.00021	-0.00183	0.002514
low LTV / high DSTI	-0.00112	0.004323	0.001472	0.007521	0.001019	0.000266	-0.00133
mid LTV / high DSTI	-0.00024	0.001472	0.01003	-0.0031	0.01013	0.00245	0.003866
low LTV / highest DSTI	-0.00527	0.007521	-0.0031	0.02291	-0.0072	0.002402	-0.00937
high LTV / low DSTI	-0.00021	0.001019	0.01013	-0.0072	0.01255	-0.00054	0.006353
high LTV / high DSTI	-0.00183	0.000266	0.00245	0.002402	-0.00054	0.01264	0.005973
High LTV / highest DSTI	0.002514	-0.00133	0.003866	-0.00937	0.006353	0.005973	0.01265

Source: bank data and authors' computation. Most covariances' values are significant: see appendix B.

Table 7 shows that the covariances among borrowers are very low in the tranches combining lower LTV or DSTI ratios. Moreover, negative covariances appear when LTV and DSTI ratios show such low values, which are the support or portfolio's diversification effects. On the contrary, covariances are much higher in the segments characterized by high levels of the two ratios. Notice that that the integration of borrowers with higher DSTI ratios in the segments seems to produce more correlated defaults than the integration of borrowers with high LTV ratios. However, observation shows negative covariances between the segment combining highest LTV and highest DSTI ratios and the other segments, what demonstrates again the existence of diversification benefits for the lender.

These diversification effects explain why the highest level of the capital ratio is not reach when the two ratios are at their highest levels, but instead in the segments with ratios' level which are close to the 33% and 100% thresholds.

To summarize, the last results tend to verify that a strict management of the credit standards, which rely on the positive interaction between borrowers, is very powerful to maintain portfolio's credit risk at a sustainable level by extracting significant diversification benefits.

Table 8 - Comparison of capital ratios using a segmentation crossing DSTI and LTV

	Economic capital ratio in % Multifactor model	Economic capital ratio in % Single factor model	Regulatory capital ratio in % Basel 2 IRB approach	share of borrowers in the whole portfolio
low LTV / low DSTI	0.17	0.66	1.96	35.5%
low LTV / high DSTI	0.11	0.77	2.39	33.5%
mid LTV / high DSTI	0.26	1.33	3.21	6.8%
low LTV / highest DSTI	0.10	0.74	1.41	3.4%
high LTV / low DSTI	0.13	1.33	2.41	6.4%
high LTV / high DSTI	0.21	1.26	2.86	11.8%
High LTV / highest DSTI	0.13	1.37	1.85	2.6%
Total	0.15	0.87		100%

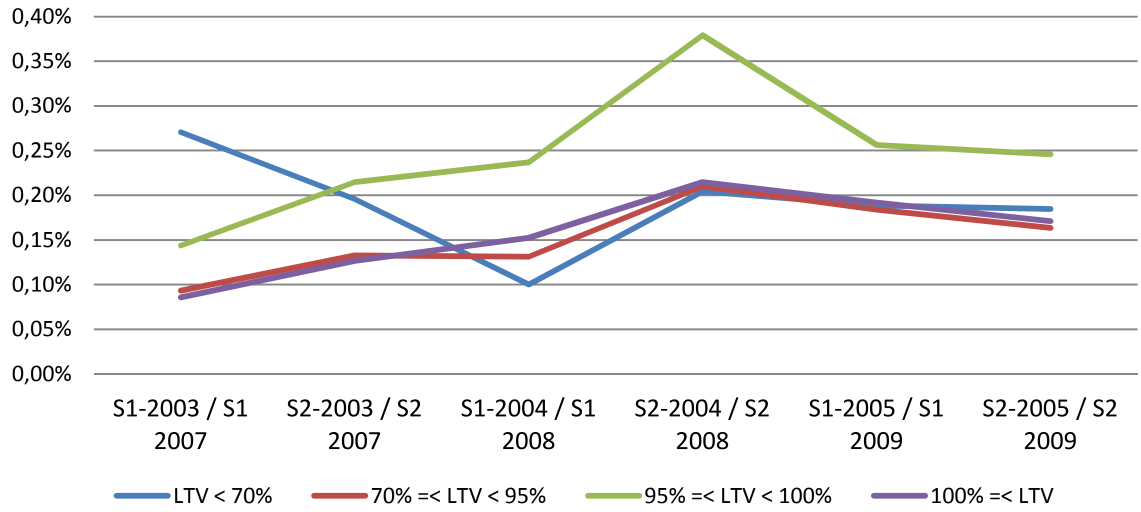
Source: bank data and authors' computation
Note: The value of economic capital for the total portfolio in the multifactor approach is the weighted average value of the four segments

5.4. Robustness check: assessing the impact of a severe recession by using a rolling windows approach

Here, we present the results of a robustness check that tries to quantify the impact of the adverse macroeconomic environment on default rates and unexpected losses. More precisely, to highlight the impact of the severe downturn of 2008-2009 on potential credit losses, we adopt a rolling window approach for the estimation of the credit risk model's parameters and the computation of economic capital ratios.

The default thresholds and the variance-covariance matrix are estimated over a rolling window of 10 semesters. The resulting set of parameters then allows as before the computation of potential credit losses each semester at the one year horizon. Thus, the simulated economic capital reflects the potential credit losses based on the conditions in the preceding ten semesters. The variations of the credit risk parameters and the resulting economic capital requirements give some insights in the impact of the bad macroeconomic environment of the 2008-2010 years on the potential credit losses. Figure 4 and 5 illustrate the results for the LTV and DSTI models.

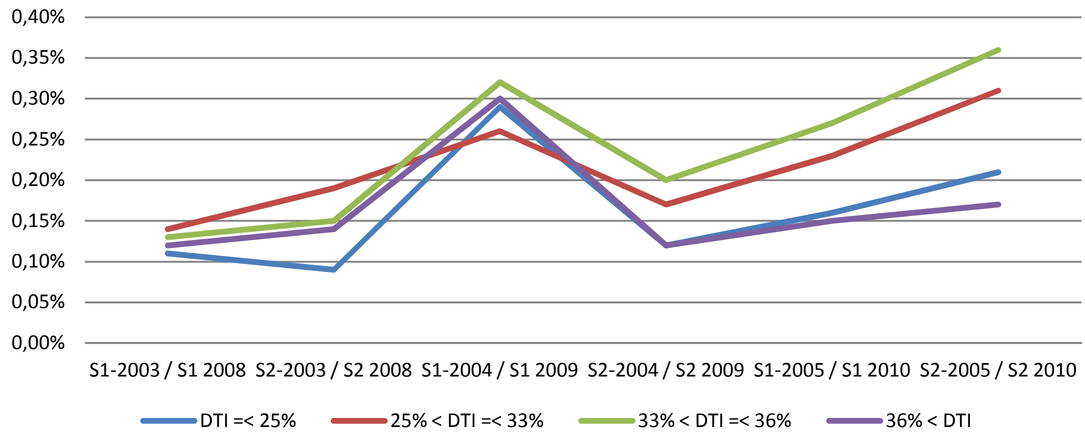
Figure 4 - Evolution of the economic capital ratio by LTV segment



Source: bank data and authors' computation

The main result of our rolling window approach is the increase of the economic capital ratio in the second semester of 2008 in the 95% to 100% LTV class, as shown in figure 5. This increase is associated with an increase in the variance parameter within this class which is multiplied by 2 in a semester and stays at a higher level the following semesters. On the contrary, in the other classes, even if the within variance also increases, their level is sufficiently low to keep the growth of the capital ratio. In addition, diversification effects across LTV segments help to reduce the capital consumption after the shock. All in all, the capital ratio stays quite stable in these LTV tranches and its level is maintained at a very low level.

Figure 5 - Evolution of the economic capital ratio by DSTI segment



Source: bank data and authors' computation

Concerning the change in the economic capital ratio by DSTI tranches (figure 5), we observe again that the levels of capital ratio do not really explode as a consequence of the severe downturn of 2008-2009. Two additional results come from the rolling window exercise. First, all tranches were impacted by the same (sudden) change and with the same intensity at the beginning of 2009. The variance in each segment almost doubles at that date. Second, economic capital ratios increase again in the first semester of 2010, but there the reaction to the shocks varies across tranches, what confirms the heterogeneity of borrowers across them. In particular, the two extreme tranches show lower growth of economic capital requirements. In fact, low covariance between these tranches and the others produce diversification benefits which help to reduce the growth of the capital ratio. Moreover, shocks do not affect heterogeneous borrowers at the same time. When we compare DSTI and LTV results, we observe that jumps do not happen in the same semester. There seems to be a lag of one semester for the 2008 downturn to produce effects on the capital ratio for DSTI tranches.

All in all, the results of the rolling window exercise confirm that even if adverse economic conditions raise the capital requirements in the portfolio's segments composed of the weaker borrowers, the housing loans market is quite resilient to macroeconomic shocks.

6. Concluding remarks and policy implication

In France, banks are conditioning lending to LTV and DSTI ratios standards that link lending to households' income and wealth. Even if we observe a loosening of such credit standards in the 2000s, default rates did not climb in France during the financial crisis period to an unsustainable level. Grounding on the French experience, this paper adopts the perspective of the lender and tries to assess the ability of LTV and DSTI caps to restrict portfolio credit risk associated to housing loans financing main residence ownership. To this aim, the paper uses a unique database combining information on the loans characteristics, the borrowers' characteristics and their ratings, including default grade. The database accounts for around one sixth of the French housing loans and covers the entire period of the 2000s. The paper also proposes a new methodology to measure the portfolio credit risk in large portfolios which consists to expand the standard single risk framework to introduce multiple sources of systematic risk. Here, additional risk factors are associated to loan standards. This multifactor methodology allows taking into account borrowers' heterogeneity and potential credit risk concentration and/or diversification effects.

Results show firstly that the individual credit risk and the portfolio credit risk tend to increase when the LTV and DSTI ratios increase. Borrowers who are more financially constrained and more exposed to systematic risk factors are those who are close to the standard caps of 33% of DSTI ratio and around 95% of LTV ratio. These results may justify at first glance the implementation of LTV and DSTI caps. But results also show that the relationship between these ratios and the lender's credit risk is not monotonic. In particular, the borrowers who are located in the upper classes of LTV and DSTI ratios are not those who generate the highest level of portfolio credit risk. In fact, these borrowers are in the upper income and wealth classes, and their probability of default is quite low. In other words, they are not as financially constrained as the borrowers in tranches of the LTV and DSTI ratios close to the common thresholds. It is information that the lenders could extract from the banking relationships. Another crucial reason is that lenders manage the two ratios simultaneously and tend to accept that borrowers cross one of the caps only if they are well below the other one. Banks are using the interplay of all credit terms to avoid to impose excessive financial constraints on the borrowers and to extract significant diversification benefits.

Consequently, in a macroprudential perspective, the calibration of LTV and DSTI ratios should consider the interaction between the different credit standards more that each standard separately. Maintaining strict credit standards such as limits in LTV and DSTI ratios help for sure to restrict the growth of excessive credit risk. But, it is not so much the implementation of limits to LTV and DSTI ratios separately than the use of the complete set of credit conditions that allows banks to exert their lending role and to give access to credit to households while maintaining portfolio's credit risk to sustainable levels. Our results show that, in a macroprudential perspective, the calibration of LTV and DSTI ratios should consider the interaction between the different components of the credit standards.

Moreover, building additional capital buffers to cover potentially procyclical additional credit risk related to LTV changes seems to be not necessary. Indeed, our comparison of the economic capital ratios (computed by using a multifactor model) and the regulatory capital ratios (using IRB Basel 2 formulas) show that the regulatory capital requirements cover widely the amount of capital that is needed to absorb the unexpected loan losses linked to high levels of the LTV or the DSTI ratios. What matters from a supervisory point of view is that the banks hold permanently the required amount of capital to cover unexpected losses in their loans portfolios. On average, the regulatory capital requirements reflect correctly the structure of credit risk by tranche of LTV and DSTI.

Finally, each housing loan market has its own characteristics. In France, housing loans finance households more than their houses and the origination process takes more the solvency of the borrowers into account than the value of the real estate goods. That explains that French banks manage simultaneously the DSTI and LTV limits. It is necessary to take account for domestic specificities before to implement strict credit standards. The results of this study show that current Basel 2 regulatory capital requirements are higher than what would be needed to cover the worst cases induced by high levels of financial constraints generated by excessive LTV or DSTI ratios. Thus, in the current state, any additional capital requirements would be in fact redundant. The present regulatory rules reflect correctly the structure of credit risk assessed by economic capital modeling.

References

- ACPR, 2014, Housing Finance in France, *Analyses et Synthèses*, n°23, juillet 2014. http://acpr.banque-france.fr/fileadmin/user_upload/acp/publications/analyses-syntheses/201407-Le-financement-de-l-habitat-en-2013.pdf
- Allen, F., and E. Carlotti, 2013, Syste*mic risk from real es*tate and macro-prudential regulation, International Journal of Banking, Accounting and Finance, vol. 5, n° 1 – 2.
- Ambrose B., M. LaCour-Little, Z. Huszar, 2005, A note on Hybrid Mortgages, Real Estate Economics 33 (4), 65-82.
- Amronim, Gene, A. Paulson, 2010. Default rates on prime and subprime mortgages: differences and similarities, Profitwise News and Views, Federal Reserve Bank of Chicago.

- Amnorin, G., J. Huang, C. Sialm, and E. Zhong, 2011, Complex mortgages, Federal Reserve Bank of Chicago, working paper.
- Bajari, P, C S. Chu, and M Park, 2008, An empirical model of subprime mortgage default from 2000 to 2007, NBER Working Paper 14625.
- BCBS, 2006, Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version, Basel Committee on Banking Supervision.
- BCBS, 2008, Range of practices and issues in economic capital modelling, Consultative document, Basel Committee on Banking Supervision.
- Bhutta, N., Dokko, J. and H. Shan, 2010, The depth of negative equity and mortgage default decisions, Financial and Economics Discussion Series working paper, Federal Reserve Board,
- Blume M.E., F. Lim, and A.C. Macklinlay, 1998, The declining credit quality of US corporate debt: myth or reality, *Journal of Finance* 53 (4), 1389-1413.
- Bucks, Brian, and K. Pence, 2008, Do borrowers know their mortgage terms? *Journal of Urban Economics* 64, 218–233.
- Calem P. S., M. LaCour-Little, 2004, Risk-based capital requirements for mortgage loans, *Journal of Banking and Finance* 28, 647-672.
- Campbell, J.Y, 2006, Household finance, *Journal of Finance*, 61, 1553-1604
- Campbell, J.Y, 2012, Mortgage market design, NBER working paper n°18339, August.
- Campbell, J.Y, and J. Cocco, 2003, Household risk management and optimal mortgage choice, *Quarterly Journal of Economics*, 118, 1449-1494.
- Campbell, J.Y, and J. Cocco, 2011, A model of mortgage default, NBER working paper n°17516, October.
- Campbell, J.Y, T. Ramadorai, B. Ranish, 2012, How do regulators influence mortgage risk? Evidence from an emerging market, CEPR discussion papers n°9136, September.
- Chan S., M. Gedal, V. Been, A. Haughwout, 2013, The role of neighbourhood characteristics in mortgage default risk: Evidence from New York City, *Journal of Housing Economics* 22, 100-118.
- Crowe C., G. Dell'Ariccia, D. Igan, and P. Rabanal, 2011, How to deal with real estate booms: Lessons from country experiences, IMF working paper, 11/91, April.
- Davis M. and S. Van Nieuwerburgh, 2014, Housing, finance and the macroeconomy, NBER working paper n°20287, July.
- Diaz-Serrano L., 2005, Income volatility and residential mortgage delinquency across the EU, Journal *of Housing Economics* 14, 153-177.
- Dietsch M., J. Petey, 2014, The credit risk implications of housing ownership promotion: The effects of public subsidies and adjustable-rate loans, working paper, University of Strasbourg.
- Ding, L., R. Quercia, W. Li, J. Ratcliffe, 2011, Risky borrowers or risky mortgages: disaggregating effects using propensity score models. Journal of Real Estate Research 33 (2), 245–278.
- Duca, J., J. Aron, J. Muellbauer, K. Murata, and A. Murphy , 2012, Credit, Housing Collateral and Consumption: Evidence from Japan, the UK and the U.S. *The Review of Income and Wealth*, September 2012.
- Duca, J., J. Muellbauer and A. Murphy, 2011, House Prices and Credit Constraints: Making Sense of the U.S. Experience, *The Economic Journal*, May 2011.
- Duca, J., J. Aron, J. Muellbauer, and A. Murphy ,2010, "Housing Markets and the Financial Crisis of 2007-09: Lessons for the Future," *Journal of Financial Stability*, December 2010.

- Egloff D., M. Leippold, S. Jöhri, C. Dalbert, 2005, Optimal importance sampling for credit portfolios with stochastic approximation, working paper.
- Ergungor O., 2010, Bank Branch Presence and access to credit in low- to moderate-income neighborhoods, *Journal of Money, Credit and Banking*, vol. 42, October, pp. 1321-1349.
- Foote, C., K. Gerardi, and P. Willen, 2008. Negative equity and foreclosure: theory and evidence, Journal of Urban Economics, vol. 64, pp, 234–245.
- Foote, C., K. Gerardi, L. Goette and P. Willen, 2009. Reducing foreclosures. NBER Macroeconomics Annual (24), 89–138.
- Frey, R., A. McNeil, 2003, Dependent defaults in models of portfolio credit risk. Journal of Risk 6 (1), 59–92.
- Furlong F. and and Y. Takhtamanova, 2012, Did the housing boom affect mortgage choice?, FRBSF Economic Letter, n°33, November.
- Glasserman P., J. Li, 2005, Importance sampling for portfolio credit risk. Management Science, 51(11), 1643–1656.
- Gordy M., 2000, A comparative anatomy of credit risk models, Journal of Banking and Finance 24 (1-2), 119-149.
- Gouriéroux C., J.P. Laurent, O. Scaillet, 2000, Sensitivity analysis of Values at Risk, *Journal of Empirical Finance* 7, 225-245.
- Guiso, L., P. Sapienza, L. Zingales, 2014, The Determinants of Attitudes towards Strategic Default on Mortgages, Journal of Finance.
- Haughwout, A., C. Mayer, and J. Tracy., 2009, Subprime Mortgage Pricing: The Impact of Race, Ethnicity and Gender on the Cost of Borrowing, Brookings-Wharton Papers on Urban Affairs: 33-63.
- Haughwout, A., R. Peach, and J. Tracy., 2008, Juvenile Delinquent Mortgages: Bad Credit or Bad Economy? Journal of Urban Economics, 64(2): 246-57.
- Hatchondo, J.C., L. Martinez, and J. M. Sanchez, 2013, Mortgage defaults and prudential regulations in a standard incomplete markets model, working paper, Federal Reserve Bank of Saint-Louis.
- Igan D.,and H. Kang, 2011, Do Loan-to-Value and Debt-Service-To-Income Limits Work? Evidence from Korea, IMF working paper, 11/297.
- IMF, 2013, Financial Sector Assessment Program Update: France, Housing prices and financial stability, Technical Note, June.
- Koijen,R., O. Van Hemert, and S. Van Nieuwerburgh, 2009, Mortgage Timing, *Journal of Financial Economics*, 93 (2, August), 292-324.
- Laufer, S., 2013, Equity extraction and mortgage default, working paper Federal Reserve Board, 2013-30.
- Lucas A., P. Klaassen, P. Spreij, S. Straetmans, 2001, An analytic approach to credit risk of large corporate bond and loan portfolios, *Journal of Banking and Finance* 25, 1635-1664.
- Mayer A., K. Pence, S.M. Sherlund, 2009, The Rise in Mortgage Defaults, *Journal of Economic Perspectives* – Volume 23, Number 1, 27 – 50.
- McNeil A., J. Wendin J, 2007, Bayesian inference for generalized linear mixed models of portfolio credit risk, *Journal of Empirical Finance* 14, 131-149.
- Mian A. and A.Sufi, (2009) The consequences of mortgage credit expansion: evidence from the 2007 mortgage default crisis, *Quarterly Journal of Economics*, Vol. 124, No. 4, November, pp. 1449–1496.
- Mian, Atif, Sufi, Amir, 2011. House prices, home equity based borrowing, and the U.S. household leverage crisis. American Economic Review 101, 2132–2156.

- Moulton, S., 2010, Originating Lender Localness and Mortgage Sustainability: An Evaluation of Delinquency and Foreclosure in Indiana’s Mortgage Revenue Bond Program. *Housing Policy Debate* 20(4): 547-583.
- Moulton S., O. and E. Ergungor, 2011, Beyond the Transaction: Depository Institutions and Reduced Mortgage Default for Low Income Homebuyers, wp.
- Quercia, R., A. Pennington-Cross, C. Yue Tian., 2012, Mortgage default and prepayment risks among moderate- and low income households. Real Estate Economics 40, S159–S198.
- Rosen R., 2011, Competition in mortgage markets: The effect of lender type on loan characteristics, Economic Perspectives, Federal Reserve Bank of Chicago, first quarter.
- Sherlund, S., 2008, The past, present and future of subprime mortgages, wp; Federal Reserve Board, September.
- Tasche, D., 1999, Risk contributions and performance measurement, Working paper, Technische Universität München.
- Tasche D., 2006, Measuring sectoral diversification in an asymptotic multi-factor framework, *Journal of Credit Risk* 2 (3), 33-55.
- Tasche D., 2009, Capital allocation for credit portfolios with kernel estimators, *Quantitative Finance* 9(5), 581-595.
- Van Hemert O., 2009, Household interest rate risk management, New York University, Stern.

Appendix A: Average characteristics of loan according to their LTV and DSTI ratios

Table 1 – Average characteristics of loan according to their LTV and/or DSTI ratios

	LTV ratio (%)	DSTI ratio (%)	Amount (euros)	Maturity (years)	share of fixed rate (%)	share in the population (%)
LTV < 70 %	43.89	20.21	70 264	16.00	75.01	37.41
70% =< LTV < 95%	80.90	25.78	94 523	19.43	62.81	35.00
95 % =< LTV < 100 %	98.25	29.83	133 734	23.66	41.10	8.69
100 % <= LTV	103.83	27.69	116 426	21.11	65.33	18.91
DSTI =< 25%	62.40	12.94	66 054	16.70	76.40	41.95
25% < DSTI =< 33%	79.59	29.41	106 097	20.25	53.68	44.46
33% < DSTI =< 36%	84.70	34.19	129 045	22.19	67.70	7.65
36% < DSTI	81.82	44.46	138 749	18.93	82.00	5.95

Source : Bank data and authors' computations

Appendix B: Estimation results: LTV, DSTI, and LTV&DSTI segmentation models

A. Goodness-of-fit measures

	LTV model	DSTI model	LTV&DSTI
-2 Res Log Pseudo-Likelihood	3131.94	2398.66	3169.08
Pseudo AIC	3151.94	2418.66	3225.08
Pseudo BIC	3159.66	2426.39	3246.71
Generalized Chi-Square	4524.26	3691.53	5058.73
Gener. Chi-Square / DF	11.97	9.77	7.60

B. Covariance parameters

	LTV		DSTI		LTV&DSTI	
Parameter	Mean	Std error	Mean	Std error	Mean	Std error
Intercept	-3.5902	0.01465	-3.5740	0.02088	-3.5437	0.01584

C. Default thresholds and probabilities

	LTV			DSTI		
Rating	Estimate	Std error	Default probability	Estimate	Std error	Default probability
1	-1.6805	0.009925	0.046430	-1.6340	0.01781	0.051129
2	-2.6862	0.01028	0.003613	-2.6327	0.01798	0.004235
3	-2.8715	0.01052	0.002043	-2.8348	0.01824	0.002293
4	-2.9922	0.01034	0.001385	-2.9699	0.01814	0.001489
5	-3.1698	0.01117	0.000763	-3.1437	0.01861	0.000834
6	-3.5902	0.01465	0.000165	-3.5740	0.02088	0.000176
7	-1.6805	0.009925	0.046430	-1.6340	0.01781	0.051129

C. Default thresholds and probabilities (continued)

	LTV&DSTI		
Rating	Estimate	Std error	Default probability
1	-1.6242	0.01151	0.052167
2	-2.6285	0.01181	0.004288
3	-2.8147	0.01216	0.002441
4	-2.9407	0.01205	0.001637
5	-3.1149	0.01275	0.000920
6	-3.5437	0.01584	0.000197
7	-1.6242	0.01151	0.052167

D. Covariance parameters tests

LTV				
Covariance Parm	Estimate	Standard Error	Z Value	Pr Z
Var(1)	0.0093	0.00281	2.68	0.0037
Var(2)	0.0103	0.00862	1.91	0.0280
Var(3)	0.0228	0.01457	2.38	0.0087
Var(4)	0.0267	0.01407	2.25	0.0121
Corr(2,1)	0.5846	0.2458	2.38	0.0174
Corr(3,1)	0.7585	0.1818	4.17	<.0001
Corr(3,2)	0.9582	0.02450	39.11	<.0001
Corr(4,1)	0.6470	0.2354	2.75	0.0060
Corr(4,2)	0.9830	0.01069	91.95	<.0001
Corr(4,3)	0.9613	0.02455	39.16	<.0001

DSTI				
Cov Parm	Estimate	Standard Error	Z Value	Pr Z
Var(1)	0.008481	0.005282	1.87	0.0308
Var(2)	0.01446	0.004785	2.10	0.0178
Var(3)	0.01444	0.005069	2.70	0.0034
Var(4)	0.01117	0.004794	1.92	0.0272
Corr(2,1)	0.8039	0.09305	8.64	<.0001
Corr(3,1)	0.8552	0.07879	10.85	<.0001
Corr(3,2)	0.9232	0.04623	19.97	<.0001
Corr(4,1)	0.8233	0.1146	7.19	<.0001
Corr(4,2)	0.9795	0.02217	44.19	<.0001
Corr(4,3)	0.8708	0.08059	10.81	<.0001

LTV & DSTI				
Cov Parm	Estimate	Standard Error	Z Value	Pr Z
Var(1)	0.01744	0.01433	1.22	0.1117
Var(2)	0.004323	0.001416	3.05	0.0011
Var(3)	0.01003	0.002937	3.42	0.0003
Var(4)	0.02291	0.01426	1.61	0.0541
Var(5)	0.01255	0.003904	3.21	0.0007
Var(6)	0.01264	0.004907	2.58	0.0050
Var(7)	0.01265	0.004862	2.60	0.0046
Corr(2,1)	-0.1288	0	.	.
Corr(3,1)	-0.01834	0	.	.
Corr(3,2)	0.2235	0	.	.
Corr(4,1)	-0.2635	0.1908	-1.38	0.1673
Corr(4,2)	0.7557	0.1025	7.37	<.0001

Corr(4,3)	-0.2047	0.1494	-1.37	0.1708
Corr(5,1)	-0.01428	0	.	.
Corr(5,2)	0.1384	0	.	.
Corr(5,3)	0.9032	0.02178	41.47	<.0001
Corr(5,4)	-0.4250	0.1460	-2.91	0.0036
Corr(6,1)	-0.1231	0.3147	-0.39	0.6958
Corr(6,2)	0.03598	0	.	.
Corr(6,3)	0.2176	0	.	.
Corr(6,4)	0.1412	0.1745	0.81	0.4185
Corr(6,5)	-0.04315	0	.	.
Corr(7,1)	0.1692	0.4313	0.39	0.6949
Corr(7,2)	-0.1791	0	.	.
Corr(7,3)	0.3432	0	.	.
Corr(7,4)	-0.5503	0.1243	-4.43	<.0001
Corr(7,5)	0.5042	0	.	.
Corr(7,6)	0.4723	0	.	.