

International dynamic risk sharing*

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Abstract

Theory predicts that under complete markets consumption changes in a given country should be related, once adjusted for the variations in real exchange rates, to consumption changes of the countries joining the risk sharing pool and not to idiosyncratic income shocks. The empirical implications of international risk sharing are usually obtained and investigated assuming “frictionless” markets. We show that by preventing forward-looking agents to adjust instantaneously to the optimal equilibrium because of market frictions (e.g. transport costs, all barriers impeding trade and factors mobility) consumption changes display a dynamic structure that is not taken into account in the prevailing literature. The lack of dynamic adjustment characterizing existing empirical analyses may help to justify, in addition to the traditional explanations based on incomplete markets and non-separable non-tradable components in the utility functions, the puzzling evidence on international risk sharing. Likelihood and regression-based methods for estimating and testing the dynamic risk sharing model are proposed. Differently from previous findings, results on a set of “core” European countries suggest that consumption data do not seem to contrast neither with the existence of integrated capital markets and risk sharing against permanent income fluctuations, nor with a gradual and interrelated across countries process of adjustment towards the equilibrium.

Keywords: Adjustment costs, Consumption risk sharing, Cointegrated VAR models, Financial market integration, Market frictions.

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1 Introduction

Common wisdom contends that under complete markets changes to country per capita consumption should be related to changes in the consumption streams of the partner countries

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joining the risk sharing agreement or to changes in aggregate (world) per capita consumption only. Conventional risk sharing tests and/or techniques aimed to measure the different channels of consumption insurance are based on this requirement, see e.g. Asdrubali et al. (1996), Asdrubali and Kim (2004) and references therein. However, several empirical tests have shown substantial departures from this proposition – the so-called ‘full risk sharing hypothesis’ (FRS) – both on individual and aggregated data – see e.g. Lewis (1996).

Standard risk sharing tests are usually based on the idea that changes to individual consumptions, once corrected for changes in aggregate consumption or consumption of the ‘leader country’ and possibly for real exchange rates, are not predictable on the basis of the available information set, see e.g. Canova and Ravn (1996). These implications arise from the well known condition that under complete markets the ex-post nominal marginal rate of substitution equalize across countries. However, the empirical evidence based on consumption data and power utility functions suggests that risks are poorly shared internationally. Also the correlations between domestic to foreign consumption and real exchange rates appear sharply below than one or even negative, as found in Backus and Smith (1993), Kollmann (1995) and Ravn (2001).

The most recurrent explanations for the observed ‘consumption correlations puzzle’ - ‘consumption home bias’ using Lewis’s (1996) terminology - in the international business cycle literature hinges on the idea that some components of utility are not separable and internationally tradeable such as leisure (Backus et al., 1992) and nontradeable goods (Backus and Smith, 1993, Stockman and Tesar, 1995)¹. However, as argued (and shown empirically) by Lewis (1996), risk sharing tests that simply correct for the presence of nontradeables do not seem to be sufficient alone to explain the lack of consumption risk sharing. A further explanation is that international financial markets are not developed enough, i.e. markets incompleteness. Also within this perspective, however, theory offers convincing arguments to doubt that incomplete asset markets alone can account for the observed low international consumption correlations². Recently Obstfeld and Rogoff (2000) provide a unified explanation of the major puzzles of international macroeconomics, including the violation of FRS and purchasing power parity (PPP), in terms of costs in goods markets (transport costs, tariffs, nontariff barriers) that impede trade, see also Ravn and Mazzega (2004) and Brandt et al. (2005).

¹Moreover, these explanations of the lack of international risk sharing can be potentially reconciled with the “equity home bias” puzzle, see e.g. Lewis (1996). The fact that “consumption home bias” is somehow related to “equity home bias” can be understood, intuitively, by observing that countries that bias their equity holdings away from foreign assets will not diversify all of their home output risk .

²Corsetti et al. (2004) show that standard international business cycle models with incomplete asset markets augmented with distribution services can account quantitatively for the high volatility of real exchange rates and their negative correlation with cross-country consumption rates.

Although full risk sharing requires frictionless markets, in practice individuals face the (dis)utility costs implied by restrictions on factory mobility as well as on trade in international goods markets. For instance, if there is a positive shock in one country, asset holdings by the other countries should in principle lead to an outflow of goods; if on the one hand restrains in capital markets can be considered negligible, on the other hand in goods markets it can be costly to ship goods and these costs might increase with the volume being shipped. Nevertheless, if these frictions and related costs are not large enough to keep consumers far from ‘frictionless’ first order conditions, the ex-post nominal marginal rate of substitution will not equalize instantaneously across countries but after a gradual process of adjustment.

The idea of the present paper is that departures from the FRS hypothesis may depend on the lack of dynamic structure characterizing the models which are traditionally implemented to test the FRS hypothesis. Specifically we show that amending standard intertemporal risk sharing models with simple exogenous costs which impede instantaneous adjustment to the optimal risk sharing position entails a dynamic structure for countries consumption changes that is not accounted in the traditional empirical analyses. Omitting such dynamics flaws standard measures of the extent of risk sharing.

The model we formalize hinges on the idea that a benevolent social planner minimizes the costs that countries face in the presence of frictions that obstacle the instantaneous achievement of FRS. The model has the following features. First, consumption changes of a given country other than depending on contemporaneous consumption changes of the partners, display an error-correcting structure involving lagged deviations from the optimal risk sharing position of (potentially) all countries in the risk sharing pool. Thus the model predicts that adjustment is interrelated across countries, i.e. shocks affecting one country in the risk sharing pool produce adjustment in all the other countries. Second, as agents are forward-looking in an environment characterized by impediments to trade and factor (especially labour) mobility, beliefs on the evolution of expected future consumption changes of the leader country and of real exchange rates of all countries affect the risk sharing allocation. Third, consistently with recent findings (see Bacchiocchi and Fanelli, 2005, and references therein), our model do not require that PPP holds among the countries belonging to the risk sharing agreement; this feature contrasts with the large majority of papers on international risk sharing tests where PPP is assumed more or less explicitly and the FRS proposition tends to be rejected. Correcting the tests of the FRS hypothesis for the dynamics implied by costs of adjustment and expectations on future market developments helps to explain why according to traditional analyses – where a dynamic structure is omitted - international risks are poorly shared.

We set out maximum likelihood (ML) and regression-based procedures for the model which

allows to assess the existence of international risk sharing as an equilibrium relation and to analyze the dynamic adjustment of consumption towards optimal levels. The proposed framework provides an alternative way, compared to e.g. Obstfeld (1989, 1994), Kollmann (1995), Canova and Ravn (1996) and Ravn (2001), to tackle the empirical analysis of risk sharing and hence to assess the degree of integration in international capital markets.

The dynamic risk sharing model is applied to a set of ‘core’ European countries that joined the European Monetary Union (EMU) in 1999. Economic intuition suggests that countries with closer economic ties, as the ones we consider in the paper, might have more efficient risk sharing mechanisms at work; in fact, by using data over a relatively long period, our results suggest that European consumption data do not seem to contrast with the existence of risk sharing as a long run phenomenon, as well as with the evidence of a dynamic process of adjustment towards equilibrium. These results contrast with the findings in e.g. Canova and Ravn (1996) and Sørensen and Yosha (1998), obtained through a different estimation method neglecting the role of dynamic adjustment.

The plan of the paper is the following. Section 2 introduces the standard international consumption risk sharing model and discusses its main implications. Section 3 provides a dynamic extension under the assumption of market frictions that prevent instantaneous adjustment to optimal risk sharing. Section 4 discusses estimation issues and in Section 5 the proposed risk sharing model is applied to investigate the extent of risk sharing among a set of ‘core’ European countries which joined the European Monetary Union in 1999. Some final remarks may be found in Section 6.

2 Model and implications

As in Canova and Ravn (1996) and Kolmann (1995) we consider a standard international business cycle model. It is assumed that a world of N countries (indexed by $i = 1, \dots, N$) exists, each country being inhabited by a infinitely lived representative agent. His/her expected lifetime utility is given by $V^i = E_t \left(\sum_{t=0}^{\infty} (\rho^i)^t U^i (C_t^i, b_t^i) \right)$, where C_t^i denotes the i -th country consumption good at time t , while $b_t^i > 0$ represents a country-specific stochastic taste shock³; ρ^i ($0 < \rho^i < 1$) denotes country i -th discount factor. Note that the goods consumed by the different countries are allowed to differ. As usual, $E_t(\cdot)$ denotes expectations conditional on all information available up to time t , Ω_t . As is standard in the literature, we further assume that the utility function $U^i(x_1, x_2)$ of country i -th representative agent is an isoelastic instantaneous period

³Within the risk sharing literature b_t^i is often interpreted as a quantity capturing factors which are beyond the control of the planner, or arguments of the utility function that interact non-separably with consumption but which are not explicitly modelled (e.g. leisure).

utility function, i.e. $U^i(x_1, x_2) = x_2 (1/\sigma^i) x_1^{\sigma^i}$ ($\sigma^i < 1$), where $1 - \sigma^i$ is a CRRA coefficient.

Without loss of generality we suppose that the N -th country of the risk sharing pool can be considered as the leader of the arrangement; we denote this country with the superscript ‘0’. Within this set-up, if a benevolent social planner allocates consumption among countries in order to maximize the expected average of country utilities (under standard budget constraints), then for each pair of countries $i, j = 1, \dots, N$ it holds

$$(\rho^i)^t U_c^i P_t^i = (\rho^j)^t U_c^j P_t^j e_t^{i/j} \quad (1)$$

where for a generic country h , U_c^h is the marginal utility of consumption, P_t^h is the price level of the country and $e_t^{i/j}$ is the nominal exchange rate between the currencies of country i and j ; the equilibrium condition (1) establishes that nominal rates of substitution are equalized across countries under FRS.

From (1) it follows that the optimal consumption streams, relative to the leader country’s consumption, are restricted as follows (Kollmann, 1995; Ravn, 2001)

$$c_t^{*i} = \theta^i c_t^0 + \delta^i r_t^{i/0} + \phi^i t + \eta_t^i, \text{ all } t \text{ and } i = 1, \dots, N - 1 \quad (2)$$

where c_t^{*i} is the optimal level of (logged) consumption in country i , c_t^0 is the (logged) consumption in the leader country, $\theta^i = (1 - \sigma^0)(1 - \sigma^i)^{-1}$ is leader country CRRA coefficient relative to country i -th CRRA coefficient, $\delta^i = (1 - \sigma^i)^{-1}$ corresponds to the intertemporal elasticity of substitution of country i , $\phi^i = \log(\rho^0/\rho^i) / (\sigma^i - 1)$, $r_t^{i/0} = \log(e_t^{i/0}) + \log(P_t^0) - \log(P_t^i)$ is the (logged) price of one unit of the leader country’s consumption good in terms of country i ’s consumption good, i.e. the logged bilateral real exchange rate between country i and the leader country⁴. Finally, η_t^i in (2) depends on the stochastic terms which enter the utility functions of the countries i and 0, i.e. the x_2 variable in $U^i(x_1, x_2)$; these terms may represent preference shocks, factors which are beyond the control of the planner, or variables which interact non-separably with consumption but which have not explicitly modelled, e.g. hours worked/leisure, government spending, real money balances and so on. Throughout it will be assumed, except where indicated, that η_t^i embodies the preference shocks of the countries i and 0 (see Kollmann 1995)

The model has strong implications on the optimal level of consumption that each country should achieve, although large part of the literature devotes attention to the growth rate version of (2) ignoring the information in the levels. According to equation (2), net of taste shocks country i -th optimal consumption equalizes a linear combination of the leader country consumption level and the real exchange rate between country i and the leader country. Moreover, the

⁴The formal derivation of (2) can be directly obtained from Kollmann (1995) and it is therefore omitted for brevity. A full proof is available from the authors upon request.

relation (2) holds jointly for all the countries belonging to the risk sharing pool. Put in different words, the model predicts that the leader country's consumption is a common factor for all the other country consumptions. Note that the θ_i coefficients do not need to be one but only to be strictly positive; values greater than 1 (lower than 1) are likely to occur when the region CRRA coefficient is lower (higher) than the average.

If the terms η_t^i , $i = 1, \dots, N - 1$, are stationary and if consumption can be well represented by means of integrated processes of order one, I(1) hereafter, then equation (2) can be seen as a cointegrating relation involving the optimal consumption level, the leader country consumption level and the real exchange rate. Therefore the linear combination

$$(c_t^i - c_t^{i*}) = (1, -\theta^i, -\delta^i, -\phi^i) \begin{pmatrix} c_t^i \\ c_t^0 \\ r_t^{i/0} \\ t \end{pmatrix} = c_t^i - \theta^i c_t^0 - \delta^i r_t^{i/0} - \phi^i t \quad (3)$$

net of the preference shock, η_t^i , must be stationary, see also Backus and Smith (1993) and Kollmann (1995), p.193-194. Furthermore, if the real exchange rate is stationary (that is, PPP holds in the long run), then the equilibrium relation involves country i -th optimal consumption level and the leader country optimal consumption and can be analyzed and estimated as a cointegration relation between c_t^i and c_t^0 .

Empirical tests based on consumption levels are usually carried out by assuming that the consumptions levels c_t^i equalize the corresponding optimal levels c_t^{i*} , described thorough the FRS proposition (2)⁵. For instance, both both Kollmann (1995) and Ravn (2001) assume that adjustment is instantaneous, i.e. $c_t^i = c_t^{i*}$, all i ⁶, implying therefore that $(c_t^i - c_t^{i*})$ is unpredictable given information available at time t . On the other hand, Canova and Ravn (1996) employ, among others, nonparametric tests for cointegration among the consumption of pairs of G7 countries, hence setting δ^i to 0 in (2) and finding little support of the FRS proposition. Implicitly, setting $\delta^i = 0$ is equivalent to assume that the real exchange rate is stationary; i.e., that PPP holds in the long run.⁷

⁵The proposition that relative consumptions and real exchange rates should be positively (and highly) correlated under FRS can be derived directly from (2) under the assumption that the i -th and the leader country have the same risk aversion coefficients and intertemporal discount rates. Indeed, with $\theta^i = 1$ and $\phi^i = 0$ and ignoring η_t^i it follows that $c_t^i - c_t^0 = \delta^i r_t^{i/0}$, where $\delta^i > 0$.

⁶On this respect, by comparing different estimates of the marginal utility growth at the international level, Brandt et al. (2005) suggest that either exchange rates are too smooth or risk is shared internationally.

⁷The majority of papers where the hypothesis of risk sharing is tested at the international level assume that PPP holds, see e.g. Crucini (1999). Obstfeld (1994) and Lewis (1996) allow for deviations from PPP in their analysis of international risk sharing by using PPP-adjusted data, finding little support for the FRS proposition.

3 Dynamic adjustment

The recent literature suggests that none of the explanations based on: (i) the presence of non-separable, non internationally tradable components in the utility function; (ii) the evaluation of the benefits/costs of diversification; (iii) incomplete asset markets, is sufficient alone to account for the lack of risk sharing at the international level (Lewis, 1996, Ravn, 2001). In this paper we recognize the importance of market imperfections: it is assumed that the adjustment of c_t^i to the optimal level c_t^{*i} implied by (2) is gradual, due to frictions characterizing the risk sharing channels.

To formalize our model, in what follows it is convenient to adopt the following matrix notation. Let $\mathbf{c}_t = (c_t^1, \dots, c_t^{N-1})'$ be the $(N-1) \times 1$ vector containing per capita consumption of the N countries except the N -th, i.e. that of leader one, and let \mathbf{c}_t^* be the vector containing the corresponding equilibrium quantities given by (2). Finally, let \mathbf{w}_t be the $N \times 1$ vector defined as $\mathbf{w}_t = (c_t^0, r_t^{1/0}, r_t^{2/0}, \dots, r_t^{N-1/0})'$, where $c_t^0 \equiv c_t^N$ and $r_t^{j/0}$ is defined as above. In the light of the FRS equilibrium relation (2) and its empirical counterpart (3), the vector of deviations of actual per capita consumption from optimal consumption, $\mathbf{c}_t - \mathbf{c}_t^*$, can be written as

$$\mathbf{c}_t - \mathbf{c}_t^* = \mathbf{c}_t - \mathbf{\Upsilon} \mathbf{w}_t - \boldsymbol{\phi} t - \boldsymbol{\eta}_t \quad (4)$$

where $\mathbf{\Upsilon}$ is a $(N-1) \times N$ matrix depending on the preference parameters θ^i and δ^i , $i = 1, \dots, N-1$. Specifically, $\mathbf{\Upsilon} = (\boldsymbol{\theta} : \text{diag}(\boldsymbol{\delta}))$, $\boldsymbol{\theta} = (\theta^1, \dots, \theta^{N-1})'$, $\boldsymbol{\delta} = (\delta^1, \dots, \delta^{N-1})'$, $\boldsymbol{\phi} = (\phi^1, \dots, \phi^{N-1})'$ and $\boldsymbol{\eta}_t = (\eta_t^1, \dots, \eta_t^{N-1})'$. For instance, with $N = 3$, $\mathbf{c}_t = (c_t^1, c_t^2)'$, $c_t^0 \equiv c_t^3$ is the consumption stream of the leader country, $\mathbf{w}_t = (c_t^0, r_t^{1/0}, r_t^{2/0})'$ and the matrix $\mathbf{\Upsilon}$ and the vector $\boldsymbol{\phi}$ take the form:

$$\mathbf{\Upsilon} = \begin{bmatrix} \theta^1 & \delta^1 & 0 \\ \theta^2 & 0 & \delta^2 \end{bmatrix}, \quad \boldsymbol{\phi} = \begin{bmatrix} \phi^1 \\ \phi^2 \end{bmatrix}. \quad (5)$$

Assume that due to market frictions (barriers to trade and international capital mobility, labor market stickiness, etc.) countries do not achieve the ‘frictionless’ first order conditions (1), however such frictions are not large enough to keep consumers completely far from (1). For the representative agents of each country being away from the utility-implied equilibrium condition (2) and varying consumption to achieve it is costly.

Assuming the perspective that deviations from the ‘frictionless’ first order conditions are transitory, we assume that a benevolent social planner re-allocates consumption streams among countries by minimizing the (dis)utility costs implied by market frictions. A stylized representation of this behavior can be formalized by the following intertemporal optimization problem:

$$\min_{\{\mathbf{c}_{t+h}\}} E_t \sum_{h=0}^{\infty} \rho^h [(\mathbf{c}_{t+h} - \mathbf{c}_{t+h}^*)' \mathbf{D}_0 (\mathbf{c}_{t+h} - \mathbf{c}_{t+h}^*) + (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})' \mathbf{D}_1 (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})] \quad (6)$$

where ρ ($0 < \rho < 1$) is a time-invariant discount factor which can be regarded as an average of all countries discount factors, \mathbf{D}_0 and \mathbf{D}_1 are $(N - 1) \times (N - 1)$ symmetric (possibly non-diagonal) positive definite matrices. There are two types of costs embedded in the present value cost function minimized in (6): the first term measures the cost of being away from the risk sharing consumption level, i.e. away from the ‘frictionless’ first order conditions (1); the second term the cost of changing consumption levels to restore equilibrium⁸. It is worth stressing that quadratic costs of adjustment are solely used for mathematical convenience; in principle, there is no reason or motivation to argue that positive and negative deviations from optimal risk sharing have the same effect on the process of adjustment. This means that the dynamic models stemming from (6) we discuss below have to be regarded as mere approximations of the actual process of adjustment that risk sharing countries undertake.

The first-order conditions for (6) read as the system of Euler equations

$$\Delta \mathbf{c}_t = \rho E_t \Delta \mathbf{c}_{t+1} - \mathbf{D}(\mathbf{c}_t - \mathbf{c}_t^*) \quad (7)$$

where $\Delta \mathbf{c}_t = \mathbf{c}_t - \mathbf{c}_{t-1}$ and the elements of the matrix $\mathbf{D} = \mathbf{D}_1^{-1} \mathbf{D}_0$ measure the relative importance of disequilibria, adjustment and cross-adjustment costs. In general, the higher adjustment costs are compared to disequilibria costs the lower will be the magnitude of the elements of D . The j -th equation of (7) (i.e. that relative to country j) is then given by

$$\Delta c_t^j = \rho E_t \Delta c_{t+1}^j - \mathbf{d}'_j(\mathbf{c}_t - \mathbf{\Upsilon} \mathbf{w}_t - \phi t) + \tilde{\eta}_t^j \quad (8)$$

where $\tilde{\eta}_t^j$ is the j -th element of $\tilde{\boldsymbol{\eta}}_t$, $\tilde{\boldsymbol{\eta}}_t = \mathbf{D} \boldsymbol{\eta}_t$, and \mathbf{d}'_j is the j -th row of \mathbf{D} ; observe that

$$\begin{aligned} \mathbf{d}'_j(\mathbf{c}_t - \mathbf{\Upsilon} \mathbf{w}_t - \phi t) &= d_{jj}(c_t^j - \theta^j c_t^0 - \delta^j r_t^{j/0} - \phi^j t) \\ &+ \sum_{i=1, i \neq j}^{N-1} d_{ji}(c_t^i - \theta^i c_t^0 - \delta^i r_t^{i/0} - \phi^i t) + \tilde{\eta}_t^j \end{aligned}$$

where d_{jj} is the j -th element on the principal diagonal of \mathbf{D} and the d_{ji} , for $j \neq i$, are the corresponding off-diagonal elements; unless $d_{ji} = 0$ (i.e. \mathbf{D} is diagonal), consumption changes in country j depends not only on its own future expected changes but also on the extent of the deviation of country j and (potentially) all the other countries from the optimal risk sharing position. Within this set-up if $d_{ji} \neq 0$, and a given country faces a departure from its optimal

⁸Actually agents might face also the costs of adjusting the speed with which changes in consumption streams are put into effect; a third cost term of the form $(\Delta \mathbf{c}_{t+h} - \Delta \mathbf{c}_{t+h-1})' \mathbf{D}_2 (\Delta \mathbf{c}_{t+h} - \Delta \mathbf{c}_{t+h-1})$ with $\Delta \mathbf{c}_{t+h} = (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})$ and \mathbf{D}_2 a symmetric $(N - 1) \times (N - 1)$ (possibly non-diagonal) positive definite matrix, could be included in (6), see e.g. Binder and Pesaran (1995) and Fanelli (2005). The paper will focus on the case of first-order adjustment costs ($\mathbf{D}_2 = \mathbf{0}$).

position, all the other countries in the risk sharing pool experience next-time period consumption variations.

The system of Euler equations (8) apparently hides the role of the variables in \mathbf{w}_t , i.e. the consumption stream of the leader country and the real exchange rates. Upon imposing a proper transversality condition the level version of (7) can be solved forward (Binder and Pesaran, 1995) as:

$$\mathbf{c}_t = \mathbf{K}\mathbf{c}_{t-1} + \sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_{(N-1)} - \rho\mathbf{K})(\mathbf{I}_{(N-1)} - \mathbf{K}) E_t \mathbf{c}_{t+h}^* \quad (9)$$

where \mathbf{K} is a $(N-1) \times (N-1)$ matrix with stable eigenvalues obtained as the (unique) solution to the second-order matrix equation

$$\rho\mathbf{K}^2 - [(1 + \rho)\mathbf{I}_{(N-1)} + \mathbf{D}]\mathbf{K} + \mathbf{I}_N = \mathbf{0}_{(N-1),(N-1)}.$$

The representation (9) highlights that for country j consumption at time t is a weighted average of consumption at time $t-1$ of all countries in the risk sharing pool and expected future values of optimal consumption which in turn depends on the variables in \mathbf{w}_t , with weights declining geometrically over time.

By using the equality $\sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_{(N-1)} - \rho\mathbf{K}) = \sum_{h=0}^{\infty} (\rho\mathbf{K})^h - \sum_{h=0}^{\infty} (\rho\mathbf{K})^{h+1}$, adding $(-\mathbf{c}_{t-1})$ to both sides and $\pm (\mathbf{I}_{(N-1)} - \mathbf{K}) \Upsilon \mathbf{w}_t$ to the right hand side, after rearranging terms and assuming that that $E_t \boldsymbol{\eta}_{t+h} = \mathbf{0}$ for $h = 1, 2, \dots$ ⁹, the model can be reparameterized in the error-correcting format

$$\begin{aligned} \Delta \mathbf{c}_t &= (\mathbf{K} - \mathbf{I}_{(N-1)})[\mathbf{c}_{t-1} - \Upsilon \mathbf{w}_{t-1} - \boldsymbol{\phi}t] \\ &\quad + \sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_{(N-1)} - \mathbf{K}) \Upsilon E_t \Delta \mathbf{w}_{t+h} + \mathbf{a} + \mathbf{v}_t \end{aligned} \quad (10)$$

where $\mathbf{v}_t = (\mathbf{I}_N - \mathbf{K})\boldsymbol{\eta}_t$ and $\mathbf{a} = (\rho\mathbf{K})(\mathbf{I}_{(N-1)} - \rho\mathbf{K})^{-1}\boldsymbol{\phi}$ is a constant. The model (10) shows that the dynamics of consumption of the countries in the risk sharing pool depends on past deviations from the optimal risk sharing position (of potentially all countries), and future expected changes of the bilateral real exchange rates and growth consumption of the reference (leader) country. The $(\mathbf{K} - \mathbf{I}_{(N-1)})$ matrix in (10) plays a role similar to that of the adjustment matrix \mathbf{D} in the system (8); indeed, the elements of \mathbf{K} are function of \mathbf{D} and, in general, if \mathbf{D} is non-diagonal, \mathbf{K} will be non-diagonal too.

3.1 Implications under VAR dynamics

Under precise conditions the model (10) can be solved for future expected values of $\Delta \mathbf{w}_t$. Assume, for example, that the process generating $\Delta \mathbf{w}_t$ can be described as a stable VAR($p-1$)

⁹This means that relative preference shocks are treated as a MDS with respect to the information set .

(for simplicity and without loss of generality we omit deterministic terms), which written in companion form reads as

$$\Delta \tilde{\mathbf{w}}_t = \mathbf{\Phi} \Delta \tilde{\mathbf{w}}_{t-1} + \tilde{\mathbf{u}}_t \quad (11)$$

where $\Delta \tilde{\mathbf{w}}_t = (\Delta \mathbf{w}'_t, \Delta \mathbf{w}'_{t-1}, \dots, \Delta \mathbf{w}'_{t-p+2})'$ and $\tilde{\mathbf{u}}_t = (\mathbf{u}'_t, \mathbf{0}')'$ are $g \times 1$ ($g = N(p-1)$), $\mathbf{u}_t \sim WN(0, \mathbf{\Sigma}_{uu})$ with covariance matrix $\mathbf{\Sigma}_{uu}$ positive definite and the matrix $\mathbf{\Phi}$ is defined as

$$\mathbf{\Phi}_{g \times g} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 & \cdots & \mathbf{\Phi}_{p-1} \\ \mathbf{I}_N & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \cdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I}_N & \mathbf{0} \end{bmatrix} \quad (12)$$

and has eigenvalues inside the unit circle in the complex plane. Let \mathbf{H}_w denote a selection matrix such that $\mathbf{H}_w \Delta \tilde{\mathbf{w}}_t = \Delta \mathbf{w}_t$; then from (11) and after conditioning with respect to the ‘observable’ information set $\mathcal{F}_t = \{\mathbf{c}_t, \mathbf{w}_t, \mathbf{c}_{t-1}, \mathbf{w}_{t-1}, \dots\}$ ($\mathcal{F}_t \subseteq \Omega_t$) and applying the law of iterated expectation, the quantity $E_t \Delta \mathbf{w}_{t+h}$ can be computed as $E_t \Delta \mathbf{w}_{t+h} = \mathbf{H}_w E(\Delta \tilde{\mathbf{w}}_{t+h} | \mathcal{F}_t) = \mathbf{H}_w \mathbf{\Phi}^h \Delta \tilde{\mathbf{w}}_t$. By substituting into (10), after some algebra the model simplifies in the expression

$$\begin{aligned} \Delta \mathbf{c}_t &= (\mathbf{K} - \mathbf{I}_{(N-1)g})[\mathbf{c}_{t-1} - \mathbf{\Upsilon} \mathbf{w}_{t-1} - \phi t] \\ &\quad + \mathbf{\Gamma}_0 \Delta \mathbf{w}_t + \mathbf{\Gamma}_1 \Delta \mathbf{w}_{t-1} + \dots + \mathbf{\Gamma}_{p-2} \Delta \mathbf{w}_{t-p+2} + \mathbf{a} + \mathbf{v}_t \end{aligned} \quad (13)$$

where the $(N-1) \times g$ matrix of parameters $\mathbf{\Gamma} = [\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, \dots, \mathbf{\Gamma}_{p-2}]$ is subject to the cross-equation restrictions¹⁰

$$vec(\mathbf{\Gamma}) = [\mathbf{I}_{(N-1)g} - \mathbf{\Phi} \otimes (\rho \mathbf{K})]^{-1} [\mathbf{H}'_w \otimes (\mathbf{I}_{N-1} - \mathbf{K})] vec(\mathbf{\Upsilon}) \quad (14)$$

and it has been assumed that $E(\mathbf{v}_t | \mathcal{F}_t) = \mathbf{v}_t$, i.e. that preference shocks are in the econometrician’s information set at time t .

For instance, coming back to the example in (5) and assuming that in (11)-(12) the number of lags is $p-1 = 1$, the quantity $\mathbf{\Upsilon} E_t \Delta \mathbf{w}_{t+h}$ of (10) is solved as (here $\mathbf{H}_w = \mathbf{I}_3$):

$$\begin{aligned} \mathbf{\Upsilon} E_t \Delta \mathbf{w}_{t+h} &= \mathbf{\Upsilon} (\mathbf{\Phi}_1)^h \Delta \mathbf{w}_t \\ &= \begin{bmatrix} \theta^1 & \delta^1 & 0 \\ \theta^2 & 0 & \delta^2 \end{bmatrix} \begin{bmatrix} {}^h \phi_{11} & {}^h \phi_{12} & {}^h \phi_{13} \\ {}^h \phi_{21} & {}^h \phi_{22} & {}^h \phi_{23} \\ {}^h \phi_{31} & {}^h \phi_{32} & {}^h \phi_{33} \end{bmatrix} \begin{pmatrix} \Delta c_t^0 \\ \Delta r_t^{1/0} \\ \Delta r_t^{2/0} \end{pmatrix} \\ &= \begin{pmatrix} (\theta^1 {}^h \phi_{11} + \delta^1 {}^h \phi_{21}) \Delta c_t^0 + (\theta^1 {}^h \phi_{12} + \delta^1 {}^h \phi_{22}) \Delta r_t^{1/0} + (\theta^1 {}^h \phi_{13} + \delta^1 {}^h \phi_{23}) \Delta r_t^{2/0} \\ (\theta^2 {}^h \phi_{11} + \delta^2 {}^h \phi_{31}) \Delta c_t^0 + (\theta^2 {}^h \phi_{12} + \delta^2 {}^h \phi_{32}) \Delta r_t^{1/0} + (\theta^2 {}^h \phi_{13} + \delta^2 {}^h \phi_{33}) \Delta r_t^{2/0} \end{pmatrix} \end{aligned}$$

¹⁰A formal proof of this result is available from the authors upon request.

where for easy of notation we denoted by ${}^h\phi_{ij}$ the ij -th elements of $(\Phi_1)^h$; this clarifies that in this set up both changes of real exchange rates $\Delta r_t^{1/0}$, $\Delta r_t^{2/0}$, other than Δc_t^0 , enter the equations relative to country 1 and 2 with ‘loading’ coefficients which depend on ρ and the elements of the \mathbf{K} matrix. More generally, the j -th equation of (13) (i.e. that relative to country j) reads as

$$\begin{aligned}
\Delta c_t^j &= (k_{jj} - 1)(c_{t-1}^j - \theta^j c_{t-1}^0 - \delta^j r_{t-1}^{j/0} - \phi^j t) \\
&+ \sum_{i=1, i \neq j}^N k_{ji}(c_{t-1}^i - \theta^i c_{t-1}^0 - \delta^i r_{t-1}^{i/0} - \phi^i t) \\
&+ \gamma_{0,jc} \Delta c_t^0 + \gamma_{0,jj} \Delta r_t^{j/0} + \sum_{i=1, i \neq j}^N \gamma_{0,ji} \Delta r_t^{i/0} \\
&+ \gamma_{1,jc} \Delta c_{t-1}^0 + \gamma_{1,jj} \Delta r_{t-1}^{j/0} + \sum_{i=1, i \neq j}^N \gamma_{1,ji} \Delta r_{t-1}^{i/0} + v_t^j
\end{aligned} \tag{15}$$

where $(\gamma_{0,jc}, \gamma_{0,j1}, \dots, \gamma_{0,jN})$ are the (opportunedly restricted) parameters of the j -th row of $\mathbf{\Gamma}_0$, $(\gamma_{1,jc}, \gamma_{1,j1}, \dots, \gamma_{1,jN})$ are the (opportunedly restricted) parameters of the j -th row of $\mathbf{\Gamma}_1$, $(k_{jj} - 1)$ is the j -th element on the principal diagonal of $(\mathbf{K} - \mathbf{I}_{(N-1)})$, k_{ji} ($i \neq j$) are the corresponding off-diagonal elements and v_t^j corresponds to the j -th elements of \mathbf{v}_t . This equation shows that consumption changes in country j not only depend on contemporaneous changes of consumption of the leader country and of the real exchange rate of all countries in the risk sharing pool and possibly their lags, but also on past deviations from the optimal risk sharing levels in country j as well as in all the other countries (provided $k_{ji} \neq 0$, $j \neq i$, i.e. \mathbf{K} non-diagonal). Moreover, in general the number of lags in (15) depends on the lags characterizing the process (11)-(12) for $\Delta \mathbf{w}_t$.

Note that the error-correcting dynamic structure of the system (13) and its equations (15) allow to explain the failure of conventional risk sharing tests. By referring to the differenced version of (2), risk sharing tests are typically aimed at establishing the orthogonality of Δc_t^j , corrected for Δc_t^0 and $\Delta r_t^{j/0}$, to the information set \mathcal{F}_t ; the equation (15) suggests that if consumers compute and update expectations through a dynamic model similar to (11), then Δc_t^j must be orthogonal with respect to the information set \mathcal{F}_t only after correcting for Δc_t^0 , $\Delta r_t^{1/0}$, ..., $\Delta r_t^{j/0}$, ..., $\Delta r_t^{N/0}$, Δc_{t-1}^0 , $\Delta r_{t-1}^{1/0}$, ..., $\Delta r_{t-1}^{j/0}$, ... and $(\mathbf{c}_{t-1} - \mathbf{\Upsilon} \mathbf{w}_{t-1})$.¹¹

¹¹Also methods aimed at measuring the risk sharing channels (Asdrubali et al., 1996) implicitly require the absence of a dynamic structure between variables. Recently, this limitation has been partially overcome by the dynamic panel model approach of Asdrubali and Kim (2004).

3.2 Implications in a more general set-up

The system (13) has been derived under a precise assumption on the process used by agents to compute expectations of exchange rates changes and the growth rate of consumption of the leader country. It is indeed assumed that the Data Generating Process for $\Delta \mathbf{w}_t$ belongs to the class of VAR processes (11). Abstracting from the fact that the exogeneity restrictions implied by (11) can be easily tested (see Section 4), the hypothesis that expectations on the short term fluctuations of exchange rates are not driven by future developments in ‘fundamentals’ is somehow restrictive, see e.g. Engel and West (2005). Observe that the link between marginal utilities and real exchange rates embodied by the equilibrium condition (1) does not necessary mean that causality runs from real exchange rates to relative consumption. Actually, it can be inferred from (1) that also marginal utilities help to forecast real exchange rates as argued in e.g. Obstfeld (1989) and Apte et al. (1996).

When the process (11) is misspecified because of the presence of feedbacks from $\Delta \mathbf{c}_t$ to $\Delta \mathbf{w}_t$, the system (13)-(14) cannot be regarded as the solution to the proposed risk sharing model. Strictly speaking, if $\Delta \mathbf{c}_t$ Granger-causes $\Delta \mathbf{w}_t$, it is not guaranteed (in the absence of a suitable set of stability restrictions) that a unique non-explosive solution exists for the risk sharing model (10) (Timmermann, 1994); moreover, even if a non-explosive solution occurs identification issues can arise in the sense that the structural adjustment parameters of (13) do not necessarily correspond to \mathbf{K} and $\mathbf{\Gamma}$.

In this situation an alternative approach can be pursued to derive testable implications of the model without specifying in detail the expectations generating system; along the lines of Engsted and Haldrup (1994), the idea is to recognize that the system (10) reads as a present value (PV) model (see e.g. Campbell and Shiller, 1987), so that it is possible to draw and adapt from the literature on this class of models.

Let $\mathbf{e}_t = [\mathbf{c}_t - \mathbf{\Upsilon} \mathbf{w}_t - \phi(t+1)]$ be the $(N-1) \times 1$ vector containing deviations of actual consumption from the optimal risk sharing position, and define the linear combination

$$\mathbf{S}_t = \mathbf{e}_t - \mathbf{K} \mathbf{e}_{t-1} + \mathbf{K} \mathbf{\Upsilon} \Delta \mathbf{w}_t.$$

By simply extending Engsted and Haldrup (1994) to the case of multiple decision variables, standard algebraic manipulations of the model (10) imply that the stationary ‘spread’

$$\boldsymbol{\xi}_t = \mathbf{S}_t - (\rho \mathbf{K})^{-1} \mathbf{S}_{t-1} + (\mathbf{I}_{(N-1)} - \mathbf{K}) \mathbf{\Upsilon} \Delta \mathbf{w}_t \quad (16)$$

must be unpredictable given information available at time $t-2$. This property of $\boldsymbol{\xi}_t$ is derived under the assumption that in (10) $E(\mathbf{v}_t | \mathcal{F}_t) = \mathbf{v}_t$ (that means that \mathbf{v}_t is observable for the econometrician exactly as it was assumed for the model (13)); however, if it is argued that

$E(\mathbf{v}_t | \mathcal{F}_t)$ could be also zero, it can be proved that the spread must be unpredictable given information dated $t - 1$ and earlier¹².

As it will be detailed in Section 4.2, the property of the spread with respect to the observable information may be used to set out a test of the dynamic risk sharing models in the situations where consumption changes and relative consumption help to predict real exchange rates and to estimate the parameters of interest.

4 Estimation and testing

The cointegration implications (3) of the risk sharing model introduced in the present paper have been already discussed in Section 2. Thus it will assumed, except where explicitly indicated, that the preference parameters in Υ and ϕ are known or fixed at their super-consistent estimates obtained in a first stage by means of cointegration techniques (see below).

Given Υ and ϕ and after fixing the average discount factor ρ to an economically plausible value, the empirical analysis of the dynamic risk sharing model introduced in Section 3 requires the estimation of the structural parameters \mathbf{K} in (10) and a test of consistency with the data. To this purpose we propose two approaches discussed respectively in Section 4.1 and Section 4.2. In the first case we deal with the maximum likelihood (ML) estimation of the dynamic risk sharing model (11)-(13), i.e. the model obtained under the assumption agents compute expectations on the consumption growth rate of the leader country and on the changes of real exchange rates by assuming that $\Delta \mathbf{w}_t = (\Delta c_t^0, \Delta r_t^{1/0}, \Delta r_t^{2/0}, \dots, \Delta r_t^{N-1/0})'$ is strongly exogenous with respect to Υ (see Section 3.1). In the second case we set out a regression-based method which hinges on the PV nature of model (10) and in particular on the property of the spread (16) regardless the expectations generating system (see Section 3.2).

4.1 Maximum likelihood approach

Define the $(2N - 1) \times 1$ vector $\mathbf{y}_t = (\mathbf{c}'_t, \mathbf{w}'_t)'$ and assume that the DGP is generated by the I(1) cointegrated model (Johansen, 1996)

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \boldsymbol{\Pi}_1 \Delta \mathbf{y}_{t-1} + \dots + \boldsymbol{\Pi}_{p-1} \Delta \mathbf{y}_{t-p+1} + \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 t + \boldsymbol{\varepsilon}_t \quad (17)$$

where $\boldsymbol{\alpha}$ is the $(2N - 1) \times r$ matrix of adjustment parameters, r the number of cointegrating relations among the elements of \mathbf{y}_t , $\boldsymbol{\beta}$ is the $(2N - 1) \times (N - 1)$ matrix containing the cointegrating vectors, $\tilde{\boldsymbol{\Pi}}_i$, $i = 1, \dots, p - 1$ are $(2N - 1) \times (2N - 1)$ matrices, $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}'_{ct}, \boldsymbol{\varepsilon}'_{wt})'$ is a MDS with

¹²We do not discuss in this context the merits/drawbacks of assuming $E(\mathbf{v}_t | \mathcal{F}_t) = \mathbf{v}_t$ or $E(\mathbf{v}_t | \mathcal{F}_t) = \mathbf{0}$ in rational expectation models; for a detailed discussion we refer to e.g. Fanelli (2005).

respect to \mathcal{F}_t with Gaussian distribution and covariance matrix

$$\mathbf{V}_\varepsilon = \begin{bmatrix} \mathbf{V}_{cc} & \mathbf{V}_{cw} \\ \mathbf{V}_{wc} & \mathbf{V}_{ww} \end{bmatrix}.$$

Given $\mathbf{y}_t = (\mathbf{c}'_t, \mathbf{w}'_t)'$ and $\varepsilon_t = (\varepsilon'_{ct}, \varepsilon'_{wt})'$, we can consider the corresponding partitions of the parameters of the VEqCM:

$$\begin{aligned} \boldsymbol{\alpha} &= \begin{pmatrix} \boldsymbol{\alpha}_c \\ \boldsymbol{\alpha}_w \end{pmatrix}, \boldsymbol{\mu}_0 = \begin{pmatrix} \boldsymbol{\mu}_{0,c} \\ \boldsymbol{\mu}_{0,w} \end{pmatrix}, \boldsymbol{\mu}_1 = \begin{pmatrix} \boldsymbol{\mu}_{1,c} \\ \boldsymbol{\mu}_{1,w} \end{pmatrix}. \\ \boldsymbol{\Pi}_i &= \begin{bmatrix} \boldsymbol{\Pi}_{c,i} \\ \boldsymbol{\Pi}_{w,i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Pi}_{cc,i} & \boldsymbol{\Pi}_{cw,i} \\ \boldsymbol{\Pi}_{wc,i} & \boldsymbol{\Pi}_{ww,i} \end{bmatrix}, \quad i = 1, \dots, p-1 \end{aligned}$$

which allows to express the partial system for $\Delta \mathbf{c}_t$ and the partial system for $\Delta \mathbf{w}_t$ given the past information \mathcal{F}_{t-1} as

$$\Delta \mathbf{c}_t = \boldsymbol{\alpha}_c(\boldsymbol{\beta}'\mathbf{y}_{t-1}) + \boldsymbol{\Pi}_{c,1}\Delta \mathbf{w}_{t-1} + \dots + \boldsymbol{\Pi}_{c,p-1}\Delta \mathbf{w}_{t-p+1} + \boldsymbol{\mu}_{0c} + \boldsymbol{\mu}_{1c}t + \varepsilon_{ct} \quad (18)$$

$$\Delta \mathbf{w}_t = \boldsymbol{\alpha}_w(\boldsymbol{\beta}'\mathbf{y}_{t-1}) + \boldsymbol{\Pi}_{w,1}\Delta \mathbf{w}_{t-1} + \dots + \boldsymbol{\Pi}_{w,p-1}\Delta \mathbf{w}_{t-p+1} + \boldsymbol{\mu}_{0w} + \boldsymbol{\mu}_{1w}t + \varepsilon_{wt} \quad (19)$$

If ε_t is Gaussian, the partial system for $\Delta \mathbf{c}_t$ conditioned with respect to $\Delta \mathbf{w}_t$ and past information \mathcal{F}_{t-1} is then given by

$$\Delta \mathbf{c}_t = \boldsymbol{\omega}\Delta \mathbf{w}_t + \boldsymbol{\delta}(\boldsymbol{\beta}'\mathbf{y}_{t-1}) + \tilde{\boldsymbol{\Pi}}_1\Delta \mathbf{w}_{t-1} + \dots + \tilde{\boldsymbol{\Pi}}_{p-1}\Delta \mathbf{w}_{t-p+1} + \tilde{\boldsymbol{\mu}}_0 + \tilde{\boldsymbol{\mu}}_1t + \tilde{\varepsilon}_{ct} \quad (20)$$

where $\boldsymbol{\omega} = \mathbf{V}_{cw}\mathbf{V}_{ww}^{-1}$, $\boldsymbol{\delta} = \boldsymbol{\alpha}_c - \boldsymbol{\omega}\boldsymbol{\alpha}_w$, $\tilde{\boldsymbol{\Pi}}_i = \boldsymbol{\Pi}_{c,i} - \boldsymbol{\omega}\boldsymbol{\Pi}_{w,i}$, $\tilde{\boldsymbol{\mu}}_h = \boldsymbol{\mu}_{h,c} - \boldsymbol{\omega}\boldsymbol{\mu}_{h,w}$, $h = 0, 1$, and $\tilde{\varepsilon}_{ct} = \varepsilon_{ct} - \boldsymbol{\omega}\varepsilon_{wt}$ (Johansen, 1996) with $E(\tilde{\varepsilon}_{ct}\varepsilon'_{wt}) = \mathbf{0}_{(N-1) \times N}$ by construction.

It can be now seen that the dynamic risk sharing model (13)-(11) represents a special case of (20)-(19). Indeed, the former can be obtained from the latter under a precise set of restrictions on both the long run and short run parameters.

As regards the long run, the vector $\mathbf{e}_t = \mathbf{c}_t - \boldsymbol{\Upsilon}\mathbf{w}_t - \boldsymbol{\phi}(t+1)$ should be stationary for the risk sharing model to hold (Section 2); therefore, the number r of cointegrating relations involving \mathbf{y}_t should not be lower than $N-1$. Furthermore, by partitioning $\boldsymbol{\beta}$ of (17) as $\boldsymbol{\beta} = (\boldsymbol{\beta}_c, \boldsymbol{\beta}_w)$, it must hold:

$$\boldsymbol{\beta}_c = \begin{pmatrix} \mathbf{I} \\ -\boldsymbol{\Upsilon}' \end{pmatrix}, \quad \boldsymbol{\mu}_{1c} = \boldsymbol{\alpha}_c\boldsymbol{\phi} \quad (21)$$

so that given $\boldsymbol{\delta} = (\boldsymbol{\delta}_c, \boldsymbol{\delta}_w)$, the quantity $\boldsymbol{\delta}(\boldsymbol{\beta}'\mathbf{y}_{t-1}) + \tilde{\boldsymbol{\mu}}_1t$ corresponds to $\boldsymbol{\delta}_c\mathbf{e}_{t-1} + \boldsymbol{\delta}_w\boldsymbol{\beta}'_w\mathbf{y}_{t-1}$ in (20), where the second addend cancels out when $r = N-1$. It is clear that a number of cointegrating relations lower than $N-1$ implies that in the long run consumption streams and real exchange rates do not conform to (3)¹³.

¹³For the sake of simplicity we do not consider in the present paper the case $r > (N-1)$.

As regards the short run, once the cointegration rank is fixed to $r = N - 1$ and β is fixed to (21) ($\beta_w = \mathbf{0}$, $\delta_w = \mathbf{0}$), the structure of dynamic adjustment implied by (11)-(13) is obtained if the following exclusion restrictions on (20) and (19) hold jointly:

$$\tilde{\Pi}_{cc,i} = \mathbf{0} \quad , \quad i = 1, \dots, p - 1 \quad , \quad \tilde{\Pi}_{cw,p-1} = \mathbf{0} \quad (22)$$

$$\alpha_w = \mathbf{0} \quad , \quad \Pi_{wc,i} = \mathbf{0} \quad , \quad i = 1, \dots, p - 1 \quad , \quad (23)$$

where $\tilde{\Pi}_i = \begin{bmatrix} \tilde{\Pi}_{cc,i} & \tilde{\Pi}_{cw,i} \end{bmatrix}$. It can be easily recognized that the latter constraints correspond to the strong exogeneity of $\Delta \mathbf{w}_t$ with respect to β .

Under (22)-(23) the dynamic risk sharing model (13) is equivalent to the unrestricted conditional VAR model (20) under the constraints

$$\delta = \mathbf{K} - \mathbf{I}_{(N-1)} \quad , \quad \omega = \Gamma_0 \quad , \quad \tilde{\Pi}_{cw,i} = \Gamma_i \quad , \quad i = 1, \dots, p - 2$$

and therefore estimation of (20) subject to (22) (provided that also (23) holds) provides the ML estimates of \mathbf{K} , Γ_0 , Γ_i , $i = 1, \dots, p - 2$. Furthermore, for fixed values of β (Υ and ϕ) and of the intertemporal discount factor ρ , a likelihood ratio (LR) or Wald test can be carried out for the cross-equation rational expectations restrictions (14), where Φ is defined exactly as in (12) (with $\Phi_i = \Pi_{ww,i}$, $i = 1, \dots, p - 1$).

Summing up, the VEqCM (17) can be used to estimate the parameters of the dynamic risk sharing model through standard likelihood methods. In principle, the number of cointegrating relation could be determined by Johansen's (1996) procedure. After the cointegration rank is determined, the constraints on the cointegration space as well as those on the deterministic drift terms as in (21) could be tested through standard LR or Wald-type statistics. However, since the model involves both a time-series and spatial dimension, it can be easily recognized that even with a relatively long span of data, it is sufficient a small number of countries, N , to the make the analysis based on $\mathbf{y}_t = (\mathbf{c}'_t, \mathbf{w}'_t)'$ cumbersome, due to the huge number of parameters to be estimated. On the other hand, the approach proposed by Pesaran et al. (2004) for modelling regional interdependencies through a "Global" VAR (GVAR) model, though appealing, is in this case not undertaken; indeed, by construction in Pesaran et al. (2004) each country-specific VAR contains a set of foreign variables constructed as weighted averages of the country specific variables with weight given by trade shares (or similar weighting schemes), whereas in our analysis each country-specific VAR involves per capita consumption of the leader country (see below).

For this reason we follow a different estimation approach sketched in the steps that follow:

- 1 Estimate the preference parameters Υ and ϕ of (4) by specifying, for each country $i = 1, \dots, N - 1$, VAR models of the form $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$ (with a linear trend in the

deterministic part) and investigating the number and structure of cointegration relations by following Johansen's (1996) procedure. This allows, for $i = 1, \dots, N - 1$, to recover super-consistent estimates of the preference parameters in Υ and ϕ .

- 2 Given the (super-consistent) estimates of Υ and ϕ and having fixed β and μ_{1c} as in (21), estimate the VEqCM (17) (hence (20)-(19)) and test the validity of the zero restrictions (22)-(23)¹⁴; if these are not supported by the data reject the form (13)-(11) of the dynamic risk sharing model, otherwise go to the step 3 below. Observe that the rejection of (13)-(11) implies that the dynamic risk sharing model obtained under the assumption of strong exogeneity of $\Delta \mathbf{w}_t$ is not supported by the data; it could happen, in fact, that the more general implications the implications described in Section 4.2
- 3 Estimate the system (20) subject to (22) and recover the ML estimates of \mathbf{K} , Γ_0 , Γ_i , $i = 1, \dots, p - 2$. For fixed values of the intertemporal discount factor ρ , check the validity of the rational expectations constraints (14).

4.2 Present value test

As observed in Section 3.2, regardless the 'true' structure of the expectations generating feedbacks and the presence of feedbacks from $\Delta \mathbf{c}_t$ (and \mathbf{e}_t) to $\Delta \mathbf{w}_t$, a general implication of the PV model (10) is that the spread

$$\xi_t = \mathbf{S}_t - (\rho \mathbf{K})^{-1} \mathbf{S}_{t-1} + (\mathbf{I}_{(N-1)} - \mathbf{K}) \Upsilon \Delta \mathbf{w}_t \quad (24)$$

is unpredictable given the information available at time $t - 2$ if the disturbance term $\mathbf{v}_t = (\mathbf{I}_N - \mathbf{K}) \boldsymbol{\eta}_t$ can be argued to belong to the observable information set F_t , and is unpredictable given the information available at time $t - 1$ if \mathbf{v}_t is not supposed to belong to F_t . This consideration suggests that if the structural parameters Υ , ρ , \mathbf{K} of the model were known, a simple (weaker) test of the dynamic risk sharing model might be constructed by regressing ξ_t on information variables dated $t - 2$ (or $t - 1$) and testing for their joint significance.

More specifically, consider the standard VAR(q) approximation for the spread ξ_t , q being sufficiently large:

$$\xi_t = \mathbf{b} + \sum_{i=1}^q \Psi_i \xi_{t-i} + \varphi_t \quad ; \quad \varphi_t \sim WN(\mathbf{0}, \mathbf{\Lambda}). \quad (25)$$

where \mathbf{b} is a $(N - 1) \times 1$ constant and Ψ_i are $(N - 1) \times (N - 1)$ matrices of parameters. Given the above VAR representation a necessary condition for the spread to be unpredictable given

¹⁴Observe that as long as the number of countries involved is around $N = 5$ or $N = 6$, the estimation of the stationary model (20)-(19) with cointegrating relations fixed is still feasible if the number of time-series observations T , is sufficiently high, see the results that follows.

information available at time $t - 2$ is that

$$\Psi_j = \mathbf{0}_{(N-1),(N-1)}, \quad j = 2, \dots, p \quad (26)$$

whereas the additional constraint

$$\Psi_1 = \mathbf{0}_{(N-1),(N-1)} \quad (27)$$

must hold if the spread is unpredictable given information available at time $t - 1$.

In principle, the above zero restrictions can be easily tested in the context of the stationary VAR(q) model for ξ_t . In practice, however, although the a priori knowledge on the average intertemporal discount factor ρ is high and the parameters in Υ can be replaced by their super-consistent estimates without affecting the asymptotics, the adjustment matrix \mathbf{K} entering the spread equation is unknown and must be estimated.

The ‘natural’ approach for estimating \mathbf{K} in (24) given Υ and ρ and the above mentioned orthogonality conditions is to apply Generalized Method of Moments (GMM) techniques. Alternatively, a regression-based approach can be undertaken. Indeed by using the definition of ξ_t and \mathbf{S}_t and imposing the zero restrictions (26), the VAR model (25) reads, after rearranging terms as,

$$\begin{aligned} \mathbf{e}_t - \Upsilon \Delta \mathbf{w}_t &= \mathbf{b} + [\mathbf{K} + (\rho \mathbf{K})^{-1} + \Psi_1] \mathbf{e}_{t-1} \\ &+ [\Psi_1 + \rho^{-1} \mathbf{I}_{N-1}] \Upsilon \Delta \mathbf{w}_{t-1} + [\rho^{-1} \mathbf{I}_{N-1} - \mathbf{K} - \Psi_1 (\rho \mathbf{K})^{-1}] \mathbf{e}_{t-2} \\ &- \rho^{-1} \Psi_1 \Upsilon \Delta \mathbf{w}_{t-2} + \rho^{-1} \Psi_1 \mathbf{e}_{t-3} + \varphi_t \end{aligned} \quad (28)$$

where we recall that ρ and Υ can be treated as known. The structure of the above multiple regression model suggests that, abstracting from the moment from the non-linear (over-identifying) restrictions characterizing the parameters Ψ_1 and \mathbf{K} , a test of the necessary condition for the dynamic risk sharing model (10) to hold can be carried out by regressing $\mathbf{e}_t - \Upsilon \Delta \mathbf{w}_t$ over a constant, \mathbf{e}_{t-1} , \mathbf{e}_{t-2} , \mathbf{e}_{t-3} , $\Upsilon \Delta \mathbf{w}_{t-1}$ and $\Upsilon \Delta \mathbf{w}_{t-2}$ and testing whether the residuals $\hat{\varphi}_t$ are generated by an uncorrelated process. If the residuals of such multiple regressions are ‘well-behaved’ it is possible to proceed by re-estimating (28) by ML subject to the non-linear restrictions

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{K} + (\rho \mathbf{K})^{-1} + \Psi_1 \\ \mathbf{A}_2 &= (\Psi_1 + \rho^{-1} \mathbf{I}_{N-1}) \\ \mathbf{A}_3 &= \rho^{-1} \mathbf{I}_{N-1} - \mathbf{K} - \Psi_1 (\rho \mathbf{K})^{-1} \\ \mathbf{A}_4 &= -\rho^{-1} \Psi_1 \\ \mathbf{A}_5 &= \rho^{-1} \Psi_1 \end{aligned}$$

where we denoted respectively by \mathbf{A}_1 , \mathbf{A}_2 , \mathbf{A}_3 , \mathbf{A}_4 and \mathbf{A}_5 the parameters associated with the regression.¹⁵

Estimation simplifies remarkably if $\boldsymbol{\xi}_t$ is assumed to be unpredictable given information available at time $t - 1$, see the discussion in Section 3.2. Now, in addition to (26) also (27) holds in the VAR (25) and the regression model (28) collapses to

$$\mathbf{e}_t - \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-\rho}^{-1} \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-1} = \mathbf{b} + [\mathbf{K} + (\rho \mathbf{K})^{-1}] \mathbf{e}_{t-1} + [\rho^{-1} \mathbf{I}_{N-1} - \mathbf{K}] \mathbf{e}_{t-2} + \boldsymbol{\varphi}_t. \quad (29)$$

and the necessary condition for the spread to be unpredictable is that the residuals $\hat{\boldsymbol{\varphi}}_t$ of the OLS regression of $\mathbf{e}_t - \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-\rho}^{-1} \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-1}$ over a constant, \mathbf{e}_{t-1} and \mathbf{e}_{t-2} are uncorrelated. Moreover, given the equation above the matrices of parameters \mathbf{B}_1 and \mathbf{B}_2 of the multiple regression

$$\mathbf{e}_t - \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-\rho}^{-1} \boldsymbol{\Upsilon} \Delta \mathbf{w}_{t-1} = \mathbf{b} + \mathbf{B}_1 \mathbf{e}_{t-1} + \mathbf{B}_2 \mathbf{e}_{t-2} + \boldsymbol{\varphi}_t \quad (30)$$

satisfy, under the null (26)-(27)

$$\mathbf{B}_1 + \mathbf{B}_2 = \rho^{-1} (\mathbf{K}^{-1} + \mathbf{I}_{N-1}) \quad (31)$$

$$\mathbf{B}_2 = \rho^{-1} \mathbf{I}_{N-1} - \mathbf{K} \quad (32)$$

so that \mathbf{B}_1 and \mathbf{B}_2 are non-linearly restricted with $(N - 1)^2$ over-identifying constraints. The estimation of (30) through ML subject to (31)-(32) delivers a LR test for the model and, if the over-identifying restrictions are supported by the data, a consistent estimate of \mathbf{K} .

5 Empirical results

In this Section we apply the proposed international risk sharing model to a set of ‘core’ European countries that joined the European Monetary Union (EMU) in 1999. Specifically, some of the major (in terms of population and GDP levels) EMU countries are considered, i.e. Germany, France, Italy, Spain, Netherlands, Belgium, Portugal and Austria; we also included the U.K. as the most important ‘non-EMU joining’ country of the European Union (EU). We use annual data over a relatively long period from 1963 to 2003 and consider Germany as the ‘leader’ country¹⁶. Data are collected from several international sources. Data on private final consumption expenditure at current prices, total population, and price deflators for final consumption expenditure are taken from National Accounts and Eurostat. Nominal exchange rates are constructed

¹⁵This approach to the estimation of \mathbf{K} , however, becomes unfeasible when the number of countries N is high.

¹⁶It could be reasonably argued that estimation results might depend on the choice of modelling Germany as the “leader” country; this choice appeared to us the “natural” alternative to the analysis of $[9!/(2!7!)] = 36$ country-pairs.

by using conversion rates between euro and former euro-zone national currencies, source: Eurostat. The real exchange rates, $rex_t^{i/0}$, are defined as $rex_t^{i/0} = ex_t^{i/0} + p_t^0 - p_t^i$, where $ex_t^{i/0}$ is the (logged) nominal exchange rate between the currency of the i -th country and the DM, p_t^0 is the (logged) private consumption deflator of Germany and p_t^i is the (logged) private consumption deflator of country i . After 1999 real exchange rates are constructed, except for the U.K., with reference to the fixed nominal exchange rates, so their variation is due to relative prices only.

Some descriptive evidence is reported in Table 1, where for each country i =France, Italy, Spain, Netherlands, Belgium, Portugal, Austria and U.K., and given 0 =Germany, the correlation between real (per capita) consumption growth (Δc_t^i) relative to Germany (Δc_t^0) and the corresponding changes of the bilateral real exchange rate ($\Delta r_t^{i/0}$), $Corr(\Delta c_t^i - \Delta c_t^0, \Delta r_t^{i/0})$ are computed. Consistent with other studies, Table 1 presents *prima facie* evidence at odds with open economy models with FRS. Indeed if the theoretical relation (2) would hold with θ^i possibly close to one and with negligible η_t^i terms, one would expect positive (and possibly high) correlations; Table 1 shows that relative consumption and real exchange rates are in almost all cases negatively correlated over the 1963-2003 period and the highest correlation is as low as 32% and involves Belgium.

The apparent lack of risk sharing among the major European countries resulting from Table 1 could be explained, in the light of the theoretical reference equation (1), by observing that each pair of investigated countries might not share the same relative risk aversion parameters, i.e. $\theta^i \neq 1$ so that the actual correlation is between $\Delta c_t^i - \theta^i \Delta c_t^0$ and $\Delta r_t^{i/0}$ ¹⁷. Another argument is that the poor correlations of Table 1 can be due to the effect of non-negligible stochastic terms η_t^i , which in the light of the model specified in Section 2 may reflect: the preference shocks of the two countries, factors which are beyond the control of the planner, arguments of the utility function that interact non-separably with consumption but which have not explicitly modelled, e.g. hours worked/leisure, government spending, real money balances, see e.g. Ravn (2001). Our alternative explanation is that the empirical counterpart of the model (1)-(2) should be regarded as long run relations between the variables in levels c_t^i , c_t^0 and $r_t^{i/0}$.

RISK SHARING IN THE LONG RUN. Following the procedure outlined in Section 4.1 (step1) we proceeded by estimating for i =France, Italy, Spain, Netherlands, Belgium, Portugal, Austria and U.K., separate VARs of the form $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$ over the 1963-2003 period, with a linear trend in the deterministic part and restricted to belong to the cointegration space. Although one could reasonably expect the presence of a structural break in 1991 due to the German unification

¹⁷Moreover, as argued in Section 3, even the correlation between $\Delta c_t^i - \theta^i \Delta c_t^0$ and $\Delta r_t^{i/0}$ might be flawed by the omission of the lagged quantities $\Delta c_{t-1}^0, \Delta r_{t-1}^{j/0}, \dots$ as well as the error-correcting deviations $(\mathbf{c}_{t-1} - \mathbf{\Upsilon} \mathbf{w}_{t-1})$, if countries face adjustment costs.

in all estimated VARs, we did not find dramatic evidence in favor of a structural change.

Before estimating the VARs we preliminarily carried out standard unit-roots tests on the time-series involved; results are reported in Table 2 and indicate that all variables appear driven by I(1) stochastic trends. This result suggests that ‘strong form’ PPP as implied by stationary real exchange rates can be sharply rejected in our framework. This result also shows that other than (3) an additional cointegrating relation given by the trivial linear combination $(0, 0, 1)\mathbf{x}_t$ should not be expected to hold in $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$.

After selecting the optimal number of lags ensuring ‘well-behaved’ VAR residuals, we tested for the number of cointegrating relations and estimated, where possible, the relation (3). Results are reported in Table 3 and Table 4. Table 3 summarizes the ‘lambda-max’ and ‘trace’ LR tests for cointegration rank (Johansen, 1996) and Table 4 the estimated relation (3) for the country pairs where a theoretically consistent cointegrating relation between c_t^i , c_t^0 and $r_t^{i/0}$ was found¹⁸. It can be noticed that for the Netherlands-Germany pair we reported the estimated relation (3) even though the tests for cointegration rank did not support the existence of long run relations; indeed, differently from the Spain-Germany and UK-Germany pairs, in that case we obtained, by forcing the cointegration rank to one, a theoretically consistent stationary relation in levels. On the other hand, the cointegration rank tests shed some doubt on the cointegration rank characterizing the VAR model for Portugal-Germany; nevertheless, in this case we were not able to identify any cointegrating relation consistent with the theory.

In summary, from the results of Table 3 and Table 4 it is possible to identify six ‘core’ EMU countries: Germany, France, Netherlands, Belgium, Austria and to some extent Italy, which seem to share risks, over the 1963-2003 period, consistently with the predictions of the model (1)-(2). For easy of reference henceforth we shall refer these countries as ‘Group 1’. On the other hand, the two Iberian countries and the U.K. seem to depart from the implications of the theory on the levels of variables. We shall refer to Spain and Portugal as ‘Group 2’.

Referring to the countries of Group 1, the estimated relations in Table 4 suggest that German real per capita consumption plays the role of a common dynamic factor; moreover for these countries we do find a significant role for the real exchange rate in the risk sharing relations. Except for the Italy-Germany country pair, all estimated coefficients exhibit signs consistent with the theory and reveal that countries do not have identical attitudes towards risk. The estimated ratio of the German coefficient relative to country i -th relative risk aversion coefficient, θ^i , ranges from 0.57 to 1.31, whereas the estimated intertemporal rate of substitution of consumption, δ^i , ranges from 0.22 to 2.81 (excluding the case of Italy-Germany). The unex-

¹⁸We carried out the empirical analysis also over sub-samples. In particular we devoted attention to the 1975-2003 and 1980-2003 sub-periods and in both cases we did not find significant differences with respect to the results reported in the tables that follow.

pected sign of the estimated δ^i in the Italy-Germany VAR model seems to indicate a violation of concavity in the CRRA utility function, albeit the χ^2 test for the hypothesis $\delta^i = 0$ leads to a p-value slightly above 0.05. The magnitude of the estimated δ^i 's in the Netherlands-Germany and Belgium-Germany pairs also points that albeit useful, the assumed CRRA specification for utility functions can be too restrictive, a common finding in the consumption literature; it can be also argued that time aggregation problem inherent in existing consumption data might have biased estimates. Furthermore, for all five country-pairs a linear trend enters significantly the cointegrating relations (see the parameter ϕ^i in (3)), highlight valuable (although small in magnitude) differences between countries intertemporal discount rates. Thus, it can be argued that for the five European country-pairs reported in Table 4 the η_t^i stochastic term appearing in the risk sharing relation (2) can be modelled as a stationary processes; this means that either preference shocks follow stationary processes, or that in this case variables omitted from the utility function that that interact non-separably with consumption (hours worked/leisure, government spending, real money balances) do not play a role. In other words, it can be ruled out for these countries the effect of non-separable terms in the utility functions.

By interpreting the results of Table 4 in the spirit of Obstfeld (1989, 1994), Backus and Smith (1993), Canova and Ravn (1996) and Ravn (2001), it can be concluded that the countries of Group 1 appear characterized by a remarkable process of financial integration and tend to insure risks against permanent income shocks. These findings contrast with Sørensen and Yosha (1998) where, using methods that ignore dynamic adjustment, poor risk sharing is detected in Europe.

On the other hand, the countries of Group 2, including the UK, do not seem to be part of the above mentioned process; clearly the lack of cointegration among c_t^i , c_t^0 and $r_t^{i/0}$ in these countries could be explained as the result of non-negligible η_t^i terms in the long run. This evidence, nevertheless, does not rule out the occurrence of international insurance mechanisms shielding against short term income fluctuations, an issue which is investigated below.

DYNAMIC ADJUSTMENT. Having established that international risk sharing as a long run phenomenon seem to characterize the EMU countries of Group 1, we are now able to investigate the short run dynamic properties of the real per capita consumptions in the light of the theoretical implications of the model outlined in Section 3. Recall that the dynamic risk sharing implies that adjustments should be interrelated, in the sense that also foreign countries' lagged temporary deviations from FRS are important for explaining future home consumption changes. One of our primary interests is to evaluate whether one of this prediction of the model find support from the data.

We start by specifying an unrestricted VECM model for $\mathbf{y}_t = (\mathbf{c}'_t, \mathbf{w}'_t)'$, $\mathbf{c}_t = (c_t^{fra}, c_t^{ita}, c_t^{net}, c_t^{bel}, c_t^{aus})'$,

$\mathbf{w}_t = (c_t^{ger}, r_t^{fra}, r_t^{ita}, r_t^{net}, r_t^{bel}, r_t^{aus})$ and with the cointegration equations (21) fixed at the super-consistent estimates obtained in Table 4. The number of lags of the VECM is set to 1 (i.e. 2 lags in the corresponding VAR in levels); standard residual-based specification tests, reported in Table 5, do not show any particular departure from the model assumptions. Furthermore, the stability of the model, for fixed cointegration relations, has also been checked through recursive techniques, showing that the short run parameters appear to be substantially stable over the considered time span.

The unrestricted VECM clearly appear to be overparametrized. In order to find a suitable reduction of the model, in Table 6 a number of sets of zero restrictions are tested (through standard Likelihood ratio tests) separately for each equation as well as at the system level. In the first row of the table, for each equation we test whether the lagged first differences of non-country specific variables (with the exception of the German consumption) can be excluded. Similarly, in the second row of the table, for each equation we test whether the lagged first differences of all variables with the exception of the German consumption and the country-specific real exchange rate can be excluded (recall that, according to (13), Δc_{t-1}^j should not appear among the regressors of the j -th equation).¹⁹ For France, Belgium and Austria, the exclusion restriction are not rejected, either with or without lagged home consumption growth among the regressors. For the Netherlands, the restrictions are not rejected provided that lagged consumption growth appears among the regressors. Only in the case of Italy the restrictions are strongly rejected, showing that Italian consumption growth seems to be characterized by a more involved dynamic structure than the other countries (which may indicate imperfection in the risk sharing mechanisms at shorter horizons, as documented by Cavaliere et al., 2005). Turning to the issue of the impact of disequilibrium terms, i.e. the impact on the short-run consumption dynamics of the deviations of actual consumption streams from optimal risk sharing levels, in Table 6, rows 3 to 5, we test for each country (as well as at the system level) whether changes to home consumption do not depend (i) on the lagged disequilibria in all countries (row 3), (ii) on the lagged home disequilibrium (row 4) and (iii) on the lagged foreign disequilibria (row 5). Overall the picture is quite clear: consumption changes *do* depend on past deviations from the optimal consumption level; furthermore, cross-adjustment terms seem to be important for most countries.

Detailed estimates of the VECM model with the exclusion restrictions tested in Table 6 imposed are reported in Table 7. For France we find strong adjustment with respect to home past

¹⁹The dynamic adjustment structure obtained in our model under quadratic adjustment costs is quite tight: we believe that the inclusion of lagged consumption growth might help to catch possible habit persistence effects which are implicitly ruled out in the solution of the model.

deviations from the optimal risk sharing position. For the Netherlands, home disequilibrium is partially significant (the associated p-value is just above 10%), and we also find evidence of inter-related disequilibrium correction mechanisms with Italy. Note that lagged consumption changes matters, hence stressing the occurrence of habit persistence phenomena even stronger than those accounted by the adjustment cost structure implied by the optimization problem (6). In the case of Belgium, we observe a significant effect of the home disequilibrium term; interestingly, we also observe a significant effect of the disequilibrium terms associated with neighbouring countries. In the case of Austria, only the own disequilibrium from FRS matters in explaining future home consumption changes. Finally, as anticipated above, changes to consumption in Italy display a quite complicate dynamic structure, which tend to be strongly affected by lagged changes to the consumption of most countries. In summary, these results shows that risk sharing, when present, is not likely to take place instantaneously; the presence of adjustment costs as well as of cross-adjustment costs seem to be a possible explanation of the observed short run dynamic patterns of consumption.²⁰ In general, the presence of cross-adjustment terms in the country specific consumption equations points that idiosyncratic shocks tend to propagate across the ‘core’ European countries engaged in the risk sharing arrangement.

Even though labour mobility can be hardly regarded as a risk sharing channel in Europe and a permanent tax-transfer system ruled by a central fiscal institution is far from being fully at work, the above results show that short run consumption fluctuations of Group 1 countries appear consistent with a gradual process of adjustment towards the outcome that would prevail under ‘frictionless’ markets.

An interesting question is whether the Group 2 countries, which are not found to share risk with the other countries in the long run, are characterized by short term movements in their consumption which are similar to those observed for the Group 1 countries. To this purpose, in Table 8 we report the estimates of a VAR model for Spain and Portugal, including exogenous regressors given by the lagged error correcting terms of the Group 1 countries estimated in Table 4. Interestingly, while for the case of Portugal changes in real per capita consumption are found to depend only on their lagged value, for the case of Spain changes in consumption are correlated to contemporaneous changes in German consumption as well as to the deviations from the optimal FRS consumption levels of all the Group 1 countries. Hence, although failing to share risks in the long run with the Group 1 countries, Spain (but not Portugal) seems to be characterized by a marked process of adjustment of its consumption which resembles – to some extent – that predicted by the proposed dynamic risk sharing model. Finally, as regards the

²⁰As previously notices, a by-product implication is traditional risk sharing tests based on growth rate specifications are usually misspecified in that they do not consider the role of the disequilibrium.

UK we did not find evidence of the existence of a connection with consumption growth rates of Group 1 countries and the corresponding bilateral real exchange rate changes.

We conclude this section by noticing that according to the estimates reported in Table 7, the zero restrictions (22) implied by the dynamic risk sharing model on the parameters of the estimated VECM can be rejected (Section 4.1, step 2); more precisely, the zero constraints in (22) result in 55 exclusion restrictions on the parameter of the system (20) and lead to a LR statistic equal to 126.49. This result is not surprising if we recognize that the dynamic risk sharing model has been derived under the restrictive assumption of quadratic adjustment costs; it may also depend, however, on the underlying assumption of exogeneity of real exchange rates (and of German consumption), which underlie the derivation of such restrictions. The results in Table 9 shows that the exogeneity assumption (23) is clearly rejected²¹, hence making formal tests of the cross-equation restriction derived in equation (14) unreliable. Tests of the dynamic implications of the risk sharing model which do not require exogeneity, see Section 4.2, are discussed next.

PRESENT VALUE TEST OF DYNAMIC RISK SHARING. We have already observed that the hypothesis of quadratic adjustment costs combined of that of strong exogeneity of real exchange rates is very tight in the present framework, so a more flexible perspective is needed to asses the empirical adequacy of the proposed dynamic risk sharing model.

As discussed in the sections 3.2 and 4.2, a general implication of the risk sharing model (10) is that the spread variable defined in (24) must be unpredictable given the information observable at time $t - 2$ or $t - 1$, depending on the assumptions made on preference shocks.

Given the annual frequency of the data it is reasonable to investigate predictability of the vector of spreads relative the Group 1 countries with respect to the information available at time $t - 1$, by checking whether the estimation through ML of the regression model (30) delivers residuals which are not correlated. In this multiple regression model the vector of deviations of observed consumption streams from the optimal FRS positions is defined as $\mathbf{e}_t = \mathbf{c}_t - \mathbf{\Upsilon}\mathbf{w}_t - \boldsymbol{\phi}(t + 1)$, where the preference parameters $\mathbf{\Upsilon}$ and $\boldsymbol{\phi}$ are fixed at their super-consistent ML estimates of Table 4. The intertemporal discount factor ρ can possibly be selected within a given grid of values.

Table 10 reports standard residuals correlation tests associated with the OLS regression of $\mathbf{e}_t - \mathbf{\Upsilon}\Delta\mathbf{w}_t - \rho^{-1}\mathbf{\Upsilon}\Delta\mathbf{w}_{t-1}$ (with ρ fixed at 0.96) over a constant, \mathbf{e}_{t-1} and \mathbf{e}_{t-2} . Residuals should be uncorrelated for the spread to be unpredictable given information at time $t - 1$. Although at the system level the test seems to reject the hypothesis of uncorrelated residuals,

²¹It is worth noting that the result that consumption helps forecasting real exchange rates confirms the finding in Apte et al. (2000).

at the single-equation level support for the null can be observed. This means that in its weakest form, i.e. without explicit reference to the implied non linear parametric constraints, the risk sharing model (10) appears consistent with the data. On the other hand, the estimation of the regression model (30) subject to the restrictions (31)-(32) entails $(N - 1)^2 = 25$ over-identification constraints which lead to LR statistic of 126.8.²²

6 Final remarks

Risk sharing can be achieved through private markets or central fiscal institutions. Actual markets, however, are characterized by frictions that impede instantaneous adjustment to the equilibrium that can be achieved in the absence of frictions. Market frictions generate costs to countries and obstacle the achievement of FRS. However, investment barriers, transport costs, tariffs and other barriers might not be large enough to impede the ex-post nominal marginal rate of substitution to equalize across countries, after a gradual process of adjustment.

This paper formalizes a stylized process of adjustment towards optimal risk sharing where a benevolent social planner minimizes the costs that countries experience by deviating from FRS due to market frictions. The resulting dynamic risk sharing equations imply interrelated adjustment across countries and give a precise role to expectations on future market developments. One of the main implications is that positive (negative) shocks affecting one country produce adjustments in the consumption streams of all the other countries belonging to the agreement. Omitting such kind of dynamics as in e.g. Kollmann (1995) and Canova and Ravn (1996) or by simply differentiating optimal risk sharing relations as in Ravn (2001), among many others, may flaw both tests and measures of the extent of international risk sharing.

Our dynamic risk sharing model is applied to a set of ‘core’ European countries which joined the EMU in 1999 over forty years span of data; one would expect integrated capital markets and shielding mechanisms against permanent income fluctuations at work for a single currency to have economic grounds (Eichengreen, 1993). Our results suggest that European consumption streams tend to conform to the implications of risk sharing as a long run phenomenon; moreover, our estimates highlight a marked process of adjustment towards equilibrium which, to some extent, can be attributed to intertemporal (dis)utility costs. It can be argued that the lack of

²²Due to the relatively high order 5×5 of the matrices of parameters involved, the estimation of (30) subject to (31)-(32) was simplified by replacing the constraints in (32) by a first-order Taylor series expansion around a 5×5 diagonal matrix; more precisely, assuming for easy of exposition that $\rho \approx 1$, the function $f(K) = K^{-1} + I$ implied by (31) was approximated linearly as $f(K) \approx f(K_0) + (-K_0)^{-1}(K - K_0)(K_0)^{-1}$, with K_0 containing the diagonal elements of the estimated adjustment coefficients $\hat{\alpha}_c$ of (20), see Table 8. Given the rejection of the over-identifying restrictions we did not report the estimated $(I_5 - \hat{K})$ matrix in Table 10.

European risk sharing found in previous studies, see e.g. Sørensen and Yosha (1998), can be largely be explained by the rich dynamic structure underlying European consumption streams.

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Tables

FRA	ITA	SPA	NET	BEL	POR	AUS	UK
-0.06	-0.29	-0.19	-0.06	0.32	-0.22	-0.08	-0.03

Table 1: PEARSON CORRELATION COEFFICIENTS BETWEEN REAL CONSUMPTION GROWTH (RELATIVE TO GERMANY) AND THE REAL EXCHANGE RATES GROWTH.

Notes: correlations are computed as $Corr(\Delta c_t^i - \Delta c_t^0, \Delta r_t^{i/0})$, $i = \text{FRA,ITA,SPA,NET,BEL,POR,AUS,UK}$; 0 = GER.

	GER	FRA	ITA	SPA	NETH	BEL	POR	AUS	UK
c_t^i	-1.24	-1.52	-1.60	-2.50	-1.95	-1.94	-0.70	-0.95	-1.78
Δc_t^i	-1.40	-2.20	-2.10	-3.14	-2.47	-2.80	-1.32	-1.23	-4.28
$r_t^{i/0}$	-2.66	-2.00	-1.57	-2.62	-2.00	-1.42	-1.44	-1.96	-1.94
$\Delta r_t^{i/0}$	-7.36	-5.66	-3.59	-5.33	-2.88	-4.54	-4.75	-3.17	-4.48

Table 2: AUGUMENTED DICKEY-FULLER TESTS ON REAL PER CAPITA CONSUMPTIONS AND REAL EXCHANGE RATES.

Notes: The tests are based on univariate $AR(p)$ models with the number of lags p selected according to the MAIC criterion of Ng and Perron (2001) under the constraint $p \leq [12(T/100)^{1/4}]$. The ADF regression for the variables in levels include a linear time trend. The 5% critical value of the ADF statistics is -3.43 for regressions with trends, -2.86 for regressions without trends.

1963-2003									Critical values	
	FRA	ITA	SPA	NETH	BEL	POR	AUS	UK	5%	10%
VAR lags	2	2	2	2	2	2	1	2		
Eigenvalues										
1	0.50	0.45	0.38	0.33	0.47	0.43	0.47	0.36		
2	0.23	0.24	0.19	0.21	0.17	0.38	0.32	0.28		
3	0.18	0.15	0.18	0.14	0.16	0.19	0.14	0.16		
Maximum eigenvalue statistics										
$r = 0$	28.53	24.70	19.33	16.59	26.08	22.88	25.77	18.68	25.43	23.11
$r \leq 1$	10.54	11.22	8.84	9.85	7.83	19.43	15.69	13.79	19.18	16.85
$r \leq 2$	8.02	6.83	8.08	6.36	7.17	8.41	6.11	7.16	12.36	10.49
Trace statistics										
$r = 0$	47.10	42.74	36.25	32.80	41.07	50.71	47.57	39.63	42.20	39.08
$r \leq 1$	18.56	18.05	16.92	16.21	14.99	27.84	21.80	20.95	25.47	22.95
$r \leq 2$	8.02	6.83	8.08	6.36	7.17	8.41	6.11	7.16	12.39	10.56

Table 3: COINTEGRATION RANK TESTS IN THE COUNTRY-SPECIFIC VAR MODELS

Notes: Each VAR is based on $x_t^i = (c_t^i, c_t^0, r_t^{i/0})'$, $i = \text{FRA,ITA,SPA,NET,BEL,POR,AUS,UK}$; 0 = GER. The estimated model contains restricted linear trend coefficients and unrestricted intercepts. Critical values of the Trace LR statistics are from Johansen (1996), Table 15.4; critical values for the maximum eigenvalue tests are from Osterwald Lenun (1992)

1963-2003	$(c_t^j - \theta^j c_t^0 - \delta^j r_t^{j/0} - \phi^j t)$
France-Germany	$c_t^{fra} - 0.57c_t^0 - 0.37r_t^{fra/0} - 0.0054t$
Italy - Germany	$c_t^{ita} - 0.88c_t^0 + 0.11r_t^{ita/0} - 0.009t$
Netherlands - Germany	$c_t^{net} - 0.72c_t^0 - 2.81r_t^{net/0} - 0.007t$
Belgium - Germany	$c_t^{bel} - 1.31c_t^0 - 2.15r_t^{bel/0} - 0.009t$
Austria - Germany	$c_t^{aus} - 0.59c_t^0 - 0.22r_t^{aus/0} - 0.015t$

Table 4: ESTIMATED RISK SHARING RELATIONS.

Notes: Estimates are based on the country-specific VAR models with cointegration rank set to 1. All coefficients are significant at the 5% level except δ^{ita} .

	FRA	ITA	NET	BEL	AUS	System
Serial Correlation, $F(2, 22)$	0.30	0.72	0.00	1.79	0.25	0.86 ^a
Normality, $\chi^2(2)$	2.28	0.53	0.73	1.14	5.04*	9.26 ^b
Heteroskedasticity, $\chi^2(32)$	30.74	35.26	35.13	37.22	34.15	485.9 ^c
Heteroskedasticity(conditional), $F(1, 22)$	0.00	0.01	0.70	0.09	0.21	—

Table 5: MISSPECIFICATION TESTS FOR THE UNRESTRICTED REDUCED FORM SYSTEM IN THE VECM FORMAT (18).

Notes: The number of lags p is set to 2; the cointegration matrix is fixed at the estimates of Table 4. * denotes significance at the 10% level. The null asymptotic distributions of the tests are as given in the table except: ^a $F(50,48)$, ^b $\chi^2(10)$, ^c $\chi^2(480)$.

	FRA	ITA	NET	BEL	AUS	System
Exclusion of lagged home consumption and all foreign variables, χ^2 (9)	10.84	22.47***	26.11***	13.66	10.60	71.60 ^a ***
Exclusion of all foreign variables, χ^2 (8)	7.67	20.05**	12.29	13.66*	10.36	68.11 ^b ***
Exclusion of all ECM terms, χ^2 (5)	12.18**	8.32*	18.66**	23.95***	16.84***	88.88 ^c ***
Exclusion of home ECM terms, χ^2 (1)	3.83**	8.79***	1.41	3.37*	8.30***	30.83 ^d ***
Exclusion of cross ECM terms, χ^2 (4)	7.27	14.39**	13.18**	14.88***	2.56	59.22 ^e ***

Table 6: TESTS OF SOME ZERO RESTRICTIONS ON THE UNRESTRICTED REDUCED FORM SYSTEM IN THE VECM FORMAT (18)

Notes: The number of lags p is set to 2; the cointegration relations are fixed at the estimates of Table 4. First row: tests of exclusion of all foreign variables and of the lagged home consumption from each equation; Second row: tests of exclusion of all foreign variables and of the lagged home consumption from each equation; Third row: tests of exclusion of all (home and foreign) lagged deviations (ECMs) from the optimal consumption levels under full risk sharing; Fourth row: tests of exclusion of the home (lagged) ECM; Fifth row: tests of exclusion of all foreign (lagged) ECMs. * denotes significance at the 10% level; ** denotes significance at the 5% level; *** at the 1% level. The null asymptotic distributions of the tests are as given in the table except: ^a χ^2 (45); ^b χ^2 (40); ^c χ^2 (25); ^d χ^2 (5); ^e χ^{10} (4).

	FRA	ITA	NETH	BEL	AUS
Δc_{t-1}^{fra}		-0.60** 0.23			
Δc_{t-1}^{ita}		0.19 0.14			
Δc_t^{net}		0.59*** 0.16	0.30** 0.11		
Δc_{t-1}^{bel}		-0.35* 0.18			
Δc_{t-1}^{aus}		0.29* 0.16			
Δc_{t-1}^{ger}	0.16 0.12	0.17 0.17	0.47** 0.18	-0.00 0.13	-0.03 0.13
$\Delta r_{t-1}^{fra/0}$	-0.12** 0.05	-0.17** 0.07			
$\Delta r_{t-1}^{ita/0}$		0.02 0.04			
$\Delta r_{t-1}^{net/0}$			-0.39*** 0.10		
$\Delta r_{t-1}^{bel/0}$				-0.15** 0.07	
$\Delta r_{t-1}^{aus/0}$					-0.09 0.12
ECM_{t-1}^{fra}	-0.30** 0.11	0.12 0.16	-0.12 0.16	0.22* 0.12	-0.04 0.13
ECM_{t-1}^{ita}	0.15 0.10	-0.43*** 0.14	0.29* 0.14	0.10 0.11	0.09 0.12
ECM_{t-1}^{net}	0.02 0.02	0.01 0.03	-0.05 0.03	-0.06*** 0.02	0.01 0.02
ECM_{t-1}^{bel}	0.02 0.03	-0.01 0.05	-0.01 0.05	-0.08* 0.04	-0.02 0.04
ECM_{t-1}^{aus}	-0.18* 0.10	-0.50** 0.15	0.16 0.15	-0.14 0.11	-0.48*** 0.13
R^2	0.66	0.72	0.73	0.61	0.60

Table 7: ESTIMATION OF THE REDUCED FORM SYSTEM IN THE VECM FORMAT (18) UNDER A SET OF ZERO RESTRICTIONS

Notes: The cointegration relations are fixed at the estimates of Table 4. The likelihood ratio test for the $m = 38$ zero restrictions is $LR = 51.604$ (p-value: 0.07).

	SPA	POR
Δc_t^{ger}	0.41*** 0.11	0.19 0.33
Δc_{t-1}^{ger}	-0.24* 0.14	
Δc_{t-1}^{spa}	-0.25** 0.11	
Δc_{t-1}^{por}		0.61*** 0.13
$\Delta r_t^{spa/0}$	0.05 0.05	
$\Delta r_{t-1}^{spa/0}$		
$\Delta r_t^{por/0}$		-0.03 0.14
$\Delta r_{t-1}^{por/0}$		0.23 0.14
ECM_{t-1}^{fra}	-0.27** 0.13	0.35 0.30
ECM_{t-1}^{ita}	-0.58*** 0.14	-0.53 0.33
ECM_{t-1}^{net}	-0.09*** 0.03	
ECM_{t-1}^{bel}	-0.07* 0.04	
ECM_{t-1}^{aus}	-0.49*** 0.12	
R^2	0.81	0.53

Table 8: ESTIMATION OF A CONDITIONAL VECM MODEL FOR THE IBERIAN COUNTRIES UNDER A SET OF ZERO RESTRICTIONS

Notes: The cointegration relations are fixed at the estimates of Table 4. The likelihood ratio test for the $m = 11$ zero restrictions is 10.53 (p-value: 0.48).

	FRA	ITA	NET	BEL	AUS	System
$r_t^{i/0}$ (Weak)	18.28***	12.97**	7.67	34.41***	21.25***	74.83 ^a ***
$r_t^{i/0}$ (Strong)	25.71***	19.15**	17.97*	42.54***	30.52***	121.33 ^b ***

Table 9: EXOGENEITY TESTS FOR REAL EXCHANGE RATES. Notes: The tests are based on the reduced form system (19). * denotes significance at the 10% level; ** denotes significance at the 5% level; *** at the 1% level. All weak (strong) exogeneity tests have $\chi^2(5)$ ($\chi^2(10)$) null asymptotic distribution except: ^a $\chi^2(25)$; ^b $\chi^2(50)$.

	FRA	ITA	NET	BEL	AUS	System
Serial Correlation, $F(2, 28)$	1.30	1.56	0.70	1.08	0.52	2.49 ^a ***

Table 10: TESTS OF THE PRESENT VALUE RESTRICTIONS.

Notes: *** denotes significance at the 1% level. The null asymptotic distributions of the tests are as given in the table except: ^a $F(50, 76)$.