Life-cycle Investing with Personal Disaster Risk*

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Abstract

This paper extends the life-cycle model by allowing for a small risk of a personal disaster with permanent effects on labour income. We calibrate the model to long-term unemployment (LTU) in the US, where it is only partially insured and is known to entail scarring effects. Despite its low probability, such risk boosts early investment in the risk-free asset. Consequently, the optimal equity portfolio share is relatively flat over the life cycle, consistent with observed investment profiles. A negligible probability of LTU or full insurance against it result in both higher optimal risk taking and equity profiles that are downward sloping in age.

Keywords: disaster risk, life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D15, E21, G11

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1 Introduction

Unemployment may lead to persistent earnings losses, that increase in its duration because of skill deterioration. The average length of unemployment spells has remarkably increased in developed economies.[1] In the United States the share of unemployed workers who were jobless for more than one year, while historically low, doubled during the Great Recession episode, reaching 24% of total unemployment in 2014, hitting all education groups (e.g., see Kroft, Lange, Notowidigdo and Katz 2016).[2] The chances of finding a job decrease in unemployment duration, together with unemployment benefits.[3]

In this paper, we investigate the effects of long-term unemployment risk on financial risk taking. We allow for a low probability of entering long-term unemployment with uncertain permanent consequences on human capital in a life-cycle model of consumption and portfolio choice. Such low probability event considerably increases the optimal portfolio share invested in the risk-free asset compared with the case of no unemployment risk. Importantly, optimal stock investment no longer decreases in age but remains remarkably flat over the whole working life, in line with evidence on US portfolios (Ameriks and Zeldes, 2004).

Our model nests the traditional life-cycle one. Thus, when the probability of LTU is zero and/or LTU is fully insured, the agents optimally reduce exposure to risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). This pattern obtains since human capital provides a hedge against shocks to stock returns, which makes financial risk bearing generally acceptable. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth. Investment then gradually declines until retirement, as human capital decreases relative to financial wealth.

[1]The magnitude of LTU varies over time and across industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993a; Davis and von Wachter, 2011) as well as countries (Machin and Manning, 1999).

[2]For instance, in 2013, the share of US unemployed workers with a high school (college) education who had been looking for work for two or more years is 12.8% (13.5%) (see Mayer, 2014).

[3]Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, to the extent that they are more likely to exit the labour force than to become re-employed. The presence of more job openings does not lead to increased employment among individuals who are jobless for more than six months, and this pattern holds across all ages, industries and education levels (Ghayad and Dickens 2012).

When LTU is instead uninsured, even if it occurs with low probability and has uncertain outcomes, the above effect is moderated by the resolution of uncertainty concerning labour and pension income as the worker safely approaches retirement age. Since the risk of long-term unemployment falls as retirement approaches, the resolution of uncertainty compensates for the hedge effect and the optimal investment in stocks is relatively flat over the life cycle.

Another implication of our model is that agents facing LTU risk have similar portfolio preferences. A small probability of personal disaster shrinks the heterogeneity of optimal portfolio choices across agents characterized by different employment histories. In the face of possible, albeit rare and of uncertain size, human capital depreciation, individuals increase precautionary savings to buffer against such possible adverse labour market outcomes. Optimal early consumption consequently falls, becoming higher during both late working years and retirement years.

We model working life careers as a three-state Markov chain driving the transitions between employment and short-term and long-term unemployment states, as in Bremus and Kuzin (2014). Careers are calibrated to broadly match observed US labour market features. Importantly, we allow for human capital erosion during unemployment. When unemployed, individuals receive benefits but simultaneously experience a reduction in the permanent component of labour income which translates to diminished future income prospects. Permanent earning losses are subsequently observed due to skill loss during long-term unemployment (Arulampalam, 2001; Schmieder, von Wachter and Bender, 2016). Importantly, we experiment with a stochastic human capital erosion conditional on long-term unemployment, to represent the uncertainty characterizing future labor market outcomes. We model the personal disaster as a Beta distribution to allow for extremely rare but potentially disastrous labour income shocks. Calibration results refer to the case when the expected human capital erosion is as low as 10%-20% of the permanent labour income component after the second year of unemployment.

This paper is not the first to explicitly connect life cycle precautionary savings to social insurance in general (Hubbard, Skinner and Zeldes, 1995) and to insurance against employment risk in particular (Low, Meghir and Pistaferri, 2010). Our analysis uncovers the link between the share of long-term unemployment risk that is left uninsured and the path of optimal equity risk taking during working years. Such previous life-cycle models with unemployment and self-insurance leave the observed age pattern of stock holding during working life largely unexplained. Some versions of the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labour income. In these models, unemployment risk affects income only during the unemployment spell and has no consequences on subsequent earnings ability (Cocco, Gomes and Maenhout, 2005), even when unemployment is persistent (Bremus and Kuzin, 2014). With no permanent consequence on subsequent earnings ability, the stock holding still counterfactually decreases with age till retirement, although the decrease is less on average than what occurs without unemployment risk. Thus, the possibility of LTU - rather than unemployment per se - restrains risk taking by young and middle-aged workers. Therefore, our model draws attention to a scenario opposite that depicted by Bodie, Merton and Samuelson (1992) and Gomes, Kotlikoff and Viceira (2008) in which the worker is able - if employed - to modify labour supply to buffer income shocks. In fact, the flexible labour supply may enhance risk-taking, thereby compressing precautionary saving and reducing consumption after retirement. However, this option is available ex post only to long-term unemployed who find a new job; what drives our results is the ex ante risk of permanently losing human capital.

Several papers already investigate alternative hypotheses that may explain the relatively flat or moderately increasing stock profile observed in the data that departs from the pattern implied by traditional life-cycle models. More conservative investments in stocks when young may be optimal if there is housing wealth (Cocco, 2004; Kraft and Munk, 2011) or if the expected labour income growth rate is sensitive to the real short-term interest rate (Munk and Sorensen, 2010). Some other prior research relates the resolution of uncertainty over working life to the flattening of the age profile of stock investment. In Bagliano, Fugazza and Nicodano (2014), such flattening depends on the presence of both another risky asset, aside from equities, and a positive correlation between stock returns and permanent labour income shocks. Hubener, Maurer and Mitchell (2016) highlight the possibility of changing family status during working age (i.e., marriage, fertility, divorce), which affects consumption both directly and through labour supply. In Chang, Hong and Karabarbounis (2017), realistic life-cycle profiles of occupational uncertainty and gradual learning about income volatility generate an age-increasing stock investment pattern. Without addressing the merits of these hypotheses, let us stress that the implications of our model concerning long-term unemployment risk are broadly consistent with the empirical evidence regarding risk taking in response to LTU. Our model predicts that financial risk taking increases the more the higher is unemployment benefit protection against long-term unemployment. In line with this evidence, Hombert et al. (2015) document that entrepreneurial risk taking increased in France after a large scale reform, that allowed unemployed entrepreneurs to retain their rights to unemployment benefits for three years in case their venture failed. Additionally, the reform mandated the unemployment insurance fund to cover any gap between their entrepreneurial revenues and their unemployment benefits, providing insurance against cash flow shortfalls. Our model also predicts that, with full protection against LTU (or negligible LTU), the optimal risk taking profile becomes downward sloping instead of flat as in the US. This is the shape of the profile for conditional stockholdings in Norway (Fagereng, Gottlieb and Guiso (2017)), where LTU has been a marginal phenomenon until 2014 (Pedersen, 2015).

The personal disaster risk in our model differs from both the individual stock market disaster in Fagereng, Gottlieb and Guiso (2017) and the aggregate economic collapse explaining asset pricing puzzles in Barro (2006). Both of these shocks hit financial wealth and may occur during retirement as well. Personal disaster risk reminds of the rare idiosyncratic disaster in Schmidt (2016) that appears to capture both the magnitude and the dynamics of the equity risk premium. Such rare personal disaster makes returns to human capital negatively skewed, a feature recently uncovered by Guvenen, Karahan, Ozkan and Song (2015). Moreover, Huggett and Kaplan (2016) show that persistency and negative skewness of earnings shocks reduce the value of human capital well below the level implied by discounting earnings at the risk-free rate and increase its stock component. In this light, our paper documents the large effects of non-normal shocks to labour income on life-cycle savings and investment, following the suggestion in Blundell (2014) of capturing higher moments and nonlinearities in shocks to labour income. Thus, our paper extends the literature on portfolio choice that has so far focused only on non-Gaussian returns to financial assets (e.g., see Guidolin and Timmerman, 2008).

The rest of the paper is organized as follows. In Section 2 we report the empirical evidence on life cycle portflio holdings. Section 3 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 4 and discuss our main results in Section 5. Section 5.3 examines the ability of the model to match the observed stockholdings in real data. Section 5 concludes the paper.

2 Life-Cycle profiles of Households Portfolios

This section builds on the method of Ameriks and Zeldes (2002) to examine the empirical relationship between age and conditional risky shares, i.e. the fraction of financial wealth held in risky assets conditional on participation to the stock market. These life-cycle investment profiles in US data will later be matched with the model-implied profiles.

We pool data from the independent cross-sectional surveys in the Survey of Consumer Finances (SCF), covering the years from 1992 to 2016. The SCF is nationally representative of households of all ages in the United States and collects detailed information on their characteristics and their investment decisions. We classify the household' financial assets into two categories: *safe* and *risky*, following Chang, Hong and Karabarbounis (2018). Safe assets include checking accounts, savings accounts, money market accounts, certificates of deposit, the cash value of life insurance, US government and state bonds, mutual funds invested in tax-free bonds and government-backed bonds, and trusts and annuities invested in bonds and money market accounts. Risky assets include stocks, stock brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate, and pension plans that are a thrift, profit-sharing, or stock purchase plan. In table 1, we report the summary statistics concerning both the households' financial assets composition and households main characteristics. We restrict the sample to households with positive financial assets and with head aged between 21 and 70.

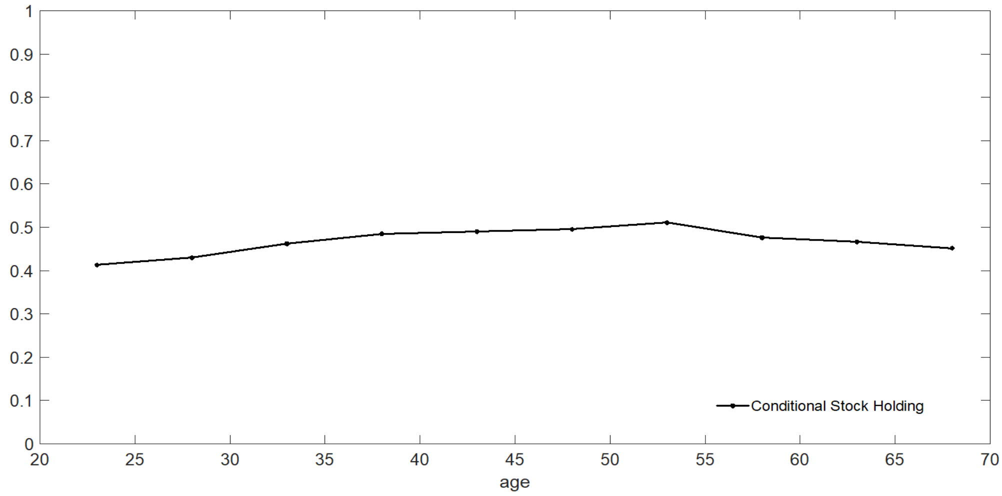
Table 1: Descriptive statistics

Wave	1992	1995	1998	2001	2004	2007	2010	2013	2016
Financial assets									
Amount ($)									
Safe	126,323	135,264	138,320	148,852	139,953	137,447	143,926	126,739	141,793
Risky	70,842	91,448	167,039	202,997	161,592	162,939	129,381	137,308	159,180
Total (Safe+Risky)	197,166	226,712	305,359	351,849	301,544	300,386	273,307	264,046	300,973
Share									
Safe	64.1%	59.7%	45.3%	42.3%	46.4%	45.8%	52.7%	48.0%	47.1%
Risky	35.9%	40.3%	54.7%	57.7%	53.6%	54.2%	47.3%	52.0%	52.9%
Participation									
Safe	43.6%	46.8%	56.3%	60.4%	56.8%	59.2%	55.1%	54.6%	53.5%
Risky	43.6%	46.8%	56.3%	60.4%	56.8%	59.2%	55.1%	54.6%	53.5%
Men	78.6%	76.8%	76.9%	77.0%	75.7%	76.5%	76.6%	75.1%	74.2%
Age	45.6	46.2	46.5	46.5	47.5	48.2	47.6	48.2	48.9
No high school	12.5%	11.5%	10.6%	10.5%	9.4%	9.1%	8.8%	7.6%	11.3%
High school	30.1%	32.6%	31.6%	31.3%	29.9%	31.4%	30.7%	29.2%	24.9%
Some college	24.2%	27.0%	27.2%	26.0%	26.3%	26.3%	26.7%	27.2%	28.4%
College	33.2%	28.8%	30.6%	32.3%	34.4%	33.2%	33.9%	36.1%	35.4%
N (households)	3906	4302	4326	4475	4526	4423	6555	6026	6261

The table reports the average composition of households financial assets and demographic characteristics across various SCF waves (1992 – 2016). The sample is restricted to households with heads aged between 21" and 70 and with a positive amount of financial assets. Nominal variables are expressed in 2015 U.S. dollars.

Over the sample period, with the aging of the baby-boom population, the average age of households heads increases from 45.6 in 1992 to 49 in 2016. *TOGLIEREI forse anche dalla tabella The proportion of households headed by men decreased by* 4%*.* Moreover, the average share of financial wealth invested in risky asset increases during the 1990s. Turning to the focus of this paper, in figure 1 we report the life cycle age profile of the average conditional portfolio share invested in risky assets. The dots represents the five-year average (from age group $21-25$ to age group $66-70$). The conditional risky share is flat over the life cycle, ranging from 40% to 49%.

Figure 1: Conditional Risky Share - SCF data



This figure displays the life cycle profile of conditional risky share of financial assets held by U.S. households grouped by five-years age classes ($21-25$,...,$66-70$). The dots represent the five-year average.

Ideally, we should distinguish the impact of age on household risk taking from that of both the calendar year and birth cohorts. However, the three effects cannot be separately identified. We therefore estimate three regression models where one effect at a time is held constant against the other two, following Ameriks and Zeldes (2002).

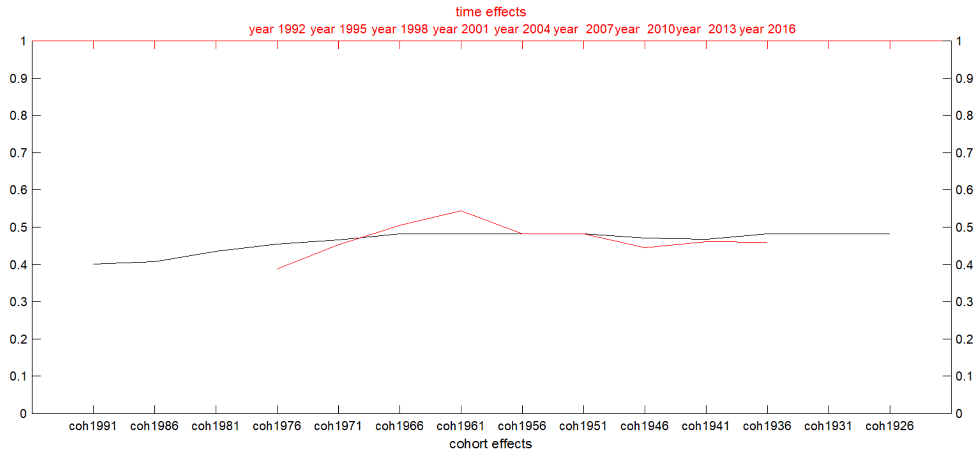
The age dummies are constructed on the basis of five-year age groups, from 21 to 70, and the reference age group is aged between 46 and 50. Similarly, the birth year cohort dummies refer to five birth-year groups (from $1924-1928$ to $1989-1993$) and we take the cohort $1953-1958$ as the reference group. Finally, the time effects refer to the years in which the surveys are collected and we take year 2004 as the reference group. In figure 2 panel a), we report the regression estimates of time and age effects based on OLS estimates with cohort effects excluded (red and black line respectively) ; panel b) reports the time and cohort effects based on OLS estimates with age effects excluded (red and black line respectively) ; finally, panel c) reports the age and cohorts effects based on OLS estimates with time effects excluded (red and black line respectively)[4]. The conditional risky share is remarkably flat across ages and cohorts in all the specifications. The time effects show a sharp increase during the 1990s and a relative slowdown after 2000, a pattern which is robust to all model specifications. These patterns are robust across education levels (see figures 3 and 4).

[4]We set to zero all the coefficients that are not statistically significant at 5% level

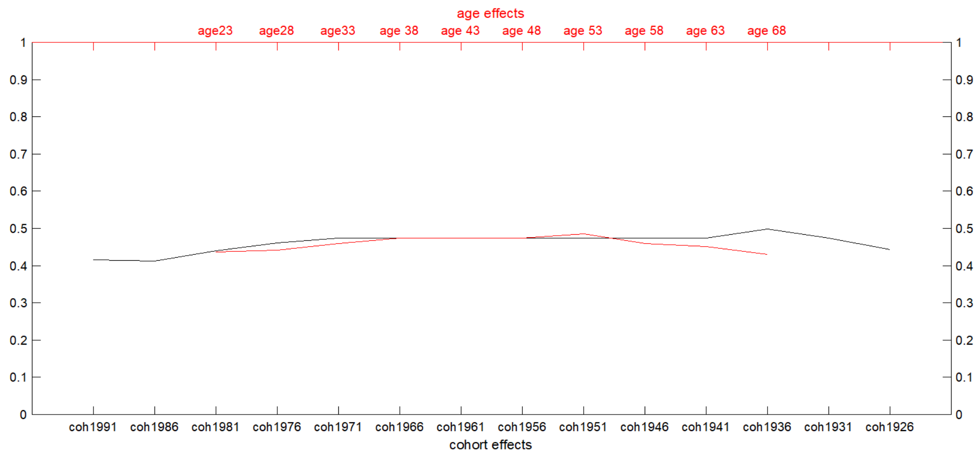
Figure 2: Age, Time and Cohort effects on Conditional Risky Share

Panel (a)

Panel (b)



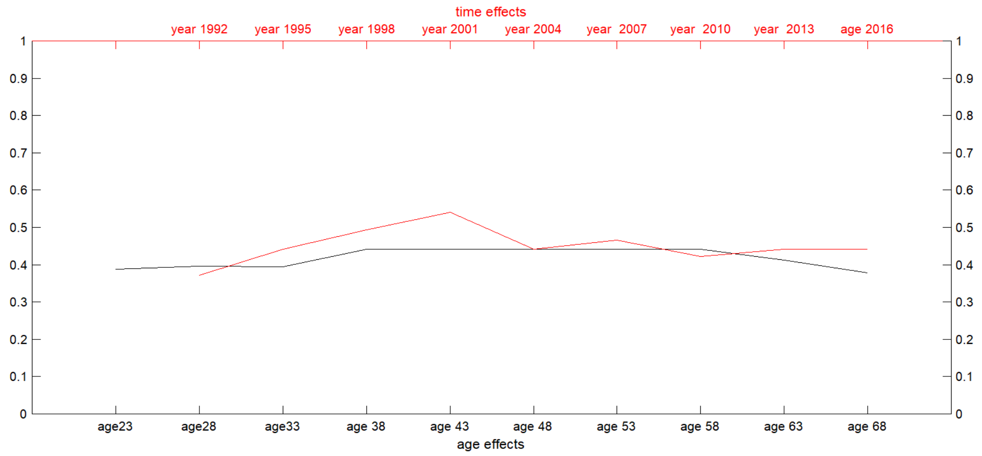
Panel (c)



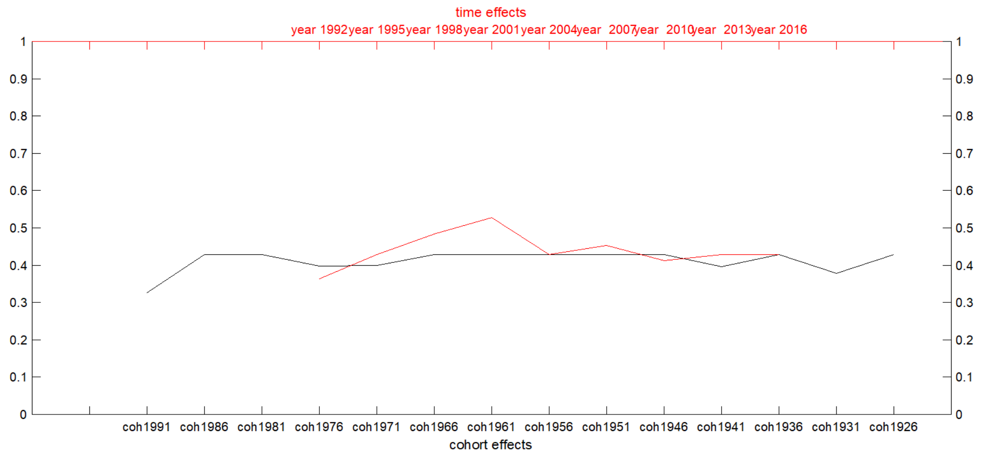
This figure displays the estimated age, cohort and time effects on conditional risky share under different model specifications. In panel (a), the cohort effect is assumed to be constant across ages and periods; in panel (b), the age effect is assumed to be constant across cohorts and periods; in panel (c), the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with head aged between 21 and 7. Coefficients not statistical significant at 5% are set to zero.

Figure 3: Age, Time and Cohort effects on Conditional Risky Share - Education level: High School Diploma or lower

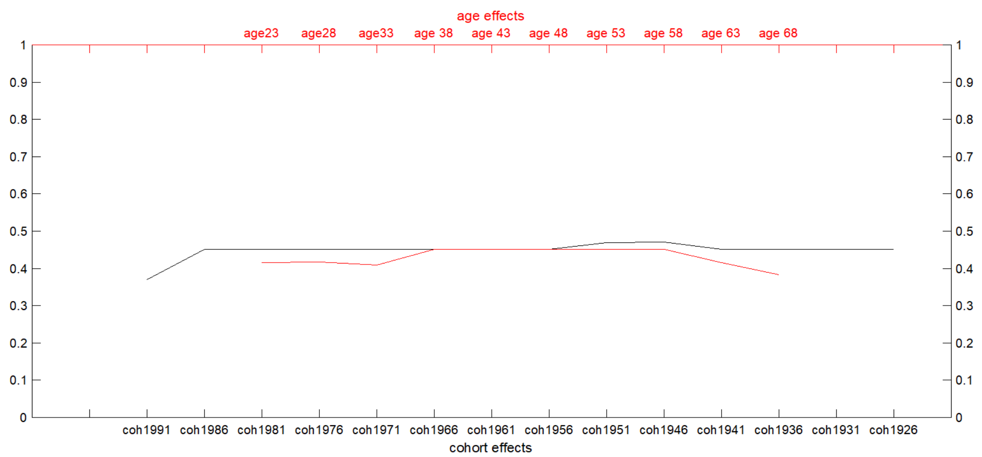
Panel (a)



Panel (b)



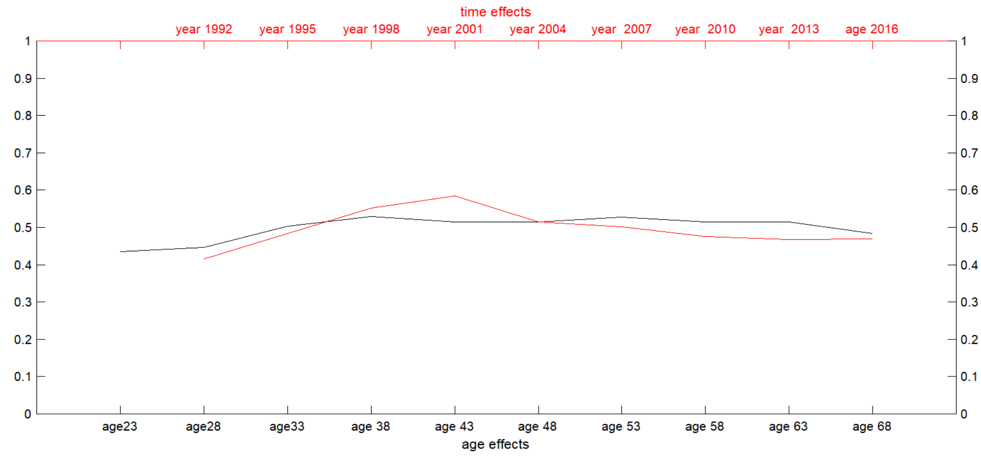
Panel (c)



This figure displays the estimated age, cohorts and time effects on conditional risky share under different model specifications. In panel a), the cohort effect is assumed to be constant across ages and periods; in panel b), the age effect is assumed to be constant across cohorts and periods; in panel c), the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with head aged between 21 and 70 who attain at most a high school diploma. Coefficients not statistical significant at 5% are set to zero.

Figure 4: Age, Time and Cohort effects on Conditional Risky Share - Education level: College Degree

Panel (a)



Panel (b)

Panel (c)

This figure displays the estimated age, cohorts and time effects on conditional risky share under different model specifications. In panel a), the cohort effect is assumed to be constant across ages and periods; in panel b), the age effect is assumed to be constant across cohorts and periods; in panel c), the time effect is assumed to be the same across ages and cohorts. SCF data from 1992 to 2016 on households with head aged between 21 and 70 and who attained a college degree. Coefficients not statistical significant at 5% are set to zero.

These results reveal that conditional risky share is almost flat in age and across cohorts. The share of wealth invested in stocks increased on average during the 1990s. However, after the recovery subsequent to the “Internet bubble” burst the average conditional risky share declined. However, the time evolution of households' behavior reflects only in part the evolution of stock market conditions. The relatively lower stock investing during the 2000s appears to reflect, at least in part, the higher households' background risk proxied by the unemployment rate and especially the long term unemployment rate. In particular, after the sharp increase in the long term unemployment rate subsequent to the Great Recession the conditional risky share remains relatively lower with respect to previous years reflecting the higher perceived background risk.

3 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t (with $p_{t_0-1} = 1$). Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=0}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\,b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \qquad (1)$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs after her death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

3.1 Labour and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labour supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term (u_1) and long-term (u_2) unemployment. Individual labour market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{nm} = \text{Prob}\,(s_{t+1} = n|s_t = m)$ with $n, m = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. Since she must experience short-term unemployment prior to becoming long-term unemployed, we set the probability of directly entering long-term unemployment at zero, $\pi_{eu_2} = 0$. Conditional on being short-term unemployed at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2} = 1 - \pi_{u_1e}$; consequently, we set $\pi_{u_1u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she is re-employed in the following period ($s_{t+1} = e$) with probability π_{u_2e} and remains unemployed with probability $\pi_{u_2u_2} = 1 - \pi_{u_2e}$.

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labour income driven by permanent and transitory shocks. In each working period, labour income Y_{it} is generated by the following process:

$$Y_{it} = H_{it}U_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_{it} = F\,(t, \mathbf{Z}_{it})\,P_{it}$ represents the permanent income component. In particular, $F\,(t, \mathbf{Z}_{it}) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. U_{it} denotes the transitory stochastic component and $\varepsilon_{it} = \log(U_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

In our set-up, which differs from that of Bremus and Kuzin (2014), labour income received by the employed individual at time t depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labour income, H_{it}. Since the empirical evidence suggests that the longer the unemployment spell the larger is the worker's human capital depreciation (Schmieder, von Wachter and Bender, 2016), we let human capital erosion increase with unemployment duration. Thus, after 1-year unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a 2-year unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labour income. In compact form, the permanent component of labour income H_{it} evolves according to

$$H_{it} = \begin{cases} F\left(t, \mathbf{Z}_{it}\right) P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

The human capital erosion parameters Ψ_1 and Ψ_2 are are modelled as stochastic variables. Indeed, the tragic human capital consequences of LTU emerged in the United States during the Great Recession, but were less evident in the past. This outcome may correspond to high (low) realizations of Ψ_2 for a large number of workers in the Great Recession (business cycle expansions) and an intermediate value in ordinary business cycle contractions.

The Beta distribution can be usefully employed to model the behavior of random variables that take values limited to finite intervals. The Beta density function is very versatile and allows to represent outcomes like proportions being defined on the continuum between 0 and 1. In the current case, random variables Ψ_j, with $j = 1, 2$, are fractions limited to the interval $[0, 1]$. We therefore assume that Ψ_j follow Beta distributions with support $[0, 1]$ and shape parameters (a_j, b_j): thus, $\Psi_j \sim Beta(a_j, b_j)$. The flexibility of the distribution allows us to concentrate nearly all the probability mass towards the most probable values of proportional human capital erosion, leaving the possibility of extremely unlikely realizations open.

The standard *Beta* distribution gives the probability density of the value of Ψ_j, with

$j = 1, 2$, on the interval (0,1):

$$f(\Psi_j : a_j, b_j) = \frac{\Psi_j^{a_j-1}(1-\Psi_j)^{b_j-1}}{B(a_j, b_j)} \tag{6}$$

where B is the beta function, thus B(a,b) plays the role of normalization constant to ensure that the total probability is 1:

$$B(a_j, b_j) = \int_0^1 t^{a_j-1}(1-t)^{b_j-1} dt \tag{7}$$

The expected value of Ψ_j, with $j = 1, 2$,is:

$$E(\Psi_j) = \frac{a_j}{a_j + b_j} \tag{8}$$

and the variance is

$$Variance(\Psi_j) = \frac{a_j b_j}{(a_j + b_j)^2(a_j + b_j + 1)} \tag{9}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income $H_{it-1} = F_{it-1}P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available: $\xi_2 = 0$. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 H_{it-1} & \text{if } \mathrm{s}_t = u_1 \\ 0 & \text{if } \mathrm{s}_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{10}$$

making the unconditional distribution of labour income no longer log-normal.

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labour income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{11}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and λ is level of the replacement rate.

3.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s, for each period. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{12}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labour income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{13}$$

where R_{it}^P is the investor's portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s)\, R^f \tag{14}$$

with α_{it}^s and $(1 - \alpha_{it}^s)$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

3.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life span, by choosing the consumption and the portfolio rules given uncertain labour income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T},\{\alpha_{it}^{s}\}_{t_0}^{T}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^{j} \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right.\right.\right.$$
$$\left.\left.\left. + \left(1-p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{15}$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right)\left(\alpha_{it}^{s} R_{t}^{s} + \left(1-\alpha_{it}^{s}\right) R^{f}\right) + Y_{it+1} \tag{16}$$

with the labour income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T},\{\alpha_{it}^{s}\}_{t_0}^{T}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_{t} \left[p_{t} V_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right.\right.$$
$$\left.\left. + \left(1-p_{t}\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{17}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labour market state $s_{it}(= e,\ u_1,\ u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit:

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^{s}\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} \right.$$
$$+ \beta \left[p_{t} \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1} | s_{it}\right) \widetilde{E_{t} V}_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right.$$
$$\left.\left. + \left(1-p_{t}\right) b \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1} | s_{it}\right) \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{18}$$

where $\widetilde{E_t V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν^s_{it+1}. The history dependence that we introduce in our set-up by making unemployment affect subsequent labour income prospects prevents having to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labour income depends on previous individual labour market dynamics we write the value function at t in each possible state as (dropping the term involving the bequest motive):

$$V_{it}(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{e,e} \\ \text{with } \; P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, u_1) \quad \text{with prob. } 1 - \pi_{e,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_1) P_{it} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_1 F_{it} P_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it}, P_{it}, u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_1) P_{it-1} \, e^{\omega_{it+1}} = P_{it} \, e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, u_2) \quad \text{with prob. } 1 - \pi_{u_1,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it}, P_{it}, u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_2,e} \\ \text{with } \; P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-2} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} V_{it+1}(X_{it+1}, P_{it+1}, u_2) \quad \text{with prob. } 1 - \pi_{u_2,e} \\ \text{with } \; P_{it+1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} \end{cases} \end{cases} \tag{19}$$

This problem has no closed form solution; therefore, we obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it} and P_{it}) by means of the standard grid search method.[5] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (18) to obtain optimal rules for consumption and portfolio shares.

4 Calibration

Parameter calibration concerns investor's preferences, the features of the labour income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause only temporary income losses, as in Bremus and Kuzin (2014), and (ii) unemployment has permanent consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US National Center for Health Statistics. With regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008) and captures an intermediate degree of risk aversion. However, Cocco, Gomes and Maenhout (2005) and Bremus and Kuzin (2014) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with an expected equity premium μ^s fixed at 0.04. The standard deviation of the return innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters.

[5]The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

Table 2: Calibration parameters

Description	Parameter	Value
Working life (max)	T	20 -65
Retirement (max)	$t_0 + K$	65 -100
Discount factor	β	0.96
Risk aversion	γ	5
Replacement ratio	λ	0.68
Variance of permanent shocks to labour income	σ^2_ω	0.0106
Variance of transitory shocks to labour income	σ^2_ϵ	0.0738
Riskless rate	r	0.02
Excess returns on stocks	μ^s	0.04
Variance of stock returns innovations	σ_s	0.157
Stock ret./permanent lab. income shock correlation	ρ_{sY}	0

	No unemployment risk	**Unemployment no disaster risk**	**Unemployment with disaster risk**
Unemployment benefits			
Short-term unemployed (ξ_1)	-	0.3	0.3
Long-term unemployed (ξ_2)	-	0	0
Human capital erosion			
Short-term unemployed (Ψ_1)	-	-	0
Long-term unemployed ($expected \Psi_2$)	-	-	0.1
–Beta distribution α	-	-	*0.1*
–Beta distribution β	-	-	*0.7*

This table reports benchmark values of relevant parameters.

4.1 Labour income and unemployment risk

The labour income process is calibrated using the estimated parameters for US households with high school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). For the high school group, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma^2_\omega = 0.0106$ and $\sigma^2_\varepsilon = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labour income, with $\lambda = 0.68$. The parameter values assumed above are maintained across all scenarios.

The resulting labour income process does not capture the evidence in Krueger, Cramer and Cho (2014) that the long-term unemployed experience a progressive declining re-employability over time and are more likely to exit the labour force. We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%.

The annual transition probabilities between labour market states are chosen to match the average annual unemployment rate in the United States:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.33 & 0 & 0.67 \end{pmatrix} \tag{20}$$

Our calibration appears quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (see the evidence on CPS data in Krueger, Cramer and Cho, 2014). Indeed, the assumed transition matrix (20) yields unconditional probabilities of being unemployed in line with what observed for the 2015 overall (5.3%) and long-term (1.7%) unemployment rates in US.

Well-established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalond and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their predisplacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their predisplacement wage. This body of evidence, combined with a probability of finding a job after being unemployed for 24 months as low as 40% (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate a substantial expected drop in human capital following a long term unemployment spell. Thus, while Ψ_1 is kept at 0,[6] Ψ_2 follows the Beta distribution with expected value of 0.12.5 standard deviation of 0.32. The calibrated distribution for Ψ_2 implies a median value for the proportional human capital erosion of about 1%, while the

[6]That is, for $a_1/b_1 \to 0$, the mean is located at the left end, $\Psi_1 = 0$. The beta distribution has a spike at the left end, $\Psi_1 = 0$, with probability 1, and zero probability everywhere else., i.e. there is 100% probability (absolute certainty) concentrated at the left end.

75^{th} percentile is about 8.5% and the 99^{th} percentile is about 95%(i.e. there is a probability of 75% of experiencing a human capital loss lower than 6.5% and a probability of 1% that the loss is extremely large of about 93%). Several studies documented effects of unemployment on future labor income and separation rates, for example, Guevenen, Karahan, Ozkan and Song (2017) find that income losses after long term unemployment are substantial and heterogeneous. In our calibration, the long term consequences of not working for a long time are modest for the majority but possibly very large in extremely rare situations.

Unemployment benefits are calibrated according to the US unemployment insurance system. In particular, considering that the replacement rate with respect to last labour income is on average low and state benefits are paid for a maximum of 26 weeks, we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0$ for the long-term unemployed. No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.[7]

For comparison, we also consider a calibration of the model without unemployment risk. This "*no unemployment risk*" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (2). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in the transition probability matrix (20) with no human capital erosion. In this "*unemployment with no disaster*" scenario, unemployment has no permanent consequences on future earnings (i.e. $\Psi_1 = \Psi_2 = 0$) but entails only a cut in current income. This case closely corresponds to the set-up studied by Bremus and Kuzin (2014), who focus only on temporary effects of long-term unemployment.

[7] Low, Meghir and Pistaferri (2010) acknowledge that layoffs are partially insured by the unemployment insurance system, while individual productivity shocks, other than major observable health shocks, are rarely insured in any formal way. As for other welfare programs, we do not model basic consumption needs and therefore overlook basic consumption insurance.

5 Results

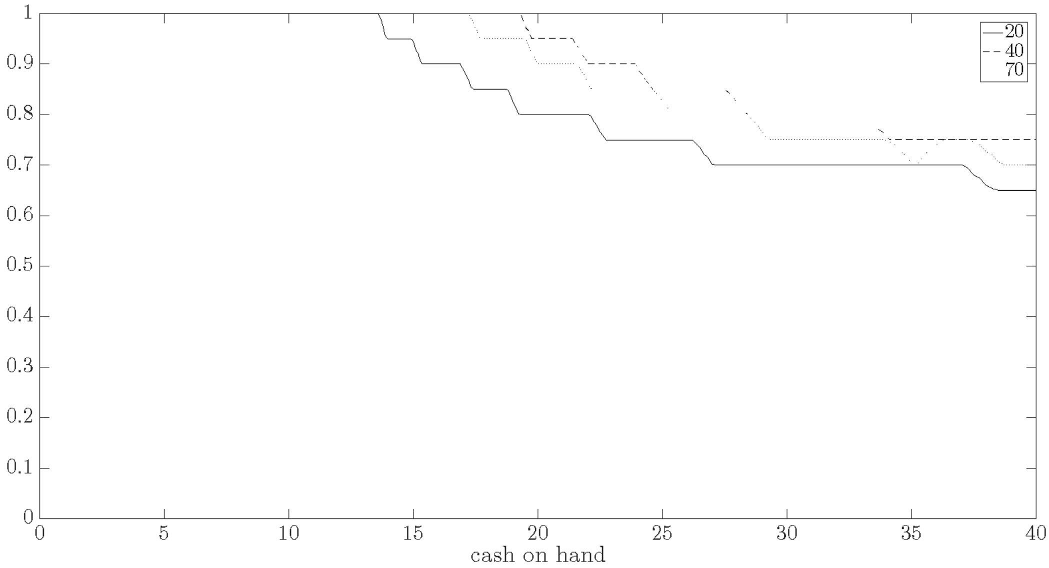
5.1 Optimal policies

Figure 6 compares investors' optimal stock shares in the standard case of "*no unemployment risk*" (panel (a)) and in our preferred scenario with "*unemployment with disaster risk*" (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labour income component of investors at three different ages (20, 40, and 70). In the case with no unemployment risk, standard life-cycle results are obtained. Labour income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

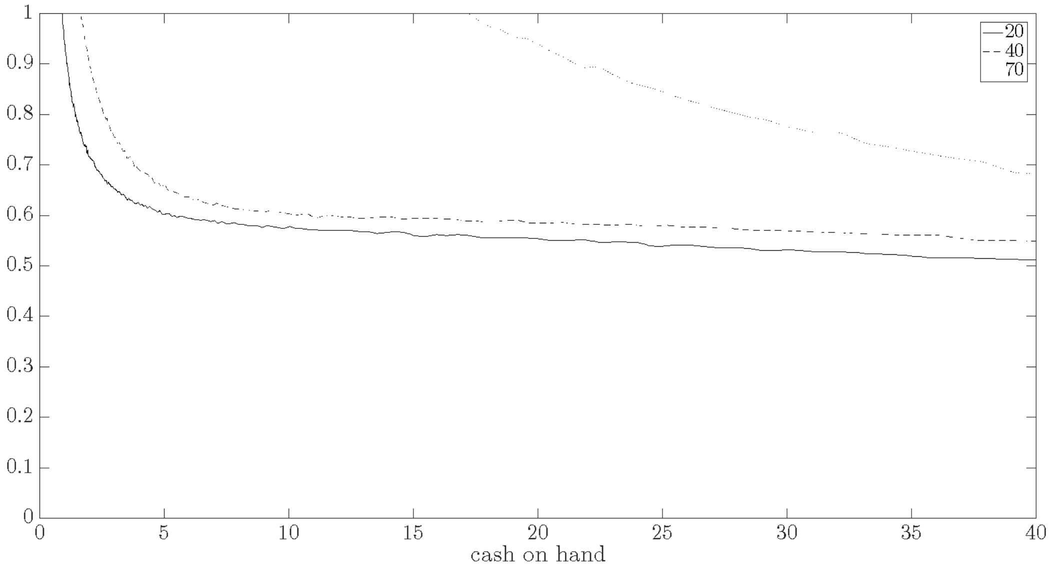
When the model is extended to allow for permanent effects of unemployment spells on labor income prospects at re-employment ("*unemployment with disaster risk*"), with the parameters governing the proportional erosion of permanent labour income set at $\Psi_1 = 0$ after one year of unemployment and at an expected $\Psi_2 = 0.125$ after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labour income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no unemployment risk.

Figure 5: Policy functions

(a) No unemployment risk



(b) Unemployment with disaster risk



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labour income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases with no unemployment risk and with unemployment disaster risk. In the latter case, the parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and expected $\Psi_2 = 0.125$. Cash on hand is expressed in ten thousands of US dollars.

5.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 7, panel (a), shows the average optimal stock shares plotted against age when unemployment risk is ignored and when it is accounted for. In the case of no unemployment risk (dotted line), the well-known result on the age profile of optimal stock portfolio shares is obtained. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When unemployment risk without human capital erosion is considered (dashed line), the optimal portfolio share of stocks still declines with age, though being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

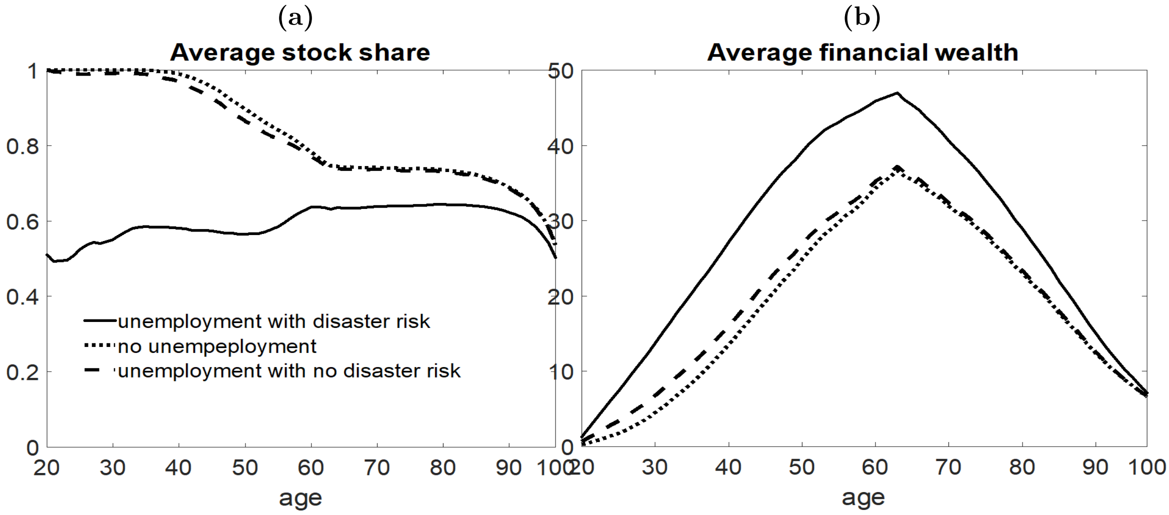
However, when long-term unemployment implies a rare but large skill erosion (solid line), the optimal stock investment is sizably reduced at any age and almost flat, at around 55-60%. The risk of permanently losing a substantial portion of future labour income prospects reduces the level of human capital and increases its riskiness. Because this effect is particularly relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young. Consequently, the age profile remains remarkably flat over the whole working life.[8] These results highlight that possible long-run consequences of unemployment significantly dampen the incentive to invest in stocks, under standard calibrations, whereas unemployment persistence, with only temporary income losses as in Bremus and Kuzin (2014), has almost no effect on the age profile of optimal portfolio composition.

The reduction in the optimal portfolio share allocated to stocks is due to higher wealth accumulation, in turn induced by larger precautionary savings.[9] Panel (b) of Figure 7 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labour market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years.

[8] The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive, common to all parametrizations considered in this paper.

[9] Love (2006) shows that higher unemployment insurance benefits reduce calibrated contributions to pension funds by the young, suggesting that precautionary savings when young is due to unemployment risk.

Figure 6: Life-cycle average profiles

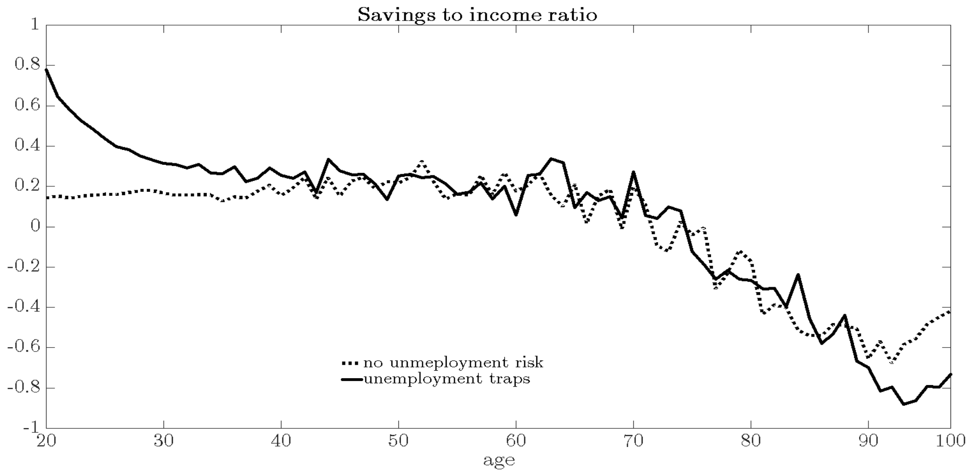


This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment with no disaseter (dashed line); unemployment disaster risk (solid line). In the latter case, the parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and expected $\Psi_2 = 0.125$. Financial wealth is expressed in ten thousands of US dollars.

Figure 8 displays the life-cycle profile of the ratio between savings and total (financial plus labour) income, comparing the case without unemployment risk to the one with unemployment traps. When the worker is 20 years old, the average propensity to save is especially high in the latter case, reaching 0.8 compared with less than 0.2 when unemployment risk is absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty as individuals age.

Consistent with these predictions, data on Norwegian households show that they engage in additional saving and in shifting toward safe assets in the years prior to unemployment, as well as in depletion of savings after the job loss (see Basten, Fagereng and Telle, 2016). Importantly, our results imply that labour market institutions targeted to long-term unemployment affect both risk taking in the equity market and precautionary saving. The expectation of a higher benefit may mitigate the adverse impact of long term unemployment on human capital, reducing the need for cautious investing and saving during working life. The variation of institutions across countries may thus generate different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data (appearing in Fagereng, Gottlieb and Guiso, 2017) may be a consequence of higher long-term unemployment benefits with respect to the US.

Figure 7: Life-cycle profiles of savings rate



This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labour income plus financial income). The two cases correspond to no unemployment risk (dotted line) and unemployment risk with traps with human capital erosion: $\Psi_1 = 0$ and Expected $\Psi_2 = 0.1$ (solid line).

5.2.1 Heterogeneity

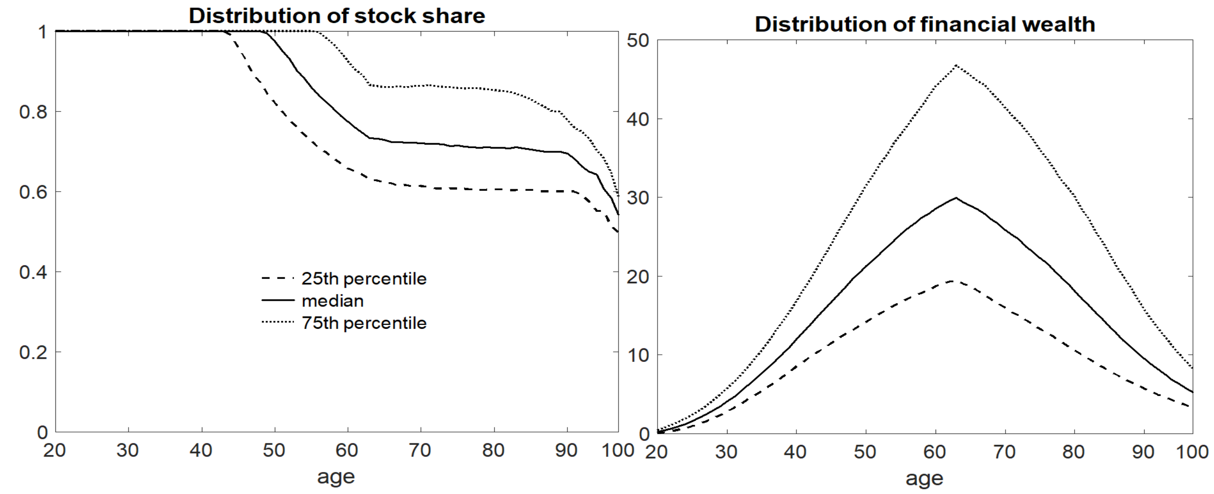
The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but large human capital depreciation, workers on average invest about 55% of their financial wealth in stocks. This average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

The case of no unemployment risk is displayed in panels (a) and (b) of Figure 9, which show the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 1, relating optimal stock shares to the amount of available cash on hand, and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

Figure 8: Life-cycle percentile profiles

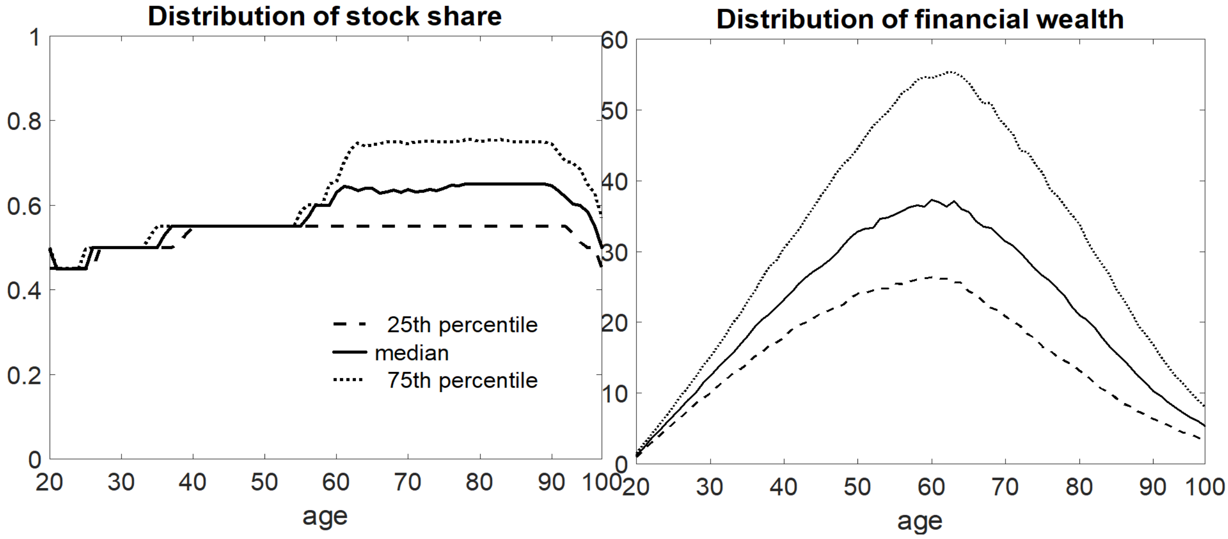
No unemployment

(a) (b)



Unemployment with disaster risk

(c) (d)



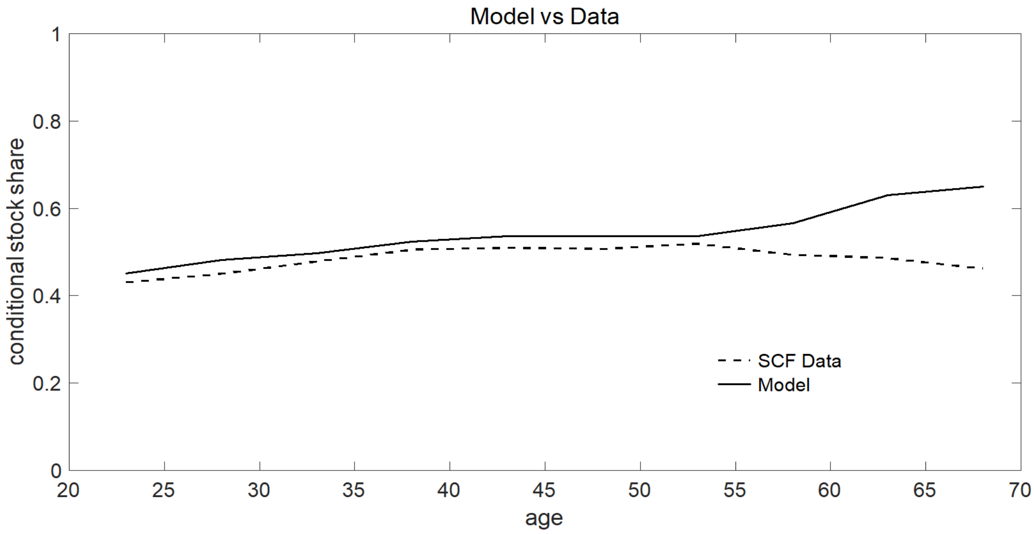
This figure displays the distribution of simulated equity share investment and financial wealth accumulation for individuals of age 20 to 100 in the case of no unemployment risk (panels (a) and (b)) and unemployment with disaster risk (panels (c) and (d)). The parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and Expected $\Psi_2 = 0.125$. Financial wealth is expressed in ten thousands of US dollars.

Panels (c) and (d) of Figure 4 display the life-cycle distribution of stock share and financial wealth for the case with uninsured long-term unemployment. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous. In particular, heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. Indeed, policy functions are relatively flat when there is long-term unemployment is uninsured (see panel (b) of Figure 1) implying that even large differences in the level of accumulated wealth result in homogeneous asset allocation choices.

5.3 Household portfolios with personal disaster risk: matching the empirical regularities

The key implication of our model is that optimal investment profiles are almost flat over the life cycle. In this section, we compare our results with conditional stockholdings for US male investors observed in the Survey of Consumer Finances data (waves from 1998 to 2016).

Figure 9: Life-cycle conditional stockholding profiles



This figure displays the life-cycle profiles of conditional stock holdings of age 20 to 100 observed in SCF data and obtained from the benchmark model with unemployment plus disaster risk. The parameters governing the human capital erosion during short-term and long-term unemployment spells are $\Psi_1 = 0$ and Expected $\Psi_2 = 0.125$.

Figure 9 compares the stock portfolio shares for stock market participants for different age classes obtained from our model with the corresponding US SCF data. The model is able to closely match the observed life-cycle pattern of equity portfolio shares conditional on participation, yielding an average value over the whole life cycle of 50%, to be compared with 54.8% in the data.[10]

6 Conclusions

This paper shows that even a small probability of experiencing human capital erosion of uncertain size generates optimal conditional stock shares in line with those observed in US data. Because of the remote possibility of a future personal disaster, younger workers face higher uncertainty concerning future income than older workers and wish to invest a higher portfolio share in the risk-free asset. These results owe to a methodological innovation in the way we model human capital erosion conditional on the occurrence of a rare disaster.

We also show that greater insurance against personal disaster risk increases risk taking especially by young workers. Our analysis thus implies that the pattern of risk taking at different ages in Target Date Funds should be related to the share of uninsured long-term employment risk. The flatter design, which is optimally associated with limited unemployment protection, should fit workers with different employment histories given the limited heterogeneity in optimal life-cycle investments induced by the threat of personal disasters.

Our analysis implies different patterns of household's financial risk taking over the life-cycle, both across cohorts and across countries, in response to the coverage of long-term unemployment. We leave this important investigation for future work.

[10] We focus on average values over the whole life cycle. As highlighted by Gomes and Michaelides (2005), participation and conditional stock holding age profiles obtained from the data are not robust to the cohort and time effects assumptions.

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