Layered Goals-Based Portfolio Optimization Utilizing the Production Properties of ESG Bias

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Abstract

Asset allocation models assume decision-makers are unbiased processors of information and, therefore, make decisions in a manner consistent with utility maximization. In reality, portfolio managers often exhibit behavioral biases when diversifying portfolios. For example, it is well-known that ESG sustainability bias (i.e., affinity bias) and loss-aversion bias implicate the portfolio decision-making process. Behavioral portfolio management (BPM) models aim to correct for these emotional biases. In addition to expressing behavioral biases, it is also likely that behavioral portfolio managers experience multiple, hierarchical, and conflicting investment performance objectives. The complex multiple objective BPM process thus extends to a multiple BPM (MBPM). In this study, we study the MBPM decision-problem using a nonlinear goal programming (NLGP) algorithm to diversify the complex behavioral portfolio. Firstly, the current research addresses the affinity for ESG sustainability by algorithmically computing three new pervasive ESG factors. Secondly, we augment the Fama and French asset valuation model to include the network of new ESG factors. After formulating a six-factor Fama and French asset valuation model, the study follows evolving research by using a shallow Bayesian neural network to estimate ESG scale effects on the production of asset returns. Lastly, we deploy the NLGP algorithm to enumerate behaviorally-inspired portfolios under alternate layered goal scenarios. Our results confirm extant findings based on mean-variance optimization while providing new insights into the range and depth of how layered goals for ESG sustainability and loss-aversion impact risk-adjusted portfolio performance.

Keywords: *Behavioral Portfolio Management, ESG Factor Estimation, Multiple Objective Optimization, Option-theoretic Shortfall, and Network Produced Asset Returns*

JEL Codes: C45, C58, C61, G11, G13

1. Introduction

Traditional asset allocation models assume economic agents act as rational and unbiased processors of relevant information. These agents can make decisions in a manner consistent with individual utility maximization. Conventional academic promotes an emphasis on standard theories such as modern portfolio theory (MPT) and the efficient market hypothesis (EMH). But, under the traditional approach, market puzzles and anomalies are not readily explained. Behavioral finance has emerged as a way to describe the interaction of psychology with financial decision-making.

Stylized findings on how the wealth management process is mostly driven by investors who make decisions based on emotions and context-sensitive heuristics endure in the literature (Das, Markowitz, Scheid, & Statman, 2010; Howard, 2014). More recent contributions to the literature seek to explain how beliefs shape portfolio choice directly within the context of a layered bias in the asset allocation process (Giglio, Maggiori, Stroebel, & Utkus, 2019). Other extensions link the multiple goal wealth management models of Das and Ostrov (2018) to the Chang, Young, and Diaz (2018) three-part behavioral portfolio management optimization that balances return estimation, return weighting, and different mental accounts. Extant research contributed by Shefrin and Statman (2000) initially viewed behavioral portfolio management (BPM) theory as an alternative to MPT. Byrne and Brooks (2008) advanced a layered view of the investment process. Under the layered view, investors build portfolios as pyramids of assets, layer by layer such that each layer is associated with a specified prioritized hierarchy of risk management goals.

Behavioral finance believes human behavior is driven by four key factors: values, personality, propensity for risk, and decision dissonance. For finance professionals – planners and advisors, portfolio managers, and institutional investors – these factors can lead to behavioral biases in investment decision-making (Baker, Filbeck, & Ricciardi, 2017). Of particular importance to this study is decision-maker susceptibility to the following biases: affinity, loss aversion (aka prospect theory), and cognitive dissonance. Affinity bias refers to the tendency to make irrational investment decisions based on beliefs and values (i.e., selecting sustainable investments when there is a strong belief that such investments are likely return-compromised). Investors who express loss aversion bias demonstrate a strong desire to avoid absolute reference point losses by prioritizing the avoidance of risk. Cognitive dissonance in investment decision-making describes the process where the decision-maker faces conflicting beliefs (i.e., investment performance goals). In the presence of multiple and possibly hierarchical beliefs, investors require a method to prioritize goal attainment with the understanding that all goals cannot be entirely and simultaneously achieved. The process of treating multiple and layered behavioral factors during the construction of an efficient investment portfolio is referred to as a multicriteria, or multiple objective behavioral portfolio model (MBPM).

The goal programming (GP) optimization method is commonly used to model multicriteria decision problems (Ogryczak, 2002). GP shifts the modeling focus from constraint limitations to prioritized goal attainment. For example, it is widely understood that there is a positive affinity bias for environmental, social, and governance (ESG) factors (Oehmke & Opp, 2020). Traditional portfolio optimization methods control for this affinity by either constraining the optimal asset diversification or reweighting asset-level metrics. The GP alternative does not require asset rescaling, and, more importantly, binding constraints are replaced with goal under-/over-achievement deviations. By layering the priority of the under-/over-achievement deviations, the investment professional can control how ESG targeting impacts returns-based performance (Pastor, Stambaugh, & Taylor, Forthcoming 2020).

The primary objective of the current study is to extend MBPM to encompass MPT with layered goal hierarchy related to both financial and behavioral investment targets. To achieve the proposed extension to MBPM, we introduce a nonlinear goal programming model (NLGP) as a solution method. NLGP makes it possible to add behavioral bias goal-constraints to the uni-objective Sharpe single-index portfolio model (SSIM). Compared to the traditional Markowitz mean-variance model (MV), the SSIM faithfully replicates the maximum rate of return (MRP) portfolio and closely replicates the global minimum variance portfolio (GMVP) (Frankfurter, Phillips, & Seagle, 1976; Sharpe, 1971). But, unlike the MV approach, the extensibility of the SSIM allows for the incorporation of multiple and layered objectives (Xidonas, Hassapis, Mavrotas, Staikouras, & Zopoundis, 2018). However, juxtaposing prevalent behavioral biases (e.g., affinity and loss-aversion) onto the MBPM necessitates a secondary aim for this study – to operationalize cognitive investor biases as a set of engineered asset valuation factors for generalized goal-target expression.

By way of example, the importance of the ESG affinity bias in portfolio construction is provided by Melas, Nagy, and Kulkarni (2017). In their study, the authors examined ESG impact by augmenting and re-sorting asset-level relationships across risk-factors. Among other findings, return-level improvements in the order of 30% were observed for ESG profile adjustments with relatively modest impact on previously targeted factor exposure. We achieve an NLGP formulation of the portfolio construction method by taking an optimization model approach. The approach relies upon estimating three new pervasive ESG factors for inclusion in specific goal-target constraint equations that are differentiated by investor priorities.

The paper proceeds as follows. In section 2, we estimate three new pervasive ESG factors to proxy ESG sustainability impact on portfolio diversification. Section 3 investigates whether these three new factors address the 'factor zoo' puzzle. This section deploys a well-followed factor disentanglement algorithm to estimate the ubiquitous standing of the newly derived ESG factors. Section 4 estimates asset returns in a networked market economy by extending recent findings on the use of shallow neural networks and production-theoretic asset pricing. Section 4 also provides an option-theoretic approach to estimate asset-level shortfall. Section 5 encapsulates the results provided in sections 3 and 4 to form the normative multiple-objective behavioral portfolio model (MBPM). Conclusions are provided in section 6.

2. Pervasive ESG Factor Estimation

The substantial literature on the creation of factor portfolios is in agreement with findings on how factor models provide three pillars of support in asset allocation models. The pillars are stated as the identification of risk premia, the identification of behavioral pricing biases, and the identification of structural impediments to efficient diversification. However, the number of relevant studies has made it difficult to pin down the economic benefit of the investigated factors. In a comprehensive examination of over 300 factors (i.e., the factor zoo), Harvey, Liu, and Zhu (2016) argue that risk factors must exhibit two characteristics. One is unpredictable variation through time. The other is the ability of the factor to explain cross-sectional return patterns. Cochrane (2011) previously argued for methods to identify prevalent and dominating risk factors. To test whether factors are priced efficiently and to overcome a data-mining bias in error specification, Ang, Liu, and Schwarz (2009) refined the factor quest by arguing for the use of stocks over portfolios. Using an augmented projection approach and principal components analysis (PCA), Lettau and Pelger (2018) successfully estimate latent factors to explain covariance and expected returns structures in equity data. The first contribution of their paper is a method to control time-variation in PCA loadings of individual stocks. A second contribution is evidence that an augmented PCA technique can lead to a superior enumeration of the optimal portfolio.

The consequences of factor identification are addressed by Pukthuanthong, Roll, and Subrahmanyam (2018), aka PRS. The PRS study presents a best practice approach for factor identification by developing a protocol for palpable risk factor extraction. The protocol takes into consideration a factor's relationship to the covariance matrix of asset returns, the priced relationship in the cross-section of returns, and the overall reward-to-risk ratio. As previously stated, a subsidiary aim of our research is to identify pervasive ESG factors. To this end, it is useful to summarize the findings of Lettau and Pelger and PRS by stating the following two assumptions supporting the research effort.

Assumption 1. Assume that excess returns follow the standard approximate factor model where the assumptions of arbitrage pricing theory are satisfied. In this case, excess return, $R_{i,t}$, has a systematic component captured by K factors and a nonsystematic, idiosyncratic component that captures asset-specific risk. Excess returns of N assets over T periods are thus described as

$$\underbrace{R}_{TxN} = \underbrace{F}_{TxK} \underbrace{\Lambda^T}_{KxN} + \underbrace{\varepsilon}_{TxN} \tag{1}$$

Where F is the matrix of unknown latent factors, and Λ is the matrix of loadings.

Assumption 2. The factors and residuals are uncorrelated; hence, the covariance matrix of the returns consists of a systematic and idiosyncratic part.

$$Var(R_t) = \Lambda Var(F_t)\Lambda^T + Var(\varepsilon_t), where, t = 1, \ldots, T \tag{2}$$

The factors drive the largest eigenvalues of $Var(R_t)$; hence, PCA is available to estimate loadings and factors. In the next section, we estimate the unknown latent factors, F, and loadings Λ, from the Refinitiv/S-Network constitutive ESG portfolios.

2.1 Portfolio Data

We refer to the three uniquely separated ESG portfolios maintained as part of the Refinitiv/S-Network ESG Best Practices Ratings and Indices (https://bit.ly/Refinitiv-SNetworks). These

Indices are designed to provide a benchmark of companies exhibiting best corporate social responsibility practices as measured by Refinitiv/S-Network's ratings schema. The ratings dynamically rank the constituent companies on Environmental, Social, and Governance performance.

In this study, we collect the company list for the three ESG large-cap portfolios, which are identified as TR*E*NVUS, TR*S*CUS, and TRC*G*VUS, respectively. Each portfolio contains P vetted securities such that $P = \{P_E, P_S, P_G\}$. Tickers with incomplete data were removed to create a research sample set $N \subseteq P$, $N = \{n_E, n_S, n_G\}$ where $n_E = 245, n_S = 245,$ and $n_G = 243$. For the market proxy, S&P 500, and all securities in *N*, we compute daily log-differenced returns (r_M = market returns; and r_i = security returns for ith security) from January 2015 through March 2018, inclusive ($T = 816$).

Following extant literature, we choose an enhanced beta estimate to capture systematic market variation in equity returns. Across all *N* securities, we implement the Vasicek (1973) adjusted market beta ($i.e., \beta_i^V, i = 1..N$). Although Hollstein and Prokopczuk (2016) find that option-implied estimators of systematic risk consistently outperform all other approaches tested on both daily and monthly datasets, Sarker (2013) and Cloete, de Jonah, and de Wet (2002) report on the efficiency and robustness of Vasicek estimators compared to using unfiltered OLS methods. In a follow-up study, Wang, Huang, and Hu (2017) demonstrate improved stock return predictability using Vasicek-adjusted betas in both the CAPM and Fama-French three-factor model (Fama & French, 1993). Accordingly, for all securities, aftermarket residuals are formed by equation (3).

$$\varepsilon_{i,t} = r_{i,t} - \alpha_{i,t} - \beta_i^v \, r_{M,t} \text{ where, } t=1..T,\ i=1..N \tag{3}$$

2.2 Latent ESG Factor Identification

Latent factors are extracted by applying PCA to the matrix of residuals for each of the three domains of securities in *N*. We begin by testing the individual residual matrix for PCA suitability. That is, we test the hypothesis that the correlation matrix for each $\boldsymbol{\varepsilon}_{n_E}, \boldsymbol{\varepsilon}_{n_S}, and\ \boldsymbol{\varepsilon}_{n_G}$ is an identity matrix. The results of applying Bartlett's test of sphericity are shown in table 1.

Table 1: Results from Bartlett's Test and the KMO Test

Domain	**Bartlett's Test of Sphericity** ***Ho*: No Common Factors;** ***Ha*: At least one common factor**	**KMO Measure of Sampling Adequacy**
Environmental	$\chi^2_{29890} = 109690.280, p < 0.001$	0.85349
Social	$\chi^2_{29890} = 105587.052, p < 0.001$	0.85222
Governance	$\chi^2_{29403} = 99281.5095, p < 0.001$	0.83165

Based on these results, the null hypothesis of no common factors is rejected at the 1% level. We conclude that an exploratory factor analysis (EFA) is statistically supportable. The Kaiser-

Meyer-Olkin (KMO) measure is also applied to the three residual matrices. The results of the KMO test indicate a high proportion of the variance in the variables is caused by the underlying factors.

Following Han (2002), we calibrate the arbitrage return-generating framework using an exploratory factor analysis (EFA) model applied to the aftermarket residuals (Jackson, 2005). Subsequently, we subject the extracted factors to an orthogonal rotation. The results obtained from the rotation corroborated extant literature as far back as the mid-1970s (Fertuck, 1975). The end-product industry effects were clearly separated (see Figure 1). To the aim of this study, we observed, for example, that the factor labeled *Banks and Bank Hldg* accounts for 46% of the aftermarket variation in the residuals. Extending these results to consider the ESG effects, we point to identifiable E- S-, and G sub-domains within the *Banks and Bank Hldg* domain. The E-, S-, and G-domains account for 16-, 16-, and 15-percent of the aftermarket variation, respectively. Similarly, for the second orthogonal factor (*Energy*), which accounts for 22% of the total aftermarket residual return variation, the ESG contribution is 8-, 8-, and 6-percent, respectively.

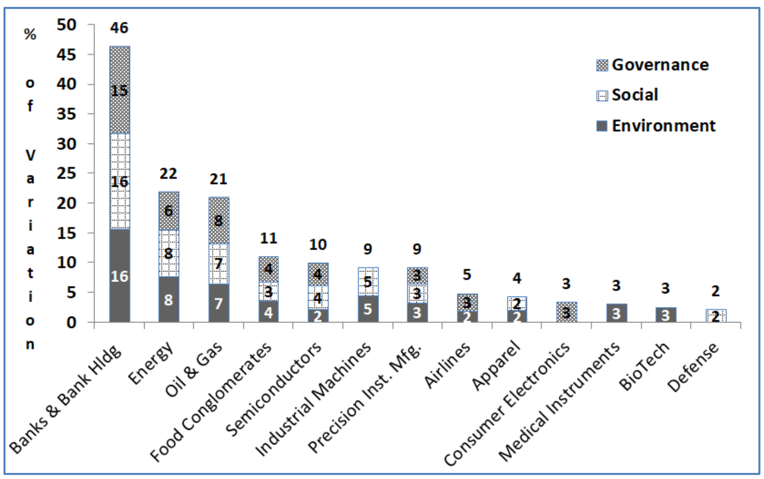


Figure 1: Percent of industry-wide aftermarket variation explained by the E, S, & G domains.

Invoking the Kaiser-Guttman criteria leads us to retain 36 factors in each of the individual domains, $C^E, C^S,$ and C^G. The percent total aftermarket variation explained is 81.14%, 80.66%, and 79.99%, respectively.

2.3 Factor Scores as Reproducible Factor Proxies

The next step in the algorithmic process is to create E, S, and G factor-based proxy variables. The index creation process requires transforming the rotated factors into hypothetical, but genuine, factor-policy variables. We compute a refined regression-based factor score estimates using SAS 9.4. The regression method proposed by Thurstone (1935) assures “maximum validity” or “highly determinate” estimates for a given analysis (Grice, 2001). Additionally, as shown by Beauducel (2007), Thurstone’s calculations can reproduce the same covariance matrix.

Although the problem of indeterminacy is resolved by the Thurston method, the scores are not correlation preserving. As amplified by Grice, the factor score estimates may be contaminated with variance from other orthogonal factors within the analysis. However, the ESG factor creation process is predicated on summing individual scores. Hence, we proceed with computing the index using the matrix of factor score estimates, ***f***. The formulae to calculate each domains index value at time *t* is as shown below:

$$\mathrm{FSI}_t^{E} = \frac{\sum f_{ti}}{C^{E}}, i = (1, \dots, C^{E}) \tag{4}$$

$$\mathrm{FSI}_t^{S} = \frac{\sum f_{ti}}{C^{S}}, i = (1, \dots, C^{S}) \tag{5}$$

$$\mathrm{FSI}_t^{G} = \frac{\sum f_{ti}}{C^{G}}, i = (1, \dots, C^{G}) \tag{6}$$

2.4 Stationarity Conditions

Continuing with the Lettau and Pelger (2018) procedure introduced in section 2.1, we evaluate the stationarity condition of the three new hypothetical factors using the Philips-Perron (PP) test. The null hypothesis for the PP test states that the series has a unit root. When applied to the factor score indices, $FSI^E, FSI^S,$ and FSI^G, we reject the respective hypotheses for trend, single mean, and zero mean. Specifically, reported results are as follows: trend (E: τ = -7.88, S: τ = -7.60 and G: τ = -9.29; all $p < 0.001$); single mean (E: τ = -7.96, S: τ = -7.65 and G: τ = -9.41; all $p < 0.002$); and zero mean (E: τ = -8.06, S: τ = -7.75 and G: τ = -9.53; all $p < 0.001$). By implication, when applied to each factor index, there is a high probability no unit root exists, a finding that each index is stationary with a zero mean.

3. The Augmented PRS Algorithm for Disentanglement

This section of the study is devoted to the disentanglement of embedded ESG factors in an investor-formed portfolio. To accomplish the task, we invoke the PRS protocol. The PRS protocol identifies factors associated with risk premia as well as 'pervasive' factors. Pervasive factors are unobserved and are extracted from the asset returns of portfolios.

3.1 Derive Pervasive ESG Factors

The pervasive factor score variables computed above utilized daily residual returns data across the three E-, S- and G-domains. As the remainder of the analysis is focused on the behavior of investor portfolios, we follow the literature and use monthly returns from this point forward (Zibri & Kukeli, 2015). Accordingly, for each ESG domain derived in section 2.3, we average the daily factor score indices, $FSI^E, FSI^S,$ and FSI^G, observations into monthly observations. The remainder of the paper relies on the monthly values of these indices.

3.2 The Investor Portfolio

The naively diversified investor portfolio in this study is owned and managed by a regional unit of the national non-profit *The Girl Scouts of the United States of America* (GSUSA). The national office transmits public policy and investment goals to its subordinate councils. In addition to earnings from current year operations, regional councils are expected to make investment decisions in a manner that is consistent with the organization's socially responsible narrative. The investor portfolio used in this study is naively diversified and is comprised of n=65 instruments representing 41 industries across 12 sectors.

3.3 Investor Portfolio Heterogeneity

The first step in the PRS algorithm is to identify an equity portfolio representing different industries with a 'good' level of heterogeneity. For the subject investor portfolio used in this research, the *Yahoo!* industry and sector classifications are as shown in figure 2.

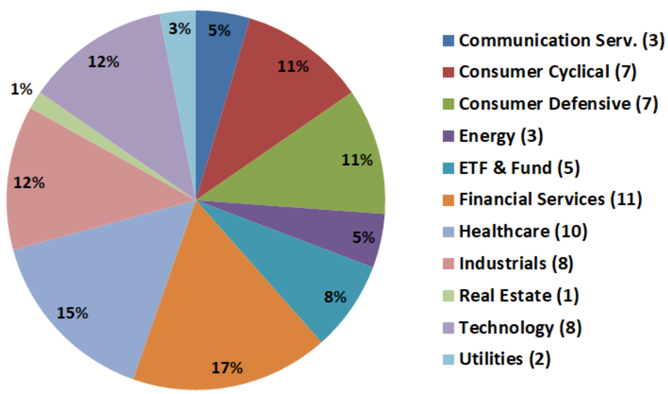


Figure 2: Diversification of investor portfolio

Next, we test the heterogeneity of the portfolio by conducting a test on the average correlation, $\bar{\rho}$, of the monthly log-differenced returns for the 65-instruments in the investor portfolio from January 2015 through March 2018, inclusive (T=39). Pollet and Wilson (2010) report that $\bar{\rho}$ has predictive power for stock market returns. In their study, the authors find returns predictability from average correlation over the periods 1963-1974, 1974-1985, and 1996-2007. With some exceptions noted, the authors report that average correlations from the late 1980s forward are between 0.15 and 0.55. For the investor portfolio of this study, the average correlation is 0.28. This finding lies within the bounds of previously reported research results.

As a further test of heterogeneity, we subject the correlation of asset returns to a Fruchterman-Reingold (FR) network analysis (Fruchterman & Reingold, 1991). The FR analysis is a force-directed network graph that distributes vertices evenly in a frame. As such, it is a useful method to examine the correlation structure. In the FR network, edges are similar in length and cross each other as little as possible. Nodes represent electrically charged particles that repulse each other when they get too close. The edges act like springs that attract connected nodes closer together. As a result, nodes are evenly distributed through the graph, and the layout is intuitive in that nodes that share more connections are closer to each other. A review of the FR network, figure 3, demonstrates the high intercorrelations among assets in the subject portfolio.

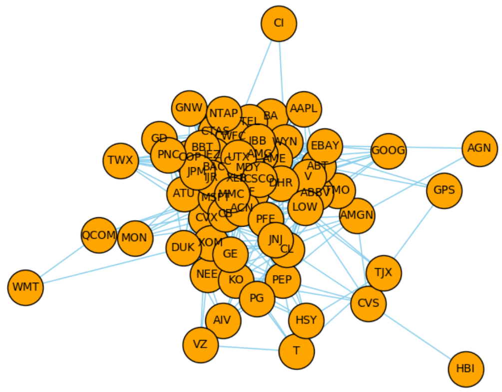


Figure 3: FR Network of Security Correlations

3.4 Extract L Principal Components

Step two of the PRS algorithm requires the extraction of L principal components from the asset return series computed from the investor portfolio. With T time-series units, we calculate the TxT matrix, $\Omega_t = \left(\frac{1}{T}\right) RR'$, where $\underbrace{R}_{Txn}$ is the return matrix. As suggested by PRS, the cutoff point for the cumulative variance explained by the principal components is set to 90%. From this procedure, 16 principal components (PC) are retained. By way of example, figure 4 displays the cross-loading of the first three principal components (PC-1, PC-2, and PC-3). The 16 eigenvectors will form the dependent domain for subsequent canonical correlation analysis.

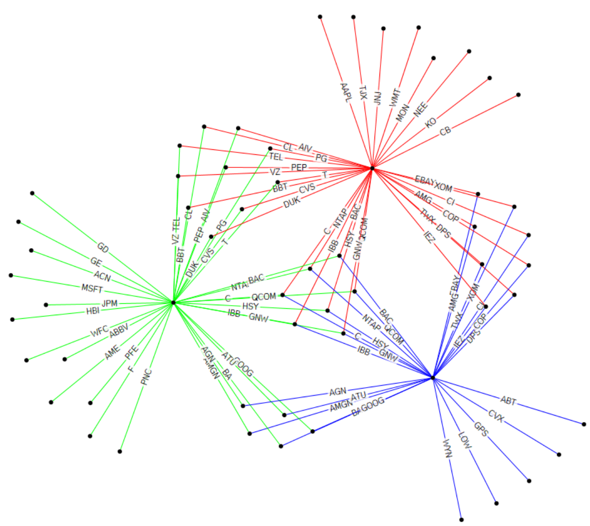


Figure 4: Excess Returns across Time.

Legend: PC-1=Green; PC-2=Blue; PC-3=Lime

3.5 The Canonical Correlation between Pervasive Factors and the Investor Portfolio

The third step in the PRS protocol requires the identification of what is expected to be pervasive factor candidates. This step was completed and discussed in sections 2.2 and 2.3. Our aim to disentangle the latent aftermarket effects due to firm investments in sustainability (E, S, and G) are represented by the previously constructed factor-policy variables (i.e., equations (4) - (6)).

The final step of the PRS algorithm, step four, requires conducting a canonical correlation analysis (CCA) between the set of pervasive ESG factors and the corresponding 16 eigenvectors of the investor portfolio. Using CCA, we investigate if a factor exhibits a significant canonical correlation with the investment portfolio's best linear combination of eigenvectors. Specifically, we examine the null hypothesis that the surrogate E, S, G pervasive factors systematically influence the movement of portfolio asset prices.

The results from the CCA analysis of the first approximate *F*-value indicate that as a group, the factor candidates are conditionally related to the covariance matrix of market returns (*Wilk's* $\lambda = 0.6411$, $F_{48,\ 63.253} = 2.0$, $p < 0.05$). The inference from the second approximate *F*-value (F=1.63; $p < 0.1$) is that the second and the third canonical correlations are equal to zero (not statistically significant). Lastly, the third approximate *F*-value (F=1.18, p-value > 0.1) suggests that the third canonical correlation is not significant.

The standardized canonical coefficients for the three-factor candidates (E = -1.1622; S = -0.5230; and G = 1.0375) indicate that the contribution to the first component (eigenvector of returns) is primarily due to the E and G domains but, in an inverse relationship. The social factor also has an inverse relationship and, comparatively, at a much smaller level. To clarify, consider the following scenario. When all other variables in the model are held constant, an asset experiencing a one standard deviation increase in monthly returns in the environmental policy area (factor) would expect a -1.1622 standard deviation decrease in the score on the first canonical variate.

In a manner consistent with the PRS algorithm, this application of CCA yields statistical evidence that the multivariate E, S, and G, factor set is pervasive and linearly correlated with the set of asset returns in the investor portfolio. In the next section, we demonstrate how pervasive factors produce asset returns.

4. Network Theory and Cognitive Biases

Powered by an array of data science methods, evolutionary financial network theory seeks to provide new insights into asset valuation interconnectedness as a source of uncertainty in systematic risk (for reviews, see Priestley and McGrath (2019) and Roukny, Battiston, and Stiglitz (2018)). For example, Erdõs, Ormos, and Zibriczky (2011) contribute evidence on how the betas of both the single- and three-factor models based on nonparametric kernel regression can explain a cross-section of stock returns. Huh (2019) eschews the use of feature engineering to estimate systematic risk parameters by deploying a deep-learning neural network. Using a bottleneck architecture, the Huh's study finds how the deep-learning network equaled but did not exceed estimation results generated by parametric factor models.

This section of the study presents a novel extension to the nonparametric estimation of asset returns[1]. First, we augment the three-factor asset pricing model to include pervasive ESG factors. Second, we estimate the parameters of the asset pricing equation by use of an interconnected information network. The extant literature provides evidence of collective, or networked, behavior among fund managers (Hong, Kubik, & Stein, 2004) as well as among individual investors (Ivkovic & Weisbenner, 2007). More recent evidence on network cross-predictability of asset returns introduces the concept of a prediction-matrix (Kelly, Malamud, & Pedersen, 2020). Based on the burgeoning literature using machine learning techniques, there is a new cornerstone for the deployment of a 'shallow' neural networks in asset valuation models. The nonparametric kernel estimation method can be used to analyze the production of return predictability in an interconnected features-model in a manner that supports behavior modeling (Ozsoylev & Walden, 2011) and belief bias (Ghosh & Roussellet, 2019).

The progression of network-based pricing models is summarized in a comprehensive study of financial institutions. Monica Billio, Caporin, Panzica, and Pelizzon (2016) extend the classic factor-based asset pricing model to include network linkages of exogenous lagged and contemporaneous links across assets. Closely related is a study by Horrace, Liu, and Patacchini (2016) in which the authors provide evidence that peer effect networks interact with production functions to transform inputs into outputs. Herskovic (2018) added to these asset valuation models by uncovering a link between equilibrium asset prices and the two network attributes that drive systematic risk – network concentration and network sparsity. Herskovic observes how a sparse asset network has fewer but stronger linkages. Under the assumption that firms experience a Cobb-Douglas shaped production technology, the Herskovic study reports innovations in the network factors are priced. Specifically, he finds the pricing factors of sparsity and concentration account for annual return spreads of 4.6% and -3.2%, respectively.

4.1 Cobb-Douglas Production Functions

A production function is a heuristic construct that describes the maximum output from alternative combinations of input factors using a given technology. The novel model we propose is a Cobb-Douglas inspired double-log estimation of asset returns based on an input mix of the three Fama-French return-generating factors along with the three FSI proxies (ESG input factors). In part, the efficient production of asset returns depends on the ability of firm managers to disaggregate noisy ESG input factors.

[1] E,S, and G pervasive factor time series and computations provided in sections 4 (K4-RANN) and 5 (NLGP) are obtained via the WinORS$_{e\text{-}AI}$ Software environment (for more information, see:The NKD Group, 2020).

4.1.1 ESG Factors and the Double-Log Production of Asset Returns

For all j firms in the investor portfolio, we expect each j^{th} firm to combine capital (k) and labor (l) to produce output using a Cobb-Douglas production technology,

$$y_j = A_j k_j^{\alpha} l_j^{1-\alpha} \tag{7}$$

Without considering a firm's age or its learning rate, we further assume $A_j = e^{\beta_j \Delta_a}$, where $\Delta_a$is a common interconnected ESG shock that affects the returns productivity of all firms and $\beta_j$is the firm-specific exposure to the common shock Δ_a. Before implementing a firm-specific production decision, firm j observes a noisy ESG signal that is unique to the firm's market exposure: $sig_j = \beta_j + \epsilon_j$ where $\epsilon_j \sim i.i.d., N(0, \frac{1}{\Delta_a}\tau^2)$. In this abstraction, the amount of noise in a firm's signal is captured by parameter τ^2. Perfect information occurs when $\tau = 0$, whereas as $\tau \rightarrow \infty$ the signal to firm j is not informative, or firm management is numb to ESG factors. When observed and controlled for, the ESG signal, sig_j, helps firm j make efficient input factor choices.

We define the model as,

$$E(r_j) = f(\boldsymbol{X}) + \varepsilon_j \,, \tag{8}$$

where $\boldsymbol{X}$ denotes the input mix ($X = X_1, \ldots, X_d$), d is the dimensionality of the factor inputs, and ε_j is a symmetric random noise term $\varepsilon_j \sim i.i.d., N(\mu, \sigma)$. For the ESG-enhanced asset returns estimation in a Cobb-Douglas framework, d = 6. We derive factor elasticity estimates under the assumption that ESG signals are fully incorporated in firm production decisions (i.e., $\tau \rightarrow 0$):

$$E(r_j) = \alpha_j + \beta_1 ln(1 + (r_M - r_f)) + \beta_2 ln(1 + FSI_E) + \beta_3 ln(1 + FSI_S) +$$
$$\beta_4 ln(1 + FSI_G) + \beta_5 ln(1 + SMB) + \beta_6 ln(1 + HML) + \epsilon_j \tag{9}$$

In equation (9), r_M is the total market portfolio return; r_f is the risk-free rate; FSI_E, FSI_S, and FSI_G, express the average monthly score for each pervasive ESG factor, respectively; SMB is the size premium, and HML is the value premium.

4.1.2 Artificial Neural Network and the Double-Log Production Function

Arreola and Johnson (2016) argue for new estimators based on modern machine learning algorithms for studies of complex observed (and statistically enumerated) datasets. In a comprehensive and comparative analysis across alternative machine learning methods, Gu, Kelly, and Xiu (2019) find that 'shallow' learning networks perform best in studies of asset return estimation. Artificial neural networks have previously provided a viable nonparametric alternative to fit nonlinear production functions and to describe the estimated technical efficiency (Santín, Delgado, & Valiño, 2007; Vouldis, Michaelides, & Tsionas, 2010). Specifically, we employ an augmented radial basis function artificial neural network (RANN) known as the K4-RANN (Dash, Kajiji, & Vonella, 2018; Kajiji, 2001). The generalization ability of the RANN for nonlinear regression is provided by Krzyẑak, Linder, and Lugosi (1996). Accordingly, this study performs kernel estimation using a RANN to map asset returns weights within the double-log framework of features.

In a generalized RANN regression, the optimal weighting values are generally extracted by applying a supervised least-squares method to a subset (training set) of the data series. The supervised learning function is stated as $\boldsymbol{y} = f(\boldsymbol{x})$ where $\boldsymbol{y}$, the output vector, is a function of the input vector $\boldsymbol{x}$ with p number of inputs. The function can be restated as:

$$f(x_i) = \sum_{k=1}^{m} w_k h_k(x) \tag{10}$$

where m is the number of basis functions (centers), h is the number of hidden units, $\boldsymbol{w}$ is the weight vector, and $i = 1...p$ where p is the number of input vectors. As shown in equation (11) the K4-RANN minimizes a modified sum of squared error (SSE) cost function.

$$\frac{\text{argmin}}{\lambda}\left(\sum_{t=1}^{T}(y_t - f(x)_t)^2 + \sum_{k=1}^{m}\lambda_k w_k^2\right) \tag{11}$$

Where, $f(x)_t$ is the model's prediction at $\boldsymbol{w}$ for all T observations. The result of applying the nonlinear K4-RANN of equation (11) is the extraction of a set of weights such that SSE is minimized while simultaneously optimizing the accuracy of the predicted fit (smoothness). The accuracy of the predicted fit is increased due to the application of a regularization parameter, λ, often termed as the weight decay parameter. The weight decay parameter is added to the error function to penalize mappings that are not smooth. When applied to the networked production function stated in equation (9), for all j in n (securities in the investor portfolio), the procedure maps the production of monthly returns, r_j, for all j-firms.

4.2 RANN Estimated ESG Returns-to-Scale

The K4-RANN weights derived from solving equation (10) are interpreted as the factor elasticity coefficients. The next section identifies the parameter settings applied to the K4-RANN algorithm. Following the discussion on parameter settings, a review of the model weights and the associated production returns-to-scale is presented in section 4.2.2.

4.2.1 Algorithmic Control Parameters

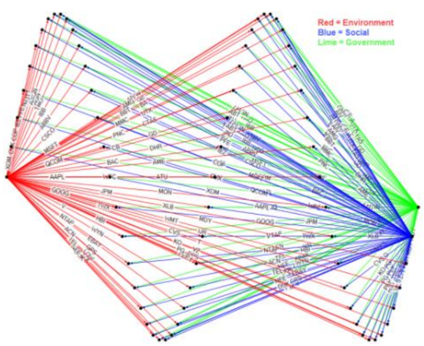
The K4-RANN[2] algorithm requires several algorithmic parameters. In this study, the model parameters were identically applied to all j pricing models. Before invoking the algorithm, all data were standardized. The underlying transfer function chosen for the K4-RANN was Gaussian. The RANN radius was uniformly set to 1.0. The error minimization rule was set to 'generalized cross-validation' (GCV). The GCV rule is known to perform well for both smooth and rough functions (Wahba, 1985).

[2] Observe a visual of the K4-RANN computation and solution in WinORS$_{e\text{-}AI}$ 2020 at http://bit.ly/K4SampleSolution.

4.2.2 Factor Elasticity Network Connectedness

The degree of asymmetric ESG connectedness among sectors and firms is shown in figure 5. The commonality in findings between the sector-limited Granger-causality networks reported by Monica Billio, Getmansky, Lo, and Pelizzon (2012) is evident. The network graph depicts the interrelated production of a standard ESG signal that binds the firms within the investor portfolio.

To understand the change in a firm's returns given a unit change in an E, S, or G factor, we interpret the K4-RANN weights as quasi-factor elasticity metrics or returns to scale (*RtS*). Before explaining the K4-RANN network weights, it is useful to view a graphical representation of the network weights. The left-side diagram (figure 5) presents an overall view of interconnectedness among the ESG pervasive factors. The right-side network chart (figure 6) shows the unique directional impact on return production from each pervasive ESG factor.



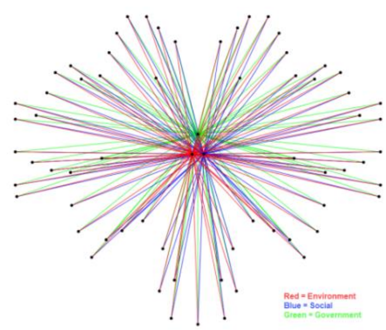


Figure 5: Interconnectedness of E, S, & G

Figure 6: Directional impact of E, S, & G

Further evidence of the proportional impact of ESG features on asset returns is provided by the K4-RANN elasticity weights (see figure 7; and table 2). For brevity, the following discussion is restricted to the asymmetric connectedness of firms in three sectors: *financial services*, *energy*, and an *ETF for oil equipment and services* (Appendix B provides complete results).

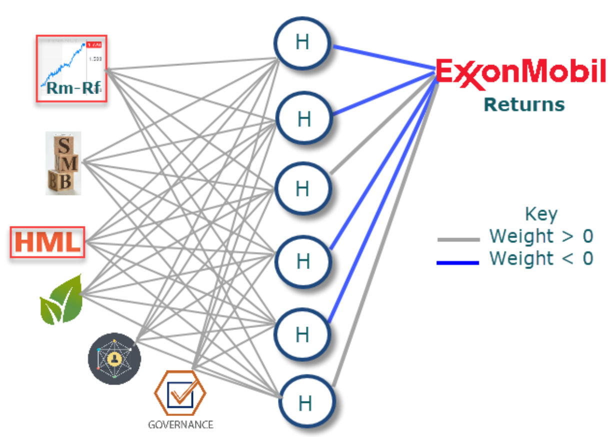


Figure 7: Network Elasticity Weights for ExxonMobil

Table 2: RANN Estimated Returns to Scale for Selected Assets

	Ticker	RtS^E	RtS^S	RtS^G	RtS
Banks – Global (Financial Services)	JPM	0.0101	0.3325	0.3058	0.6484
	WFC	-0.1957	0.2673	0.2562	0.3278
	BAC	0.0808	0.2226	0.1862	0.4896
	C	-0.2620	-0.0870	-0.1650	-0.5140
Oil&Gas (Energy)	XOM	-0.0943	-0.0388	0.7870	0.6538
	CVX	-0.2114	0.5529	0.7629	1.1045
	COP	0.4275	0.3749	0.7556	1.5580
iShares Oil Equip & Services ETF	IEZ	0.1224	0.0645	0.7072	0.8942

For Bank of America (BAC), the weights for E, S, and G are each positive (0.0808; 0.2226; and, 0.1862, respectively). Conversely, each weight for Citicorp (C) is negative (-0.2620; -0.0870; -0.1650). There is clear evidence of how the common ESG signal is decomposed into a unique '*Financial Services*' signal that is asymmetrically differentiated among individual sector-related firms. Given changes in sustainability performance, the expectation is for BAC to experience a positive increase in returns. While for the same change in sustainability investments, Citicorp will likely experience depreciated returns. Of further interest is the G dimension. Except for Citicorp, all firms displayed in table 2 are expected to benefit by firm responses to government sustainability changes. The asymmetric ESG weights within this sector offer further evidence of the need to understand the contribution of pervasive ESG factor variation to the returns producing process. In the next section, we extend the MBPM to include a dynamic loss-aversion metric.

4.3 Dynamic Option-Theoretic Shortfall Estimation

The layered bias premise of this research requires a dynamic risk measure to proxy for 'loss aversion' bias. We limit our focus to the popular frequency-based conditional value-at-risk (CVaR) metric of Rockafeller and Uryasev (2002). CVaR captures the conditional expectation of losses in top (100 – β)% over a given investment horizon (e.g., β = 0.95 or 0.99).

$$CVaR_{\alpha}(X) = \frac{1}{\alpha}\int_{-\infty}^{X} VaR_{\beta}\, d\beta \tag{12}$$

CVaR is a coherent risk measure (Artzner, Delbaen, Eber, & Heath, 1999). When CVaR is used in the context of portfolio risk minimization, the metric can be expressed as a continuous and convex function with respect to the optimization variables in a convex program (Krokhmal, Palmquist, & Uryasev, 2002; Rockafellar & Uryasev, 2002; Rockafellar, Uryasev, & Zabarankin, 2006). CVaR in linear and multiple goal optimization models is also in evidence. Ogryczak (2002) was one of the first to contribute evidence on the incorporation of CVaR in a goal constraint. Kaminski, Czupryna, and Szapiro (2009) extended this line of research by providing a CVaR-based goal programming portfolio selection method to account for investor risk attitudes.

The implementation of CVaR (and VaR) is dependent on knowing the exact statistical distribution of market parameters. Often, these parameters are characterized by sampling error. There is a significant strain of literature devoted to the calculation of CVaR and its associated sensitivities. For example, Hong, Jeff, and Liu (2011) provide a detailed review of the performance of Monte Carlo methods used to estimate VaR and CVaR (including sensitives). Hsieh, Liao, and Chen (2014) extend the use of Monte Carlo methods by providing a fast algorithm to estimate VaR and CVaR. By contrast, Yao, Li, and Lai (2013) employ nonparametric estimation of CVaR when applied to the portfolio selection problem.

In this paper, we adopt the put-option market algorithm of Barone-Adesi (2016). Under the Barone-Adesi plan, for a given α, we estimate CVaR for an optionable asset by capturing the instantaneous spot price (*S*), the risk-free rate (*r*), and time to expiration *(T)*. Then for a given near-the-money strike (X_{put}), the algorithm calculates p = $BSOPM_{(put)}$ using the Black-and-Scholes price approximation. The algorithm proceeds by restating CVaR as the expected dollar loss beyond VaR given *S*. As such, it is affected by fatness in the tail of the distribution of *S*. In the Barone-Adesi model CVaR for a given security is stated as:

$$CVaR = \frac{1}{\alpha}\int_{-\propto}^{X_{put}} L(S)f(S)dS \qquad (13)$$

$$CVaR = e^{rT}\frac{p}{\alpha} + VaR \qquad (14)$$

5. Modeling the Goal-Directed and Behaviorally-Biased Investor Portfolio

In this section, we formalize a nonlinear goal programming (NLGP) model to formulate the MBPM. Multiobjective optimization methods are naturally extensible and capable of handling hierarchical and conflicting objectives simultaneously. Solution algorithms for multiobjective models embrace a wide variety of methodologies, from evolutionary algorithms (Kapiamba, Ulungu, & Mubenga, 2015) to goal programming. When the NLGP decision problem states no more than two hierarchical goals, the solutions generated can form a Pareto optimal front. But, by design, NLGP scalability permits hierarchical goals to exceed two (i.e., a complex GP model); hence, choosing an optimal compromise solution is no longer a trivial task (Ruotsalainen, 2010).

Upon surveying over 40 articles on heuristic and exact solution methods, Mokhtar, Shib, and Mohamad (2014) reported that GP applications dominated applications applied to portfolio optimization. Extant examples of alternative NLGP applications across various disciplines appear in Saber and Ravindran (1993) and Miettinen (1998). A recent application to portfolio optimization appears in Dash and Kajiji (2014). In this study, NLGP is used to dynamically solve a two-stage stochastic model that invokes the minimum variance (stock-index) hedge ratio to stabilize returns of a diversified equity portfolio. The first-stage solution exploited function separability to achieve mean-variance efficiency. The second-stage solution invoked a binary control of hedging based on first-stage outcomes.

The question naturally arises about the fitness of NLGP to model financial cognitive dissonance. We provide perspective in the next section.

5.1 Layered Goal Optimization and Financial Cognitive Dissonance

We adapt the separable-based hierarchical goal program of Dash and Kajiji (2014) as the method to model the real-valued MBPM.

$$NLGP = Min\ Z = \sum_{k=1}^{K} P_k[\delta h^-, \delta h^+]$$

$$\text{S.T. } Ax + Ih^- - Ih^+ = b$$
$$x, b, h^-, h^+ \geq 0,$$

where Z quantifies k prioritized objectives. That is, resource allocations must be achieved in a prioritized sequence, or layers, where $P_1 > P_2 > \cdots > P_K$. Within goal k, δ is a constant term to indicate scale preference; A is an m x n matrix of technological coefficients; x is the n-component column vector of decision variables; b is an m-component vector of goal targets; and, $h^- and\ h^+$are m-component column vectors of goal over- and under-achievement, respectfully. Lastly, define x^* as the solution that satisfies all hierarchical priority levels as much as possible.

Specifically, the layered goals-based algorithm seeks to solve for the lexicographically smallest goal vector $(a_1x_1, a_2x_2, \ldots, a_nx_n)$ give $\Gamma_k = \{x | Ax + h^- - h^+ = b;\ x \geq 0\}$. The goal program proceeds by solving the first linear program (LP), $LP_1 \equiv Min\{Z_1x | x \in \Gamma_1\}$, with optimal solution at x^*. Each K-1 program with its associated immediately follows the solution x^*. Stated succinctly, the LP seeks to satisfy all hierarchical priorities as much as possible such that $Min\ Z \equiv MLP_k = Min[Ax | x \in \Gamma]$, implying $\forall x \in \Gamma_k = \{x \in\ \Gamma_k | A_kx = A_{k-1}x^*, k = 1,2, \ldots K\}$.

5.2 Financial Cognitive Dissonance and Multiobjective Portfolio Efficiency

In behavioral finance, the theory of cognitive dissonance explains why, as decision-makers, investors feel internal tension and anxiety when subjected to conflicting investment beliefs. According to Sharma (2014), cognitive dissonance is a psychological phenomenon that is not easily measurable; hence, to reduce financial cognitive dissonance, the decision-maker should ladder (aka bridge) thoughts across two or more goals (models). One approach to lowering dissonance is to change behavior. However, because this is often very difficult to achieve, a more common approach seels to spread-apart-alternatives – a process of increasing the attractiveness of one alternative while decreasing the attractiveness of other options (Jedryka & Szapiro, 2002; McLeod, Feb 05, 2018). In pre-emptive GP using constraints, goals are ranked (i.e., spread-apart) from most important to least important. The method satisfies the first-order goal then comes as close as possible to fulfilling the second-order goal. The procedure continues in this fashion until all lower-order goals are satisfied as much as possible; or, stated differently, a set of non-dominated solutions is achieved.

Before developing the spread-apart goal hierarchy of the MBPM, it is useful to cite extant literature to establish the ability of the SSIM to approximate the Pareto optimal solutions of the traditional Markowitz mean-variance model.[3] Historical confirmations provide evidence on how SSIM generated solutions consistently replicate the maximum rate of return portfolio but produce slightly inefficient asset diversification outcomes for the global minimum variance portfolio (GMVP) (Frankfurter et al., 1976; Sharpe, 1971). Following these models, we begin the current NLGP application using the SSIM by stating a bi-objective quadratic goal programming model. Priority one (P_1) is set to minimize deviations from the expected portfolio return level and priority two (P_2) targets the minimization of portfolio variance through efficient diversification of assets.

Building upon the bi-objective SSIM, we examine five alternative models where each is formulated as a complex GP. To position the models, we use a two-step approach. The first step is to define the structural goal equations of the bi-objective asset diversification model. The second step is to tie the model together by introducing the goal constraints that express cognitive dissonance as such conflict relates to investment tastes and beliefs.

5.2.1 Canonical Bi-objective MBPM

The canonical goal constraint equations for the traditional MBPM follow.

$$NL1\colon \sum_{j=1}^{n+1} \varepsilon_j^2 x_j - h_1^+ = 0 \tag{15}$$

$$Beta\colon \sum_{j=1}^{n} \beta_j^V x_j = \beta_M \tag{16}$$

$$Full\ Inv\colon \sum_{j=1}^{n} x_j = 1.0 \tag{17}$$

$$E(r)\colon \sum_{j=1}^{n} r_j x_j + h_4^- - h_4^+ = R_{R_p} \tag{18}$$

[3] It is well known that Markowitz mean-variance optimization often leads to unbalanced portfolios that are optimal in-sample but perform poorly out-of-sample. Our study does not attempt to circument this problem. For recent comments on out-of-sample mean-variance performance, see Fernandes et al. (2020).

Here, $\sum_{j=1}^{n} r_j x_j$ is the fraction of the available capital invested in the expected return of asset *j*. Equation (15) and (16) frame the unsystematic and systematic risk goals. Equation (15) expresses the variance of the idiosyncratic risk (ε_j) for the *n* investment securities as well as the variance of returns for the market proxy, σ^2, as the *n+1* security. The canonical form of the SSIM requires the portfolio beta to equal the weighted sum of the individual security beta coefficients as represented by equation (16). Equation (17) forces the portfolio to be fully invested. Equation (18) is a goal constraint. R_{R_p}, is used to set the required return for the efficient portfolio.

5.2.2 ESG Affinity Bias

Individual security responses to pervasive ESG systemic risk production factors are modeled in equations (19) through (21). These goal constraints equate the *j-th* securities contribution to sustainable investing in return-to-scale units.

$$E_{FSI}: \sum_{j=1}^{n} RtS_j^E\, x_j + h_5^- - h_5^+ = RtS_p^E \tag{19}$$

$$S_{FSI}: \sum_{j=1}^{n} RtS_j^S\, x_j + h_6^- - h_6^+ = RtS_p^S \tag{20}$$

$$G_{FSI}: \sum_{j=1}^{n} RtS_j^G\, x_j + h_7^- - h_7^+ = RtS_p^G \tag{21}$$

Here RtS_p^E, RtS_p^S RtS_p^G= 1.0; or, constant returns-to-scale for each sustainable dimension.

5.2.3 Loss-Aversion Bias and CVaR

The goal expression of the dynamically estimated option-priced CVaR for each *j-th* security is expressed in equation (22).

$$CVaR: \sum_{j=1}^{n} CVaR_j + h_8^- - h_8^+ = 0.0 \tag{22}$$

Here, $CVaR_j$ is as computed in equation (14).

5.2.4 Layered Goals and the MBPM

This section of the study demonstrates the NLGP's domain of applicability as an efficient MBPM processor. Specifically, we tie the structural equations to various model characterizations of the following behavioral biases – mean-variance (M1); loss-aversion bias (M2); 'brown' ESG affinity bias targeting (M3); 'green' affinity bias targeting; and mean-variance with low-priority loss-aversion targeting (M5).

$$M1: Min\ Z = \{P_1[h_4^-], P_2[h_1^+]\} \tag{23}$$

$$M2: Min\ Z = \{P_1[h_4^-], P_2[h_8^+], P_3[h_1^+]\} \tag{24}$$

$$M3: Min\ Z = \{P_1[h_4^- + h_5^-], P_2[h_1^+ + h_6^-], P_3[h_8^+ + h_7^-]\} \tag{25}$$

$$M4: Min\ Z = \{P_1[h_4^- + h_5^- + h_6^- + h_7^-], P_2[h_1^+], P_3[h_8^+]\} \tag{26}$$

$$M5: Min\ Z = \{P_1[h_4^-], P_2[h_1^+], P_3[h_8^+]\} \tag{27}$$

Mean-Variance Targeting. Equation (23) presents the mean-variance targeting model, M1. The first two priority levels state the bi-objective mean-variance optimization. Although there are no other prioritized goals, the goal statements in equations (18) through (22) continue to bind the model decision space.

ESG Affinity and confirmation biases. Scalability across more than two goals makes choosing an optimal compromise solution a non-trivial task. The best available evidence implicates a need to seek layered goals and efficiently diversified ESG-focused funds for risk mitigation, alpha production, resilience to negative exogenous shocks (Cerqueti, Ciciretti, Dalò, & Nicolosi, 2020).[4] To this end, models M3 and M4, equations (25) and (26), are crafted from the equilibrium analysis of (Pastor et al., Forthcoming 2020). Among other findings, this study reports a fund separation effect based on investor risk aversion. That is, strongly aggressive ESG investors adopt portfolios with a green tilt (e.g., model M4) and investors with weaker ESG goal taste angle for a brown tilt (e.g., model M3). In equilibrium, the tilts are larger when risk aversion tends towards risk tolerance. The study surmises that stronger ESG tastes can lead to riskier expected returns, especially when risk aversion is low, and the average ESG taste is high.

The common structure across M3 and M4 is prioritized attainment for network scale bias in ESG portfolio returns; or ESG goal taste. Model M3 prioritizes ESG affinity as sub-goals across two-different priority levels. The first-priority goal has two sub-goals. One is to minimize deviation from the portfolio expected rate of return target. The second sub-goal within priority one is an environment returns-to-scale target (E). The second-level priority level also states two sub-goals – portfolio variance and the returns-to-scale objective for the social policy impact (S). The third priority accounts for sub-goals on governance (G) and CVaR. Because the ESG goal targets spread across three hierarchical priorities, the model is considered 'brown.'

Model M4 is decidedly representative of the ESG 'green' investor. As with model M3, deviation from portfolio expected return is expressed within priority one. However, this priority level also includes sub-goals for all three ESG targets. Following the traditional model statement, priority two minimizes portfolio expected variance. Added to the structure is a third priority to control for the CVaR deviation 'loss-aversion' goal target.

[4] For additional evidence on confirmation bias, see US SIF (The Forum for Sustainable and Responsible Investing) Foundation's 2018 *Report on US Sustainable, Responsible and Impact Investing Trends*. https://www.ussif.org/trends .

Loss-Aversion Targeting. After the prioritization of the portfolio return objective, model M2 prioritizes the loss aversion bias goal after seeking portfolio return as a priority one goal. The third priority goal seeks the traditional efficient diversification of assets. M2 is stated in a manner that is consistent with the Rickenberg (2020) representation of the 'tail risk targeting' strategy. Rickenberg provides evidence that higher Sharpe ratios, better drawdown protection, and higher utility gains for loss-averse investors are achievable by risk-targeting and switching between volatility and CVaR targeting during periods of a bear-market regime. In the context of this study, the Rickenberg strategy would imply risk-averse investors should implement model M2 (equation (24)) or model M5 (equation (27)) based on investor's alternating risk attitudes. As risk aversion alternates to risk tolerance, the investors should consider rebalancing their portfolio by following model M1 diversification strategies.

5.3 Efficient Portfolios

As a prelude to the analysis of goal hierarchy across models and for consistency in our discussion, we utilize extant terminology for efficient portfolio features from Sanghvi and Dash (1978).

Corner Portfolio: An efficient set is comprised of corner portfolios. A corner portfolio is delineated whenever there is a change to the slope of the efficient set. Alternatively, an observed characteristic of adjacent corner portfolios is the addition, deletion, or significant asset rebalancing across two adjacent corner portfolios.

Maximum Rate-of-Return Portfolio: The corner portfolio with the highest expected rate of return is referred to as the 'maximum-rate-of-return' portfolio (MRP).

Global Minimum-Variance-Portfolio: The portfolio with the smallest expected risk is the 'global minimum variance portfolio' (GMVP).

Core Securities: Core securities are those individual assets that appear in corner portfolios.

5.4 Core Security Attributes

Table 3 presents the market metrics for the core securities of the client portfolio. Return and risk serve as inputs to the MBPM quadratic optimization problem. CVaR is obtained from section 4.3 above. Market ESG elasticity estimates and the associated return-to-scale are from section 4.2.

Table 3: Market Characteristics of Core Securities

Tickers		Return	Risk	CVaR	RtS^E	RtS^S	RtS^G	RtS
NEE	NextEra Energy, Inc.	1.24%	3.27%	18.79%	0.196	0.140	-0.272	0.064
DUK	Duke Energy Corp.	0.08%	3.84%	11.00%	0.096	-0.318	-1.059	-1.281
TJX	The TJX Co, Inc.	0.53%	3.01%	9.84%	-0.081	-0.021	-0.157	-0.258
CVS	CVS Health Corp	-0.82%	4.59%	8.09%	-0.778	0.387	1.128	0.737
KO	Coca-Cola Co.	0.19%	2.33%	5.13%	-0.217	-0.160	0.103	-0.274
PG	Procter & Gamble Co.	-0.36%	3.61%	9.07%	-0.575	0.760	-0.341	-0.155
PEP	PepsiCo, Inc.	0.39%	2.58%	11.24%	0.408	0.828	0.058	1.294
DPS	Dr. Pepper Snapple	1.33%	4.34%	14.96%	-0.193	1.192	0.734	1.733
F	Ford Motor Co.	-0.77%	5.10%	2.87%	-0.081	-0.021	-0.157	-0.258
EBAY	eBay, Inc.	1.23%	5.57%	4.54%	-0.553	0.043	0.234	-0.276
HBI	Hanesbrands, Inc.	-0.92%	6.07%	3.45%	-1.949	0.691	0.561	-0.696
WMT	Walmart, Inc.	0.11%	4.44%	10.02%	1.781	1.094	1.057	3.931
VZ	Verizon Comm. Inc.	0.30%	4.43%	5.48%	-0.725	1.795	0.157	1.227
BA	The Boeing Co.	2.74%	5.25%	33.80%	0.105	0.981	1.007	2.093
QCOM	Qualcomm, Inc.	-0.99%	6.83%	7.46%	2.315	0.037	0.524	2.876
V	Visa, Inc.	1.55%	3.20%	13.32%	0.400	0.243	1.043	1.685
LOW	Lowe's Companies	0.56%	5.16%	9.89%	-0.363	1.187	1.689	2.513
CTAS	Cintas Corp.	1.96%	3.50%	17.06%	-0.243	0.445	0.169	0.371

5.5 Comparative Efficient Sets

For comparative purposes, we also enumerate an equally weighted client portfolio (aka, EqWG). As shown by equation (28), the equally weighted portfolio requires adding n additional constraints:

$$\sum_{j=1}^{n} x_j = (1/n) \qquad where \quad x_j \begin{cases} 1 & if(i=j) \\ 0 & if(i \neq j) \end{cases}. \tag{28}$$

Reference is made to figure 8. The traditional mean-variance (dark blue) and SSIM (maroon) efficient sets are obtained by application of Lemke's complementary slackness algorithm (Cottle, Pang, & Stone, 1992). For reference, corner portfolio solutions are displayed on the mean-variance efficient set. As expected, the material difference between the mean-variance and SSIM efficient set occurs at and around the expected return level of the GMVP. For comparative purposes, superimposed are efficient sets for five alternative layered goal priority structures. The MBPM efficient sets are color-coded as follows: M1-light blue; M2-green; M3-black; M4 red; and M5-light green. Lastly, the gray parabola with zero risk tangency provides a theoretical visual for all efficient set solutions[5].

[5] Observe a visual of the MBPM/NLGP computation and solution in WinORS$_{e\text{-}AI}$ 2020 at http://bit.ly/NLGPSampleSolution.

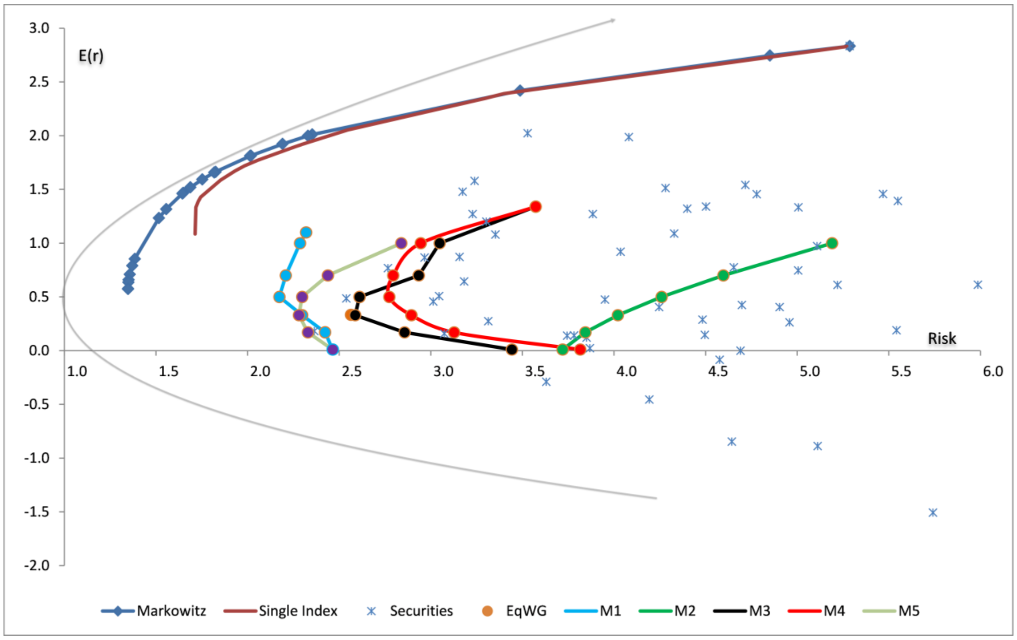


Figure 8: Comparative Efficient Frontiers

As previously stated, the mean-variance efficient set, M1, dominates the other MBPM efficient sets. This finding reinforces simulation-based findings provided by Pfiffelmann, Roger, and Bourachnikova (2016). This study demonstrates how most BPM portfolios and BPM portfolios with diminishing sensitivity and loss-aversion targets are mean-variance efficient but with a higher level of risk. Given the structural goal equations, the efficient set traces out a conic curve beyond the GMVP point. Model M5 is model M1 with a third-priority goal targeting loss-aversion, and, as expected, model M5 is dominated by model M1.

Moreover, as implicated by the Rickenberg (2020) study, there is a cross-over effect from CVaR targeting to variance targeting. As investors exhibit higher levels of risk tolerance, they should adopt the diversification plans offered by model M1. We offer support for this observation by comparing Sharpe ratios for portfolios with an expected return of 0.70% across the efficient alternative sets. The ratios are presented in tables 4 – 8. For model M1, the Sharpe ratio at this return level is 31.69% versus 28.70% for model M5. For the corresponding GMVP portfolios (expected return of 0.01%), the respective Sharpe ratios are the same at 0.41%.

By contrast, loss-aversion risk targeting, model M2, produces an efficient set that is dominated by all other MBPM efficient sets. The shape of the conic curve for this model is consistent with the risk-averse investor seeking portfolio opportunities from its MRP to the respective GMVP. Comparative Sharpe ratios enable further confirmation. For the GMVP, the Sharpe ratio is lower compared to models M1 and M5 at 0.27%. For the portfolio with an expected return of 0.70%, we find a lower Sharpe ratio (15.23%) compared to those produced for models M1 and M5.

The two alternate ESG affinity bias efficient sets, M3 and M4, provide empirical confirmation of the hypotheses offered in the contemporary literature. The 'green' investor model, M4, demonstrates that higher portfolio returns are available, albeit with higher risk levels, for the strongly aggressive ESG investor. By contrast, the 'brown' ESG investor only earns higher portfolio returns at low rates of expected return.

The Sharpe ratio analysis supports observational findings for models M3 and M4. Both models produce an identical Sharpe ratio of 37.50%. At an expected return level of 0.70%, green model M4 renders a Sharpe ratio of 25.04%. At the same return level, the 'brown' ESG portfolio produced a Sharpe ratio of 23.85%. However, for the GMVP portfolios, the 'brown' ESG portfolio generated a higher Sharpe ratio (0.29%) versus the lower 'green' ESG targeted portfolio (0.26%).

Table 4: M1 Efficient Set: Mean-Variance Targeting

Tickers	**M1-1**	**M1-2**	**M1-3**	**M1-4**	**M1-5**	**M1-6**	**M1-7**
NEE		20.00%	20.00%	20.00%	25.00%	25.00%	28.26%
DUK		20.00%	20.00%	20.00%	25.00%	25.00%	25.00%
TJX		7.78%	19.65%	20.00%	8.63%		2.02%
CVS		12.22%	0.35%				
KO	67.08%	20.00%	20.00%	20.00%	25.00%	8.75%	
PG	32.92%	20.00%	20.00%				
PEP				18.42%			
DPS				1.58%	6.89%	24.62%	25.00%
F							
EBAY							
HBI							
WMT							
VZ							
BA							
QCOM							
V							
LOW							
CTAS					9.48%	16.63%	19.72%
Portfolio Return	0.0100%	0.1700%	0.3300%	0.5000%	0.7000%	1.0000%	1.1000%
Portfolio Risk	2.4651%	2.4230%	2.2982%	2.1742%	2.2090%	2.2868%	2.3200%
Sharpe Ratio	0.41%	7.02%	14.36%	23.00%	31.69%	43.73%	47.41%

Model-1: P_1=Return; P_2=Risk

Table 5: M2 Efficient Set: Loss-Aversion Targeting

Tickers	M2-1	M2-2	M2-3	M2-4	M2-5	M2-6
NEE						
DUK						
TJX						
CVS						
KO						
PG						
PEP						
DPS						
F	31.62%	26.55%	21.49%	16.11%	9.78%	0.29%
EBAY	40.97%	48.77%	56.56%	64.84%	74.58%	89.19%
HBI	27.41%	24.68%	21.95%	19.05%	15.64%	10.52%
WMT						
VZ						
BA						
QCOM						
V						
LOW						
CTAS						
Portfolio Return	0.0100%	0.1700%	0.3300%	0.5000%	0.7000%	1.0000%
Portfolio Risk	3.7200%	3.8440%	4.0213%	4.2603%	4.5972%	5.1907%
Sharpe Ratio	0.27%	4.42%	8.21%	11.74%	15.23%	19.27%

Model-2: P_1=Return; P_2=CVaR; P_3=Risk

Table 6: M3 Efficient Set: Brown Tilt ESG Affinity Targeting

Tickers	M3-1	M3-2	M3-3	M3-4	M3-5	M3-6	M3-7
NEE				2.26%	11.80%	8.18%	
DUK							
TJX		20.00%	20.00%	20.00%			
CVS							
KO							
PG	26.27%						
PEP	7.34%	13.56%	21.30%	5.59%			
DPS			5.45%	20.00%	25.37%	12.03%	
F							
EBAY							
HBI							
WMT	63.92%	43.08%	48.47%	51.39%	57.28%	55.10%	53.42%
VZ	2.07%	15.09%	4.78%				
BA				0.76%	5.55%	24.69%	46.58%
QCOM		8.27%					
V							
LOW							
CTAS							
Portfolio Return	0.0100%	0.1700%	0.3300%	0.5000%	0.7000%	1.0000%	1.3400%
Portfolio Risk	3.4432%	2.8571%	2.5877%	2.6113%	2.9347%	3.0486%	3.5733%
Sharpe Ratio	0.29%	5.95%	12.75%	19.15%	23.85%	32.80%	37.50%

Model-3: P_1=Return & E; P_2=Risk & S; P_3=CVaR & G

Table 7: M4 Efficient Set: Green Tilt ESG Affinity Targeting

Tickers	M4-1	M4-2	M4-3	M4-4	M4-5	M4-6	M4-7
NEE							
DUK							
TJX		19.77%	20.00%	14.74%			
CVS	12.74%	8.13%	4.30%				
KO							
PG							
PEP	5.29%						
DPS			10.05%	15.04%	23.22%	13.97%	
F							
EBAY							
HBI							
WMT	80.54%	58.13%	55.53%	54.07%	56.11%	54.21%	53.42%
VZ	1.43%	4.25%					
BA					3.55%	21.93%	46.58%
QCOM							
V			0.70%	7.16%	13.64%	9.77%	
LOW		9.73%	9.42%	8.99%	3.48%	0.12%	
CTAS							
Portfolio Return	0.0100%	0.1700%	0.3300%	0.5000%	0.7000%	1.0000%	1.3400%
Portfolio Risk	3.8165%	3.1279%	2.8945%	2.7747%	2.7954%	2.9460%	3.5733%
Sharpe Ratio	0.26%	5.43%	11.40%	18.02%	25.04%	33.94%	37.50%

Model-4: P_1=Return, E, S, G; P_2=Risk; P_3=CVaR

Table 8: M5 Efficient Set: Mean-Variance with Loss Aversion Targeting

Tickers	M1-1	M1-2	M1-3	M1-4	M1-5	M1-6
NEE			13.24%	29.50%	48.63%	77.32%
DUK						
TJX						
CVS						
KO	67.08%	96.10%	86.76%	70.50%	51.37%	22.68%
PG	32.92%	3.90%				
PEP						
DPS						
F						
EBAY						
HBI						
WMT						
VZ						
BA						
QCOM						
V						
LOW						
CTAS						
Portfolio Return	0.0100%	0.1700%	0.3300%	0.5000%	0.7000%	1.0000%
Portfolio Risk	2.4651%	2.3302%	2.2808%	2.2980%	2.4387%	2.8392%
Sharpe Ratio	0.41%	7.30%	14.47%	21.76%	28.70%	35.22%

Model-5: P_1=Return; P_2=Risk; P_3=CVaR

5.6 Quantifying Dissonance across Layered Goal Attainment

Quantifying financial cognitive dissonance is made possible by a review of layered goal achievement. Goal target achievement is a study of the under- and over-achievement levels as captured by the variables h^- and h^+, respectively. Table 9 presents model comparisons of goal attainment across the five study models. When considering the impact of loss-aversion bias, the focus is placed on models M2 and M5. We note that model M2 – a model designed to target loss-aversion bias as the top-level goal – overachieved the CVaR goal of 0.0 by 0.0421 units. By contrast, model M5 expressed the loss-aversion goal at the third priority level. When compared to model M2, the M5 model experienced higher overachievement at 0.1175 units.

ESG affinity goal targets solve with a zero level of under- and over-achievement for both the 'green' model (M4) and the 'brown' model (M3). Table 10 data provides insight into the performance of these two models. Recall, model M3 spread the ESG sustainability targets over three different priority levels, whereas M4 expressed all sustainable ESG targets within the first-priority layer. Model M3 experienced third-priority underachievement of 18.49. Comparably, underachievement for model M4 – the green model – is zero. Priority layer two exposes a negligible amount of underachievement for models M3 (0.0010) and M4 (0.0012).

Table 9: Goal Attainment by Model

Constraints	**RHS**	**M1-5** Under (h-)	**M1-5** Over (h+)	**M2-5** Under (h-)	**M2-5** Over (h+)	**M3-5** Under (h-)	**M3-5** Over (h+)	**M4-5** Under (h-)	**M4-5** Over (h+)	**M5-5** Under (h-)	**M5-5** Over (h+)
NL1	0.0000	n/a	0.0005	n/a	0.0032	n/a	0.0010	n/a	0.0012	n/a	0.0005
Beta	0.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
Full Inv	1.0000	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
E(r)	0.0070	0.0000	n/a	0.0000	n/a	0.0000	n/a	0.0000	n/a	0.0000	n/a
E_{FSI}	1.0000	1.0159	0.0000	1.7251	0.0000	0.0000	0.0000	0.0000	0.0000	1.0159	0.0000
S_{FSI}	1.0000	1.0143	0.0000	0.8618	0.0000	0.0000	0.0000	0.0000	0.0000	1.0143	0.0000
G_{FSI}	1.0000	1.0793	0.0000	0.7527	0.0000	0.1849	0.0000	0.0000	0.0000	1.0793	0.0000
CVaR	0.0000	0.0000	0.1175	0.0000	0.0421	0.0000	0.1363	0.0000	0.1246	0.0000	0.1175

Note: The constraints are defined in equations (15) through (22)

Table 10: Underachievement of Priorities

Tickers	**M1-5**	**M2-5**	**M3-5**	**M4-5**	**M5-5**
Priority 1	None	None	None	None	None
Priority 2	0.0003	1.7251	0.0010	0.0012	0.0005
Priority 3	n/a	0.0032	0.1849	None	1.0159

Note: The priorities for each model are as defined in equations (23) through (27)

6. Summary and Conclusions

In this study, we examined a new approach to BPM when the decision problem is layered with multiple, hierarchical, and conflicting goals. We specifically focused on the behavioral emotions of the affinity bias for sustainable investments and loss-aversion bias. Previous BPM seeking to incorporate sustainability goals mainly focused on single-index and ranking methods. Our study introduced a new dimension to sustainability and BPM. We implemented the PRS algorithm to produce three new pervasive factors to represent the ESG dimensions. Once created, the study examined the network impact of the three ESG indexes on asset returns by using the K4-RANN to estimate the weights of a six-factor Fama and French production-theoretic asset valuation model. The K4-RANN-based production theoretic model provided factor elasticity estimates as well as network scale returns. These outputs formed the valuation goal constraints that encapsulated ESG affinity bias within the proposed BPM model extension. Similarly, asset-level loss-aversion metrics were obtained from market data. We invoked a put-option structured model to estimate CVaR specific estimates for the statement of a loss-aversion goal constraint.

Lastly, the study demonstrated why it is possible to model investor cognitive dissonance – decision-making involving layered and conflicting behaviors – as a goal program. Previous findings introduced a factor-based NLGP as a decision-structure to replicate the mean-variance portfolio optimization process when there are two or fewer investment priorities. We confirmed extant findings on comparative NLGP and mean-variance optimization and then extended the

New contributions about the complexity of investor goal layering were elicited from the five models investigated in this study. First, and as expected, the benchmark standard mean-variance and SSIM portfolio models dominated all behavioral portfolio model diversification strategies. Second, Sharpe performance ratios for ESG affinity-bias portfolios were impacted by layered investor goal priorities. Investor's expressing a 'green' sustainability tilt experienced differential performance characteristics than those representing a more 'brown' sustainability tilt. After comparing risk-adjusted return results across modeled portfolios, we found return dampening effects introduced by the adoption of ESG goals. But, importantly, we also report evidence on how sustainability bias can be partially overcome by switching from 'brown' to 'green' layered goals at different expected return levels.

The modeling effort also provided confirmatory insights on how to emphasize 'loss aversion' investor bias. We confirmed extant literature that argues 'loss-aversion' bias is likely to produce smaller Sharpe performance ratios. We were also able to confirm that a shift from prioritizing loss-aversion (CVaR) to targeting traditional variance minimization leads to improved Sharpe ratios. These preliminary findings notwithstanding, there are several important areas where new research is needed. The time-variation of the new ESG factors received attention but deserves a more detailed study. Also, the examination of alternate priority structure models is a worthy undertaking for future research. Lastly, the application of a machine learning method to the estimation of a production-theoretic model of asset returns is relatively new but is gaining traction in the literature. A more extensive study on the use of the K4-RANN should provoke robust results in the area of networked asset return production. The application results provided here extend our understanding of the importance of modeling layered goals in MBPM. For the rational investor, the multiobjective model proved capable of solving the efficient portfolio selection problem while meeting biased goals as closely as possible.

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Forthcoming.

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Appendix A

ESG Sustainable Impact Dimensions

Environmental	Social	Governance
Environment Policy	Human Rights	Corporate Governance
Environment Performance	Labor Standards	Code of Ethics
Climate Change	Health and Safety	Bribery and Corruption
Nuclear Energy	Employee Development	Death Penalty
Biodiversity	Supply Chain Standards	Military Expenses

Source: Invesco, Vigeo Eiris, https://www.invesco.com/corporate/about-us/esg

Appendix B

Results from the K4-RANN Analysis – Page 1

		Rm-Rf	SMB	HML	RtS^E	RtS^S	RtS^G	RtS	MSE
XOM	Exxon Mobil Corporation	-1.746	-0.737	0.585	-0.094	-0.039	0.787	0.654	0.00020
CVX	Chevron Corporation	-1.532	-0.001	0.554	-0.211	0.553	0.763	1.104	0.00028
COP	ConocoPhillips	-0.034	0.279	1.000	0.428	0.375	0.756	1.558	0.00032
IEZ	iShares US Oil Eqp & Srv ETF	-0.457	0.040	0.408	0.122	0.065	0.707	0.894	0.00012
PFE	Pfizer Inc	-0.284	0.016	0.000	-0.163	0.128	0.087	0.051	0.00000
JNJ	Johnson & Johnson	-1.407	-0.519	-0.518	-0.265	-0.159	-0.097	-0.520	0.00007
ABT	Abbott Laboratories	-0.034	0.266	-0.224	0.058	0.071	0.439	0.567	0.00002
CI	Cigna Corporation	-1.670	-2.011	-0.582	0.341	0.125	0.236	0.701	0.00046
AMGN	Amgen Inc	-0.340	0.008	-0.629	-0.661	0.000	0.316	-0.345	0.00006
AGN	Allergan plc	-0.712	-0.218	0.013	-0.819	-0.432	0.336	-0.915	0.00018
TMO	Thermo Fisher Scientific Inc	0.079	0.085	0.085	0.075	0.064	0.136	0.275	0.00000
IBB	iShares Nasdaq Biotech ETF	0.044	0.157	-0.026	-0.057	0.162	0.256	0.361	0.00009
ABBV	AbbVie Inc	0.925	-0.533	-1.805	0.196	0.134	0.778	1.108	0.00055
CSCO	Cisco Systems, Inc	0.612	0.715	0.048	0.729	0.924	0.998	2.652	0.00016
MSFT	Microsoft Corporation	0.180	0.223	0.063	0.756	0.125	0.642	1.524	0.00006
QCOM	QUALCOMM Incorporated	-0.223	-0.773	-1.052	2.315	0.037	0.524	2.876	0.00087
AAPL	Apple Inc	-0.338	0.279	-0.261	0.604	0.133	0.026	0.762	0.00005
GOOG	Alphabet Inc	-0.818	-0.311	-0.976	0.204	-0.372	0.562	0.395	0.00010
V	Visa Inc	-0.152	0.072	-0.045	0.400	0.243	1.043	1.685	0.00004
NTAP	NetApp, Inc	0.123	0.746	0.598	0.966	0.829	0.729	2.523	0.00052
ACN	Accenture plc	-0.067	-0.097	-0.261	0.185	0.159	0.186	0.530	0.00000
TEL	TE Connectivity Ltd	0.073	0.089	-0.012	0.194	0.049	0.159	0.402	0.00000
NEE	NextEra Energy, Inc	-0.106	0.287	0.193	0.196	0.140	-0.272	0.064	0.00001
DUK	Duke Energy Corporation	-0.667	-0.172	0.032	0.096	-0.318	-1.059	-1.281	0.00006
F	Ford Motor Company	-0.747	0.007	0.131	-0.081	-0.021	-0.157	-0.258	0.00004
TJX	The TJX Companies, Inc	0.513	0.673	0.567	0.477	0.712	0.623	1.812	0.00005
LOW	Lowe's Companies, Inc	-0.480	-1.682	-0.248	-0.363	1.187	1.689	2.513	0.00053

Results from the K4-RANN Analysis – Page 2

		Rm-Rf	**SMB**	**HML**	RtS^{E}	RtS^{S}	RtS^{G}	RtS	**MSE**
GPS	The Gap, Inc	0.063	0.236	-0.014	0.270	0.822	0.226	1.318	0.00021
EBAY	eBay Inc	0.659	-0.248	-0.556	-0.553	0.043	0.234	-0.276	0.00009
WYN	Wyndham Worldwide Corp	-0.079	-0.135	-0.158	0.007	0.074	0.253	0.335	0.00001
HBI	Hanesbrands Inc	-0.951	-0.565	-0.675	-1.949	0.691	0.561	-0.696	0.00059
TWX	Time Warner Inc	0.034	-0.108	-0.044	-1.196	-0.031	0.058	-1.169	0.00009
JPM	JPMorgan Chase & Co	0.044	-0.061	-0.247	0.010	0.333	0.306	0.648	0.00002
WFC	Wells Fargo & Company	-0.694	-0.603	-0.595	-0.196	0.267	0.256	0.328	0.00008
BAC	Bank of America Corp.	0.070	0.037	-0.108	0.081	0.223	0.186	0.490	0.00002
CB	Chubb Limited	-0.663	-0.908	-0.787	-0.446	-0.541	-0.402	-1.389	0.00006
PNC	PNC Financial Srv Grp, Inc.	0.109	-0.115	-0.816	-0.270	0.472	0.330	0.532	0.00006
MMC	Marsh & McLennan Co. Inc	-0.306	-0.294	-0.350	-0.330	-0.326	-0.327	-0.983	0.00001
BBT	BB&T Corporation	0.171	0.080	-0.154	0.084	0.402	0.553	1.039	0.00003
AMG	Affiliated Managers Group, Inc	-0.500	-0.031	-0.553	-0.242	0.216	-0.014	-0.040	0.00008
AIV	Apartment Investment and Management Company	-0.937	-0.105	0.061	-0.125	-0.230	-0.707	-1.062	0.00005
GNW	Genworth Financial, Inc	-0.116	-0.069	-0.101	-0.101	-0.088	-0.098	-0.287	0.00003
C	Citigroup Inc	-0.261	-0.345	-0.353	-0.262	-0.087	-0.165	-0.514	0.00004
GE	General Electric Company	-2.113	-0.936	-0.534	-2.518	-1.083	-0.297	-3.897	0.00110
BA	The Boeing Company	0.491	-0.341	-0.131	0.105	0.981	1.007	2.093	0.00018
UTX	United Technologies Corp.	-0.104	-0.049	-0.139	-0.017	0.098	0.243	0.324	0.00000
CTAS	Cintas Corporation	-0.186	0.382	-0.196	-0.243	0.445	0.169	0.371	0.00002
GD	General Dynamics Corporation	0.054	-0.046	-0.451	-0.629	-0.309	0.232	-0.706	0.00002
DHR	Danaher Corporation	-0.193	0.250	0.026	0.721	-0.166	0.802	1.358	0.00004
AME	AMETEK, Inc	0.082	0.224	-0.196	0.282	0.372	0.421	1.075	0.00002
ATU	Actuant Corporation	-0.430	-0.243	-0.102	-0.256	-0.137	-0.221	-0.615	0.00004
MON	Monsanto Company	0.059	0.011	0.158	-0.076	-0.015	0.148	0.057	0.00000
XLB	Materials Sector SPDR Fund	-0.333	-0.211	-0.209	-0.112	-0.077	0.114	-0.076	0.00001
WMT	Walmart Inc	-0.882	-2.064	-0.237	1.781	1.094	1.057	3.931	0.00054

Results from the K4-RANN Analysis – Page 3

		Rm-Rf	SMB	HML	RtS^E	RtS^S	RtS^G	RtS	MSE
CVS	CVS Health Corporation	-1.367	-1.106	0.019	-0.778	0.387	1.128	0.737	0.00027
KO	The Coca-Cola Company	-1.112	-0.550	-0.164	-0.217	-0.160	0.103	-0.274	0.00002
PG	Procter & Gamble Company	-1.820	-0.726	-0.685	-0.575	0.760	-0.341	-0.155	0.00018
PEP	PepsiCo, Inc	-1.878	-0.687	-1.071	0.408	0.828	0.058	1.294	0.00010
HSY	The Hershey Company	-1.231	-0.243	-0.570	-0.215	0.308	-0.370	-0.277	0.00012
CL	Colgate-Palmolive Company	-0.519	-0.309	-0.275	-0.309	-0.167	-0.193	-0.669	0.00001
DPS	Dr Pepper Snapple Group, Inc	2.540	0.059	0.333	-0.193	1.192	0.734	1.733	0.00043
VZ	Verizon Communications Inc	-0.867	-0.629	0.034	-0.725	1.795	0.157	1.227	0.00027
T	AT&T Inc	-0.296	-0.324	-0.498	-0.703	0.847	-0.194	-0.050	0.00007
IJR	iShares S&P SmallCap ETF	-0.482	0.064	-0.157	-0.170	-0.114	0.114	-0.170	0.00001
MDY	SPDR S&P MIDCAP 400 ETF	-0.868	0.059	-0.344	0.034	-0.014	0.370	0.391	0.00002