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Abstract

We propose a model where risk appetite is determined by the interplay of the default put option and the down-and-out call (DOC) option, pricing the franchise value, and the market value of tangible assets. The bank manager takes incremental decisions maximizing his objective function, i.e. the sum of the two options, adjusting jointly the level of leverage, assets and franchise value volatility and the policy rate. Risk appetite is given by the first order derivatives. Since the franchise value is non-observable, we consider risk appetite also as the latent variable in a state space model. Its dynamic moves in line with the previous estimation and is persistent. We show that regulators should tune their recommendations depending on the targeted cluster, since the driver of risk appetite alternates between the two options depending on the cluster and on the underlying variable considered and that the optimal policy rate is higher with respect to the existing one.

Keywords: risk appetite, risk-free rate, default put option, down-and-out call option, franchise value, assets and franchise' volatility, leverage.

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1 Introduction

The main goal of our research is to understand bank's risk appetite and the role played by the monetary policy in shaping it. We find what are the main drivers for risk appetite and their impact on the bank market and franchise values. Bank risk appetite is determined by the interplay of the default put option, PUT^{def} , and the franchise value, i.e. the net present value of non-observable bank's growth opportunities, priced through a down-and-out call (DOC) option. The franchise value is not directly observable: how can we evaluate this figure? We rely on Barone-Adesi et al. (2014) pricing it through the DOC option and we propose to estimate it implicitly from the equity market value, extending the standard structural models. Risk appetite is determined by the manager objective function which we propose it to be the ratio between the sum of the two options and the market value of the tangible assets, i.e. $\frac{DOC+PUT^{def}}{MVA}$. We propose two ways to reach the goal of shaping risk appetite. First, we suggest a two steps optimization problem, second, we assess risk appetite in a state space model.

In the first step of the optimization problem, we estimate the non observable quantities, namely the franchise value and the market value of the assets. We estimate numerically the optimal value of the franchise value together with the market value of the assets. The optimization procedure is based on the non-linear least squares estimator. We discriminate banks with and without franchise value and subsequently we perform a cluster analysis based on leverage. Given the industry in which we perform our analysis, we define leverage as the ratio between from one side, the market value of the assets and the franchise value, and on the other side, the market value of equity, which is the denominator, as in Kalemli-Ozcan et al. (2012). In the second step, we optimize the objective function, given by the sum of the two options, evaluating risk appetite simultaneously with respect to leverage, volatility and the policy rate. The first-order derivatives of this function determines bank and monetary policy risk appetite. The determinant of the hessian matrix tells us in which direction the manager optimizes this function. In our model, both the bank manager and the monetary policy have to align their policy optimizing the risk appetite in line with the regulators prescriptions. The monetary policy has to set the rho² equal to zero (policy rate-driven risk appetite) considering also the manager risk management policy. The latter sets the vega³ equal to zero (volatility-driven risk appetite) simultaneously with the derivative with respect to the leverage (leverage-driven risk appetite). We stress the importance of performing

²Objective function sensitivity with respect to change in the policy rate.

³Objective function pricing sensitivity with respect to change in the implied volatility.

jointly this optimization, meaning that the monetary policy maker has to cooperate with the bank manager in order to align their policy and to allow for an effective objective function optimization. This is equivalent to say that also second order and joint derivatives are considered from a technical point of view and the sign of the determinant of the hessian matrix gives us the direction of the bank and monetary policy strategy. The pricing of the two options is designed to have different impact on the appetite for risk of our bank depending on the cluster we focus on and the variable we consider. The regulators should tune their recommendations depending on the targeted cluster in order to be effective. This is an element of primary interest because regulation does not differentiate enough in the banking industry and flat recommendations do not fit all the peculiarities we find in clustering the industry. Furthermore, the impact can be counterproductive, given that differences among clusters are relevant and consequences can go in an opposite direction with respect to what is intended by the regulator. Our specification of objective function returns a three-dimensional perspective and addresses the main instruments of regulation and monetary policy. Thus, it can be a useful instrument for the regulator, allowing for a more comprehensive understanding of the joint impact of the three optimizing variables.

Our specification for the objective function, and consequently for risk appetite, is not observable in the market because of the role played by the franchise value. Thus we want to understand the goodness of our estimation. We propose a state space model, where the state variable is the objective function of the bank and we model it as an $AR(1)$ with a coefficient ϕ smaller than one in absolute value ($|\phi| < 1$). The observed variable is the implied assets and franchise value volatility. Conforming to the existent literature, the measurement equation is non-linear, and for simplicity we adopt a quadratic specification. To accommodate this non-linearity feature we use a non-linear extension of the Kalman filter. We perform our filtering procedure through the extended Kalman filter (EKF, Haykin (2001)). The filtering is used to estimate the model parameters by pseudo-maximum likelihood (PML) and to understand the underlying dynamic of the latent state. Regulation sets constraints on leverage assets' riskiness and the monetary policy plays a role in determining the liquidity. All those elements impact objective function as defined in our framework.

Our empirical sample consists of 1436 listed US banks and the time span considered is 1980-2014. We perform a cluster analysis in order to accommodate for the main differences across the industry. First of all we distinguish between banks with franchise value and without. We find that about the 17% of the banks in the sample do not have franchise value at least in one year of the time span considered. Second, we cluster our sub-samples into three categories depending on leverage. The main results for the sub-sample of banks without

franchise value are easy to predict since the put option is the only player in the objective function optimization and in determining the shape of risk appetite. More interestingly, we assess the sub-sample of banks with a portfolio of growth opportunities and we cluster it as follows: we have (i) 4829 “overcapitalized” banks (cluster 21), with an actual average leverage of 6.3299; (ii) 3298 “average capitalized” banks (cluster 22), with 12.9144 and (iii) 1117 “undercapitalized” banks (cluster 23) with 22.0996.

We perform a sensitivity analysis with respect to the three optimizing variables in order to understand the shape of risk appetite moving one of the three variables given the optimal quantities for the other two. Furthermore, we disentangle which option is the main driver among the three clusters. Considering leverage, the risk appetite is determined first by the default put option, then by the DOC one. There is a difference in the leverage-driven risk appetite at a cluster level related to the positioning of the peak. On the volatility side, both the options contribute in shaping the objective function but the default put option is an early operator with respect to the DOC one. As the leverage increases, the volatility-driven risk appetite is smaller, since the risk appetite peaks goes to the left-hand side. Concerning the policy rate, the shape of risk appetite is a concave function in all the three cases, with minor differences among the clusters. The DOC option drives the shape at the beginning leaving the place to the put one afterwards. In cluster 21, the bell-shape is quite symmetric, instead in the other two it is right-skewed. Empirically, we always find that the estimated policy rate is higher relative to the actual one. Increasing the leverage, the optimized objective function naturally increases (since it is partly determined by the franchise value). The risk appetite is assessed through the behaviour of its three main drivers and the associated shape is quite different among the clusters.

Considering the state space model, we call the estimation of the state variable, obtained through the PML, *RAPML*. We compare it to the volatility-driven estimate for risk appetite obtained in the two-steps optimization and we call it *RAM*. We compare the evolution of our two estimators, the one given in the optimization section and the other one obtained through the filtering procedure. The estimations produce coherent results especially at an aggregate level, in the decade 1997-2007. Some differences emerge when looking at cluster level. The *RAPML* is very persistent due to a large coefficient ϕ in absolute value and, interestingly, we show that the major driver in the measurement equation is given by the quadratic term. Looking at clusters, we can see that the *RAPML* variability is very low compared to the other one and cluster 23 presents the most similar results with respect to the aggregate perspective.

The paper proceeds as follows. Section 2 introduces a literature review. Section 3 presents

the model and the pricing of the options. The two step optimization problem is described in Section 4. Section 5 analyses the filtering model and the PML estimation. Section 6 shows the empirical results. Section 7 concludes. Further material is given in the appendix.

2 Literature review

This paper is related to several different strands of the literature. First of all, the building blocks of the literature about structural models are considered. Second, regulation issues are reviewed from both a theoretical and an empirical perspective. Third an overview on growth opportunities evaluation issues is presented.

Our paper is based on the seminal works by Black and Scholes (1973) and Merton (1973a), where the liabilities of a company are seen as an European option written on the assets of a firm. In the case of Merton (1973a), the capital structure of a firm is composed by a zero-coupon bond, as debt, and equity. At the beginning of the period, debtholders hold a portfolio consisting of the face value of debt and a short position on a European put option. Instead, equityholders hold a European call option on the market value of assets, with strike equal to the face value of debt. Under the non arbitrage assumption, the price of this option is equal to the market value of equity. Default can happen only at maturity and standard Black-Scholes world assumptions⁴ hold.

Several studies extend the original model considering the assumptions by Black and Scholes. Black and Cox (1976b) and Longstaff and Schwartz (1995) allows for default also prior to maturity. Merton (1977, 1978) examines default risk in banks, with several issues that were addressed by recent literature. In those cases, equity is considered as a barrier option and the default event is triggered at the first hitting time of an exogenously determined barrier.

The endogenization of the default threshold, proposed by Leland and Toft (1996), provides alone not a clear improvement with respect to the standard Merton model, unless a jump component is introduced, as in Leland (2006). Our study is more related to Brockman and Turtle (2003), who introduce in equity path dependency, i.e. equity can be knocked out whenever a legally binding barrier is breached.

The model we propose relies on regulatory principles: Basel III indicates a bank is insolvent if the common equity tier one (from now on CET1) is below 4.5%. It is true that in

⁴Such as perfect markets, continuous trading, constant volatility, deterministic and constant interest rates, infinite liquidity and Ito dynamics for the process of the market value of the assets in place

some countries banks, that would be declared insolvent for Basel III, still run their assets. Thus, a possible extension to this model would consider the interplay between an exogenous default barrier set by the regulator and the endogenous one chosen by the bank, highlighting an important weakness in monitoring by the regulator.

Hugonnier and Morellec (2014) propose a dynamic model of banking assessing the impact of the main instruments in Basel III. From one side, they show that liquidity requirements impact only on the short run increasing potentially default risk. On the other side, leverage requirements decrease default risk and increase growth opportunities of the bank, on the long-run, which is partly in line with our findings. Additionally, raising equity requirements make the loss to be borne by shareholders and the distance to default increases (see e.g. Admati and Hellwig (2013)).

Berger and Bouwman (2009) provide an empirical application, in order to answer a theoretical question on the relationship between capital and liquidity. They study the link between value and liquidity and this is relevant for our franchise value issue. They find that banks, creating more liquidity, have significantly higher market-to-book and price-earnings ratio.

From a social point of view, Hugonnier and Morellec (2014) provide a measure for social benefits of regulation, that together with the liquidity dimension could be related to the present work. DeAngelo and Stulz (2014) show the reasons of an increase in bank leverage over the last 150 years. They highlight that high bank leverage, per se, does not necessarily cause systemic risk. They warn regulators, putting too high leverage constraints, since regulated banks loose in terms of competitiveness with respect to unregulated shadow banks.

Looking at regulatory barriers, Episcopos (2008) considers a wealth transfer from stockholders to the insurer. He shows that stockholder incentives to perform asset substitution diminish, when the regulatory barrier is increased. Concentrating on too-big-to-fail banks, Lucas and McDonald (2006) build their modelling of the public guarantee in a Sharpe (1976) and Merton (1977) framework, where the insurance is a put option on the assets' value. For a firm with guaranteed debt, equity value has another component with respect to the standard call option on the operating assets: the public guarantee. It is assumed to accrue to equity holders, since it is equivalent to writing a put option, from the government point of view. Hovakimian and Kane (2000) assess the effectiveness of capital regulation at U.S. commercial banks on a ten year window (1985-1994). They find that capital discipline do not prevent large banks from shifting risk onto the safety net. Bank capitalization is necessary to make the franchise value and managerial risk aversion strong enough to cope with the fact that risk remains mispriced at the margin by the Federal Deposit Insurance Corporation (FDIC)

as in Gorton and Rosen (1995).

Another main ingredient in our study is franchise value, which is the net present value of future growth opportunities. Arnold et al. (2013) propose a switching regime framework for evaluating the growth option. One of their main results consists in showing that growth opportunities require less leverage and, as we explain in Section 2, this is the case also in our model. Instead, Marcus (1984) and Li et al. (1996) develop an option-pricing model to show how charter value can counterbalance moral-hazard-induced risk incentives. Martinez-Miera and Repullo (2010) individuate a *U*-shape relation between franchise value and the risk of bank failure, in case of lower interest rate charged by banks due to a competitive environment.

The underlying framework, for our model, is given by Froot and Stein (1998) who found the rationale for risk management arises from the concavity of the franchise value. The market value of equity is assessed building on Babel and Merrill (2005). In their model, the franchise value and the default put option accrue to equityholders. Barone-Adesi et al. (2014) argue that the risk appetite of financial intermediaries is determined by the interplay of default put option and growth opportunities.

3 Research methodology

3.1 The model

The subject of our study is a bank held by shareholders who benefit from limited liability. They discount cash flows at a constant interest rate.

The structure of the balance sheet, in book values, is given as follows. The bank owns a portfolio of risky assets and liquid reserves, and is financed by insured deposits, risky debt and equity. On the left hand side of the balance sheet, risky assets are relative illiquid due to informational problems (see e.g. Hugonnier and Morellec (2014) and Froot and Stein (1998)). For the while, assuming there are no costs of raising funds, liquidity reserves do not play a role. On the right hand side of the balance sheet, the focus of the analysis is on risky debt and equity, instead deposits are seen as a relative stable source of financing for the bank (see Hanson et al. (2014)).

Going to market values, debt is seen as a portfolio of cash plus a short position in a put option on firm value as in Merton (1974) and equity as a call option on assets as in Black and Scholes (1973). In our model, we focus on the interplay between the standard default put option and the down-and-out call option that accrue to shareholders.

Main assumptions and model description

In this subsection, we introduce the main assumptions of our model, building on the fundamental work of Black and Scholes (1973) and Merton (1974), and the following insights by Babbel and Merrill (2005) and Barone-Adesi et al. (2014).

The setting of the underlying model deals with continuous time, with initial date $t = 0$ and terminal date $t = T$. No frictions, like transaction costs, taxes and costs of raising funds, nor limits on short sales are considered and no riskless arbitrage opportunities exist. Agents are risk-neutral and there are no conflicts of interest between shareholders and managers. The focus of the project is to understand how the regulator should set appropriate risk-taking incentives, given that the bank is maximizing its end of the period equity market value.

Initially, shareholders contribute the entire equity of the bank and, subsequently, consider operating a debt-equity swap at t_0 , where debt has face value FV^{SD} . The proceeds from debt issue are invested in the assets in place and future growth opportunities that at time T are worth $A(T)$ and $Fr(T)$, respectively.

The default can occur only at the end of the period, T , in case liabilities exceed assets. The value of the assets at time t is given by:

$$A(t) = A(0) \exp \left(\mu_{At} - \frac{\sigma_A^2}{2} t + \sigma_A B_t \right), \quad (1)$$

where B_t is a standard Brownian motion defined on (Ω, \mathcal{F}, Q) , so that

$$d \ln(A(t)) = \left(\mu_{At} - \frac{\sigma_A^2}{2} t \right) dt + \sigma_A dB_t, \quad (2)$$

where the drift, μ_t , is time-varying and σ is constant.

For simplicity, we fix the risk-free rate and dividend issues equal to zero. Limited liability holds, equity is a call option on the value of net tangible assets at maturity, i.e. $A(T)$. The option form is the standard one, $E(T) = \max(A(T) - FV^{SD}, 0)$, where FV^{SD} is the face value of standard debt. The default put, that accrues to shareholders, materializes in case at T the value of the assets are smaller than the face value of debt: $Put^{def}(T) = \max(FV^{SD} - A(T), 0)$. In case of TBTF banks, debt becomes riskless and the put value is provided by public guarantee, acting as subsidy to shareholders. The net present value of future growth opportunities of the bank at the terminal date is $Fr(T)$. Future growth opportunities materialize only at the end of the period, T , but the franchise value might

vanish previously, as soon as the liabilities exceeds the asset value in $0 \leq t \leq T$, that is when

$$\tau_{Fr=0} = \inf \{t \geq 0 : MVA(t) \leq MV^{SD}\}, \quad (3)$$

where, MVA is the market value of the assets in place and MV^{SD} is the market value of debt outstanding. This is slightly different with respect to Demsetz et al. (1996) or Jones et al. (2011), because in their model this value is lost in case of bankruptcy. The franchise value follows the same dynamics as the assets value:

$$d \ln(Fr(t)) = \left(\mu_{Frt} - \frac{\sigma_{Fr}^2}{2} t \right) dt + \sigma_{Fr} dB_t, \quad (4)$$

and the two are correlated, with correlation coefficient ρ , where $-1 \leq \rho \leq 1$.

The bank's balance sheet at time zero can be summarized as follows:

<u>ASSETS</u>	<u>LIABILITIES AND NET WORTH</u>
	Deposits: $D(0)$ Short term liabilities: $SL(0)$ Long term Liabilities: $LL(0)$
Tangible Assets: $MVA(0)$	
Default Put Option: $Put^{Def}(0)$ DOC Option: $DOC(0)$	
Intangible Assets: $Put^{Def}(0) + DOC(0)$	Total Liabilities: $D(0) + L(0)$
	Shareholder Equity: $MVE(0)$
TOTAL: $MVA(0) + Put^{Def}(0) + DOC(0)$	TOTAL: $D(0) + L(0) + MVE(0)$

Table 1: Bank balance sheet at time zero.

At time zero, equity market value exceeds the capital supplied by the shareholders and this difference comes from the value at time zero of the franchise value and the option shareholders have to default, where the value at time zero of those options is given in the following sections. The terminal claim on the DOC option and equity are respectively:

$$DOC(T) = \begin{cases} F(T) & \text{if } MVA(T) \geq (L(T) + D(T)) \\ 0 & \text{otherwise} \end{cases} ; \quad (5)$$

$$MVE(T) = \begin{cases} MVA(T) + Fr(T) + Put^{def}(T) - L(T) - D(T) & \text{if } MVA(T) \geq (L(T) + D(T)) \\ (L(T) + D(T)) - MVA(T) & \text{if } MVA(T) < (L(T) + D(T)) \end{cases} . \quad (6)$$

In the expressions above, we show that the franchise value come to fruition in case the tangible value of the assets do not fall below the contemporaneous value of the liabilities and deposits. We comment on the barrier in the context of the DOC pricing. Furthermore, in case the franchise value do not vanish, the put option is out of the money and the shareholders do not exercise the put option and its present value is still given by the option price that can be potentially exercised in the future. The opposite is true when the tangible value of the assets is eroded.

3.2 Pricing the default option

Following the reasoning in Barone-Adesi et al. (2014), bank shareholders are long on the default option, which the manager has to maximize acting on behalf of the shareholders. The pricing formula for the value of the put option at time zero together with the DOC one we present in the following section section, considers the franchise value as major ingredient both in the underlying value and in the volatility. The franchise value, explained subsection 2.3, has to be taken into account in the market value of the assets. This is necessary in order to prevent potential arbitrage opportunities, that could arise otherwise, buying the bank and selling short the tangible assets and the franchise value, if this last one would not be considered. The put option is a convex, decreasing function of the asset value and is maximized when the value of the liabilities, as well as the riskiness of the assets is magnified. This means it is a a driver for risk-taking. The underlying is given by $MVA(0)$, the value of the net tangible assets at the beginning of the period, and Fr the franchise value. The strike price is the market value for straight debt, MV^{SD} ⁵. T is the time to maturity, σ_{A+Fr} is the volatility of both the assets and the franchise value and rf is the policy rate. Our pricing for the default put option is given by:

⁵We proxy the market value for the straight debt with the KMV model (KMV corporation).

$$\begin{aligned}
Put^{def} &= MV^{SD} N(-d_2) + \\
&- (MVA(0) + Fr(0)) N(-d_1), \\
&\text{with } \{\tau_{Fr=0} > T\}, \\
\text{where } d_1 &= \left(\frac{\ln\left(\frac{MVA(0)+Fr(0)}{MV^{SD}}\right) + \left(rf + \frac{\sigma_{A+Fr}^2}{2}\right)T}{\sigma_{A+Fr}\sqrt{T}} \right), \\
\text{and } d_2 &= d_1 - \sigma_{A+Fr}\sqrt{T}
\end{aligned} \tag{7}$$

In absence of growth opportunities, the pricing formula goes back to the standard one. This option push the bank manager to adopt a risk-taking policy. We present further information about the sensitivity of the default put option with respect to volatility, leverage and policy rate in the optimization section. In the case we are dealing with too big to fail (TBTF) banks, the default put option value coincide with the government bailout program, since the government will rescue the bank as a whole. In the case the banks are not considered TBTF, the option payoff is greater than the government subsidize, since it covers only the deposit value.

3.3 Pricing the DOC option, in presence of non-observable underlying

At time T , $F(T)$ represents a portfolio of positive NPV growth opportunities. Before maturity, the expected value of $Fr(T)$ is embedded in the value of the risky assets of the bank and is the franchise value which is given by the DOC option (see Barone-Adesi et al. (2014)). This option is a down and out call, with a pricing formula that is slightly different from a mathematical point of view with respect to the standard one in Black-Scholes framework (Merton (1973a)), but it confers a much different economic interpretation, where $Fr(0)$ constitutes the value of potential growth net of investment cost in the case the bank does not opt for default. Since investment costs are already considered in $Fr(0)$, the strike for this option is set to zero. The barrier is given by the market value of standard debt. This option is priced in an european framework given that the franchise value comes to fruition only

at maturity⁶, but it is path dependent. In case the barrier is breached before maturity the option expires and the franchise value is driven immediatly to zero. The relation is given as follows:

$$\begin{aligned}
 DOC (Fr(0), MVA(0), MV^{SD}) &= Fr(0) \left[N(v_1) + \right. \\
 &\quad \left. - \left(\frac{MV^{SD}}{MVA(0)+Fr(0)} \right)^{2\lambda} N(y_1) \right] \\
 &\quad \text{with } \{\tau_{Fr=0} > T\}, \\
 \text{where} \quad \lambda &= \frac{rf + \frac{\sigma_{A+Fr}^2}{2}}{\sigma_{A+Fr}^2} \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 v_1 &= \frac{\ln\left(\frac{MVA(0)+Fr(0)}{MV^{SD}}\right)}{\sigma_{A+Fr}\sqrt{T}} + \lambda\sigma_{A+Fr}\sqrt{T} \\
 y_1 &= \frac{\ln\left(\frac{MV^{SD}}{MVA(0)+Fr(0)}\right)}{\sigma_{A+Fr}\sqrt{T}} + \lambda\sigma_{A+Fr}\sqrt{T}
 \end{aligned}$$

The franchise value, $Fr(0)$, is not directly observable. However, we present below how to estimate it in the framework of our model. Indeed, the value of the DOC option is a part of the market value of the assets, where the remaining is given by the tangible assets. The term in parenthesis gives the "pricing" probability that the intermediary will survive long enough for the growth opportunities to come to fruition. We show in the following sections when the DOC prevails over the default put one in determining the shape of objective function and consequently the one of risk appetite.

4 The optimization problem for risk appetite

Risk appetite is a non-negative real number that describes investor's appetite for risk, with higher values corresponding to a greater degree of aggression. The reciprocal is risk aversion that is the attitude of investors toward risk. Risk appetite is commonly defined as the level and type of risk a firm is able and willing to assume in its exposures and business activities, given its business objectives and obligations to stakeholders. We build our model in a von Neumann-Morgenstern utility function framework (Von Neumann and Morgenstern (1944)),

⁶that is equivalent to say that we can exercise it only at maturity

where risk aversion is described by a concave utility function, risk-neutral investors have a linear utility function, and risk prone investors have a convex utility function. We prefer to understand the appetite for risk of the bank, rather than concentrate on risk-taking, because, in the definition we propose, the default put option promote seeking (convex shape) instead the DOC one is designed to refrain the bank to undertake excessive risk (concave shape). A rising risk appetite implies that investors are willing to hold riskier assets. Since it is not possible to observe directly risk appetite, we need to understand how it is determined and where to extract information about its manifestation. From both a risk-management and a regulation point of view, it is a priority to infer some information about objective function in the banking system.

We define the manager and monetary policy maker objective function (*O.f.*) as the ratio between the sum of the two options and assets' market value:

$$O.f. := \frac{DOC(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t}) + PUT^{def}(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{MVA}. \quad (9)$$

We propose this definition for the objective function because the two options accrues to the bank market value and involves both the bank manager and the monetary policy maker given that the two options are determined by leverage, volatility and the policy rate. The bank manager has decision power over the first two, instead the monetary policy maker decides over the third one. The risk appetite is determined in the optimization problem we present in Section 4.2 and is given by the first order derivatives and the determinant of the hessian matrix.

The optimization problem is twofold because in the first step we estimate the franchise value and the market value of the assets that are not observable in the market, but are embedded in the equity market value and are key in order to perform the second one, where we look for the optimal level of leverage, assets' volatility and policy rate that simultaneously optimize the objective function.

4.1 First step ingredients

In the first step, the goal is to estimate the unobservable franchise value and the market value of the assets. We argue that our unobservable quantities are embedded in the equity

market value. We model the bank equity market value as the sum of the call option on the assets, the default put option and the franchise value considering the value of the bank at the time in which the franchise value comes to fruition.

We solve the following system of equation symultaneously:

$$\begin{cases} MVE_{i,T}(Fr_{i,T},MVA_{i,T},\sigma_{(A+Fr)_{i,T}})= & Call(Fr_{i,T},MVA_{i,T},\sigma_{(A+Fr)_{i,T}})+Fr_{i,T}+Put^{def}(Fr_{i,T},MVA_{i,T},\sigma_{(A+Fr)_{i,T}}), \\ \sigma_{MVE_{i,T}}MVE_{i,T}= & \sigma_{(A+Fr)_{i,T}}(MVA_{i,T}+Fr_{i,T})N(d_{1i,T}), \end{cases} \quad (10)$$

where $N(d_{1,T}) = \frac{\frac{1}{2}\sigma_{(A+Fr)_{i,T}}^2 T + \ln\left(\frac{A(0)}{MVS D}\right)}{\sigma_{A,T}\sqrt{T}}$. This extension of the Merton specification allows us to consider the franchise value both at the underlying and implied volatility level. Since the equity market value incorporates the information regarding both the assets market value and the franchise value, consequently the implied volatility estimated in this model refers to the one considering both the assets and the franchise value. At the beginning of the period we do not have information regarding the franchise value, so that we consider its price through the DOC option. We solve this problem through the nonlinear least squares criterion function, for each bank at any time t on the whole time span considered, optimizing the distance between the data concerning the equity market value and the model extended accomodating for both the default put option and the DOC one. We perform a step by step optimization, building on the Bellman's Principle of Optimality (Bellman (1952)), applied also in Merton (1973b). In order to perform this, we build on the following error function:

$$\begin{cases} e_{1,i,t}= & MVE_{i,t}-(Call(Fr_{i,t},MVA_{i,t},\sigma_{(A+Fr)_{i,t}})+DOC(Fr_{i,t},MVA_{i,t},\sigma_{(A+Fr)_{i,t}})+Put^{def}(Fr_{i,t},MVA_{i,t},\sigma_{(A+Fr)_{i,t}})), \\ e_{2,i,t}= & \sigma_{MVE_{i,t}}MVE_{i,t}-\sigma_{(A+Fr)_{i,t}}(MVA_{i,t}+Fr_{i,t})N(d_{1i,t}), \end{cases} \quad (11)$$

where $\{i\}_1^n$ is the bank identificator and $\{t\}_1^m$ the year considered. In this specification, we perform our analysis at the beginning of the period, because we need to estimate the major ingredients for the objective function optimization. The non linear least square function is the following:

$$\underset{(Fr_{i,t},MVA_{i,t},\sigma_{A_{i,t}})}{\operatorname{arg\,min}} \quad \sum_{j,i,t=1}^{2,n,m} [e_{j,i,t}^2], \quad (12)$$

where we have inserted what will be the solution value, $(Fr_{i,t}, MVA_{i,t}, \sigma_{(A+Fr)_{i,t}})$, that

optimize the sum of the squared deviations, which are the nonlinear least squares estimators. Thus, the optimal quantities $MVA_{i,t}^*$, $\sigma_{Ai,t}^*$ and $Fr_{i,t}^*$ are derived and we can proceed to the next step optimizing the objective function⁷. At this step, empirically, we proceed in our first clustering distinguishing among banks with franchise value and without.

4.2 Second step

In this step, we optimize the objective function to cope with the regulator standard indications concerning leverage and assets'volatility. The bank manager and the monetary policy maker have to cooperate in order to achieve this goal even if standard regulation focuses only on the bank without caring about monetary issues. In our model instead, we propose a powerful set of instruments incorporating also the monetary policy perspective. On one side, the manager has to optimize the objective function of the bank, modifying its exposure to risky assets and adjusting bank's leverage at time zero, operating always for allowing the franchise value to come to fruition at time T . The shape of risk appetite is assessed through the determinant of the hessian matrix in a three-dimensional perspective. We propose a volatility-driven risk appetite, as well as a leverage-driven one and a policy rate-driven one. The outline of those optimal quantities differs among clusters depending on which option drives the behaviour in that specific case. This is an element of primary interest because regulation does not differentiate enough in the banking industry and flat recommendations do not fit all the peculiarities we find in clustering the industry. Furthermore, the impact can be counterproductive, given that differences among clusters are relevant and consequences can go in an opposite direction with respect to what is intended by the regulator.

In our framework, where the objective function is driven by the two options, the monetary policy maker shapes his risk appetite setting ρ equal to zero, focusing on the sensitivity of the objective function with respect to the policy rate (policy rate-driven risk appetite). The decision variables over which, instead, the manager has discretionary power at time zero are volatility and leverage. On the bank manager side, the shape of risk appetite is determined setting equal to zero ν (volatility-driven risk appetite) and the first order derivative with respect to the leverage (leverage-driven risk appetite)⁸. All of those first order derivatives are obtained given optimal values for the other two variables. Our optimization

⁷As we explain in the following step, we perform the optimization at each time step t , following Bellman (1952) and Merton (1973b), in order to allow the franchise value of the bank to come to fruition at time T .

⁸In standard literature, it does not exist a "greek letter" identifying the sensitivity of an option price with respect to leverage.

procedure goes beyond what presented till now. We accomodate for a joint optimization, where the three optimal quantities are estimated simulatenously. The optimization variables are the leverage, the assets'volatility and the policy rate, so our theta in this case is: $\Theta_{i,t} := (lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})$. The optimization problem is:

$$\underset{(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{arg \max} \left[\frac{\delta O f_{i,t}(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{\delta lev_{i,t} \delta \sigma_{A_{i,t}+Fr_{i,t}} \delta rf_{i,t}} \right] \quad (13)$$

In this framework our three-dimensional risk appetite (R.A.) is given by:

$$\begin{aligned} \text{leverage} - \text{driven} \quad R.A. : & \left[\frac{\delta O f_{i,t}(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{\delta lev_{i,t}} \right] = 0 \mid_{\sigma_{A_{i,t}+Fr_{i,t}}^*, rf_{i,t}^*} \\ \text{volatility} - \text{driven} \quad R.A. : & \left[\frac{\delta O f_{i,t}(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{\delta \sigma_{A_{i,t}+Fr_{i,t}}} \right] = 0 \mid_{lev_{i,t}^*, rf_{i,t}^*} \\ \text{policy} - \text{rate} - \text{driven} \quad R.A. : & \left[\frac{\delta O f_{i,t}(lev_{i,t}, \sigma_{A_{i,t}+Fr_{i,t}}, rf_{i,t})}{\delta rf_{i,t}} \right] = 0 \mid_{lev_{i,t}^*, \sigma_{A_{i,t}+Fr_{i,t}}^*} \end{aligned} \quad (14)$$

Numerically, we use the methodology developed by Byrd et al. (1995) which allows box constraints, that is each variable can be given a lower and/or upper bound. The initial value must satisfy the constraints. This uses a limited-memory modification of the BFGS quasi-Newton method (Broyden (1970); Fletcher (1970); Goldfarb (1970); Shanno (1970)). The algorithm always achieve the finite convergence.

In presence of interest rate risk, diversification provides an additional risk management opportunity. Indeed, if the interest rate and asset risk exposures are of similar magnitude, and if these risks are uncorrelated, then one would expect diversification to be very important, especially if franchise values are high. In this case we perform a pointwise optimization since we are interested in the parameters that optimize the objective function of each bank on the whole time span. The optimal objective function do not have theoretial bounds, but we focus on $0 \leq O.f_{i,t} \leq 1$, since it is hard to find empirically a bank having the sum of the two options greater than the market value of the assets (our normalizing quantity). Although it is well known what is the behaviour of the default put option with respect to the three variables assessed, the same is not straightforward for the DOC pricing and, consequently, for our specification of objective function. In the appendix we show the derivation of both the first order derivatives and the cross ones, taken into account given the simultaneous

approach. In order to understand which option is the main driver for the objective function we need to perform a cluster analysis. Section 6 presents the main theoretical results before showing the empirical ones, thus it will become clearer the shape of the objective function and the consequent risk appetite one. Our differentiation among clusters is crucial for setting effective regulatory recommendations, because flat rules miss the peculiarities of the different patterns of objective function we could appreciate in the clustering. In section 6 we present results both aggregated and clustered, pointing out the importance of more accurate analysis in this domain.

5 Risk appetite in a state space model

The objective function and consequently risk appetite are not directly observable since the franchise value is the key ingredient but it is not observable in the data. The proposal is to assess this issue in a state space framework. In general, it is possible to describe a generic state space model by a state equation and a measurement equation. The first one determines the dynamics of unobserved state variables and the second links the state variable to some observables.

In this section, we concentrate on the volatility-driven risk appetite. Thus, we model risk appetite as the latent driver of assets' volatility that is the most reasonable observable manifestation, given our definition of volatility-driven risk appetite. Due to the non-linear relation between assets' volatility and risk appetite, that is determined by the default put option and the unobservable franchise value, priced through the DOC option, we use a non-linear extension to Kalman filter. In order to capture this non-linear dynamic, we perform our filtering procedure through the extended Kalman filter (EKF, Haykin (2001)). We propose a quadratic measurement, for simplicity, in order to accommodate for the non-linearity. Other techniques are available as the quadratic Kalman filter (QKF): a new filtering and smoothing technique for non-linear state space models developed by Monfort et al. (2013).⁹ We prefer the extended Kalman filter to the quadratic one, due to the "brute force" of imposing a quadratic relation, because we want to evaluate our measurement specification in a non-linear way. Further work will consider to compare results with the unscented Kalman filter (UKF) Haykin (2001).

⁹They find that the extended and unscented Kalman filter underperformed the quadratic one by 70% in terms of higher root mean square errors (RMSEs), in case the transition equation is linear and the measurement one is quadratic.

First, we propose a linear-quadratic state space model, where the goal is to address the non linearity exposed in section 3.

In this work, the linear-quadratic state space model is defined by the following two equations. The transition equation is:

$$R_t = \phi R_{t-1} + \epsilon_t, \tag{15}$$

where the objective function $\{R_t\}_{t=0}^{t=T}$ is the unobservable variable and it is modeled as an AR(1) with a coefficient ϕ that has to be smaller than one in absolute value ($|\phi| < 1$). It is expected to be quite large in absolute value for capturing persistency the bank have in objective function no matter what is the economic situation. The equation is linear in R_{t-1} .

The measurement equation is:

$$\sigma_t = \alpha + \lambda_0 R_t + \lambda_1 R_t^2 + \eta_t, \tag{16}$$

where σ_t is the implied assets'volatility. The equation is quadratic in R_t .

The components α, ϕ, λ_i could depend on σ_{t-1} ; the two error terms ϵ_t, η_t are Gaussian, independent, zero mean, unit variance-covariance matrix. The filtering is used to estimate the model parameters by pseudo-maximum likelihood (PML) and to understand the underlying dynamic of the latent state.

5.1 Implementation

We test the ability of the filtering procedure to capture the dynamics of the latent process with a simulation exercise because we have no information about the true values. We simulate the objective function according to the transition equation in the time span (1980-2014), then we perform the simulation on the observation part of the model. Subsequently, we apply the filtering technique to recover the process for the risk appetite.

We call X the state vector, which here is univariate and only includes R_t .

We initialize X and its variance at the unconditional mean and variance:

$$\bar{X}_t = 0 \tag{17}$$

$$P\bar{X}_t = \frac{\sigma(\epsilon_t)^2}{(1 - \phi^2)} \quad (18)$$

The prediction state and its variance are given by the following relations:

$$\bar{X}_t = \phi\bar{X}_t \quad (19)$$

$$P\bar{X}_t = \phi^2 P\bar{X}_t + \sigma(\epsilon_t)^2 \quad (20)$$

The filtered state (*Sfilt*) at this stage is given by:

$$Sfilt_t = \bar{X}_t \quad (21)$$

The update of the measurement (\bar{Y}) that for us only includes σ_t here is called *Yobs_t* is the following:

$$\bar{Y}_t = \alpha + \lambda_0\bar{X}_t + \lambda_1\bar{X}_t^2 \quad (22)$$

and the model implied values of the measurement (*Ymod*) is:

$$Ymod_t = \bar{Y}_t \quad (23)$$

The prediction error is:

$$\nu_t = Yobs_t - \bar{Y}_t \quad (24)$$

Our Jacobian matrix has the following form:

$$C_t = \lambda_0 + 2\lambda_1\bar{X}_t \quad (25)$$

$$S_t = C_t^2 P \bar{X}_t + \sigma (\eta_t)^2 \quad (26)$$

The Kalman gain is given by:

$$K_t = \frac{P \bar{X}_t C_t}{S_t} \quad (27)$$

and the update of the state is:

$$\bar{X}_t = \bar{X}_t + K_t \nu_t \quad (28)$$

$$P \bar{X}_t = (1 - K_t C_t) P \bar{X}_t \quad (29)$$

To estimate model parameters, $\theta = [\phi; \alpha; \lambda_0; \lambda_1; \sigma(\epsilon); \sigma(\eta)]'$, we define the log-likelihood for each time t , assuming normally distributed observation errors, as:

$$l_t(\theta) = -\log(S_t) - \frac{\nu_t^2}{S_t}. \quad (30)$$

where ν_t and S_t are the prediction error of the measurement series and the covariance of the measurement series, respectively, obtained from the EKF. Model parameters are chosen to maximize the log-likelihood of the data on the time span:

$$\hat{\theta} \equiv \arg \max_{\Theta} \mathcal{L}(\theta, \{\sigma_t\}_{t=0}^{t=T}), \quad (31)$$

with

$$L(\theta, \{\text{Opt}_t\}_{t=0}^{t=T}) = \sum_{t=0}^T l_t(\theta), \quad (32)$$

where T denotes the number of time periods in the sample of estimation, that coincides with the bank survival period in our dataset.

6 Results

6.1 Who drives our three-dimensional risk appetite? A simulation exercise

In our definition of the objective function, two options play a role. When the bank does not have any consistent portfolio of growth opportunities, the DOC option is worthless so that the default put option is the only determinant of the objective function and of the risk-appetite. In this case, it is well known what is the impact of the optimizing variables and consequently what is the shape for risk appetite. But what happens when the bank has an embedded franchise value? In this case in a theoretical framework it is not clear which option has the main impact on the objective function, theoretically which is the main driver and also the optimization procedure is not trivial. We assess this issue for the banks in the sample having growth opportunities at stake clustered by leverage¹⁰. We cluster our sub-sample of banks with consistent franchise value into three categories: “overcapitalized” banks (cluster 21), with an actual average leverage¹¹ of 6.3299, “average capitalized” banks (cluster 22), with 12.9144 and “undercapitalized” banks (cluster 23) with 22.0996. Those figures are in line with standard literature, given our definition of leverage that is the ratio between assets’ market value and franchise value and equity market value.

We simulate the option prices, and consequently risk appetite value, building on win-sorized average data per cluster. We perform a sensitivity analysis with respect to the three optimization variables in order to understand the shape of the risk appetite. We assess the shape of risk appetite moving one variable, given the optimal quantities for the other two. We can see how the behaviour of risk appetite changes among different clusters when considering leverage and volatility. Interestingly, when looking at the policy rate, the objective function shape is similar in the three clusters. The patterns are entirely presented in the Appendix.

Leverage

The put option value is a decreasing function of leverage because when assets and franchise value increase the default put value decreases. The DOC option price is easy to see that is an increasing function of leverage. In the first cluster (21, i.e. “overcapitalized” banks), the risk appetite is determined first by the default put option, then by the DOC one. There is a difference in the leverage-driven risk appetite at a cluster level related to the positioning of the peak. The smaller values for leverage where we have the peak are in cluster 22.

¹⁰We present our clustering empirical analysis in Section 6.3.

¹¹Actual leverage is computed at market values.

Volatility

When considering volatility, the put options price is an increasing function, instead the DOC one is flat and relative high for smaller volatility values and decreasing afterwards. The risk appetite shape is determined for smaller values of sigma by the default put option and by the DOC one for larger values of our variable. As the leverage increases, going from cluster 21 to 23, the peak for the volatility-driven risk appetite is smaller, since the objective function peaks goes to the left-hand side.

Policy rate

When assessing the policy rate change impact on the two options prices we show that the default put one is a decreasing function, instead the DOC option is an increasing function. Risk appetite shape is a concave function in all the three cases, with minor differences. This is a good news for the monetary policy maker, because, once it is understood our risk appetite specification, the impact it has changing the rate has a clear direction. In the case of cluster 21, the two options have almost the same impact in determining objective function, the curve is almost symmetric. In the cluster 22 and 23, the main driver is the default put option since the risk appetite shape is skewed to the right.

6.2 Empirical results at aggregate level

Our empirical sample consists of 1436 listed US banks and the sample period is 1980-2014. Balance sheet items are taken from COMPUSTAT and considered on an annual basis. Market prices from the Center for Research in Security Prices (CRSP). Price data are taken on a monthly basis to accommodate the constant volatility hypothesis.

Summary statistics for both the input and the results at aggregate level are presented in the appendix. We perform our optimizations with several initial values in order to check we have results numerically stable. Furthermore, we calculated the confidence intervals. We show those results in the appendix as well. We find that the estimate of the risk-free rate is slightly higher with respect to the actual one on the whole time span. This result is even stronger looking at banks which can count on a portfolio of growth opportunities (see later, cluster analysis). The following figure shows the evolution of both the actual policy rate (on the right y axis) and the optimal one we estimated (on the left y axis).

Optimal risk-free rate vs actual risk-free rate evolution

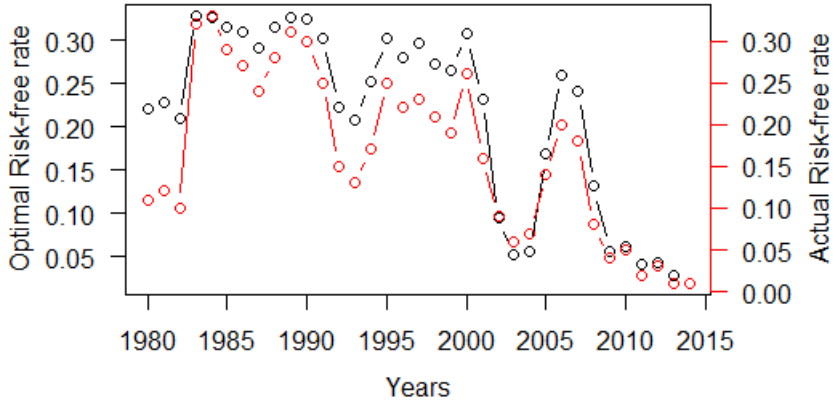


Figure 1: Optimal versus Actual policy rate evolution.

We show in the next figure that our average evaluation of the objective function aggregated on the whole sample per year and the optimal average quantities for the policy rate-, volatility- and leverage-driven risk appetite. Aggregated results loose a lot in terms of interpretability and meaning. In this aggregate dimension the objective function seems to follow the leverage and volatility patterns. On the policy rate side, the two move in opposite directions. This means that during periods of higher interest rates our objective function is relative low and this is the effect of the default put option over the DOC one. Considering periods of lower policy rates as signaling a crisis, we can see that our measure for risk appetite is driven relative higher (that is the case after 2010). In our model the manager chooses the optimal level of leverage, asset's volatility at the beginning of the period on the basis of past information so the present action has an impact on the following period. Our manager's policy considers the franchise value in its potential status at time t , but is backward looking, in the sense that builds on past information. The pattern is not straightforward to be interpreted, but in the time span considered, especially recently, during periods of lower interest rates optimal assets' volatility is relative higher because our manager has to shift the bank investments to riskier assets in order to perform earnings. Viceversa, during periods of relative higher interest rates we can see a flight to quality, because the bank investing in the risk-free asset is already achieving a satisfactory performance. This last consideration becomes clearer and more evident when considering clusters of banks with growth opportunities.

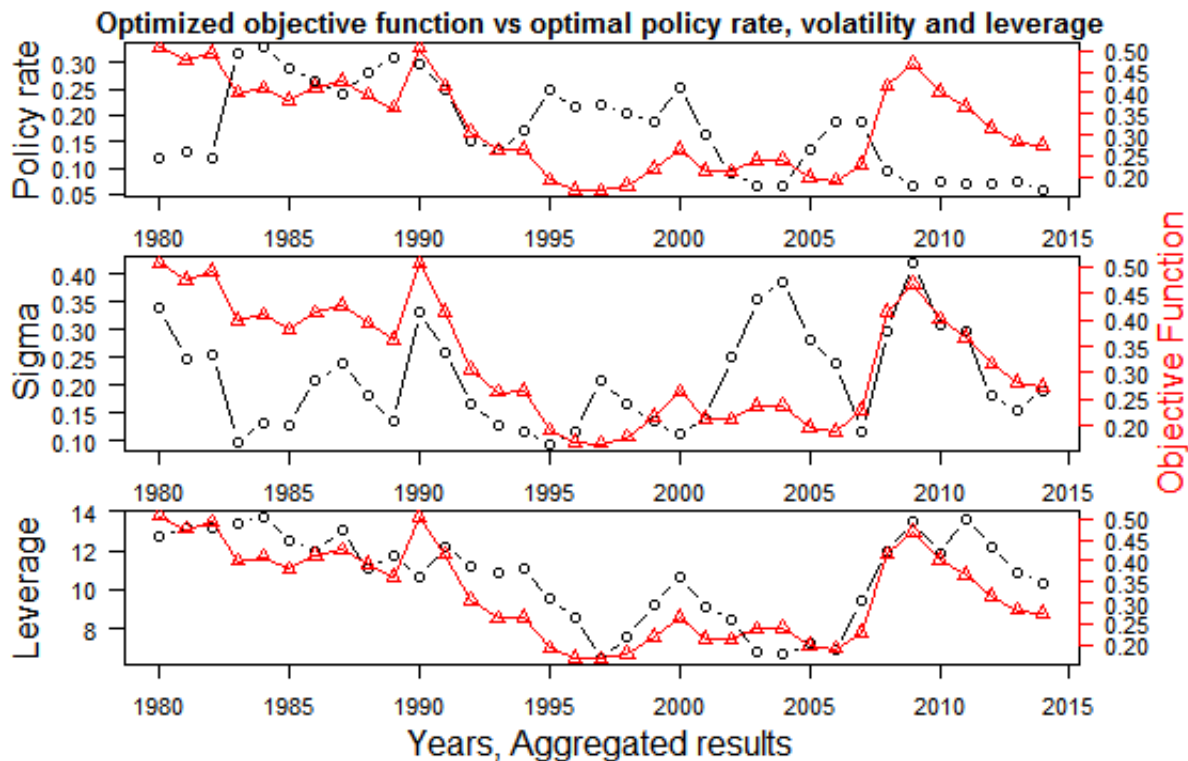


Figure 2: objective function versus optimal variables evolution.

6.3 Empirical results in a cluster analysis

Results differ a lot when considering our clustering analysis. We perform a two step-clustering, since we first distinguish between banks with franchise value and without, second we cluster the two subsets of banks with respect to the leverage. The sub-sample of banks without franchise value accounts for about the 17% of the whole sample. It is categorized as follows: (i) “over-capitalized” banks (cluster 11), with an actual average leverage of 5.0644 and the sub-sample is given by 377 banks, (ii) “average capitalized” banks (cluster 12) counting also 377 banks, with 11.4448 and (iii) “under-capitalized” banks (cluster 13) with 22.5164 where we can find 426 banks. We cluster our sub-sample of banks with consistent franchise value into the same three categories: we have (i) 4829 “under-capitalized” banks (cluster 21), with an actual average leverage of 6.3299; (ii) 3298 “average capitalized” banks (cluster 22), with 12.9144 and (iii) 1117 “over-capitalized” banks (cluster 23) with 22.0996¹². The

¹²The sum of the number of banks in each cluster exceed the total amount of banks, because some banks move across different clusters in the time span considered.

input variables for our optimization and the results are presented cluster by cluster in the appendix, here we present the main results and their implications. The population of banks is not uniformly distributed across the clusters, this has an impact on quality of estimates of the sub-sample of banks which do not have growth opportunities and the sub-sample with franchise value present more similar results with respect to the aggregated analysis we discussed in the previous section. Instead, the clustering analysis performed in a standard framework provides similar average level of leverage among the clusters.

In the following set of pictures, we can see that our estimates for the policy rate is always higher than the actual one. The greater spread across the two are present in the sub-sample of banks which have a portfolio of growth opportunities, because risk appetite in this context is driven upwards by the franchise value, since a portfolio of growth opportunities is always risky. The sub-sample of banks without franchise value tracks more the actual risk-free rate and those more pronounced swings are given by the driving effect of the DOC option in the clusters counting on franchise value. Banks in cluster 23, with the greatest level of leverage, ask for a remarkably higher optimal policy rate especially in the last five years where the actual risk-free rate set by the regulator was at its minima levels.

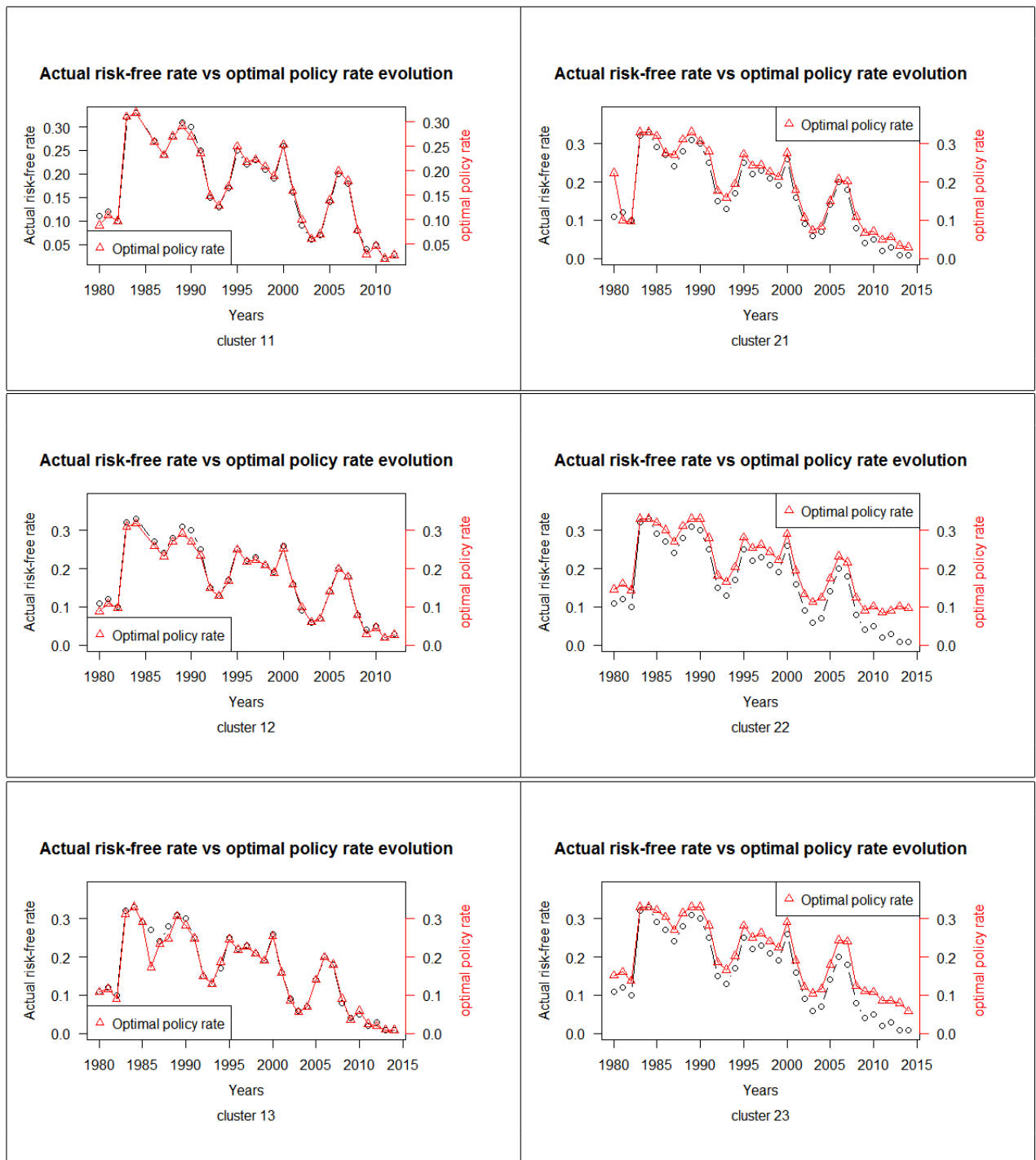


Table 2: Actual risk-free rate vs Optimal policy rate, cluster by cluster.

Anticipating what we show in the following set of figures, this together with a relative low volatility lead to a very low objective function, since the bank manager optimizes his

strategy investing in the risk-free asset.

In the next set of pictures we present the resulting optimal estimates for the optimized objective function and its corresponding variables-driven risk appetite. Clusters 11 and 12 show very similar patterns for the evolution of objective function and the three optimizing variables. The estimate appears to be slightly higher and stable because only the put option plays a role in determining the shape of the objective function. Only cluster 13 differentiates from the other two with respect to the optimized objective function. With greater levels of leverage, we can see that the differences among clusters diminishes: clusters 13 and 23 path is more similar. In clusters without the franchise value volatility-driven risk appetite is relative much higher with respect to the optimal average one present in cluster with franchise value. This result is not easily interpretable, since the franchise value volatility accrues to the assets one. Apparently, there is a diversification effect that impacts positively on the total volatility-driven risk appetite. This is confirmed by the data, since it becomes even more evident when leverage, and consequently the franchise value, increases. As we stated above, clusters 21, 22 and 23 point out results much more in line with the aggregated ones, being the greater sub-sample in terms of number of banks involved. In those clusters objective function moves a lot and each cluster present its pattern. The objective function optimal average values increases with the clusters, since we find that in cluster 21 the maximum level of objective function is about 20% instead in cluster 23 it approaches 80%, even if in cluster 22 the objective function decreases along the time span at least till 1997.

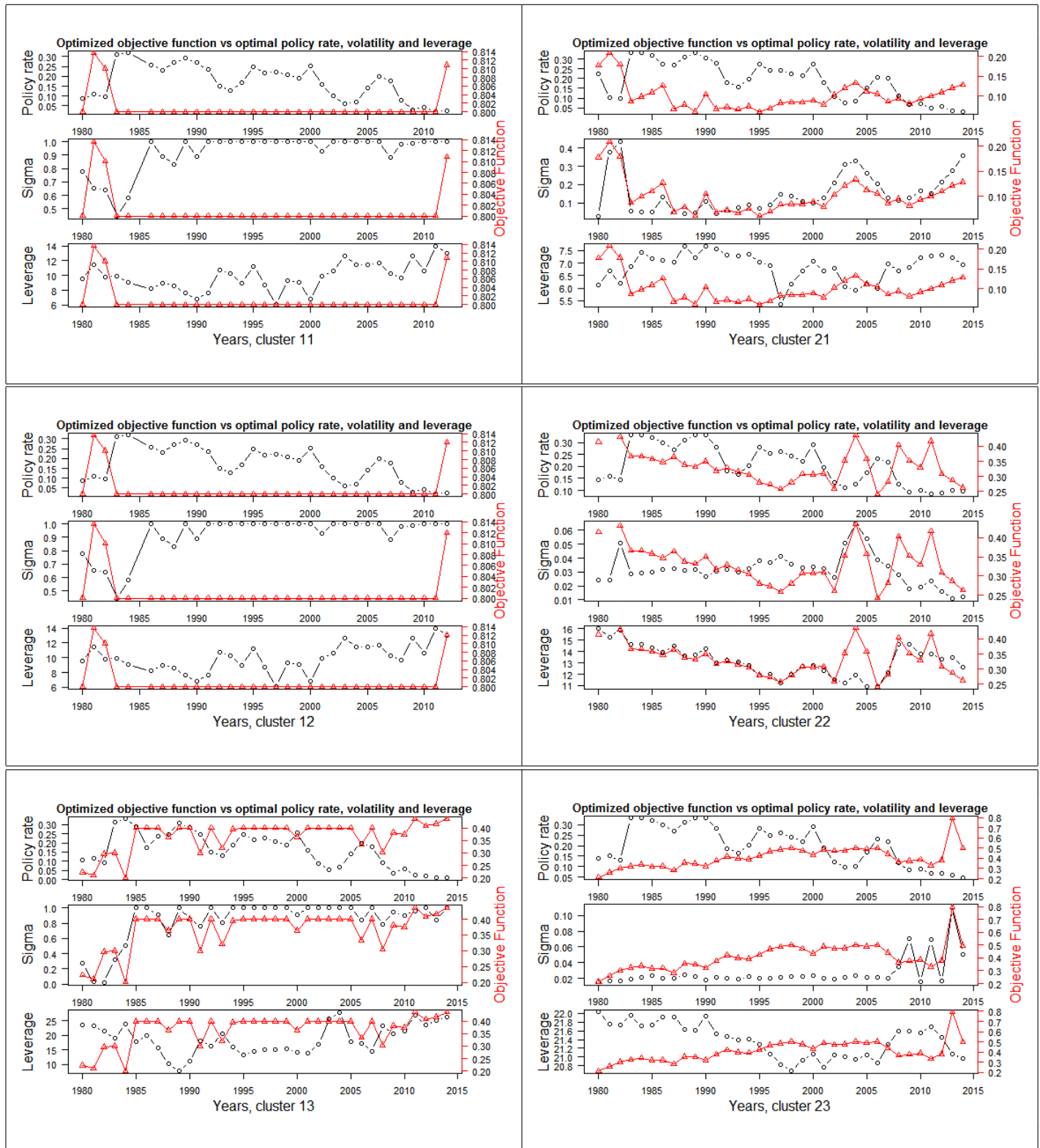


Table 3: optimized objective function evolution against its optimal determinants , cluster by cluster.

In this empirical analysis, we show that undercapitalization not always harms profitable growth opportunities, it does in cluster 21 and 23, where the optimized objective function moves in an opposite direction with respect to leverage, instead in cluster 22 they move together at least before 2000.

6.4 Results from the Extended Kalman Filter

In this section we present the results of the filtering estimates at an aggregate level and for cluster 21, 22 and 23. We do not consider the sub-sample of banks without growth opportunities because their volatility is not moving enough to allow significant estimates, anyway we do not lose much information because the greater majority of banks is assessed. We plot the evolution of our estimates for the estimated volatility-driven risk appetite, from the optimization problem, and the estimated coming from the filtering procedure in the following figure. We call the estimation of the state variable, obtained through the PML, RAPML, and the estimation of the volatility-driven risk appetite, resulting from the optimization problem, RAM. The estimations produce coherent results after 2000, instead before, at an aggregate level, they move in opposite directions. Further work will consider hypothesis testing, in order to understand whether the two estimations produce really similar results, against the hypothesis they are different.

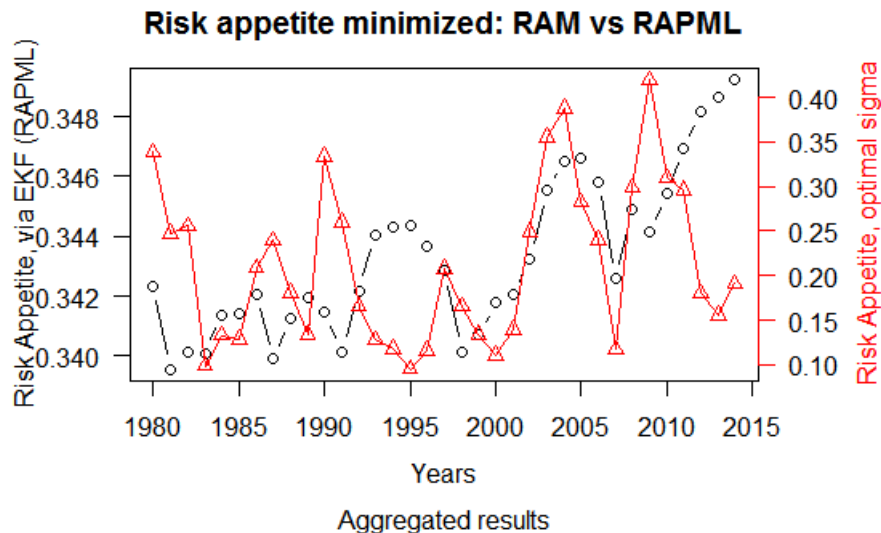


Figure 3: RA PML estimate vs objective function evolution

The summary statistics, also in this case, are presented in the appendix. As expected,

objective function appears to be a persistent variable. The first quantile for the AR(1) coefficient is negative, meaning that for those banks objective function moves in opposite directions one period after another. The coefficient α for the measurement present both positive and negative results showing that we have both convexity and concavity in our point estimate of RAPML. The other parameters (λ_0 and λ_1) are also both positive and negative and interestingly the shape is driven by the quadratic term since the median of λ_0 is equal to zero.

In the majority of crisis years, our risk appetite estimate peaks. This can be explained partly with the relative higher leverage adopted during those periods. We notice that after a bust in the economic cycle and a peak in our estimate of risk appetite, our variable decreases sharply with almost the same intensity with which it increased in the previous period. This issue becomes clearer with the PML estimation results presented in the following section. At aggregate level, we find the intensity parameter ϕ of the AR(1) process to be quite large in absolute values and with negative sign. Future work should consider to understand whether a negative sign in this parameter is an indicator of a peak or a bust in the economic cycle.

At a cluster level, we show in the following table interesting dynamics. The range of the objective function estimate through the PML in the filtering procedure is almost the same for the three clusters and is in line with the one estimated at aggregate level. However, the pattern changes significantly among the three clusters. In the first cluster we can see that the outline of the two estimates move together a part from the most recent years where they go in opposite directions. Cluster 22 shows the most relevant differences among the two estimates. Even if the range where the estimate through PML moves is quite tiny, the pattern is increasing, instead the one for the objective function estimate obtained through the optimization is decreasing. In cluster 21 the dynamics of the two estimates is similar starting from 2000. In cluster 23, the two estimates move in opposite directions, this is maybe due to the fact that the RAPML do not embody the optimal quantities of leverage- and policy rate-driven risk appetite. However, once again, this cluster analysis adds precious information with respect to the one proposed at aggregate level. The non-linearity issue treated with a quadratic perspective in the filtering procedure at aggregate levels works well in mimicking the impact of the two options on objective function, but at cluster level it is not always the case.

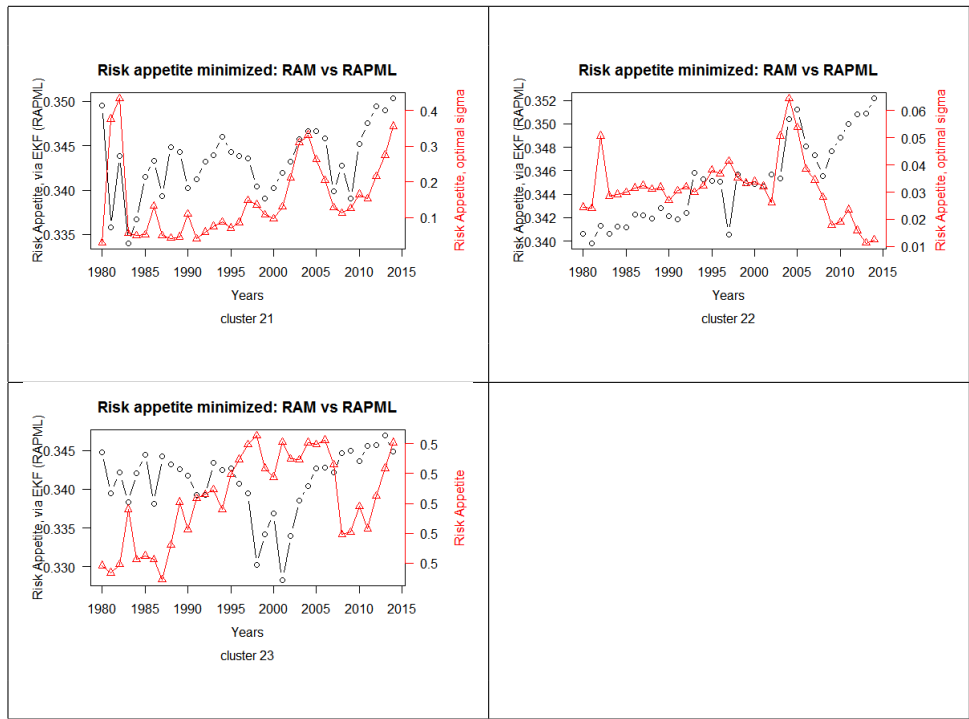


Table 4: Actual risk-free rate vs Optimal policy rate, cluster by cluster.

7 Conclusions

In this paper, we investigate the shape of the risk appetite of our bank and the role played by the monetary policy in framing it. Bank objective function and its risk appetite are determined by the interplay of the default option and the down-and-out call (DOC) option, pricing the franchise value, i.e. the net present value of non-observable bank's growth opportunities. We define the objective function as the ratio between the sum of the two options' prices and the market value of the tangible assets. Our major contribution consists in assessing risk appetite in three dimensions, allowing also the monetary policy to play a role on risk appetite and to work jointly with the bank manager in the optimization of the objective function.

First, we estimate the franchise value, and we discriminate banks with and without franchise value. Second, at the beginning of each period, we optimize the objective function adjusting simultaneously the level of leverage, volatility and the policy rate. In order to set the optimal values, we have to consider both the bank manager decisions and the monetary policy. The monetary policy maker sets rho equal to zero considering also the manager risk management policy (policy rate-driven risk appetite). The bank manager sets the vega equal to zero (volatility driven-risk appetite) simultaneously with the derivative with respect to the leverage (leverage-driven risk appetite), considering also the monetary policy maker strategy. Those three optimizations are conditional to the other two optimal quantities.

The risk appetite, volatility driven one, is not directly observable in the market, thus we want to understand the goodness of our estimation. We propose a state space model, where the state variable is the risk appetite of the bank and we model it as an $AR(1)$ with a coefficient ϕ that we find it to be large in absolute value, indicating a persistency in the latent variable. The measurement proposed is a non linear function, where the observable part is given by the implied assets' volatility. We use the extended Kalman filter, given that our specification for the measurement is quadratic, and we estimate the model parameters with pseudo-maximum likelihood (PML). At an aggregate level, we show the major driver in the measurement equation is given by the quadratic term and the underlying dynamic of the latent state moves in line with the previous estimation of objective function. Its dynamic moves in line with the previous estimation and is persistent. Furthermore, the non-linear shape of the measurement is mainly driven by the quadratic term and works well in mimicking the impact of the two options on the risk appetite at an aggregate level, but not so well in the cluster analysis. This is due to the lack of information that we incorporate in the filtering procedure.

We test our optimizations on a sample of 1436 banks, listed in the US, over 1980-2014.

We find that the optimal risk-free rate is higher with respect to the existing one in the whole sample period. A clustering analysis is necessary in order to understand the shape of risk appetite, which is its underlying main driver and what is the impact of changes in the optimizing variables. We show that the impact of the single variable on risk appetite is not always the same among the clusters, this is a result of both structural differences among the clusters and the joint impact of the other variables that are simultaneously optimized. Empirically, we always find that the estimated policy rate is higher relative to the actual one. The objective function is magnified for higher values of leverage, which is straightforward given our specifications. We find different patterns among the clusters and this imposes a cluster analysis in order to understand risk appetite behaviour. The monetary policy maker has to cooperate with the bank manager in order to align their policy and to allow for an effective risk appetite optimization. We show that regulators should tune their recommendations depending on the targeted cluster, since the driver of risk appetite alternates between the two options depending on the cluster and on the underlying variable considered, given the other two. Our three dimensional risk appetite specification could be an effective instrument for the regulator because it comprehends the three most important dimensions for shaping risk appetite in presence of franchise value. It is determined by the joint optimization, thus we need to condition on two optimal quantities in order to optimize with respect to the third one.

Furthermore, introducing the franchise value in the specification of risk appetite, we propose an incentive for the manager to adopt a policy long-term oriented. The choice of the proper incentives in order to boost this long-term oriented perspective is left to future work.

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Appendix

A: Who drives the risk appetite? A simulation exercise

In the following table we perform a sensitivity analysis to change in the three optimization variables. We comment in Section 6 how the shape of risk appetite differs among the clusters.

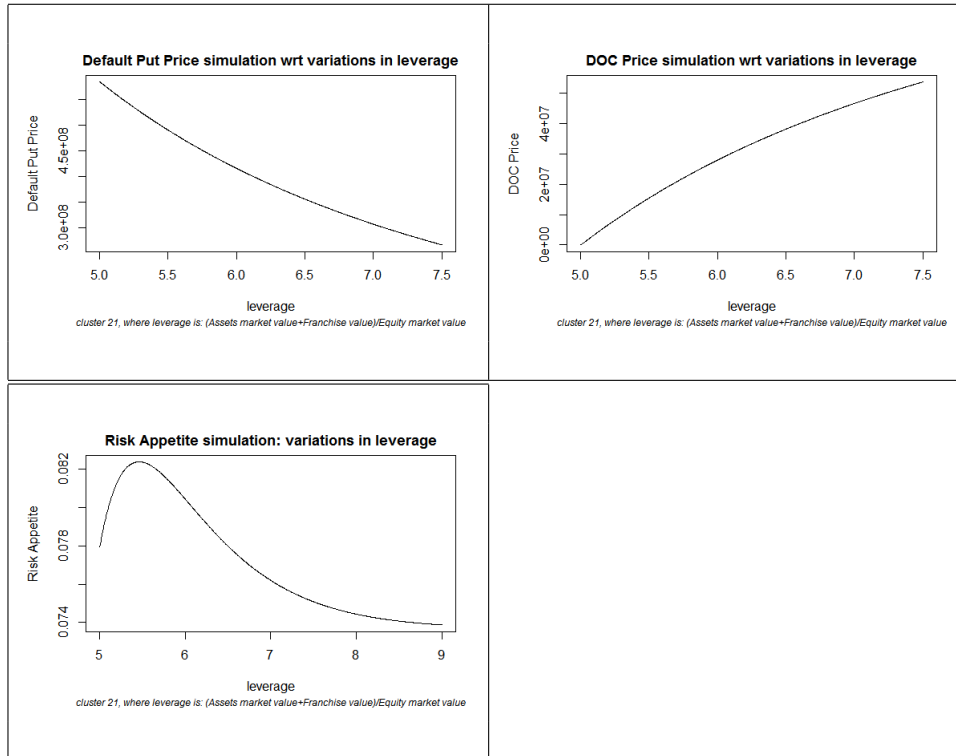


Table 5: Cluster 21: Sensitivity to change in leverage.

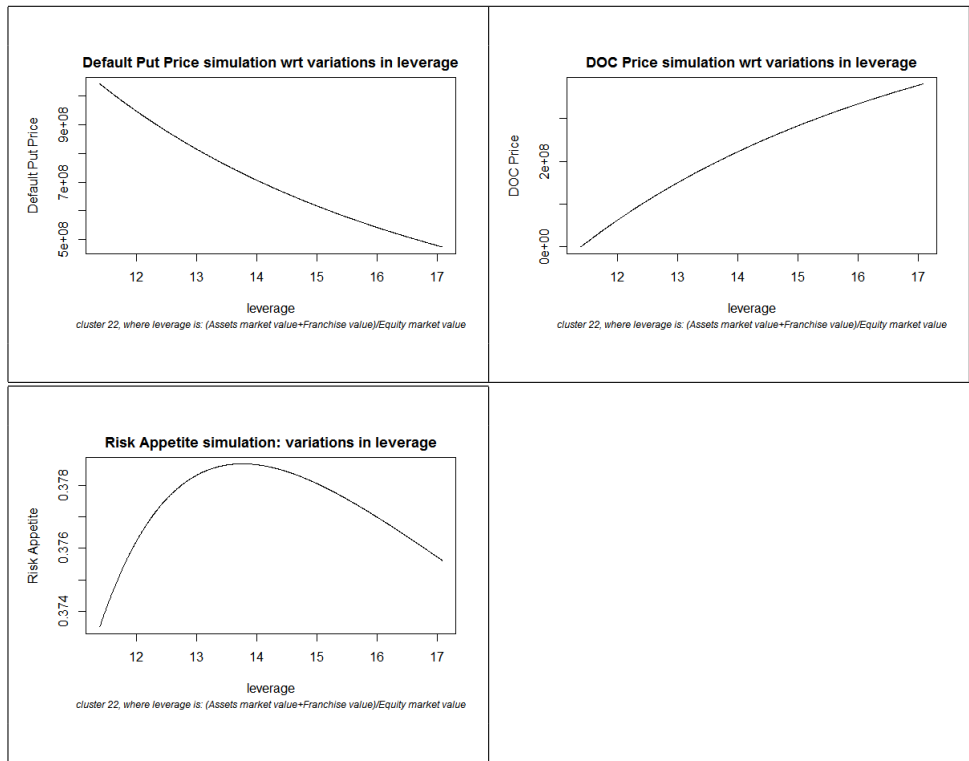


Table 6: Cluster 22: Sensitivity to change in leverage.

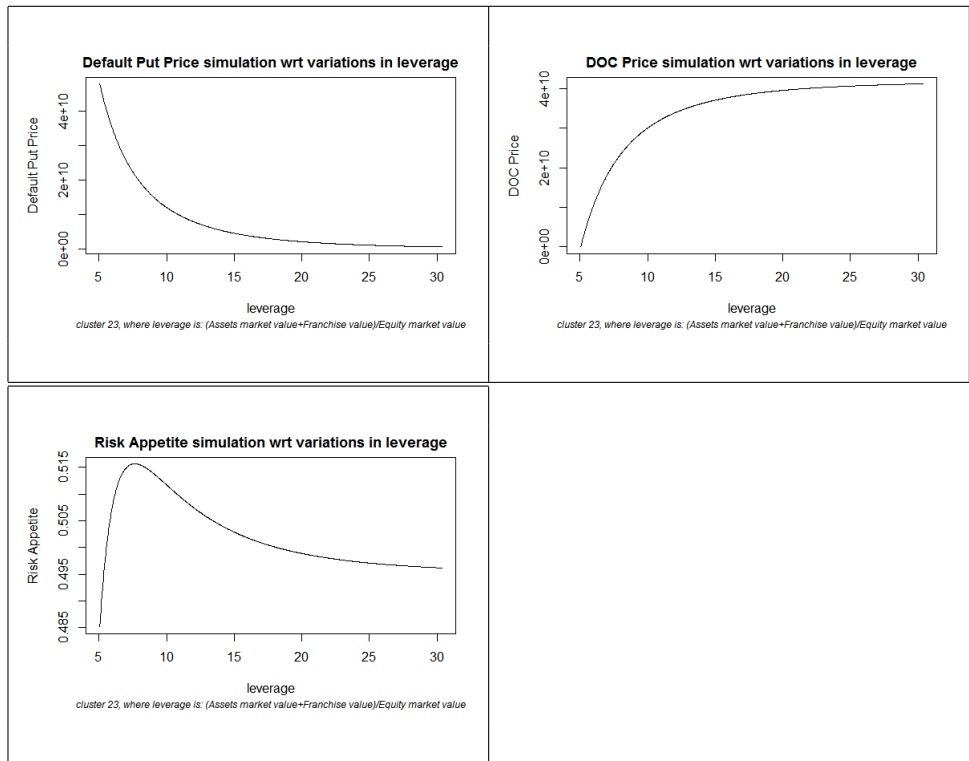


Table 7: Cluster 23: Sensitivity to change in leverage.

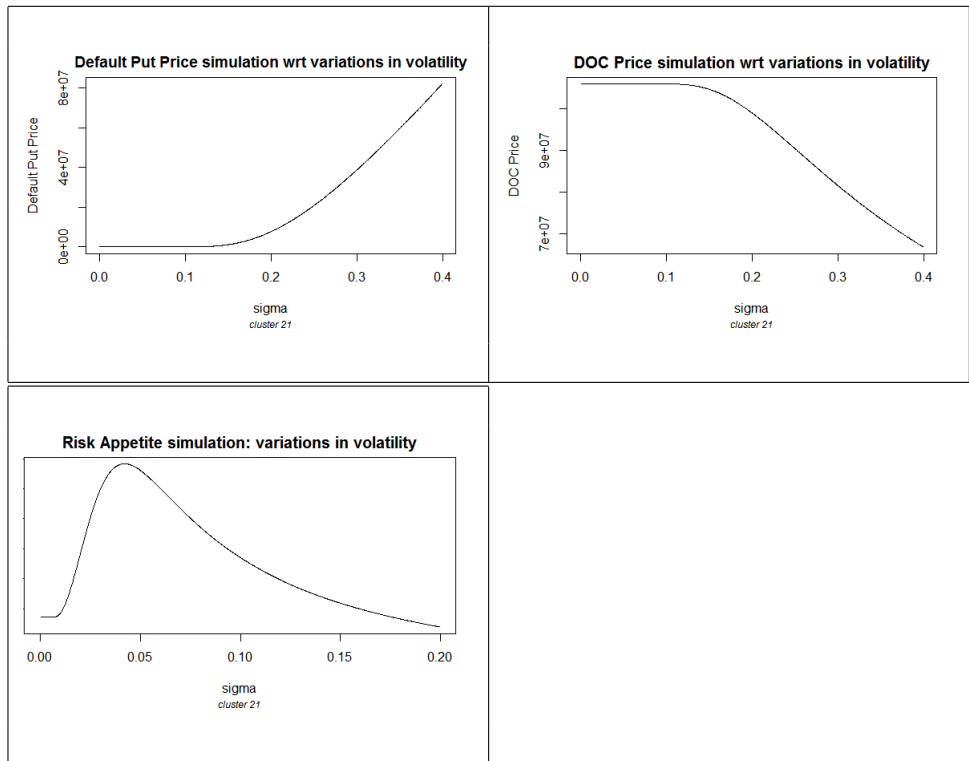


Table 8: Cluster 21: Sensitivity to change in volatility.

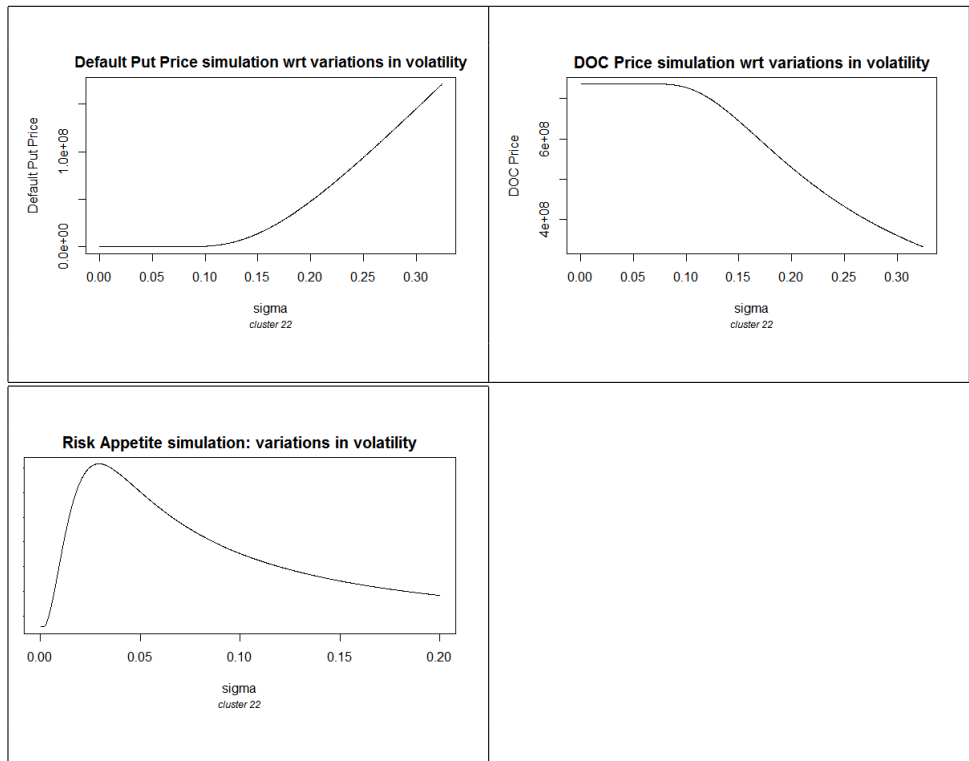


Table 9: Cluster 22: Sensitivity to change in volatility.

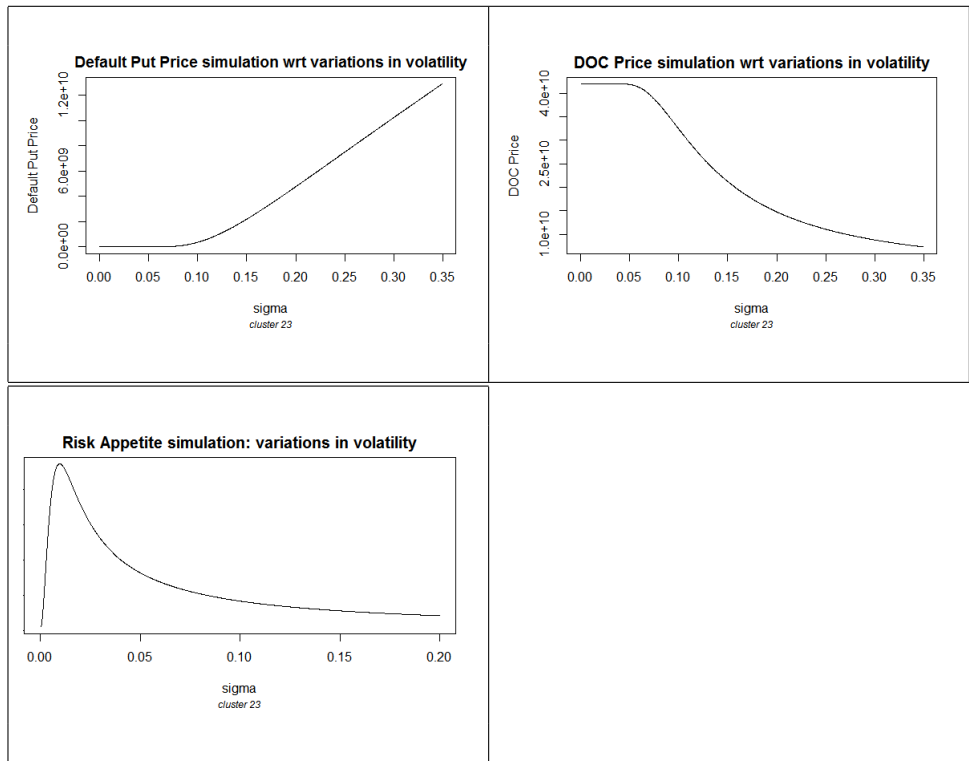


Table 10: Cluster 23: Sensitivity to change in volatility.

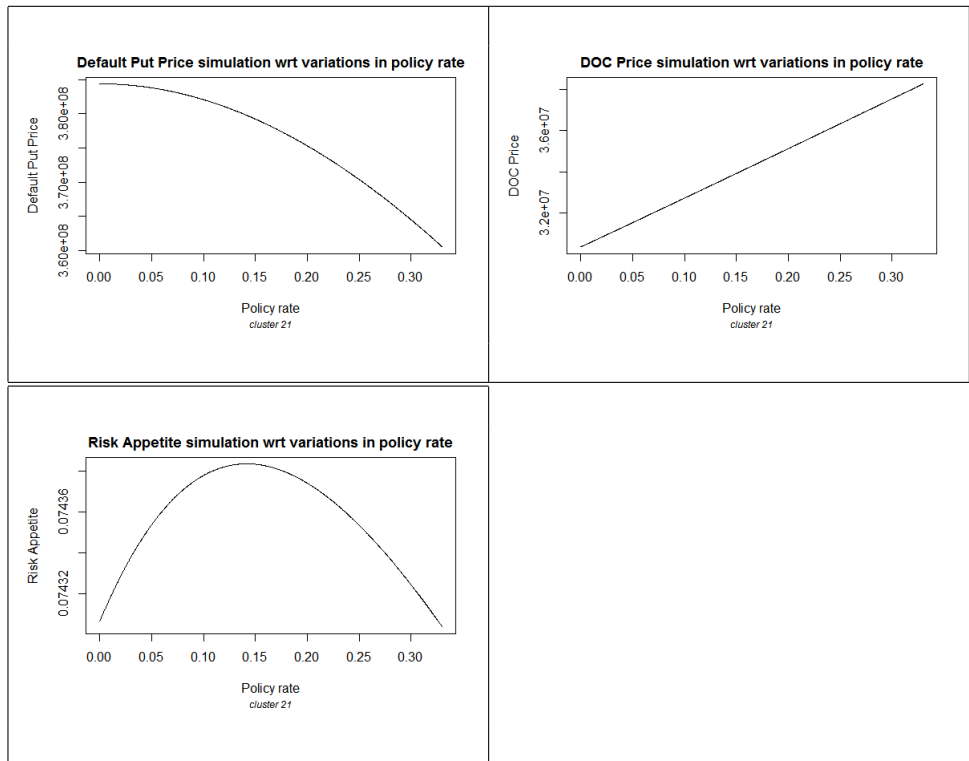


Table 11: Cluster 21: Sensitivity to change in policy rate.

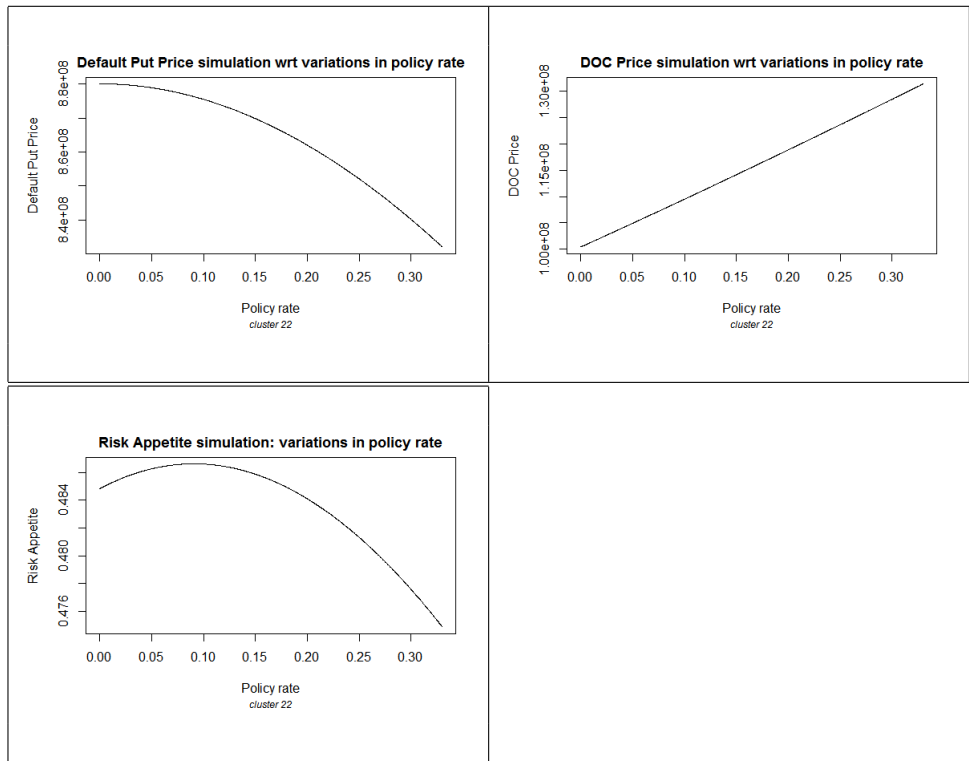


Table 12: Cluster 22: Sensitivity to change in policy rate.

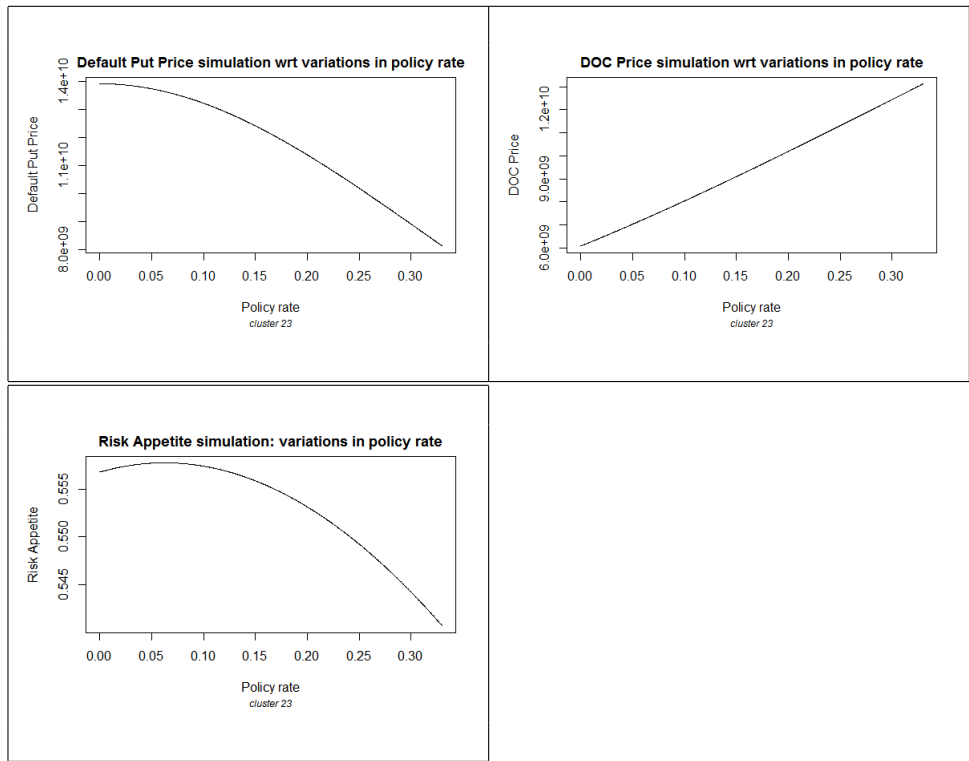


Table 13: Cluster 23: Sensitivity to change in policy rate.

B: Optimization with different initializations and Confidence intervals

We perform our optimizations with several initial values¹³ and we propose a sensitivity analysis that shows a persistency of our results.

¹³Those initial values are generated randomly respectively from the most likely candidate as originating distributions.

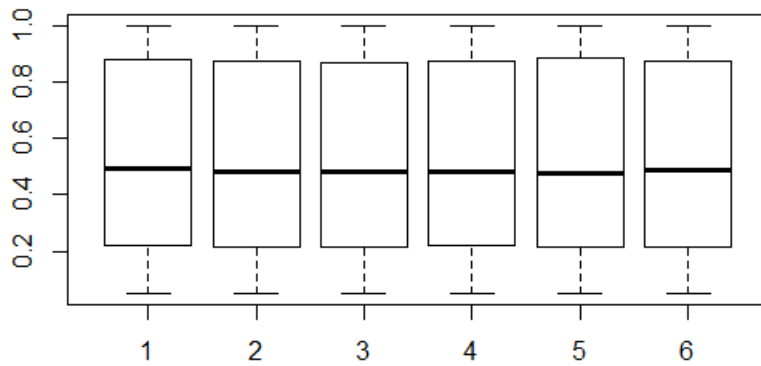


Figure 4: Boxplot of the optimal leverage that optimizes objective function with six different initializations.

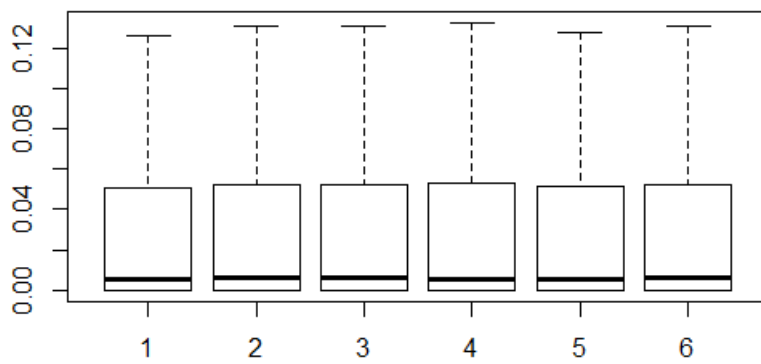


Figure 5: Boxplot of the optimal assets' vol that optimizes objective function with six different initializations

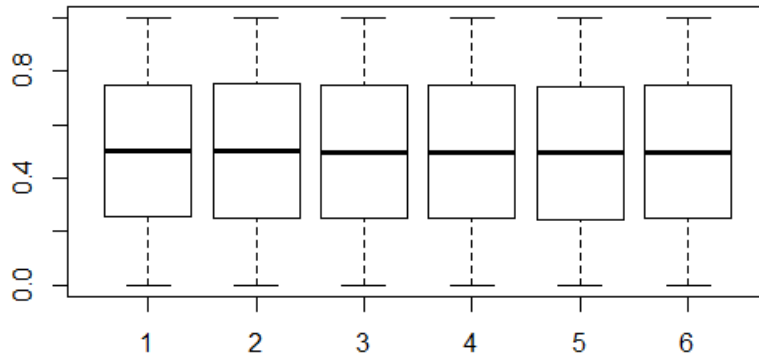


Figure 6: Boxplot of the minimum objective function with six different initializations.

We report in the following table the confidence intervals for the estimated optimal variables.

	cluster	Lower bounds	Upper bounds	number of observations
Average (Franchise Value (NPV)) (in US\$)	11	0	0	377
	12	0	0	377
	13	0	0	426
	21	84993572	127005948	4829
	22	445654089	1026307845	3298
	23	30534603975	53488697248	1117
Average (Assets (MV)) (in US\$)	11	2361739205	3622469328	377
	12	2361739205	3622469328	377
	13	2388433535	3601983217	426
	21	1264784888	1614005232	4829
	22	1356303083	2723281243	3298
	23	6.1935e+10	107789105918	1117
Average (Optimal leverage)	11	10.11474	10.64324	377
	12	10.11474	10.64324	377
	13	19.52399	20.94874	426
	21	6.393547	6.733862	4829
	22	12.82122	13.52006	3298
	23	21.30842	21.46068	1117
Average (Optimal volatility)	11	0.883556	1	377
	12	0.883556	1	377
	13	0.8408021	0.9701865	426
	21	0.1646186	0.1899505	4829
	22	0.02845392	0.03283247	3298
	23	0.0279483	0.03224904	1117
Average (Optimal policy rate)	11	0.1004694	0.1170219	377
	12	0.1004694	0.1170219	377
	13	0.09923235	0.1198637	426
	21	0.1573507	0.1763942	4829
	22	0.1946735	0.2118842	3298
	23	0.1708415	0.1899149	1117

Table 14: Confidence intervals for average optimal estimates.

C: Summary statistics of the optimization figures at aggregate level and cluster by cluster

First of all we present summary statistics of our input variables: end-of-year equity market value, its monthly volatility adjusted on an annual basis, the risk-free rate, existent in the market in the time span considered, the market value of debt, calculated according to KMV model in order to account for the value that triggers the franchise value of the bank and the leverage defined as the ratio between the sum of the market value of the assets (MVA) and the franchise value (Fr) and the equity market value ¹⁴(MVE).

Variable\Summary statistics	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Equity market value (in US\$)	1.922e+05	4.324e+07	1.012e+08	1.050e+09	2.784e+08	2.339e+11	7522590845
Equity volatility (annualized)	0.0610	0.2553	0.3073	0.3484	0.3991	0.4234	0.1474
Risk-free rate	0.0100	0.0700	0.1600	0.1521	0.2200	0.3300	0.0863
Debt market value (in US\$)	3.140e+05	3.971e+08	8.661e+08	1.812e+10	2.263e+09	2.782e+12	124251669021
Leverage ($MVA + Fr$) / MVE	0.07	6.07	8.73	10.79	14.26	42.59	6.17

Table 15: Model inputs summary statistics.

Those are the inputs used to estimate the franchise value first and consequently to proceed in our objective function maximization. We provide in the following table, the summary statistics for the results of our two-steps optimization at aggregate level: the net present value (NPV) of the franchise value, the market value (MV) of the assets, the parameters optimizing pointwise the objective function (leverage, assets' volatility and the optimal risk-free rate).

¹⁴Leverage data are in line with findings in Kalemli-Ozcan et al. (2012).

Variable\Summary statistics	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Franchise Value (NPV) (in US\$)	0.000e+00	2.651e+06	6.906e+07	6.155e+09	2.447e+08	1.638e+12	52728416739
Assets (MV) (in US\$)	0.000e+00	3.524e+08	7.620e+08	1.340e+10	1.879e+09	3.275e+12	105383415766
Optimal leverage	0.000237	5.884000	8.579000	9.942000	13.890000	18.410000	5.0767
Optimal volatility	0.00010	0.02489	0.03709	0.21280	0.05910	1.000	0.3756529
Optimal risk-free rate	0.00010	0.07808	0.17000	0.15830	0.23000	0.33000	0.0951796

Table 16: Results summary statistics.

In the table below we present the input variables for our optimization and the results are presented cluster by cluster.

Summary statistics/ Variable	cluster	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Equity market value (in US\$)	11	6.314e+05	1.478e+07	3.473e+07	2.318e+08	1.262e+08	2.063e+09	456639609
	12	6.314e+05	1.478e+07	3.473e+07	2.318e+08	1.262e+08	2.063e+09	456639609
	13	1.922e+05	1.123e+07	3.551e+07	1.441e+08	1.315e+08	2.000e+09	289979782
	21	4.015e+06	6.398e+07	1.380e+08	2.697e+08	3.263e+08	2.027e+09	328450812
	22	2.409e+06	3.063e+07	6.328e+07	1.970e+08	1.277e+08	1.705e+10	612017210
	23	1.420e+06	6.938e+07	2.000e+09	6.169e+09	5.562e+09	1.881e+11	16087635115
Equity volatility (annualized)	11	0.2234	0.3299	0.4292	0.4591	0.5486	0.7000	0.1715
	12	0.2234	0.3299	0.4292	0.4591	0.5486	0.7000	0.1715
	13	0.2326	0.3153	0.4179	0.4868	0.5783	0.7125	0.2247
	21	0.0610	0.2421	0.2909	0.3247	0.3656	0.5214	0.1346
	22	0.1104	0.2612	0.3119	0.3519	0.4106	0.6341	0.1406
	23	0.06766	0.26820	0.31300	0.34130	0.36800	0.4582	0.1236
Risk-free rate	11	0.0200	0.0500	0.0800	0.1151	0.1600	0.3300	0.0831
	12	0.0200	0.0500	0.0800	0.1151	0.1600	0.3300	0.0831
	13	0.0100	0.0300	0.0500	0.1128	0.2200	0.3300	0.1019
	21	0.0100	0.0700	0.1600	0.1488	0.2100	0.3300	0.0768
	22	0.0100	0.0900	0.1700	0.1649	0.2500	0.3300	0.0907
	23	0.0100	0.0600	0.1400	0.1473	0.2400	0.3300	0.0967
Debt market value (in US\$)	11	6.368e+07	8.437e+08	2.600e+09	1.123e+11	3.497e+10	2.353e+12	309739663195
	12	6.368e+07	8.437e+08	2.600e+09	1.123e+11	3.497e+10	2.353e+12	309739663195
	13	6.294e+07	9.663e+08	3.519e+09	7.738e+10	6.077e+10	2.782e+12	230386133312
	21	3.140e+05	3.528e+08	7.269e+08	1.298e+09	1.562e+09	6.722e+10	1983973062
	22	3.404e+07	3.432e+08	7.009e+08	2.767e+09	1.520e+09	2.281e+11	10495203151
	23	4.025e+07	1.563e+09	1.026e+10	7.460e+10	3.651e+10	2.046e+12	258150800966
Leverage ¹⁵ $(MVA + Fr) / MVE$	11	7.560	9.366	11.490	5.0644	11.5203	15.570	2.378418
	12	7.560	9.366	11.490	11.440	13.530	15.570	2.378418
	13	15.66	17.39	20.70	22.52	28.74	30.00	5.440212
	21	1.004	5.163	6.394	6.330	7.571	9.019	1.540092
	22	9.023	10.320	12.090	12.910	15.140	20.110	3.03266
	23	20.15	21.00	21.00	22.10	22.71	30.11	1.953871

Table 17: Model inputs summary statistics, cluster by cluster.

Summary statistics/ Variable	cluster	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
Franchise Value (NPV) (in US\$)	11	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0
	21	1.000e+00	4.159e+06	3.963e+07	1.060e+08	1.117e+08	2.708e+09	207429079
	22	3.000e+00	8.162e+07	1.686e+08	7.360e+08	3.878e+08	7.116e+10	2866880794
	23	3.000e+00	2.401e+08	1.380e+10	4.299e+10	3.893e+10	1.317e+12	112679424840
Assets (MV) (in US\$)	11	7.617e+06	1.725e+08	3.797e+08	2.992e+09	1.351e+09	2.979e+10	6224644098
	12	7.617e+06	1.725e+08	3.797e+08	2.992e+09	1.351e+09	2.979e+10	6224644098
	13	5.573e+06	2.526e+08	7.431e+08	2.995e+09	2.545e+09	4.312e+10	5991698567
	21	0.000e+00	4.004e+08	8.174e+08	1.439e+09	1.754e+09	1.457e+10	1724217036
	22	2.648e+07	2.851e+08	5.888e+08	2.040e+09	1.281e+09	1.740e+11	6749226012
	23	3.619e+07	1.304e+09	2.926e+10	8.676e+10	7.786e+10	2.634e+12	225098742682
Optimal leverage	11	0.006045	8.404000	10.400000	10.380000	12.440000	15.450000	2.609345
	12	0.006045	8.404000	10.400000	10.380000	12.440000	15.450000	2.609345
	13	0.01365	14.92000	19.47000	20.24000	27.07000	30.00000	7.034474
	21	0.000378	5.306000	6.605000	6.564000	7.925000	11.570000	1.680249
	22	0.00285	10.40000	12.24000	13.17000	15.44000	37.23000	3.4504
	23	0.0022	21.0000	21.0000	20.7000	22.9600	45.1500	5.919417
Optimal volatility	11	0.1062	1.0000	1.0000	0.9466	1.0000	1.0000	0.203542
	12	0.1062	1.0000	1.0000	0.9466	1.0000	1.0000	0.203542
	13	0.008906	1.000000	1.000000	0.900800	1.000000	1.000000	0.2891098
	21	0.00010	0.03823	0.04861	0.08640	0.10749	1.00000	0.3244621
	22	0.00010	0.02282	0.02890	0.03049	0.03609	1.00000	0.04637664
	23	0.00010	0.01559	0.01901	0.02833	0.02290	1.00000	0.09868465
Optimal risk-free rate	11	0.002889	0.043790	0.078350	0.108700	0.157700	0.318600	0.08172531
	12	0.002889	0.043790	0.078350	0.108700	0.157700	0.318600	0.08172531
	13	0.00010	0.02289	0.05228	0.10950	0.20980	0.33000	0.1018636
	21	0.0001	0.0900	0.1900	0.1669	0.2400	0.3300	0.0940241
	22	0.0400	0.1392	0.2009	0.2033	0.2800	0.3300	0.08497489
	23	0.0001	0.1000	0.1900	0.1888	0.2800	0.3300	0.09778638

Table 18: Model results summary statistics, cluster by cluster.

D: Extended Kalman Filter estimates

The table below shows the summary statistics for the model parameters at aggregate level in the above table and for cluster 21, 22 and 23 in the table below.

Variable\Summary statistics	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	sd
ϕ	-0.9990	-0.5048	0.1750	0.0582	0.5880	0.9990	0.6720
$\sigma(\epsilon)$	0.001	0.001	0.0340	0.0531	0.0790	0.2000	0.0605
α	-0.1270	0.0060	0.0130	0.0162	0.0220	0.2120	0.0191
λ_0	-0.9990	-0.0985	0	0.0062	0.1520	0.9990	0.3686
λ_1	-0.9990	-0.2415	0.9000	0.4113	0.9990	0.9990	0.7889
$\sigma(\eta)$	0.001	0.001	0.001	0.0047	0.0070	0.1020	0.0074

Table 19: Results summary statistics at aggregate level.

Variable\Summary statistics	cluster	Mean
ϕ	21	-0.076
	22	0.5
	23	0.5
$\sigma(\epsilon)$	21	0.38
	22	0.5
	23	0.5
α	21	0.5
	22	-0.3
	23	0.17
λ_0	21	0.0010
	22	0.5
	23	0.69
λ_1	21	-0.4470
	22	0.9
	23	0.5
$\sigma(\eta)$	21	0.5
	22	0.9
	23	0.5

Table 20: Results summary statistics for cluster 21, 22 and 23.

E: Rho, vega, derivative with respect to leverage and the joint derivatives

diff(Of(lev,MVA,sig_A_Fr,T,Fr,rf),lev);

$$\begin{aligned}
 & (MVA + Fr) \left(\frac{\operatorname{erf} \left(\frac{0.5 \operatorname{sig_A_Fr}^2 T + rf + \log(\operatorname{lev})}{\sqrt{2} \operatorname{sig_A_Fr} \sqrt{T}} \right)}{2} + \frac{1}{2} \right) \\
 & + \frac{(MVA + Fr) e^{-\frac{(0.5 \operatorname{sig_A_Fr}^2 T + rf + \log(\operatorname{lev}))^2}{2 \operatorname{sig_A_Fr}^2 T}}}{\sqrt{2} \sqrt{\pi} \operatorname{sig_A_Fr} \sqrt{T}} \\
 & - \frac{(MVA + Fr) e^{-\frac{(-0.5 \operatorname{sig_A_Fr}^2 T - rf + \log(\operatorname{lev}))^2}{2 \operatorname{sig_A_Fr}^2 T}}}{\sqrt{2} \sqrt{\pi} \operatorname{lev} \operatorname{sig_A_Fr} \sqrt{T}} \\
 & + Fr \left(\frac{2 \operatorname{lev}^{\frac{2(\operatorname{sig_A_Fr}^2 + rf)}{\operatorname{sig_A_Fr}^2} - 1} (\operatorname{sig_A_Fr}^2 + rf) \left(\frac{\operatorname{erf} \left(\frac{\operatorname{sig_A_Fr}^2 + rf + \frac{\log(\operatorname{lev})}{\operatorname{sig_A_Fr}}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right)}{\operatorname{sig_A_Fr}^2} \right. \\
 & \left. - \frac{\operatorname{lev}^{\frac{2(\operatorname{sig_A_Fr}^2 + rf)}{\operatorname{sig_A_Fr}^2} - 1} e^{-\frac{\left(\frac{\operatorname{sig_A_Fr}^2 + rf + \frac{\log(\operatorname{lev})}{\operatorname{sig_A_Fr}}}{2} \right)^2}}{\sqrt{2} \sqrt{\pi} \operatorname{sig_A_Fr}} - \frac{e^{-\frac{\left(\frac{\operatorname{sig_A_Fr}^2 + rf - \frac{\log(\operatorname{lev})}{\operatorname{sig_A_Fr}}}{2} \right)^2}}{\sqrt{2} \sqrt{\pi} \operatorname{lev} \operatorname{sig_A_Fr}}}{\sqrt{2} \sqrt{\pi} \operatorname{lev} \operatorname{sig_A_Fr}} \right)
 \end{aligned}$$

diff(Of(lev,MVA,sig_A_Fr,T,Fr,rf),sig_A_Fr);

$$\begin{aligned}
& \frac{lev (MVA + Fr) \left(\frac{1.0 \sqrt{T}}{\sqrt{2}} - \frac{0.5 sig_A_Fr^2 T + rf + \log(lev)}{\sqrt{2} sig_A_Fr^2 \sqrt{T}} \right) e^{-\frac{(0.5 sig_A_Fr^2 T + rf + \log(lev))^2}{2 sig_A_Fr^2 T}}}{\sqrt{\pi}} \\
& - \frac{(MVA + Fr) \left(-\frac{-0.5 sig_A_Fr^2 T - rf + \log(lev)}{\sqrt{2} sig_A_Fr^2 \sqrt{T}} - \frac{1.0 \sqrt{T}}{\sqrt{2}} \right) e^{-\frac{(-0.5 sig_A_Fr^2 T - rf + \log(lev))^2}{2 sig_A_Fr^2 T}}}{\sqrt{\pi}} \\
& + Fr \left(-lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} \log(lev) \left(\frac{4}{sig_A_Fr} - \frac{4 (sig_A_Fr^2 + rf)}{sig_A_Fr^3} \right) \left(\frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) \right. \\
& \left. - \frac{lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} \left(-\frac{sig_A_Fr^2 + rf}{sig_A_Fr^2} - \frac{\log(lev)}{sig_A_Fr^2} + 2 \right) e^{-\frac{\left(\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr} \right)^2}{2}}}{\sqrt{2} \sqrt{\pi}} \right. \\
& \left. + \frac{\left(-\frac{sig_A_Fr^2 + rf}{sig_A_Fr^2} + \frac{\log(lev)}{sig_A_Fr^2} + 2 \right) e^{-\frac{\left(\frac{sig_A_Fr^2 + rf}{sig_A_Fr} - \frac{\log(lev)}{sig_A_Fr} \right)^2}{2}}}{\sqrt{2} \sqrt{\pi}} \right)
\end{aligned}$$

diff(Of(lev,MVA,sig_A_Fr,T,Fr,rf),rf);

$$\begin{aligned}
& \frac{lev (MVA + Fr) e^{-\frac{(0.5 sig_A_Fr^2 T + rf + \log(lev))^2}{2 sig_A_Fr^2 T}}}{\sqrt{2} \sqrt{\pi} sig_A_Fr \sqrt{T}} + \frac{(MVA + Fr) e^{-\frac{(-0.5 sig_A_Fr^2 T - rf + \log(lev))^2}{2 sig_A_Fr^2 T}}}{\sqrt{2} \sqrt{\pi} sig_A_Fr \sqrt{T}} \\
& + Fr \left(-\frac{2 lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} \log(lev) \left(\frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right)}{sig_A_Fr^2} - \frac{lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} e^{-\frac{\left(\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr} \right)^2}{2}}}{\sqrt{2} \sqrt{\pi} sig_A_Fr} + \frac{e^{-\frac{\left(\frac{sig_A_Fr^2 + rf}{sig_A_Fr} - \frac{\log(lev)}{sig_A_Fr} \right)^2}{2}}}{\sqrt{2} \sqrt{\pi} sig_A_Fr} \right)
\end{aligned}$$

$$\frac{d^{lev}}{dlev^{lev}} \left(Fr \left(+ \frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} - \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) \right)$$

`diff(Of(lev,MVA,sig_A_Fr,T,Fr,rf),sig_A_Fr,sig_A_Fr);`

$$\begin{aligned} & \frac{d^{sig_A_Fr}}{d sig_A_Fr^{sig_A_Fr}} \left(lev (MVA + Fr) \left(\frac{\operatorname{erf} \left(\frac{0.5 sig_A_Fr^2 T + rf + \log(lev)}{\sqrt{2} sig_A_Fr \sqrt{T}} \right)}{2} + \frac{1}{2} \right) \right) - \frac{d^{sig_A_Fr}}{d sig_A_Fr^{sig_A_Fr}} \left((MVA + Fr) \left(\frac{\operatorname{erf} \left(\frac{-0.5 sig_A_Fr^2 T - rf + \log(lev)}{\sqrt{2} sig_A_Fr \sqrt{T}} \right)}{2} + \frac{1}{2} \right) \right) \\ & + \frac{d^{sig_A_Fr}}{d sig_A_Fr^{sig_A_Fr}} \left(+ Fr \left(-lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} \left(\frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) + \frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} - \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) \right) \end{aligned}$$

`diff(Of(lev,MVA,sig_A_Fr,T,Fr,rf),rf,rf);`

$$\begin{aligned} & \frac{d^{rf}}{d r f^{rf}} \left(lev (MVA + Fr) \left(\frac{\operatorname{erf} \left(\frac{0.5 sig_A_Fr^2 T + rf + \log(lev)}{\sqrt{2} sig_A_Fr \sqrt{T}} \right)}{2} + \frac{1}{2} \right) \right) \\ & + \frac{d^{rf}}{d r f^{rf}} \left(- (MVA + Fr) \left(\frac{\operatorname{erf} \left(\frac{-0.5 sig_A_Fr^2 T - rf + \log(lev)}{\sqrt{2} sig_A_Fr \sqrt{T}} \right)}{2} + \frac{1}{2} \right) + Fr \left(-lev \frac{2 (sig_A_Fr^2 + rf)}{sig_A_Fr^2} \left(\frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} + \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) + \frac{\operatorname{erf} \left(\frac{\frac{sig_A_Fr^2 + rf}{sig_A_Fr} - \frac{\log(lev)}{sig_A_Fr}}{\sqrt{2}} \right)}{2} + \frac{1}{2} \right) \right) \end{aligned}$$