WORKING PAPER n.02.02

Aprile 2002

FIGARCH models: stationarity, estimation methods and the identification problem

M. Caporin[a]

a. Università Ca' Foscari, Venice.

FIGARCH models: stationarity, estimation methods and the identification problem

Massimiliano Caporin
Università "Ca' Foscari" di Venezia

First version November 2001
This version March 2002

Abstract

In this study we extend the analysis of Baillie, Bollerslev and Mikkelsen (1996) and Bollerslev and Mikkelsen (1996) on the estimation and identification problems with a FIGARCH specification for the conditional variance. We first deal with the stationarity issue, then we show that the well-known Lee and Hansen (1994) proof for consistency of GARCH(1,1) quasi maximum likelihood estimators cannot be extended to the FIGARCH case, therefore we suggest an alternative approach. We show then the power of different information criteria in distinguishing among short and long memory specifications for the conditional volatility. In the same situation traditional tests for residual correlation and ARCH effects fail to discriminate across models.

Keywords: FIGARCH, long memory, identification.

1 Introduction

With the availability of high frequency data for financial markets analysis there has been an increase in studies dealing with the persistence of shocks both on the mean and on the variance of financial instrument returns. There were also numerous findings of persistence in financial markets, in particular in the volatility, see among others, Breidt, Crato and De Lima (1998). Starting from these points Baillie, Bollerslev and Mikkelsen (1996) introduced a new process, the FIGARCH, generalizing the well known GARCH to allow for persistence in the conditional variance. They presented a detailed Montecarlo analysis showing, empirically, the consistence and asymptotic normality of the quasi maximum likelihood estimators for the parameters of interest, giving also some insight in the theory, suggesting that the Lee and Hansen (1994) proof for consistency of GARCH(1,1) could be easily extended to the FIGARCH case. Moreover in a second paper by Bollerslev and Mikkelsen (1996), another Montecarlo study is presented, dealing in that case with the identification of FIGARCH orders.

In our study we will extend these analysis in different directions: at a first stage, section 2, we introduce the FIGARCH model, as an extension of the IGARCH, mentioning another formulation, due to Chung (2001), slightly different from the one of Baillie et al. (1996), in that section we will also deal with the stationarity and ergodicity of FIGARCH. We consider then the estimation problem in section 3, showing that the Lee and Hansen proof cannot be used in the FIGARCH context, and we suggest an alternative approach due to Jeantheau (1998). In the last part of this study we will present a detailed Montecarlo analysis, showing the power of different identification criteria and tests in discerning between long and short memory in conditional volatility. We will show that, on a long memory data generating process, the information criteria of Akaike, Hannan-Quinn, Shibata and Schwarz can clearly identify long memory but cannot help in specifying the orders of the process. With the same approach we show that the traditional tests of normality and for the presence of autocorrelation and ARCH effects, when applied to the standardized residuals, fail to discriminate among short and long memory processes, and are therefore useless in the identification analysis.

2 The FIGARCH processes

In this study we assume, unless differently specified, that the following representation hold for the mean process:

$$y_t = \mu_t + \varepsilon_t \tag{1}$$

where for simplicity $\mu_t = 0$, I^{t-1} represent the information set up to time $t-1$ and $\varepsilon_t | I^{t-1} \sim iid\left(0, \sigma_t^2\right)$: the mean is assumed to be zero and the residuals, conditional to the information set up to time $t-1$, are identically distributed with mean zero and time-dependent variances.

Following Engle (1982) and Bollerslev (1986) we specify a GARCH(p,q) model for the variance, that is:

$$\varepsilon_t = z_t \sigma_t \tag{2}$$

where $E\left[z_t | I^{t-1}\right] = 0$, $Var\left[z_t | I^{t-1}\right] = 1$ and σ_t is defined by

$$\sigma_t^2 = \omega + \alpha(L)\,\varepsilon_t^2 + \beta(L)\,\sigma_t^2 \tag{3}$$

where L is the lag operator, $\alpha(L) = \sum_{i=1}^{q} \alpha_i L^i$, $\beta(L) = \sum_{j=1}^{p} \beta_j L^j$. The stationarity of this process is achieved when the following restriction is satisfied: $\alpha(1) + \beta(1) < 1$.

Defining $\upsilon_t = \varepsilon_t^2 - \sigma_t^2$ this process may be conveniently rewritten as an ARMA(m,p) process

$$[1 - \alpha(L) - \beta(L)]\,\varepsilon_t^2 = \omega + [1 - \beta(L)]\,\upsilon_t \tag{4}$$

with $m = \max\{p, q\}$. From this formulation, allowing for the presence of a unit root in $1 - \alpha(L) - \beta(L)$, Bollerslev and Engle (1986) defined the IGARCH(p,q) process:

$$(1 - L)\,\phi(L)\,\varepsilon_t^2 = \omega + [1 - \beta(L)]\,\upsilon_t \tag{5}$$

where $\phi(L) = \sum_{i=1}^{m-1} \phi_i L^i$ and is of order $m - 1$. For a comprehensive survey on GARCH processes, refer to Bollerslev, Engle and Nelson (1994). Even if flexible, and with numerous extensions to include particular characteristics found in the markets, such as asymmetric behavior, switching regime and news impact, GARCH processes are not able to adequately explain the various finding on persistence (or long memory) in the volatility of financial instruments returns. With persistence we refer to the slow decaying of the autocorrelation function of a time series: in case of persistence, or long memory, the usual exponential decaying, typical of an ARMA representation, turn to a much slower hyperbolically decaying. The first works in this field were the one of Granger and Joyeux (1980) and Hosking (1981) who introduced the ARFIMA processes. This long-memory model is represented as:

$$(1 - L)^d\,\Phi(L)\,(x_t - \mu) = \Theta(L)\,\varepsilon_t \tag{6}$$

where the usual AR and MA polynomial are used $\Phi(L) = \sum_{i=1}^{\overline{p}} \overline{\phi}_i L^i$, $\Theta(L) = \sum_{i=1}^{\overline{q}} \overline{\theta}_i L^i$, with the standard restrictions on parameters to ensure stationarity and invertibility of the process. The main difference is in the integration parameter, d, which here is allowed to assume values in $\mathbb{R}$. In this situation the integration operator $(1 - L)^d$ may be factorized as follow:

$$(1 - L)^d = \sum_{i=0}^{\infty} \pi_i L^i \tag{7}$$

$$\pi_i = \prod_{0 \leq k \leq i} \frac{k - 1 - d}{k}$$

Stationarity and invertibility of the process are obtained constraining d to the range (-$\frac{1}{2}$,$\frac{1}{2}$), while long memory is attained only in the positive region of the previous range. Moreover as the d parameter tends to the upper stationary limit the memory (the correlation with past observations) of the process increase. For a general stationary ARFIMA(p,d,q) process the following relation hold as k$\longrightarrow \infty$:

$$\rho_k \sim C k^{2d-1} \tag{8}$$

that is, the autocorrelation function is approximate by a constant $C > 0$ that may depend on the parameters of the model but not on k, and by an hyperbolic term. The convergence to zero of this autocorrelation is slower than in the ARIMA processes. Using a parallel with ARMA and ARFIMA processes Baillie et al. (1996) extended the IGARCH case allowing the integration coefficient (here previously restricted to the usual dichotomy $\{0,1\}$ to vary in the range $[0,1]$. The FIGARCH(p,d,m)[1] process is defined as follow:

$$(1-L)^d \phi(L)\varepsilon_t^2 = \omega + [1-\beta(L)]\upsilon_t \tag{9}$$

where $\phi(L) = \sum_{i=1}^{m-1} \phi_i L^i$ is of order $m-1$. Baillie et al. (1996) claimed that, extending the arguments of Nelson (1990), the FIGARCH(p,d,m) process, even if not weakly stationary is ergodic and strictly stationary. Unfortunately this is not so easy to verify, we will deal with this problem in a following subsection. Its major feature is connected with the impulse response analysis, which have in this case an hyperbolic decay, typical of long memory models. This mean that the impact of the innovation lie between the exponential decaying, typical of any GARCH, and the infinite persistence, typical of any IGARCH. Davidson (2001) gave some insight on the memory properties of the FIGARCH, pointing out that the degree of persistence of the FIGARCH model operate in the opposite direction of the ARFIMA one: as the d parameter get closer to zero, the memory of the process increase. This is due to the inverse relation between the integration coefficient and the conditional variance: the memory parameter act directly on the squared errors, not on the σ_t^2, this particular behavior may also influence the stationarity properties of the process, again Davidson (2001). These observations are strictly related to the impulse response analysis on the effect of a shock on a system driven by a FIGARCH process. In such a system a shock in time t, should be interpreted as the difference between the squared mean-residuals in time t and the one-step-ahead forecast of the variance of time t, made in time $t-1$. This shock is exactly the innovation in the ARMA representation of the FIGARCH process, that is

$$\varepsilon_t^2 = \omega + [1-\beta(L)]\left[(1-L)^d \phi(L)\right]^{-1} \upsilon_t \tag{10}$$

The shock may be also interpreted as an unexpected volatility variation, or, as the forecast error of the variance (remember that the squared residuals are a proxy for the variance and that the time t variance depend on time $t-1$ information set and may be viewed as a one-step-ahead forecast). Rearranging the FIGARCH equation as in Baillie et al. (1996), expanding the polynomial in the lag operator, it is easy to see that the coefficient of this polynomial converge to zero at a rate $O\left(j^{-d-1}\right)$ this mean that the memory of the process increase ad d gets closer to 1 (Baillie et al. (1996) obtained the wrong sign claiming the same memory property valid for the ARFIMA).

Analyzing in detail ARFIMA and FIGARCH processes Chung (2001) noted that the claimed parallel between the two was not complete: in the ARFIMA case the long memory operator is applied to the constant but this is not true in the FIGARCH model; then in ARFIMA processes $d \in \left(-\frac{1}{2}, \frac{1}{2}\right)$, while in FIGARCH $d \in [0,1]$. In his work Chung suggested an alternative parametrization: starting from (4) we can rewrite the GARCH(p,q) using the value of the

[1] Here we prefer using m instead of q, as BBM, BM and CH do, to avoid possible confusion in the orders of the process: m refer to the order of $\phi(L)$ while q refer to $\beta(L)$

unconditional variance $\sigma^2 = \omega / (1 - \alpha(1) - \beta(1))^2$as:

$$[1 - \alpha(L) - \beta(L)] \left(\varepsilon_t^2 - \sigma^2\right) = [1 - \beta(L)] \upsilon_t$$

and from this equation the alternative formulation is straightforward:

$$(1 - L)^d \phi(L) \left(\varepsilon_t^2 - \sigma^2\right) = [1 - \beta(L)] \upsilon_t \tag{11}$$

However, in this formulation, the interpretation of the parameter σ^2 is not clear: does it represent the unconditional variance as claimed by Chung, or is simply a constant for the squared observations? In this work we will not pursue this point. In his work Chung presented also some Montecarlo results exploiting the different convergence rates of estimators of d above and below the cutoff value of $\frac{1}{2}$. For a more comprehensive discussion on the parametrization and on the structure and behavior of the models refer to the cited paper of Chung (2001). In the remainder we will refer to (9) as FIGARCH I or simply FIGARCH and to (11) as FIGARCH II. The two processes can be conveniently rewritten, exploiting the relation $\upsilon_t = \varepsilon_t^2 - \sigma_t^2$, respectively as:

$$\sigma_t^2 = \omega / [1 - \beta(1)] + \left\{1 - [1 - \beta(L)]^{-1} (1 - L)^d \phi(L)\right\} \varepsilon_t^2 \tag{12}$$

$$\sigma_t^2 = \sigma^2 + \left\{1 - [1 - \beta(L)]^{-1} (1 - L)^d \phi(L)\right\} \left(\varepsilon_t^2 - \sigma^2\right) \tag{13}$$

sometimes these equations are referred as the ARCH(∞) representation. In both FIGARCH I and II, parameters have to fulfill some restrictions to ensure positivity of conditional variances. Here we present the two different sets of sufficient conditions, valid for the FIGARCH(1,d,1), suggested, respectively, by Baillie et al. (1996) and Chung. As noted by Chung (2001), both sets are admissible for FIGARCH I and II, however they are not equivalent and there may exist a set of parameters value that satisfy one set of conditions and not the other. Baillie et al. derived a group of two sets of inequalities

$$\begin{aligned} \beta - d \quad &\leq \quad \phi \leq \frac{2-d}{3} \\ d\left(\phi - \frac{1-d}{2}\right) \quad &\leq \quad \beta(d - \beta + \phi) \end{aligned} \tag{14}$$

while Chung express the restriction with a unique set

$$0 \leq \phi \leq \beta \leq d \leq 1 \tag{15}$$

Restrictions for lower order models can be derived directly from the one presented, while for higher order models restrictions on parameters cannot be so easily computed and are not presented.

[2]From the ARMA representation of GARCH model $\varepsilon_t^2 = \frac{\omega}{1-\alpha(1)-\beta(1)} + \frac{1-\beta(L)}{1-\alpha(L)-\beta(L)} \upsilon_t$ and taking axpectations on both sides we get the value of the unconditional variance

3 Stationarity of FIGARCH I and II

Baillie et al. (1996) were quoting Nelson (1996) for proving the stationarity of the FIGARCH model they proposed, but only limited to the case $p = 1$ and $m = 0$. They claimed that stationarity could be verified with a dominance type argument between the sequence of coefficients of the ARCH(∞) representations of the FIGARCH(1,d,0) and of an appropriately chosen IGARCH(1,1). However as noted by Mikosch and Starica (2001) this "proof" is questionable: how can we bound an hyperbolically decaying sequence of coefficients with an exponential one? This way seem therefore inapplicable. Some insight on the stationarity of this model is due to Davidson (2001) who pointed out that some of the particular relations that hold for FIGARCH may be due to the inverse memory relation. Again referring to Mikosch and Starica (2001) we want to stress an ambiguous point in the Baillie et al. (1996) work: they were defining the FIGARCH model using the ARMA formulation of a general GARCH and then imposing a long memory integration operator $(1-L)^d$, however this methodology in not completely correct since in this derivation the innovation process ν_t depend on the process we are trying to define, therefore we are building a noise sequence that depend on a process defined using that noise sequence! Moreover the ARMA formulation of a FIGARCH process can be derived once a stationary solution is given. The best way to define a FIGARCH model seems therefore the use of a much general approach or formulation, in detail the ARCH(∞) processes, as defined by Robinson (1991):

$$\sigma_t^2 = \tau + \sum_{k=1}^{\infty} \psi_k \varepsilon_{t-k}^2 \qquad (16)$$
$$\tau > 0 \qquad \psi_k \geq 0$$

The FIGARCH structure can be imposed with an adequate formulation of the coefficients in the infinite ARCH expansion. Given this representation the stationarity of the FIGARCH processes can be proved recalling the stationarity conditions for a generic ARCH(∞) process and trying to figure out if the coefficient structure of the FIGARCH can meet this requirements via its ARCH(∞) formulation. The main works in this area are the one of Giraitis, Kokoszka and Leipus (2000), Kazakevicius and Leipus (1999 and 2001), and Zaffaroni (2001).

The first paper, Giraitis et al. (2000) present a condition for the existence of a stationary solution of an ARCH(∞) process, giving the following theorem:

Theorem 1 *(rearranged from Giraitis et al. (2000), page 6, theorem 2.1): given $\varepsilon_t = z_t \sigma_t$ and (16), a stationary solution with finite first moment $E(\varepsilon_t)$ exist if $E(z_t^2) < \infty$ and $E(z_t^2)\sum_{k=1}^{\infty}\psi_k < 1$. If the constant $\tau = 0$ unique stationary solution is $\varepsilon_t = 0$. If $E(z_t^4) < \infty$ and $\left[E(z_t^4)\right]^{1/2}\sum_{k=1}^{\infty}\psi_k < 1$ the stationary solution is unique. (See the cited paper for the proof).*

The stationary solution follow a Volterra series expansion of the form

$$\varepsilon_t^2 = \tau z_t^2 \sum_{l=0}^{\infty} \sum_{h_1<h_2<.....<h_l<l}^{\infty} \psi_{l-h_1}\psi_{h_1-h_2}.....\psi_{h_{l-1}-h_l} z_l^2 z_{h_1}^2.....z_{h_l}^2 \tag{17}$$

This formulation impose a moment condition on the square of the observations and rule out long memory a priori, in fact for any value of d, we have: $(1-L)^d = \sum_{j=0}^{\infty} \pi_j(d) = 1 + \sum_{j=1}^{\infty} \pi_j(d) = 0$. Therefore this result is inapplicable in our case. An extension of this methodology is due to Kazakevicius and Leipus (1999 and 2000), who reformulate the existence and stationarity conditions for an ARCH(∞) in a form similar to the one given by Bougerol and Picard (1992) for the GARCH(p,q) model, that is using a top Lyapunov exponent γ, defined as follows:

$$\gamma = \lim_{n\to\infty} n^{-1} \log \|A_1 A_2 .....A_n\| \tag{18}$$

where the matrices A_j depend on the parameters and on the structure of the process (see the cited papers for an example). The main result of Kazakevicius and Leipus (1999) is summarized in the following theorem:

Theorem 2 *(adapted and rearranged from Kazakevicius and Leipus (1999)): given $\varepsilon_t = z_t\sigma_t$ and (16), if $E(\log z_t^2)$ is well defined $\gamma \leq 0$ is a necessary and $\gamma < 0$ is a sufficient for the existence of an ARCH(∞) process. If for any strictly stationary sequence $(h_i, i \geq 1)$ of non-negative random variables such that $\sum_{i=1}^{\infty} \psi_i h_i < \infty$ we have*

$$\lim_{n\to\infty} \sum_{i=1}^{\infty} \psi_{i+n} h_i = 0 \qquad a.s.$$

and the top Lyapunov exponent γ is negative then (17) is the unique strictly stationary solution. If $\gamma = 0$ there is no solution at all. (See the cited paper for the proof).

In this theorem there are no moment conditions on the standardized errors but there is an integrability conditions and a limit condition on the coefficients of the ARCH(∞) expansion. This result was then used by Kazakevicius and Leipus (2000) to assess the existence and stationarity of the FIGARCH model. The main result is in the following theorem:

Theorem 3 *(adapted and rearranged from Kazakevicius and Leipus (2000)) If a) $E|\log z_t^2| < \infty$ and b) for some $k > 1$ we have $\sum_{i=1}^{\infty} \psi_i k^i < \infty$ then the top Lyapunov exponent γ is strictly negative, therefore the ARCH(∞) exist as well as a stationary solution. If assumption b) is not satisfied the Lyapunov exponent is identically equal to zero.*

Assumption b) simply require that the coefficients of the ARCH(∞) decay at an exponential rate, when this is not the case, as in FIGARCH, the existence of the ARCH(∞) as well as of a stationary solution become questionable. At the end of this excursus among this first group of papers we want to stress a point: the condition for the existence of a stationary solution, imposed through a Lyapunov exponent is a necessary one, therefore a possible less restrictive condition, a sufficient one, may exist. The results of the previous papers did not considered a general approach but came to the FIGARCH analysis only indirectly, imposing conditions that are not fulfilled by FIGARCH processes.

We will now analyze the results of Zaffaroni (2000) to prove the strict stationarity and ergodicity of FIGARCH(p,d,m). The proof is really a corollary to the following theorem of Zaffaroni. Given this setup:

consider the ARCH(∞) model of (16), then if we assume that $\gamma = E\left(\ln z_t^2\right)$ is well defined (even unbounded) and we set

$$\lambda = \begin{cases} \frac{\gamma}{2} & \gamma < 0 \\ \frac{3(\gamma+\delta)}{2} & \gamma \leq 0 \end{cases}$$

for any constant $\delta > 0$ we have the following

Theorem 4 *Zaffaroni 2000, Theorem 2, page 6) Let* $\underline{\sum}_M = \sum_{k=1}^{M} \psi_k$, $\overline{\sum}^M = \sum_{k=M+1}^{\infty} \psi_k$, $\kappa = E\left(z_t^2\right)$. *Assume that a) 0<τ<∞ and b) for at least one* $0 < M < \infty$

$$\max\left[e^{\lambda}\underline{\sum}_M + \kappa\overline{\sum}^M, e^{\lambda}\overline{\sum}^M + \kappa\underline{\sum}_M\right] < 1$$

then for the ARCH(∞) model, for any t, $\tau \leq \sigma_t^2 < \infty$ *a.s. and* σ_t^2 *is strictly stationary and ergodic, with a well-defined nondegenerate probability measure on* $[\tau, \infty)$. *Sufficient conditions to satisfy assumption b) are*

$$e^{\lambda}\sum_{k=1}^{\infty}\psi_k < 1 \quad , \quad \kappa\sum_{k=1}^{\infty}\psi_k \leq 1$$

and $\psi_k\psi_j > 0$ *for at least two* $k \neq j$.

Proof. See Zaffaroni (2000). ■

The power of this theorem is that it does not require any moment condition, apart the integrability condition on the squared residuals as in Kazakevicius an Leipus (1999, 2000), moreover it does not require any strict condition on coefficients allowing mild explosive behaviors as well as hyperbolic decaying. Given this result we can restate the following

Corollary 5 *(adapted from Zaffaroni (2000) Remark 2.2) For 0<d≤1, q≥0, p≥0 and with adequate restrictions on coefficients that ensure positivity of conditional variances, the FIGARCH(p,d,q) I and II are strictly stationary and ergodic if* $\gamma = E\left(\ln z_t^2\right) < 0$.

Proof. for both (12) and (13) we have the following polynomial for the ARCH(∞) representation of the models

$$\lambda(L) = 1 - [1 - \beta(L)]^{-1} (1 - L)^d \phi(L) = \sum_{i=1}^{\infty} \lambda_k L^k$$

if the coefficient satisfies restrictions that ensure positivity of conditional variances $\lambda_k \geq 0 \ \forall k$, and positive for at least one $k \geq 1$, $0 < \omega / [1 - \beta(1)] < \infty$ or $0 < \sigma^2 < \infty$. Then noting that

$$\left.(1 - L)^d\right|_{L=1} = \left.\sum_{i=0}^{\infty} \pi_i L^i\right|_{L=1} = \sum_{i=0}^{\infty} \pi_i = 0 \tag{19}$$

since $\pi_0 = 1$, $\pi_i < 0 \ \forall i > 0$ and $\lim_{k \to \infty} \sum_{i=1}^{k} \pi_i = -1$. So we can write

$$\lambda(1) = \sum_{i=1}^{\infty} \lambda_k = 1 - [1 - \beta(1)]^{-1} \phi(1) \sum_{i=0}^{\infty} \pi_i = 1$$

Using then the fact that $\gamma < 0$ and plugging in the condition of Zaffaroni $\gamma/2 < 0$ we have

$$e^{\gamma/2} \sum_{i=1}^{\infty} \lambda_k = e^{\gamma/2} < 1$$

this complete the proof. ■

We can note that the FIGARCH(p,d,m) is strictly stationary and ergodic under the assumption of normality of the standardized residuals, this can be easily proved by the strict concavity of the log function using Jensen inequality. We want to stress now another point: in GARCH processes is of common use the assumption that the standardized residuals follow a Student distribution, this to capture the fact that the tails of the empirical distributions of financial market returns are thicker than in the normal case. Under the assumption of a T-distribution for z_t we have to check the condition $E\left(\ln z_t^2\right) < 0$, to prove the strict stationarity and ergodicity of the FIGARCH. The square of a T distribution with n degrees of freedom follow an $F(1, n)$ distribution. The evaluation of the expected value was carried out numerically, and the results show that increasing the degrees of freedom, the expected value converge to zero but from above. From this we can state that the FIGARCH(p,d,m) is not strictly stationary under the assumption of a T-distribution for the standardized residuals.

Turning now to the analysis of the FIGARCH specification suggested by Chung (2001): in this case, given the structure of the model we can rewrite it as

$$\begin{aligned} \sigma_t^2 &= [1 - \beta(L)]^{-1} (1 - L)^d \phi(L) \sigma^2 + \left\{1 - [1 - \beta(L)]^{-1} (1 - L)^d \phi(L)\right\} \varepsilon_t^2 = \\ &= \left\{1 - [1 - \beta(L)]^{-1} (1 - L)^d \phi(L)\right\} \varepsilon_t^2 \end{aligned}$$

given the relation (19). This violate one of the assumptions of the Zaffaroni's theorem, the presence of a positive constant and the result cannot be applied. In this situation the previous work of Nelson (1990) shows that the only stationary solution when the constant is null is that the conditional variance itself is null. This result was also derived by Giraitis, Kokoszka and Leipus (1998). This was not noted by Chung (2001), but probably could be observed in a well defined Montecarlo experiment, simulating a long time series reducing in such a way the effect of truncation in the ARCH(∞). In fact we suppose that the approximation induced by the truncation create a stationary solution as in an ARCH(p) model with very high p.

Again referring to Zaffaroni (2000), a direct application of Theorem 3, page 9, show, using previous results, that FIGARCH(p,d,m) is not covariance stationary as IGARCH processes.

4 Estimation of FIGARCH models

In this work we mainly refer to previous analysis and different papers for simulating and estimating techniques, concentrating on their asymptotic behavior, showing the inapplicability of the well known Lee and Hansen proof.

The main problem in obtaining a simulated realization of a FIGARCH process is related with the long memory component. In fact its infinite representation require a necessary truncation in the recursive formulae derived from (12) and (13). Following Baillie et al. (1996), Chung (2001) and Teyssière (1996) we will mirror to some extent their simulation and estimation procedure, introducing for the simulation of a FIGARCH model (case I), some initial values and setting them to the constant, as it appear in equation (12). For the simulation of a series of length T, we will generate $2000 + T$ observations, to avoid dependance from initial values. In the estimation procedure we have to introduce a truncation given that we will use the ARCH(∞) representation for our purposes. In this framework there are two different approaches: the first apply a fixed truncation value say m, for the whole series, while an alternative is to use all data point available, so with an increasing truncation point. Baillie et al. were suggesting the use of a truncation value set to 1000, while Chung was considering the whole information availbale. In this study we will follow the approach of Chung, because by this method all the available information is used.

For the estimation of these kind of processes the mainly used technique is the Quasi Maximum Likelihood, maximizing with respect to the parameters of interest the following log-likelihood function:

$$Q\left(\theta; \{\varepsilon_t\}_{t=1\ldots T}\right) = -\frac{1}{2}\log\left(2\pi\right) - \frac{1}{2}\sum_{t=1}^{T}\left[\log\sigma_t^2 + \varepsilon_t^2/\sigma_t^2\right] \tag{20}$$

where T is the sample size, σ_t^2 follow in this study either (12) or (13), and θ represent the set of parameters. We will also refer, as normal practice in this field, to $\varepsilon_t/\sigma_t = z_t$ as to the standardized residuals. In estimating the different models we will use a truncation value equal to the information set, as in simulation, and we will introduce with our FIGARCH DGP a pre-sample equal to the sample variance of ε_t. The QMLE follow asymptotically a normal distribution with variance-covariance matrix dependant on the Hessian of parameter estimates and on the cross product of gradient at the optimum. In this work we refer to the QML formulation given by Bollerslev and Woolridge (1992), the same used by Baillie et al. in their paper, for comparability of results.

Baillie et al. (1996) claimed that the result of Lee and Hansen (1996), which shows the consistence and asymptotic normality of the QMLE for IGARCH(1,1) processes, "...extends directly to the FIGARCH(1,d,0) model through a dominance-type argument...", unfortunately this is not correct. Taking the lemma numbers as in Lee and Hansen, the cornerstone of their proof is in the possibility of bounding the ratio between the conditional volatility computed with the true parameters and the one computed with the estimated parameters. This is established in Lemma 4.(4) and 4.(5). This result is then used repeatedly to assess boundness of other ratios between conditional variances and then of their expected value used to prove the boundness of the likelihood function. The point is that lemma 4.(4) is no more valid with a FIGARCH(p,d,q) DGP, resulting in an unbounded ratio. We will go through the Lee and Hansen proof to verify this claim, to avoid confusion we will maintain their notation

Lemma 6 *(Lee and Hansen Lemma 4. page 34)*

(4) If $\beta \leq \beta_0$, $\frac{{}_{\epsilon}\sigma_t^2}{{}_0\sigma_t^2} \leq K_l \equiv \frac{\omega_u}{\omega_0} + \frac{\alpha_u}{\alpha_0} < \infty$ *a.s.*

(5) If $\beta \geq \beta_0$, $\frac{{}_{\epsilon}\sigma_t^2}{{}_0\sigma_t^2} \leq H_u \equiv \frac{\omega_0}{\omega l} + \frac{\alpha_0}{\alpha l} < \infty$ *a.s.*

where ${}_{\epsilon}\sigma_t^2$ represent the conditional variance with the estimated parameters and ${}_0\sigma_t^2$ the true conditional variance, whose parameters are denoted by ω_0 and α_0 , while for estimated parameters Lee and Hansen derived a bound depending on the upper (lower) limits of the compact parameter space ω_u and α_u (ω_l and α_l). Moreover they also splitted the parameter space for the β deriving two bounds depending on the relation between the estimated and the true value. This result is then used in deriving the bounds used in verifying the boundness of likelihood function and then for consistency and asymptotic normality. This result is therefore necessary for all the proof, and we are going to plug in the demonstration the FIGARCH(1,d,0) instead of the GARCH(1,1).

Proof. we can go through the proof both using the standard FIGARCH representation or with the ARCH(∞) formulation. Both formulation are equivalent, and we present counterproof of non-consistency using the two representations. Start with the standard one, plugging it in Lemma 4.(4)

$$\frac{{}_{\epsilon}\sigma_t^2}{{}_0\sigma_t^2} = \frac{\omega + \beta\sigma_{t-1}^2 + (d-\beta)\,\varepsilon_{t-1}^2 + \sum_{i=2}^{\infty}(-\pi_i)\,\varepsilon_{t-i}^2}{\omega_0 + \beta\sigma_{t-1}^2 + (d_0-\beta_0)\,\varepsilon_{t-1}^2 + \sum_{i=2}^{\infty}[-\pi_i\,(d_0)]\,\varepsilon_{t-i}^2}$$

where we dropped for convenience the subscripts of the conditional variance. Substituting now repeatedly the conditional variance with its expression, back to the past infinity, we get the following representation

$$\frac{{}_{\epsilon}\sigma_t^2}{{}_{0}\sigma_t^2} = \frac{\frac{\omega}{1-\beta} + (d-\beta)\sum_{i=0}^{\infty}\beta^i\varepsilon_{t-1-i}^2 + \sum_{j=0}^{\infty}\beta^j A_j}{\frac{\omega_0}{1-\beta_0} + (d_0-\beta_0)\sum_{i=0}^{\infty}\beta_0^i\varepsilon_{t-1-i}^2 + \sum_{j=0}^{\infty}\beta_0^j \hat{A}_j}$$

where $A_j = \sum_{i=2}^{\infty}(-\pi_i)\,\varepsilon_{t-i-j}^2$ and $\hat{A}_j = \sum_{i=2}^{\infty}[-\pi_i(d_0)]\,\varepsilon_{t-i-j}^2$. Using the fact that all quantities are positive we can rewrite as

$$\frac{{}_{\epsilon}\sigma_t^2}{{}_{0}\sigma_t^2} \leq \frac{\omega}{1-\beta}\frac{1-\beta_0}{\omega_0} + \frac{d-\beta}{d_0-\beta_0}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta_0}\right)^i + \sum_{j=0}^{\infty}\left(\frac{\beta}{\beta_0}\right)^j\frac{A_j}{\hat{A}_j}$$

focus now on the last term in the formula

$$\frac{A_j}{\hat{A}_j} = \frac{\sum_{i=2}^{\infty}(-\pi_i)\,\varepsilon_{t-i-j}^2}{\sum_{i=2}^{\infty}[-\pi_i(d_0)]\,\varepsilon_{t-i-j}^2} \leq \sum_{i=2}^{\infty}\frac{\pi_i}{\pi_i(d_0)}$$

using again the fact that all terms are positive. Noting that for large M we can use the Stirling approximation on the coefficients we have

$$\pi_k = \frac{\Gamma(k-d)}{\Gamma(-d)\,\Gamma(k+1)} \sim k^{-d-1} \quad \text{for } k > M$$

then from the last summation

$$\sum_{i=2}^{\infty}\frac{\pi_i}{\pi_i(d_0)} \geq \sum_{i=M}^{\infty}\frac{\pi_i}{\pi_i(d_0)} \sim \sum_{i=M}^{\infty}\frac{i^{-d-1}}{i^{-d_0-1}} = \sum_{i=M}^{\infty} i^{d_0-d} = \infty$$

Last equality follow from the fact that for $d_0 > d$ we have a succession of term greater than 1, diverging to infinity, for $d_0 < d$ we have a generalized harmonic succession again diverging. The approximation may be taken as closed as required, but the important point is that this imply that

$$\frac{A_j}{\hat{A}_j} = \infty \Rightarrow \frac{{}_{\epsilon}\sigma_t^2}{{}_{0}\sigma_t^2} = \infty$$

and we cannot so easily bound the ratio as in Lee and Hansen. This result may be interpreted reasoning on the asymptotic decaying of the coefficients. In the GARCH(1,1) case coefficients decay exponentially to zero, while in the FI-GARCH(1,d,0) the convergence is hyperbolic, so a dominance type argument, such as in the claim of Baillie, Bollerslev and Mikkelsen cannot be used, we cannot dominate an hyperbolic decaying succession by an exponentially decaying one, since we can always find a point in which the exponential cross the hyperbolic one and stay below in the infinity. This show the first possibility, but Lee and Hansen carried out the proof using the ARCH(∞) formulation, the same Baillie et al. (1996) were referring to in their paper. Since both representation can be obtained one from the other, the result does not change. Using ARCH(∞) formulation we have

$$\frac{{}_{\epsilon}\sigma_t^2}{{}_{0}\sigma_t^2} = \frac{\frac{\omega}{1-\beta} + \sum_{i=1}^{\infty}\lambda_i\varepsilon_{t-i}^2}{\frac{\omega_0}{1-\beta_0} + \sum_{i=1}^{\infty}\lambda_i(d_0)\,\varepsilon_{t-i}^2}$$

where $\lambda(L) = 1 - \left[(1-\beta L)^{-1}(1-L)^d\right]$ and using positivity of all coefficients and quantities we have

$$\frac{{}_\epsilon\sigma_t^2}{{}_0\sigma_t^2} \leq \frac{\omega}{1-\beta}\frac{1-\beta_0}{\omega_0} + \sum_{i=1}^{\infty}\frac{\lambda_i}{\lambda_i(d_0)}$$

and we focus on the terms of the last summation

$$\frac{\lambda_i}{\lambda_i(d_0)} = \sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i \frac{\pi_{j-i}}{\pi_{j-i}(d_0)}$$

the evaluation of the ratio between the coefficient of the long memory integration part can be carried out only defining the relation between the true value of d and its estimate. Observing Figure 1, that plot the coefficients $\pi_i(d)$ for $d \in (0.01, 0.99)$ and $i = 2, 3...1000$, we can deduce that

$$\begin{cases} a) \; \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} > 1 & \frac{1}{2} \leq d \leq d_0 < 1 \\ b) \; \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} < 1 & \frac{1}{2} \leq d_0 \leq d < 1 \\ c) \; \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} > 1 & 0 \leq d \leq d_0 < \frac{1}{2} \quad j-i > M \\ d) \; \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} < 1 & 0 \leq d_0 \leq d < \frac{1}{2} \quad j-i > M \end{cases}$$

then for case a)

$$\sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} \geq \sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i = \left[1-\left(\frac{\beta}{\beta_0}\right)^i\right]\left[1-\frac{\beta}{\beta_0}\right]^{-1}$$

and plugging back

$$\sum_{i=1}^{\infty}\frac{\lambda_i}{\lambda_i(d_0)} \geq \left[1-\frac{\beta}{\beta_0}\right]^{-1}\sum_{i=1}^{\infty}\left[1-\left(\frac{\beta}{\beta_0}\right)^i\right] = \infty$$

and the ratio cannot be bounded.

For case b)

$$\sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} \leq \sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i = \left[1-\left(\frac{\beta}{\beta_0}\right)^i\right]\left[1-\frac{\beta}{\beta_0}\right]^{-1}$$

$$\sum_{i=1}^{\infty}\frac{\lambda_i}{\lambda_i(d_0)} \leq \left[1-\frac{\beta}{\beta_0}\right]^{-1}\sum_{i=1}^{\infty}\left[1-\left(\frac{\beta}{\beta_0}\right)^i\right] = \infty$$

again an unbounded relation. Cases c) and d) are a bit complex, in fact the relation showed are true only after M coefficients, the previous M-1 have the inverse relation. But we can find such M, for which the following relations hold

$$\sum_{j=0}^{i}\left(\frac{\beta}{\beta_0}\right)^i \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} \geq \sum_{j=M+1}^{i}\left(\frac{\beta}{\beta_0}\right)^i \frac{\pi_{j-i}}{\pi_{j-i}(d_0)} \gtrless \left(\frac{\beta}{\beta_0}\right)^{M+1}\left[1-\left(\frac{\beta}{\beta_0}\right)^{i-M}\right]\left[1-\frac{\beta}{\beta_0}\right]^{-1}$$

using the same substitutions as before, and where the last relation depend on cases c) and d). Then using

$$\sum_{i=1}^{\infty} \frac{\lambda_i}{\lambda_i(d_0)} \geq \sum_{i=M+1}^{\infty} \frac{\lambda_i}{\lambda_i(d_0)}$$

and plugging back we are in the same situation as in case a) or b) obtaining an unbounded relation for case c), while for case d) we have

$$\sum_{i=1}^{\infty} \frac{\lambda_i}{\lambda_i(d_0)} \geq \sum_{i=M+1}^{\infty} \frac{\lambda_i}{\lambda_i(d_0)} \leq \infty$$

a bound may exist but cannot be found by this way. ■

Given that Lemma 4.(4) of Lee and Hansen is not consistent with FIGARCH(1,d,0), then following their proof alsoLemma 4.(5) and Lemma 6.(1) give non-bounded relations. Therefore we cannot split the parameter space as in page 35 and Lemma 5, 7 and 8 are no more valid, breaking down all the proof for the consistence. Moreover also the proof of asymptotic normality break down because is built on the bound used to prove the consistence. A similar result can be obtained also for the FIGARCH(1,d,1) with a non-bounded solution for likelihood function ratios.

Given that this approach cannot be pursued for the proof of the consistence of the QMLE for FIGARCH, we follow the works of Jeantheau (1998) that suggest a different set of conditions that imply consistency of QMLE. Here is the set of assumptions required by Jeantheau and the theorem:

1. Compactness: Θ is compact
2. Ergodicity: $\forall \theta_0 \in \Theta$ the model considered admits a unique strictly stationary and ergodic solution
3. Lower bound: there exist a deterministic constant $c > 0$ such that $\forall t, \forall \theta_0 \in \Theta, \sigma_t^2 \geq c$
4. Logarithmic moments: $\forall \theta_0 \in \Theta, E_{\theta_0}\left[\left|\log\left(\sigma_t^2\right)\right|\right] < \infty$
5. Identifiability: the conditional variance is such that $\forall \theta \in \Theta, \forall \theta_0 \in \Theta, \sigma_t^2(\theta) = \sigma_t^2(\theta_0)\, a.s. \Rightarrow \theta = \theta_0$
6. Continuity: the conditional variance is a continuous functions of the parameters

Theorem 7 *(Jeantheau 1998, page 72, theorem 2.2)*

Under assumptions 1-6, the QMLE of the parameters is strongly consistent, that is to say

$$\hat{\theta}_T \stackrel{T\to\infty}{\to} \theta_0 \; a.s$$

Under the FIGARCH DGP: assumption 1 is common in GARCH framework and is embodied in the restrictions that ensure positivity, we have only to add an upper bound for the constant; assumption 2 and 3 are derived by the corollary to the theorem of Zaffaroni; assumption 6 depend on the structure of the model and is also satisfied. There remain to prove only assumptions 4 and 5, for the first we state the following result, while for the identification we assume it a priori:

Lemma 8 *If σ_t^2 follow a FIGARCH structure, $\omega + \beta \geq 1$ and the model is correctly identified, $E_{\theta_0}\left[\left|\log\left(\sigma_t^2\right)\right|\right] < \infty$*

Proof. Considering that

$$\sigma_t^2 = \frac{\omega}{1-\beta} + \sum_{i=1}^{\infty} \lambda_i \varepsilon_{t-i}^2$$

under the assumption that $\omega + \beta \geq 1$ we can write

$$\sigma_t^2 \geq 1 \rightarrow \log\left(\sigma_t^2\right) \geq 0 \rightarrow E_{\theta_0}\left[\left|\log\left(\sigma_t^2\right)\right|\right] = E_{\theta_0}\left[\log\left(\sigma_t^2\right)\right]$$

then we have

$$E_{\theta_0}\left[\log\left(\sigma_t^2\right)\right] < \log E_{\theta_0}\left[\sigma_t^2\right]$$

and given that

$$\sigma_t^2 = \frac{\omega}{1-\beta} + \sum_{i=1}^{\infty} \lambda_i \sigma_{t-i}^2 z_{t-i}^2 = \frac{\omega}{1-\beta} + \frac{\omega}{1-\beta} \sum_{i=1}^{\infty} \lambda_i z_{t-i}^2 + \sum_{i=1}^{\infty} \lambda_i z_{t-i}^2 \sum_{j=1}^{\infty} \lambda_j \sigma_{t-j}^2 z_{t-j}^2 = ...$$

$$z_t \sim iidN\left(0,1\right) \rightarrow z_t^2 \sim iid\chi^2\left(1\right) \rightarrow E\left[z_t^2\right] = 1$$

$$\sum_{i=1}^{\infty} \lambda_i = 1$$

we can therefore prove

$$E_{\theta_0}\left[\sigma_t^2\right] = \sum^{\infty} \frac{\omega}{1-\beta} = \infty$$

$$E_{\theta_0}\left[\log\left(\sigma_t^2\right)\right] < \log E_{\theta_0}\left[\sigma_t^2\right] = \infty$$

■

So we have verified the 4th condition, we cannot find the upper bound but only say that the logarithmic moment under a restriction on the parameter space and the assumption of normality of standardized errors is finite. Assuming that the model is correctly identified we have also the condition 5, and the theorem of Jeantheau hold.

We showed consistency in a constrained region for the QMLE estimator, a complete proof, removing the hypothesis on admissible parameter space in not available at the moment. Another important point must be specified: Jeantheau theorem deal with pointwise consistency, not with uniform consistency, given that he used in the proof a pointwise uniform law of large number to show convergence of the estimator. A more general proof of consistency necessary need a generic uniform law of large numbers that will ensure convergence to the correct parameter set in the whole parameter space.

5 The identification problem

In this section we deal with the identification problem. Given a real time series, whose conditional variances appear to be time dependent, possibly showing also long memory, how can we choose the best parametrization? In this introductory part we define criteria and tests that will be used in the following experimental analysis.

Information criteria are a well known method for model identification and specification. Even if they are constructed with different approaches their structure is at the end very similar. All are based on the maximum likelihood evaluated at the optimum, and on a penalization term that depend on the number of parameters and on sample length. Defining $\hat{Q}\left(\widehat{\theta};\{\varepsilon_t\}_{t=1\ldots T}\right)$ as the QML evaluated at the optimal point $\widehat{\theta}$, T as the sample length and k as the number of parameters in the specification used, the four information criteria we use have the following representation[3]:

Akaike IC	$-2\frac{\hat{Q}\left(\widehat{\theta};\{\varepsilon_t\}_{t=1\ldots T}\right)}{T}+2\frac{k}{T}$
Hannan-Quinn IC	$-2\frac{\hat{Q}\left(\widehat{\theta};\{\varepsilon_t\}_{t=1\ldots T}\right)}{T}+2\frac{k\log[\log(T)]}{T}$
Schwarz IC	$-2\frac{\hat{Q}\left(\widehat{\theta};\{\varepsilon_t\}_{t=1\ldots T}\right)}{T}+2\frac{\log(k)}{T}$
Shibata IC	$-2\frac{\hat{Q}\left(\widehat{\theta};\{\varepsilon_t\}_{t=1\ldots T}\right)}{T}+\log\left(\frac{T+2k}{T}\right)$

As we can see the only difference is in the penalization term. One point arise: all these information criteria are built in general models, and with general assumptions, but can we apply them in presence of conditional heteroskedasticity? The answer comes from the work of Sin and White (1996) who provided general sufficient conditions on the penalization term, that guarantee the correct selection of the model choosing the with the lower information criteria.

[3] Akaike (1973), Hannan-Quinn (1979), Schwarz (1978) and Shibata (1980)

In this study we will also make use of different tests: for normality, correlation and ARCH effects. All of these tests are performed on the standardized (SR) and/or squared-standardized residuals (SSR):

$$\begin{aligned} SR &= \hat{z}_t = \frac{\varepsilon_t}{\hat{\sigma}_t\left(\hat{\theta}\right)} \\ SSR &= \hat{z}_t^2 \end{aligned} \tag{21}$$

For normality we will compute, on SR, the Jarque-Bera normality test, whose distribution, under the null hypothesis of normality is a χ^2 with 2 degree of freedom. This test is based on sample skewness and kurtosis of SR, with the following equation:

$$JB = \frac{1}{T}\left[\left(\frac{\hat{\mu}_3^2}{3}\right) + \frac{(\hat{\mu}_4 - 3)^2}{24}\right] \to \chi^2(2) \tag{22}$$

where T is the sample length, $\hat{\mu}_3$ and $\hat{\mu}_4$ are respectively sample skewness and sample kurtosis.

The tests for the presence of correlation are ran both on SR and SSR, using the Box-Pierce Q-statistic. This imply the computation of empirical autocorrelations up to an order l, on which the test will be based. Under the null hypothesis of no correlation the test based on l autocorrelation is distributed as a $\chi^2(l-k)$ where k is the number of parameters of the specification. The test has the following equation:

$$\begin{aligned} Q_l &= \frac{T}{T+2}\sum_{i=1}^{l}\frac{\hat{\rho}_{SR}(i)}{T-1-i} \to \chi^2(l-k) \\ Q_l^2 &= \frac{T}{T+2}\sum_{i=1}^{l}\frac{\hat{\rho}_{SSR}(i)}{T-1-i} \to \chi^2(l-k) \end{aligned} \tag{23}$$

and $\hat{\rho}_{SR}(i)$ $[\hat{\rho}_{SSR}(i)]$ are the sample autocorrelations of [squared]standardizes residuals.

Finally for testing on the presence of residual ARCH effects we use the Engle lagrange multiplier test (LM). This test is based on a regression of squared residuals on its lags up to a value l. Under the null hypothesis of no ARCH effect the LM statistic is again distributed as a χ^2 now with l degrees of freedom. Given a regression on squared standardized residuals:

$$z_t^2 = \mu_0 + \sum_{i=1}^{l}\mu_i z_{t-i}^2 \tag{24}$$

the test statistic is computed using the R^2 of this regression, as

$$LM_l = TR^2 \to \chi^2(l) \tag{25}$$

Lets now turn to the basic aim of this study: that is to verify the power of information criteria and of a group of tests in distinguishing between short and long memory in conditional variances. We will also try to verify the ability of these procedures in discriminating the orders of FIGARCH specifications. This analysis will be executed with a Montecarlo approach. We will deal with eight different data generating processes, according to the following table

Table 4.1 - Simulating DGP

DGP	μ	ω	d	β	ϕ
FIGARCH(1,d,1)	0	0.01	0.8	0.5	0.3
FIGARCH(1,d,1)	0	0.01	0.8	0.5	0.05
FIGARCH(1,d,0)	0	0.01	0.8	0.5	0
FIGARCH(0,d,0)	0	0.01	0.8	0	0
FIGARCH(1,d,1)	0	0.01	0.4	0.3	0.2
FIGARCH(1,d,0)	0	0.01	0.4	0.3	0
FIGARCH(0,d,0)	0	0.01	0.4	0	0
GARCH(1,1)	0	0.01	0	0.65	0.3 (α)

Where the models follow these specifications:

$$\begin{aligned} y_t &= \mu + \varepsilon_t \\ FIGARCH &\rightarrow \sigma_t^2 = \omega + \left[1 - (1-\beta L)^{-1} (1-L)^d (1-\phi L)\right] \varepsilon_t^2 \\ GARCH &\rightarrow \sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 \end{aligned}$$

As you can see we have chosen two different values of the long-memory coefficient, combining these with adequate values of the other parameters that satisfy the positivity restriction. The choice of these two levels was done to verify if there are differences between DGP with long memory below 0.5 or above. In fact Chung (2001) reported a different rate of converge of the QMLE for the parameter d, depending on this cutoff value. For each model three sample length are analyzed: T=500, T=1000 and T=2000. This in order to verify the consistence of the identification analysis. Moreover for each model 1000 replications are run. We will now describe the different steps of the algorithm we used. At a first stage a return series is simulated, according to a GARCH, IGARCH or FIGARCH data generating process for its conditional variances, as in table 3.1, in all cases the mean of the returns is assumed to be identically equal to zero. The simulating algorithm generate for each series t+2000 observation, saving only the last T, this to avoid dependence from initial values and to reduce at a minimum effect the approximation induced by truncation in the ARCH(∞) representation (this only for FIGARCH DGPs). Then, on each simulation trial, five different models are estimated: FIGARCH(1,d,1), FIGARCH(1,d,0) FIGARCH(0,d,0), GARCH(1,1) and IGARCH(1,1). We used the optimization routine included in the FANPAC package for GAUSS, Non-linear-programming (NLP), and the BFGS algorithm. We did not used the BHHH algorithm, even if it is more precise, because BFGS results to be faster with our procedures and we focused on identification and parameter estimation, not on Hessian computation, which turns to be better with BHHH, see Lombardi (2001) for a discussion on this topic. The estimation procedure required is based on the approach of Teyssière (1996) who suggest, in presence of long memory in the conditional variances, to add a pre-sample set equal to the unconditional variance of the series. This should help in obtaining consistence in the estimate of the long memory parameter. Following this approach we used a pre-sample of 2000 observation, and also a truncation in the ARCH(∞) representation set to 1000, for the series with T=1000, 2000 for T=2000, and to 500 for T=500. With the fitted parameters log-likelihoods are computed and used for information criteria computation, while standardized residual are the source for all other tests. We introduced also a GARCH(1,1) DGP because we want to verify if, in cases of no-long memory, but with a GARCH(1,1) process close to an IGARCH, there is possibility of misspecification of the model choosing a long memory specification on a short memory DGP. In this case another point arise: observing (4) and (9) we can note that, if $d = 0$, the FIGARCH structure does not always collapse on a GARCH model (in the case $d = 1$ FIGARCH trivially collapse on an IGARCH). We must distinguish between the three different FIGARCH parametrization we used: for the FIGARCH(1,d,1) when $d = 0$, the model collapse on a GARCH structure such as $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + (\phi - \beta)\,\varepsilon_{t-1}^2$ and given restrictions on parameters imposed this can be a GARCH(1,1) but not an IGARCH(1,1), since $\phi << 1$; the result is different for a FIGARCH(1,d,0), in this case the resulting GARCH should be $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 - \beta\varepsilon_{t-1}^2$ definitely not a GARCH model; finally the FIGARCH(0,d,0) cannot collapse on any GARCH structure. This shows that FIGARCH and GARCH are not always nested models as claimed by Baillie, Bollerslev and Mikkelsen (1996). In the simulation analysis we will also observe the effect of such a problem. From these observations derive also the presence of a FIGARCH(1,d,1) DGP, to verify if there is possibility of misspecification to a GARCH(1,1) assuming $d = 0$. In the following tables this legend hold for model correspondence in the remaining of the chapter:

Table 4.2: model symbols

Symbol	Model type
I	FIGARCH(1,d,1)
II	FIGARCH(1,d,0)
III	FIGARCH(0,d,0)
IV	GARCH(1,1)
V	IGARCH(1,1)

Identification criteria The results on the power and consistency of identification criteria are contained in Tables from 3.3 to 3.10. All of these panels are divided into six blocks: the first to the fourth are selection frequencies with each of the four information criteria (IC): Akaike (AIC), Hannan-Quinn (HQ), Schwarz (BIC) and Shibata (SH); the fifth is the selection based only on the Log-likelihood, to test the effect of the penalizing terms in the IC; the last block was introduced to replicate the choice of the researcher, who choose the model observing in the meantime different IC. In this last situation the selection is made with the 4 IC together considered, the best model is the one that minimize most of the IC. Given that we have four IC, we may have double counts, so the sum of the frequencies of the sixth row may be above one.

Analyzing in details all the tables a group of considerations arise:

First of all we stress on the main result: all information criteria, even if with different power, can clearly discriminate between long memory structures and short memory ones. In all of the tables the sum of selection frequencies of long memory models against GARCH specification increase with the sample length up to values ranging from 75% to more than 99%. This show consistency of IC identification and confirm the results of the preliminary analysis of Bollerslev and Mikkelsen (1996), extending that to allow for different IC and comparing directly long and short memory;

As expected the selection frequency depend on the parametrization and on parameters value. Compare for example table (3.4) a FIGARCH(1,d,1) with table (3.6), FIGARCH(0,d,0) or table (3.5), another FIGARCH(1,d,1): selection frequencies vary, at T=2000, for the IGARCH(1,1) from 0.001% to 22%, depending on the model specification (1,d,1) or (0,d,0) and long memory parameter value 0.8 or 0.4. This also depend on the IC we are using, the Hannan-Quinn perform poorly (compared with the others, but not in absolute sense) with high values of d, while all IC seem to be influenced by the structure of the model when d=0.4 (the case of high persistence), they show good discriminating power but in the (1,d,0) case.

Consider now the selection of the orders, answering to the question of model specification choice: here parameter values have a stronger impact. We can observe different behavior with d=0.8 or d=0.4. In the first case the correct specification is chosen but in the FIGARCH(1,d,1) case where we are shifted to the (1,d,0) choice, as if the parsimonious model give very similar results to the correct one. This may be caused by parameter values for example when $\phi = 0.05$ but observing tables (3.4) and (3.5) we see that this effect is not vanishing. When d=0.4 we are not able to select correctly the orders p and q but in the (1,d,0) case. Here we can also observe an high variability among the IC, however none of them can give us a very clear identification of the correct model (none result in q frequency higher than 75%).

Move now to the log-likelihood value at the optimum: here the best choice is most of the time the (1,d,1) specification, and this show what is the effect of the penalizing term, moving choice, in some cases, to the correct model

Consider the selection based on 4 IC together considered, here no improvement can be made. We can observe that these choices are always consistent with the AIC results, seem that all IC agree at least on the model selected via

Akaike criterion, possibly suggesting that this could be the best criterion in identifying the long memory presence

A final observation on consistence: if we observe all tables, including the cases where IC cannot correctly identify the FIGARCH orders,we note that all IC have a growing mass, increasing consequently sample length, both on long memory specifications and specially on the correct generator, while the selection frequencies of GARCH and IGARCH generally decrease. This let us conjecture that the identification problem could be solved or at least reduced with sample of more than 2000 observations.

Correlation, normality and ARCH effect tests On these test the results turn to be really useless, they show no power in discriminating both among long and short memory specifications and within different values of p and q. All the results are reported in tables from (3.11) to (3.58). However we can observe that in all cases, even if the model is misspecified in all tests the null hypothesis is accepted most of the times. One observation on correlation test: when computing the test on squared standardized residuals with the (0,d,0) specification the null hypothesis is rejected with an higher frequency, this is due to a bias in the estimate on the d parameter, that is underestimated, probably depending on the misspecification.

Additional analysis Given the previous results we also tried to combine the two methods of model selection, information criteria and test, in order to mimic the choice of a researcher that choose the best model observing both IC and test computed on different specifications. Given this approach we built another group of tables with selection frequencies, that are based on the results of tests and IC. We gave a point to the model with the higher IC, and a point for each tests that were accepting the null hypothesis, then the best model is the one with higher point. This analysis, however does not improve the results obtained with IC selection, therefore we did not report here the table,that are available from the author on request.

6 Conclusions

In this paper we gave some intuition on finding the asymptotic properties of the quasi maximum likelihood estimators for the FIGARCH(m,d,q) models, given that we cannot use the Lee and Hansen (1994) approach as previously suggested by Baillie, Bollerslev and Mikkelsen (1996). In the second part of the paper we focused on the identification problem extending the analysis of Bollerslev and Mikkelsen (1996) to a wider class of generators and considering a greater number of tests and information criteria. We simulated different models and with different sample length, and after estimating with QMLE (the Montecarlo analysis of Baillie et al. (1996) is still valid and show consistency and asymptotic normality of the estimators) we computed different information criteria and tests. Our study show that all information criteria can clearly distinguish between long and short memory data generating processes, and the performances improve with the sample length. However we are not able to discriminate between different specifications of the FIGARCH, none of the information criteria can identify the true generator in all cases, results depend on sample length, and on parameters value. We can observe that a great impact is given by the long memory parameter. As in Bollerslev and Mikkelsen (1996) we can observe that tests have no power in distinguishing among long and short memory generators.

References

[1] Baillie, R.T and T. Bollerslev, H.O. Mikkelsen, 1996, *Fractionally integrated generalized autoregressive conditional heteroskedasticity*, Journal of Econometrics 74, 3-30

[2] Bollerslev, T., 1986, *Generalized autoregressive conditional heteroskedasticity,* Journal of Econometrics 31, 307-327

[3] Bollerslev, T., R.F. Engle and D.B. Nelson 1994, *ARCH models*, in R.F. Engle and D. McFadden, eds., Handbook of Econometrics, Vol 4, Elsevier Science B.V. Amsterdam

[4] Bollerslev, T and H.O. Mikkelsen, 1996, *Modeling and pricing long memory in stock market volatility*, Journal of Econometrics 73, 151-184

[5] Breidt, F.J., N. Crato and P. de Lima, 1998, *The detection and estimation of long memory in stochastic volatility,* Journal of Econometrics 83, 325-348

[6] Chung, Ching-Fan, 2001, *Estimating the fractionally integrated GARCH model*, National Taiwan University discussion paper

[7] Davidson, J., 2001, *Moment and memory properties of linear conditional heteroskedastic models*, working paper

[8] Engle, R.F., 1982, *Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation,* Econometrica 50, 987-1008

[9] Engle, R.F. and T. Bollerslev, 1986, *Modelling the persistence of conditional variances,* Econometric reviews 5, 1-50

[10] Gallo, G.M. and Lombardi M.J., 2001, *Analytic Hessian matrices and the computation of FIGARCH estimates,* working paper

[11] Giraitis, L., Kokoszka, P. and Leipus R., 2000, *Stationary ARCH models: dependence structure and central limit theorem*, Econometric theory, 16, 3-22

[12] Granger C.W.J. and R. Joyeux, 1980, *An introduction to long-memory time series and fractional differencing*, Journal of Time Series Analysis, 1, 15-30

[13] Hosking J.M.R., 1981, *Fractional differencing*, Biometrika, 68(1), 165-176

[14] Kazakevicius, V. and Leipus, R. 1999, *On stationarity in the ARCH(*∞*) model*, Econometric theory, forthcoming

[15] Kazakevicius, V. and Leipus, R. 2001, *Stationarity in the integrated ARCH models*, working paper

[16] Jeantheau T. 1998, *Strong consistency of estimators for multivariate ARCH models*, Econometric theory, 14, 70-86

[17] Lee, Sang-Won and B.E. Hansen, 1994, *Asymptotic theory fir the GARCH(1,1) quasi-maximum likelihood estimator*, Econometric Theory 10, 29-52

[18] Mikosch, T. and Starica, C., 2000, *Long range dependence effects and ARCH modelling*, Preprint

[19] Nelson D.B., 1990, *Stationarity and persistence in the GARCH(1,1) model*, Econometric theory, 6, 318-334

[20] Robinson, P. M., 1991, *Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression*, Journal of Econometrics, 47, 67-84

[21] Sin, Chor-Yiu and H. White, 1996, *Information criteria for selecting possibly misspecified parametric models*, Journal of Econometrics 71, 207-225

[22] Teyssière, G, 1996, *Double long-memory financial time series*, GREQAM working paper

[23] Zaffaroni P., 2000, *Stationarity and memory of ARCH models*, STICERD publications EM/2000/383

Table 4.3

Frequency of model selection: DGP FIGARCH(1,d,0)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.5$ $\phi = 0$ $d = 0.8$						
MODEL	N	I	II	III	IV	V
	500	0.004	0.253	0.017	0.137	0.589
AIC	1000	0.027	0.557	0.002	0.124	0.290
	2000	0.073	0.799	0.000	0.059	0.069
	500	0.000	0.111	0.036	0.049	0.804
HQ	1000	0.000	0.360	0.010	0.046	0.584
	2000	0.009	0.732	0.000	0.036	0.223
	500	0.091	0.403	0.003	0.241	0.262
BIC	1000	0.169	0.602	0.000	0.158	0.071
	2000	0.281	0.640	0.000	0.065	0.014
	500	0.005	0.255	0.017	0.138	0.850
SH	1000	0.027	0.559	0.002	0.125	0.287
	2000	0.073	0.799	0.000	0.059	0.069
	500	0.475	0.208	0.000	0.306	0.011
LL	1000	0.612	0.215	0.000	0.172	0.001
	2000	0.740	0.197	0.000	0.061	0.002
	500	0.004	0.253	0.017	0.137	0.011
4IC	1000	0.027	0.557	0.002	0.124	0.290
	2000	0.073	0.799	0.000	0.059	0.069

Table 4.4

Frequency of model selection: DGP FIGARCH(1,d,1)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.5$ $\phi = 0.05$ $d = 0.8$						
MODEL	N	I	II	III	IV	V
	500	0.007	0.249	0.275	0.089	0.380
AIC	1000	0.056	0.555	0.005	0.090	0.294
	2000	0.174	0.696	0.000	0.053	0.077
	500	0.000	0.097	0.351	0.027	0.525
HQ	1000	0.004	0.412	0.025	0.048	0.511
	2000	0.028	0.732	0.000	0.028	0.212
	500	0.185	0.341	0.150	0.177	0.147
BIC	1000	0.311	0.494	0.001	0.130	0.064
	2000	0.497	0.441	0.000	0.056	0.006
	500	0.007	0.251	0.274	0.091	0.377
SH	1000	0.057	0.554	0.005	0.090	0.294
	2000	0.177	0.693	0.000	0.053	0.077
	500	0.661	0.135	0.001	0.203	0.000
LL	1000	0.716	0.138	0.000	0.145	0.001
	2000	0.849	0.100	0.000	0.051	0.000
	500	0.007	0.249	0.275	0.089	0.380
4IC	1000	0.057	0.555	0.005	0.090	0.294
	2000	0.174	0.696	0.000	0.053	0.077

Table 4.5

Frequency of model selection: DGP FIGARCH(1,d,1)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.5$ $\phi = 0.3$ $d = 0.8$						
MODEL	N	I	II	III	IV	V
	500	0.005	0.228	0.468	0.054	0.245
AIC	1000	0.068	0.501	0.329	0.046	0.056
	2000	0.251	0.553	0.180	0.011	0.005
	500	0.000	0.073	0.573	0.011	0.343
HQ	1000	0.001	0.312	0.529	0.022	0.136
	2000	0.036	0.572	0.368	0.008	0.016
	500	0.230	0.339	0.222	0.124	0.085
BIC	1000	0.452	0.351	0.144	0.048	0.005
	2000	0.660	0.272	0.058	0.009	0.001
	500	0.005	0.232	0.467	0.055	0.241
SH	1000	0.069	0.500	0.329	0.046	0.056
	2000	0.254	0.552	0.178	0.011	0.005
	500	0.705	0.149	0.000	0.144	0.002
LL	1000	0.813	0.135	0.004	0.048	0.000
	2000	0.878	0.112	0.001	0.009	0.000
	500	0.005	0.228	0.468	0.054	0.245
4IC	1000	0.068	0.506	0.325	0.042	0.059
	2000	0.251	0.553	0.180	0.011	0.005

Table 4.6

Frequency of model selection: DGP FIGARCH(0,d,0)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0$ $\phi = 0$ $d = 0.8$						
MODEL	N	I	II	III	IV	V
	500	0.006	0.288	0.075	0.106	0.525
AIC	1000	0.031	0.061	0.871	0.017	0.020
	2000	0.033	0.085	0.879	0.002	0.001
	500	0.000	0.117	0.113	0.037	0.733
HQ	1000	0.005	0.013	0.948	0.006	0.028
	2000	0.003	0.021	0.973	0.001	0.002
	500	0.175	0.398	0.024	0.201	0.202
BIC	1000	0.141	0.212	0.618	0.025	0.004
	2000	0.139	0.231	0.626	0.004	0.000
	500	0.006	0.294	0.073	0.106	0.521
SH	1000	0.031	0.062	0.870	0.017	0.020
	2000	0.033	0.086	0.878	0.002	0.001
	500	0.588	0.154	0.003	0.249	0.006
LL	1000	0.490	0.393	0.089	0.027	0.001
	2000	0.458	0.426	0.111	0.005	0.000
	500	0.006	0.288	0.075	0.106	0.525
4IC	1000	0.031	0.061	0.871	0.017	0.020
	2000	0.033	0.085	0.879	0.002	0.001

Table 4.7

Frequency of model selection: DGP FIGARCH(1,d,0)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.3$ $\phi = 0$ $d = 0.4$						
MODEL	N	I	II	III	IV	V
	500	0.006	0.295	0.056	0.156	0.487
AIC	1000	0.019	0.633	0.021	0.290	0.037
	2000	0.052	0.788	0.000	0.159	0.001
	500	0.000	0.135	0.109	0.071	0.685
HQ	1000	0.001	0.594	0.048	0.253	0.104
	2000	0.005	0.822	0.000	0.162	0.011
	500	0.134	0.397	0.015	0.263	0.191
BIC	1000	0.147	0.551	0.010	0.290	0.002
	2000	0.236	0.624	0.000	0.139	0.001
	500	0.006	0.299	0.053	0.158	0.484
SH	1000	0.021	0.631	0.021	0.290	0.037
	2000	0.052	0.788	0.000	0.159	0.001
	500	0.518	0.170	0.000	0.303	0.009
LL	1000	0.548	0.186	0.000	0.266	0.000
	2000	0.669	0.199	0.000	0.131	0.001
	500	0.006	0.295	0.056	0.156	0.487
4IC	1000	0.019	0.633	0.021	0.290	0.037
	2000	0.052	0.788	0.000	0.159	0.001

Table 4.8

Frequency of model selection: DGP FIGARCH(1,d,1)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.3$ $\phi = 0.2$ $d = 0.4$						
MODEL	N	I	II	III	IV	V
	500	0.007	0.258	0.134	0.136	0.465
AIC	1000	0.061	0.304	0.497	0.134	0.021
	2000	0.125	0.490	0.348	0.037	0.000
	500	0.000	0.104	0.187	0.055	0.654
HQ	1000	0.008	0.182	0.686	0.103	0.021
	2000	0.021	0.338	0.608	0.033	0.000
	500	0.174	0.339	0.055	0.248	0.184
BIC	1000	0.309	0.336	0.218	0.137	0.000
	2000	0.458	0.399	0.107	0.036	0.000
	500	0.008	0.260	0.133	0.140	0.459
SH	1000	0.062	0.306	0.494	0.134	0.004
	2000	0.127	0.488	0.348	0.037	0.000
	500	0.557	0.146	0.000	0.293	0.004
LL	1000	0.747	0.121	0.000	0.132	0.000
	2000	0.874	0.093	0.000	0.033	0.000
	500	0.007	0.258	0.134	0.136	0.465
4IC	1000	0.061	0.304	0.497	0.134	0.004
	2000	0.125	0.490	0.348	0.037	0.000

Table 4.9

Frequency of model selection: DGP FIGARCH(0,d,0)						
$\mu = 0$ $\omega = 0.01$ $\beta = 0.0$ $\phi = 0$ $d = 0.4$						
MODEL	N	I	II	III	IV	V
	500	0.004	0.272	0.051	0.115	0.558
AIC	1000	0.046	0.297	0.518	0.134	0.005
	2000	0.143	0.479	0.356	0.022	0.000
	500	0.000	0.116	0.079	0.044	0.761
HQ	1000	0.003	0.166	0.720	0.087	0.024
	2000	0.032	0.347	0.602	0.018	0.001
	500	0.148	0.389	0.013	0.242	0.208
BIC	1000	0.306	0.319	0.234	0.139	0.002
	2000	0.424	0.439	0.117	0.002	0.000
	500	0.004	0.277	0.051	0.116	0.552
SH	1000	0.046	0.297	0.518	0.134	0.005
	2000	0.146	0.478	0.354	0.022	0.000
	500	0.553	0.144	0.000	0.292	0.011
LL	1000	0.782	0.090	0.001	0.127	0.000
	2000	0.863	0.120	0.001	0.016	0.000
	500	0.004	0.272	0.051	0.115	0.558
4IC	1000	0.046	0.297	0.518	0.134	0.005
	2000	0.143	0.479	0.356	0.022	0.000

Table 4.10

Frequency of model selection: DGP GARCH(1,1)						
$\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
MODEL	N	I	II	III	IV	V
	500	0.000	0.054	0.040	0.421	0.485
AIC	1000	0.004	0.070	0.004	0.700	0.222
	2000	0.003	0.032	0.000	0.894	0.071
	500	0.000	0.013	0.074	0.198	0.715
HQ	1000	0.000	0.032	0.021	0.466	0.481
	2000	0.000	0.023	0.000	0.765	0.212
	500	0.028	0.107	0.014	0.665	0.186
BIC	1000	0.032	0.076	0.001	0.836	0.055
	2000	0.022	0.023	0.000	0.943	0.012
	500	0.000	0.055	0.039	0.427	0.479
SH	1000	0.006	0.070	0.004	0.701	0.219
	2000	0.003	0.032	0.000	0.894	0.071
	500	0.147	0.076	0.000	0.770	0.007
LL	1000	0.101	0.028	0.000	0.870	0.001
	2000	0.040	0.007	0.000	0.953	0.000
	500	0.000	0.054	0.040	0.421	0.485
4IC	1000	0.004	0.070	0.004	0.700	0.222
	2000	0.003	0.032	0.000	0.894	0.071

Table 4.11

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.869	0.931	0.961	0.928	0.966
Q(5)	1000	0.880	0.939	0.962	0.939	0.968
	2000	0.905	0.955	0.971	0.954	0.977
	500	0.943	0.962	0.964	0.959	0.973
Q(10)	1000	0.947	0.962	0.966	0.964	0.974
	2000	0.957	0.968	0.977	0.970	0.978
	500	0.955	0.968	0.967	0.969	0.977
Q(20)	1000	0.951	0.965	0.962	0.964	0.972
	2000	0.966	0.974	0.981	0.974	0.983
	500	0.962	0.973	0.960	0.973	0.974
Q(50)	1000	0.970	0.974	0.968	0.976	0.981
	2000	0.981	0.986	0.985	0.984	0.990
	500	0.953	0.963	0.962	0.962	0.966
Q(100)	1000	0.972	0.975	0.967	0.973	0.977
	2000	0.985	0.986	0.986	0.984	0.986
	500	0.942	0.978	0.607	0.973	0.985
$Q^2(5)$	1000	0.949	0.979	0.298	0.972	0.984
	2000	0.958	0.978	0.028	0.968	0.983
	500	0.967	0.983	0.638	0.978	0.987
$Q^2(10)$	1000	0.982	0.987	0.297	0.983	0.988
	2000	0.980	0.990	0.025	0.981	0.985
	500	0.966	0.984	0.703	0.983	0.988
$Q^2(20)$	1000	0.987	0.989	0.430	0.988	0.993
	2000	0.988	0.989	0.083	0.984	0.983
	500	0.969	0.979	0.808	0.979	0.985
$Q^2(50)$	1000	0.980	0.982	0.637	0.975	0.980
	2000	0.987	0.991	0.212	0.980	0.985
	500	0.972	0.982	0.862	0.975	0.976
$Q^2(100)$	1000	0.977	0.978	0.730	0.972	0.971
	2000	0.988	0.988	0.367	0.973	0.979

Table 4.12

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.665	0.805	0.854	0.800	0.884
Q(5)	1000	0.698	0.819	0.873	0.818	0.900
	2000	0.705	0.838	0.901	0.833	0.910
	500	0.804	0.870	0.882	0.867	0.911
Q(10)	1000	0.828	0.878	0.895	0.877	0.923
	2000	0.854	0.903	0.916	0.908	0.945
	500	0.863	0.897	0.891	0.900	0.917
Q(20)	1000	0.882	0.903	0.902	0.905	0.928
	2000	0.881	0.908	0.921	0.907	0.948
	500	0.886	0.897	0.893	0.899	0.916
Q(50)	1000	0.902	0.920	0.911	0.911	0.927
	2000	0.911	0.930	0.925	0.932	0.944
	500	0.878	0.896	0.893	0.894	0.903
Q(100)	1000	0.907	0.914	0.905	0.909	0.923
	2000	0.942	0.949	0.949	0.944	0.950
	500	0.826	0.911	0.402	0.890	0.946
$Q^2(5)$	1000	0.830	0.909	0.139	0.890	0.944
	2000	0.831	0.901	0.006	0.864	0.914
	500	0.904	0.937	0.422	0.930	0.949
$Q^2(10)$	1000	0.917	0.949	0.162	0.929	0.956
	2000	0.907	0.918	0.011	0.898	0.924
	500	0.923	0.935	0.532	0.929	0.953
$Q^2(20)$	1000	0.933	0.950	0.244	0.941	0.961
	2000	0.935	0.938	0.051	0.930	0.932
	500	0.914	0.929	0.657	0.921	0.934
$Q^2(50)$	1000	0.944	0.946	0.435	0.924	0.938
	2000	0.950	0.957	0.105	0.921	0.936
	500	0.923	0.930	0.732	0.918	0.927
$Q^2(100)$	1000	0.934	0.936	0.557	0.914	0.922
	2000	0.945	0.949	0.216	0.897	0.912

Table 4.13

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.890	0.959	0.977	0.960	0.991
Q(5)	1000	0.893	0.957	0.969	0.961	0.970
	2000	0.906	0.957	0.967	0.960	0.982
	500	0.960	0.980	0.981	0.977	0.986
Q(10)	1000	0.952	0.973	0.977	0.972	0.984
	2000	0.954	0.970	0.974	0.973	0.986
	500	0.970	0.979	0.985	0.981	0.987
Q(20)	1000	0.970	0.983	0.981	0.984	0.988
	2000	0.968	0.979	0.981	0.982	0.983
	500	0.958	0.965	0.969	0.963	0.971
Q(50)	1000	0.969	0.977	0.977	0.975	0.981
	2000	0.973	0.976	0.977	0.974	0.979
	500	0.950	0.962	0.963	0.953	0.956
Q(100)	1000	0.966	0.970	0.976	0.964	0.965
	2000	0.972	0.980	0.980	0.980	0.983
	500	0.923	0.977	0.849	0.962	0.979
$Q^2(5)$	1000	0.959	0.979	0.442	0.971	0.986
	2000	0.953	0.971	0.088	0.959	0.976
	500	0.947	0.983	0.832	0.971	0.978
$Q^2(10)$	1000	0.977	0.984	0.463	0.982	0.988
	2000	0.981	0.982	0.078	0.979	0.985
	500	0.950	0.982	0.869	0.959	0.969
$Q^2(20)$	1000	0.971	0.980	0.581	0.976	0.983
	2000	0.979	0.983	0.120	0.977	0.970
	500	0.949	0.979	0.914	0.962	0.965
$Q^2(50)$	1000	0.986	0.989	0.745	0.980	0.988
	2000	0.983	0.982	0.376	0.953	0.962
	500	0.949	0.974	0.925	0.957	0.961
$Q^2(100)$	1000	0.980	0.981	0.810	0.965	0.969
	2000	0.977	0.981	0.553	0.951	0.957

Table 4.14

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.682	0.821	0.893	0.825	0.910
Q(5)	1000	0.703	0.837	0.888	0.843	0.903
	2000	0.715	0.864	0.906	0.859	0.914
	500	0.838	0.888	0.917	0.897	0.935
Q(10)	1000	0.830	0.896	0.910	0.898	0.935
	2000	0.835	0.896	0.905	0.899	0.931
	500	0.874	0.905	0.922	0.907	0.932
Q(20)	1000	0.877	0.906	0.924	0.906	0.937
	2000	0.866	0.905	0.916	0.915	0.935
	500	0.888	0.911	0.911	0.913	0.920
Q(50)	1000	0.915	0.925	0.923	0.918	0.931
	2000	0.902	0.920	0.924	0.921	0.936
	500	0.874	0.884	0.902	0.881	0.892
Q(100)	1000	0.913	0.927	0.927	0.918	0.927
	2000	0.918	0.925	0.926	0.920	0.928
	500	0.802	0.913	0.692	0.878	0.918
$Q^2(5)$	1000	0.851	0.913	0.239	0.891	0.943
	2000	0.873	0.896	0.028	0.853	0.898
	500	0.877	0.927	0.720	0.908	0.938
$Q^2(10)$	1000	0.915	0.938	0.261	0.930	0.946
	2000	0.909	0.926	0.027	0.895	0.919
	500	0.904	0.946	0.756	0.909	0.933
$Q^2(20)$	1000	0.935	0.945	0.372	0.929	0.943
	2000	0.929	0.930	0.096	0.912	0.915
	500	0.904	0.939	0.841	0.893	0.900
$Q^2(50)$	1000	0.943	0.949	0.558	0.913	0.924
	2000	0.929	0.923	0.213	0.899	0.909
	500	0.909	0.933	0.859	0.895	0.906
$Q^2(100)$	1000	0.924	0.930	0.668	0.908	0.914
	2000	0.949	0.949	0.339	0.886	0.899

Table 4.15

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.890	0.963	0.979	0.957	0.981
Q(5)	1000	0.863	0.951	0.973	0.950	0.977
	2000	0.880	0.961	0.974	0.957	0.979
	500	0.946	0.970	0.980	0.971	0.984
Q(10)	1000	0.932	0.963	0.973	0.970	0.980
	2000	0.936	0.965	0.980	0.968	0.978
	500	0.961	0.973	0.973	0.975	0.981
Q(20)	1000	0.957	0.978	0.979	0.975	0.981
	2000	0.966	0.980	0.986	0.982	0.986
	500	0.958	0.968	0.977	0.965	0.974
Q(50)	1000	0.960	0.975	0.974	0.967	0.974
	2000	0.967	0.982	0.982	0.977	0.982
	500	0.963	0.973	0.976	0.966	0.971
Q(100)	1000	0.964	0.975	0.978	0.971	0.972
	2000	0.970	0.978	0.981	0.974	0.980
	500	0.892	0.975	0.943	0.952	0.969
$Q^2(5)$	1000	0.886	0.969	0.928	0.896	0.934
	2000	0.860	0.932	0.834	0.658	0.716
	500	0.905	0.979	0.934	0.958	0.967
$Q^2(10)$	1000	0.901	0.976	0.921	0.919	0.941
	2000	0.888	0.952	0.852	0.782	0.816
	500	0.906	0.978	0.959	0.956	0.966
$Q^2(20)$	1000	0.907	0.980	0.947	0.912	0.927
	2000	0.900	0.968	0.906	0.777	0.814
	500	0.915	0.983	0.959	0.951	0.960
$Q^2(50)$	1000	0.904	0.979	0.966	0.889	0.902
	2000	0.899	0.970	0.944	0.756	0.788
	500	0.907	0.971	0.947	0.953	0.955
$Q^2(100)$	1000	0.908	0.980	0.965	0.902	0.908
	2000	0.902	0.973	0.955	0.766	0.780

Table 4.16

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.678	0.843	0.898	0.841	0.910
Q(5)	1000	0.658	0.817	0.880	0.828	0.902
	2000	0.668	0.814	0.895	0.824	0.908
	500	0.814	0.889	0.918	0.885	0.931
Q(10)	1000	0.798	0.881	0.910	0.888	0.936
	2000	0.806	0.873	0.910	0.874	0.923
	500	0.850	0.901	0.915	0.900	0.922
Q(20)	1000	0.850	0.899	0.923	0.896	0.928
	2000	0.845	0.894	0.918	0.890	0.920
	500	0.884	0.913	0.926	0.907	0.922
Q(50)	1000	0.886	0.917	0.927	0.896	0.914
	2000	0.878	0.905	0.921	0.895	0.906
	500	0.881	0.902	0.911	0.899	0.907
Q(100)	1000	0.895	0.914	0.922	0.904	0.913
	2000	0.908	0.925	0.933	0.904	0.914
	500	0.781	0.908	0.842	0.843	0.896
$Q^2(5)$	1000	0.764	0.898	0.785	0.715	0.782
	2000	0.721	0.818	0.647	0.371	0.442
	500	0.835	0.919	0.863	0.878	0.897
$Q^2(10)$	1000	0.832	0.911	0.814	0.803	0.832
	2000	0.807	0.865	0.711	0.553	0.601
	500	0.859	0.938	0.893	0.875	0.898
$Q^2(20)$	1000	0.842	0.921	0.848	0.796	0.824
	2000	0.841	0.905	0.771	0.553	0.601
	500	0.859	0.928	0.892	0.882	0.894
$Q^2(50)$	1000	0.854	0.928	0.886	0.777	0.801
	2000	0.852	0.917	0.842	0.553	0.589
	500	0.865	0.926	0.904	0.883	0.886
$Q^2(100)$	1000	0.865	0.935	0.908	0.791	0.805
	2000	0.847	0.914	0.873	0.605	0.634

Table 4.17

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.899	0.955	0.972	0.956	0.980
Q(5)	1000	0.878	0.957	0.983	0.964	0.982
	2000	0.838	0.948	0.975	0.946	0.974
	500	0.944	0.968	0.972	0.968	0.981
Q(10)	1000	0.924	0.972	0.984	0.970	0.984
	2000	0.915	0.968	0.986	0.964	0.986
	500	0.962	0.973	0.977	0.976	0.982
Q(20)	1000	0.945	0.977	0.988	0.972	0.981
	2000	0.950	0.979	0.985	0.976	0.984
	500	0.970	0.974	0.970	0.973	0.978
Q(50)	1000	0.963	0.980	0.983	0.979	0.982
	2000	0.961	0.976	0.978	0.976	0.978
	500	0.957	0.966	0.961	0.961	0.964
Q(100)	1000	0.968	0.975	0.981	0.973	0.977
	2000	0.967	0.982	0.983	0.973	0.977
	500	0.943	0.981	0.721	0.976	0.990
$Q^2(5)$	1000	0.571	0.985	0.989	0.893	0.935
	2000	0.507	0.978	0.992	0.730	0.792
	500	0.976	0.993	0.725	0.988	0.991
$Q^2(10)$	1000	0.584	0.979	0.991	0.868	0.892
	2000	0.515	0.981	0.985	0.659	0.714
	500	0.976	0.990	0.784	0.980	0.990
$Q^2(20)$	1000	0.591	0.982	0.987	0.794	0.832
	2000	0.513	0.981	0.985	0.659	0.714
	500	0.976	0.987	0.858	0.974	0.984
$Q^2(50)$	1000	0.626	0.988	0.989	0.783	0.817
	2000	0.517	0.982	0.981	0.525	0.558
	500	0.967	0.976	0.879	0.970	0.974
$Q^2(100)$	1000	0.658	0.980	0.980	0.838	0.854
	2000	0.524	0.986	0.985	0.604	0.631

Table 4.18

H_0 5%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.679	0.830	0.884	0.830	0.912
Q(5)	1000	0.655	0.850	0.927	0.856	0.932
	2000	0.633	0.822	0.901	0.822	0.898
	500	0.815	0.872	0.893	0.869	0.917
Q(10)	1000	0.787	0.881	0.918	0.875	0.914
	2000	0.760	0.878	0.924	0.866	0.911
	500	0.859	0.898	0.896	0.890	0.930
Q(20)	1000	0.845	0.898	0.923	0.893	0.914
	2000	0.827	0.913	0.938	0.909	0.932
	500	0.884	0.904	0.910	0.904	0.915
Q(50)	1000	0.892	0.918	0.934	0.903	0.916
	2000	0.884	0.932	0.943	0.926	0.934
	500	0.885	0.895	0.896	0.891	0.901
Q(100)	1000	0.898	0.922	0.930	0.904	0.916
	2000	0.904	0.932	0.941	0.906	0.921
	500	0.817	0.901	0.539	0.901	0.955
$Q^2(5)$	1000	0.483	0.897	0.949	0.723	0.794
	2000	0.433	0.899	0.936	0.480	0.563
	500	0.896	0.926	0.563	0.931	0.952
$Q^2(10)$	1000	0.532	0.929	0.945	0.695	0.754
	2000	0.483	0.928	0.942	0.428	0.504
	500	0.917	0.936	0.630	0.920	0.939
$Q^2(20)$	1000	0.547	0.918	0.940	0.620	0.678
	2000	0.479	0.928	0.935	0.335	0.383
	500	0.945	0.954	0.741	0.931	0.944
$Q^2(50)$	1000	0.572	0.938	0.947	0.650	0.686
	2000	0.483	0.923	0.933	0.380	0.416
	500	0.920	0.927	0.786	0.902	0.913
$Q^2(100)$	1000	0.593	0.929	0.934	0.712	0.736
	2000	0.490	0.927	0.929	0.457	0.485

Table 4.19

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.927	0.969	0.979	0.967	0.986
Q(5)	1000	0.893	0.958	0.982	0.960	0.984
	2000	0.893	0.944	0.964	0.944	0.971
	500	0.959	0.969	0.973	0.970	0.980
Q(10)	1000	0.948	0.960	0.966	0.963	0.977
	2000	0.945	0.968	0.979	0.970	0.980
	500	0.972	0.979	0.979	0.979	0.986
Q(20)	1000	0.957	0.964	0.972	0.966	0.981
	2000	0.969	0.977	0.982	0.976	0.988
	500	0.971	0.976	0.975	0.972	0.978
Q(50)	1000	0.968	0.975	0.976	0.977	0.980
	2000	0.987	0.991	0.987	0.990	0.994
	500	0.966	0.967	0.970	0.968	0.971
Q(100)	1000	0.964	0.968	0.967	0.968	0.977
	2000	0.986	0.988	0.982	0.986	0.991
	500	0.958	0.979	0.733	0.971	0.987
$Q^2(5)$	1000	0.974	0.992	0.530	0.920	0.951
	2000	0.977	0.985	0.095	0.861	0.932
	500	0.976	0.986	0.724	0.985	0.993
$Q^2(10)$	1000	0.981	0.986	0.585	0.951	0.958
	2000	0.984	0.986	0.131	0.915	0.915
	500	0.987	0.989	0.812	0.988	0.992
$Q^2(20)$	1000	0.986	0.987	0.652	0.973	0.974
	2000	0.984	0.983	0.221	0.943	0.929
	500	0.978	0.985	0.867	0.976	0.977
$Q^2(50)$	1000	0.983	0.983	0.765	0.961	0.966
	2000	0.987	0.989	0.398	0.955	0.934
	500	0.968	0.971	0.878	0.967	0.972
$Q^2(100)$	1000	0.974	0.979	0.819	0.945	0.957
	2000	0.987	0.986	0.532	0.952	0.946

Table 4.20

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.699	0.869	0.914	0.869	0.939
Q(5)	1000	0.673	0.819	0.895	0.823	0.908
	2000	0.677	0.833	0.895	0.825	0.899
	500	0.846	0.900	0.908	0.901	0.932
Q(10)	1000	0.822	0.874	0.898	0.873	0.930
	2000	0.830	0.880	0.895	0.881	0.923
	500	0.894	0.915	0.922	0.921	0.934
Q(20)	1000	0.865	0.889	0.904	0.892	0.927
	2000	0.887	0.913	0.939	0.911	0.953
	500	0.911	0.915	0.923	0.917	0.929
Q(50)	1000	0.897	0.911	0.902	0.910	0.928
	2000	0.928	0.942	0.941	0.943	0.960
	500	0.896	0.906	0.910	0.906	0.917
Q(100)	1000	0.891	0.895	0.900	0.893	0.914
	2000	0.926	0.936	0.930	0.932	0.947
	500	0.844	0.928	0.531	0.896	0.945
$Q^2(5)$	1000	0.866	0.928	0.292	0.792	0.862
	2000	0.887	0.928	0.021	0.672	0.807
	500	0.915	0.946	0.525	0.934	0.959
$Q^2(10)$	1000	0.925	0.934	0.337	0.857	0.859
	2000	0.930	0.938	0.036	0.762	0.738
	500	0.937	0.957	0.648	0.947	0.961
$Q^2(20)$	1000	0.940	0.942	0.439	0.889	0.874
	2000	0.937	0.944	0.126	0.818	0.785
	500	0.936	0.941	0.747	0.925	0.933
$Q^2(50)$	1000	0.927	0.931	0.576	0.889	0.893
	2000	0.943	0.945	0.217	0.849	0.803
	500	0.915	0.925	0.790	0.897	0.907
$Q^2(100)$	1000	0.916	0.913	0.644	0.868	0.882
	2000	0.948	0.947	0.336	0.850	0.832

Table 4.21

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.892	0.949	0.974	0.944	0.983
Q(5)	1000	0.887	0.957	0.980	0.955	0.988
	2000	0.909	0.961	0.980	0.962	0.986
	500	0.950	0.972	0.970	0.973	0.981
Q(10)	1000	0.944	0.975	0.982	0.975	0.986
	2000	0.965	0.977	0.987	0.982	0.990
	500	0.961	0.970	0.972	0.974	0.983
Q(20)	1000	0.966	0.977	0.983	0.975	0.990
	2000	0.975	0.985	0.988	0.983	0.995
	500	0.960	0.968	0.970	0.967	0.974
Q(50)	1000	0.975	0.977	0.979	0.976	0.984
	2000	0.976	0.978	0.983	0.975	0.984
	500	0.968	0.971	0.972	0.964	0.970
Q(100)	1000	0.973	0.976	0.977	0.974	0.985
	2000	0.979	0.982	0.986	0.979	0.986
	500	0.954	0.980	0.758	0.978	0.986
$Q^2(5)$	1000	0.970	0.981	0.948	0.936	0.904
	2000	0.982	0.984	0.941	0.773	0.695
	500	0.979	0.984	0.768	0.987	0.989
$Q^2(10)$	1000	0.980	0.979	0.954	0.948	0.903
	2000	0.987	0.988	0.939	0.844	0.619
	500	0.982	0.986	0.810	0.987	0.991
$Q^2(20)$	1000	0.988	0.989	0.970	0.954	0.935
	2000	0.990	0.988	0.956	0.872	0.745
	500	0.977	0.980	0.865	0.971	0.974
$Q^2(50)$	1000	0.988	0.989	0.976	0.946	0.952
	2000	0.992	0.993	0.967	0.858	0.858
	500	0.968	0.972	0.895	0.962	0.962
$Q^2(100)$	1000	0.987	0.984	0.973	0.937	0.939
	2000	0.988	0.988	0.965	0.844	0.877

Table 4.22

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.684	0.809	0.876	0.817	0.900
Q(5)	1000	0.678	0.817	0.891	0.824	0.904
	2000	0.691	0.839	0.909	0.846	0.924
	500	0.813	0.878	0.901	0.874	0.925
Q(10)	1000	0.800	0.864	0.903	0.882	0.930
	2000	0.835	0.897	0.937	0.904	0.952
	500	0.872	0.905	0.904	0.901	0.928
Q(20)	1000	0.854	0.886	0.913	0.890	0.939
	2000	0.882	0.922	0.932	0.918	0.949
	500	0.894	0.905	0.899	0.902	0.920
Q(50)	1000	0.904	0.915	0.919	0.911	0.941
	2000	0.914	0.925	0.933	0.918	0.939
	500	0.891	0.897	0.890	0.895	0.905
Q(100)	1000	0.902	0.913	0.916	0.908	0.923
	2000	0.918	0.923	0.930	0.914	0.929
	500	0.853	0.924	0.581	0.915	0.946
$Q^2(5)$	1000	0.870	0.920	0.957	0.775	0.740
	2000	0.888	0.919	0.817	0.508	0.409
	500	0.918	0.936	0.594	0.932	0.951
$Q^2(10)$	1000	0.913	0.931	0.868	0.851	0.731
	2000	0.924	0.937	0.839	0.631	0.300
	500	0.938	0.945	0.669	0.942	0.950
$Q^2(20)$	1000	0.937	0.947	0.895	0.860	0.804
	2000	0.939	0.937	0.851	0.709	0.529
	500	0.923	0.932	0.766	0.904	0.924
$Q^2(50)$	1000	0.952	0.949	0.916	0.841	0.845
	2000	0.953	0.950	0.897	0.688	0.654
	500	0.916	0.922	0.798	0.900	0.908
$Q^2(100)$	1000	0.946	0.946	0.915	0.826	0.836
	2000	0.948	0.947	0.909	0.685	0.722

Table 4.23

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.879	0.942	0.964	0.941	0.968
Q(5)	1000	0.904	0.964	0.984	0.964	0.987
	2000	0.891	0.943	0.973	0.954	0.980
	500	0.937	0.963	0.962	0.961	0.972
Q(10)	1000	0.947	0.963	0.972	0.965	0.980
	2000	0.943	0.963	0.983	0.970	0.988
	500	0.954	0.974	0.976	0.970	0.982
Q(20)	1000	0.966	0.976	0.982	0.974	0.986
	2000	0.966	0.977	0.986	0.975	0.989
	500	0.962	0.971	0.968	0.968	0.977
Q(50)	1000	0.976	0.978	0.986	0.979	0.988
	2000	0.973	0.975	0.982	0.974	0.982
	500	0.961	0.964	0.962	0.965	0.971
Q(100)	1000	0.972	0.976	0.983	0.972	0.976
	2000	0.972	0.973	0.974	0.966	0.977
	500	0.965	0.984	0.735	0.978	0.991
$Q^2(5)$	1000	0.977	0.988	0.956	0.932	0.893
	2000	0.982	0.984	0.942	0.752	0.676
	500	0.978	0.991	0.741	0.992	0.994
$Q^2(10)$	1000	0.988	0.988	0.966	0.960	0.913
	2000	0.988	0.984	0.934	0.826	0.600
	500	0.973	0.982	0.799	0.985	0.988
$Q^2(20)$	1000	0.986	0.990	0.965	0.950	0.934
	2000	0.980	0.982	0.946	0.868	0.750
	500	0.975	0.982	0.857	0.977	0.981
$Q^2(50)$	1000	0.980	0.979	0.964	0.926	0.928
	2000	0.979	0.978	0.954	0.844	0.830
	500	0.973	0.975	0.904	0.967	0.970
$Q^2(100)$	1000	0.984	0.984	0.970	0.922	0.935
	2000	0.974	0.972	0.949	0.821	0.852

Table 4.24

H_0 5%: DGP FIGARCH(9,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.649	0.815	0.872	0.818	0.891
Q(5)	1000	0.693	0.828	0.909	0.834	0.919
	2000	0.684	0.832	0.896	0.838	0.910
	500	0.809	0.872	0.898	0.875	0.915
Q(10)	1000	0.823	0.873	0.908	0.879	0.925
	2000	0.828	0.881	0.913	0.889	0.925
	500	0.860	0.887	0.886	0.889	0.912
Q(20)	1000	0.869	0.903	0.920	0.902	0.940
	2000	0.894	0.916	0.917	0.919	0.925
	500	0.892	0.906	0.912	0.907	0.921
Q(50)	1000	0.901	0.919	0.926	0.911	0.929
	2000	0.894	0.908	0.916	0.898	0.924
	500	0.886	0.895	0.904	0.886	0.905
Q(100)	1000	0.913	0.922	0.931	0.914	0.932
	2000	0.913	0.921	0.926	0.91	0.933
	500	0.873	0.929	0.518	0.922	0.959
$Q^2(5)$	1000	0.891	0.928	0.861	0.804	0.735
	2000	0.878	0.905	0.812	0.503	0.388
	500	0.915	0.945	0.558	0.938	0.959
$Q^2(10)$	1000	0.936	0.945	0.883	0.870	0.742
	2000	0.928	0.929	0.826	0.618	0.316
	500	0.928	0.947	0.646	0.932	0.948
$Q^2(20)$	1000	0.934	0.937	0.891	0.852	0.803
	2000	0.936	0.930	0.822	0.697	0.523
	500	0.928	0.943	0.742	0.920	0.934
$Q^2(50)$	1000	0.932	0.929	0.902	0.819	0.809
	2000	0.930	0.929	0.880	0.670	0.642
	500	0.932	0.942	0.798	0.914	0.918
$Q^2(100)$	1000	0.936	0.940	0.908	0.824	0.836
	2000	0.925	0.924	0.881	0.651	0.696

Table 4.25

H_0 1%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.897	0.957	0.969	0.952	0.978
Q(5)	1000	0.910	0.963	0.979	0.960	0.990
	2000	0.898	0.949	0.970	0.944	0.979
	500	0.955	0.973	0.972	0.971	0.985
Q(10)	1000	0.955	0.975	0.973	0.973	0.986
	2000	0.949	0.968	0.966	0.964	0.982
	500	0.967	0.981	0.978	0.975	0.989
Q(20)	1000	0.969	0.978	0.977	0.976	0.987
	2000	0.963	0.976	0.975	0.972	0.982
	500	0.968	0.974	0.966	0.971	0.977
Q(50)	1000	0.976	0.983	0.985	0.983	0.987
	2000	0.976	0.981	0.985	0.981	0.989
	500	0.963	0.963	0.964	0.963	0.968
Q(100)	1000	0.971	0.975	0.981	0.974	0.977
	2000	0.976	0.982	0.984	0.980	0.983
	500	0.955	0.977	0.739	0.976	0.988
$Q^2(5)$	1000	0.958	0.987	0.489	0.990	0.993
	2000	0.939	0.972	0.129	0.983	0.982
	500	0.975	0.984	0.740	0.981	0.989
$Q^2(10)$	1000	0.973	0.985	0.523	0.986	0.993
	2000	0.965	0.976	0.128	0.982	0.978
	500	0.981	0.982	0.799	0.984	0.991
$Q^2(20)$	1000	0.983	0.990	0.669	0.989	0.995
	2000	0.966	0.979	0.250	0.984	0.981
	500	0.981	0.982	0.877	0.983	0.986
$Q^2(50)$	1000	0.984	0.989	0.773	0.986	0.991
	2000	0.980	0.985	0.452	0.987	0.983
	500	0.978	0.980	0.903	0.986	0.984
$Q^2(100)$	1000	0.980	0.985	0.837	0.988	0.987
	2000	0.978	0.983	0.625	0.991	0.981

Table 4.26

H_0 5%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.685	0.833	0.897	0.831	0.906
Q(5)	1000	0.717	0.850	0.896	0.834	0.925
	2000	0.695	0.842	0.886	0.830	0.909
	500	0.815	0.870	0.889	0.861	0.923
Q(10)	1000	0.848	0.905	0.912	0.900	0.942
	2000	0.823	0.885	0.904	0.878	0.931
	500	0.882	0.910	0.904	0.905	0.933
Q(20)	1000	0.895	0.917	0.913	0.905	0.932
	2000	0.874	0.908	0.914	0.904	0.931
	500	0.907	0.917	0.917	0.913	0.930
Q(50)	1000	0.916	0.930	0.936	0.926	0.947
	2000	0.904	0.922	0.924	0.916	0.938
	500	0.900	0.911	0.919	0.904	0.923
Q(100)	1000	0.910	0.920	0.928	0.919	0.932
	2000	0.923	0.934	0.933	0.931	0.942
	500	0.842	0.914	0.517	0.908	0.955
$Q^2(5)$	1000	0.816	0.907	0.281	0.907	0.948
	2000	0.730	0.835	0.042	0.902	0.893
	500	0.904	0.936	0.542	0.940	0.954
$Q^2(10)$	1000	0.904	0.933	0.306	0.937	0.940
	2000	0.873	0.902	0.047	0.935	0.904
	500	0.920	0.942	0.643	0.938	0.948
$Q^2(20)$	1000	0.936	0.953	0.447	0.953	0.958
	2000	0.887	0.905	0.123	0.926	0.901
	500	0.932	0.942	0.752	0.944	0.945
$Q^2(50)$	1000	0.938	0.945	0.609	0.952	0.950
	2000	0.934	0.944	0.259	0.949	0.937
	500	0.930	0.937	0.806	0.934	0.938
$Q^2(100)$	1000	0.944	0.950	0.706	0.948	0.943
	2000	0.927	0.937	0.411	0.945	0.930

Table 4.27

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.905	0.909	0.816	0.900	0.902
JB	1000	0.898	0.900	0.760	0.887	0.890
	2000	0.887	0.885	0.648	0.863	0.865

Table 4.28

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.814	0.816	0.705	0.810	0.811
JB	1000	0.809	0.806	0.623	0.792	0.790
	2000	0.771	0.772	0.489	0.741	0.739

Table 4.29

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.893	0.904	0.861	0.887	0.881
JB	1000	0.892	0.896	0.793	0.878	0.877
	2000	0.886	0.890	0.711	0.874	0.875

Table 4.30

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.783	0.800	0.744	0.774	0.773
JB	1000	0.788	0.785	0.675	0.769	0.770
	2000	0.797	0.793	0.566	0.780	0.779

Table 4.31

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.858	0.886	0.874	0.865	0.862
JB	1000	0.839	0.878	0.854	0.820	0.815
	2000	0.814	0.876	0.856	0.764	0.763

Table 4.32

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.745	0.779	0.758	0.758	0.750
JB	1000	0.736	0.780	0.754	0.712	0.709
	2000	0.702	0.754	0.734	0.628	0.623

Table 4.33

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.886	0.887	0.828	0.882	0.885
JB	1000	0.747	0.877	0.879	0.800	0.799
	2000	0.614	0.873	0.872	0.670	0.670

Table 4.34

H_0 5%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.800	0.805	0.704	0.792	0.792
JB	1000	0.630	0.783	0.778	0.678	0.676
	2000	0.485	0.750	0.750	0.544	0.544

Table 4.35

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.904	0.905	0.843	0.899	0.893
JB	1000	0.938	0.936	0.882	0.922	0.915
	2000	0.929	0.927	0.863	0.918	0.910

Table 4.36

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.811	0.815	0.719	0.796	0.797
JB	1000	0.847	0.849	0.746	0.835	0.822
	2000	0.839	0.839	0.740	0.825	0.803

Table 4.37

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.904	0.906	0.827	0.890	0.897
JB	1000	0.919	0.919	0.914	0.893	0.888
	2000	0.913	0.910	0.902	0.897	0.879

Table 4.38

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.793	0.795	0.725	0.793	0.791
JB	1000	0.834	0.831	0.823	0.799	0.788
	2000	0.833	0.830	0.817	0.771	0.743

Table 4.39

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.889	0.895	0.809	0.885	0.887
JB	1000	0.926	0.926	0.924	0.904	0.885
	2000	0.931	0.927	0.922	0.880	0.860

Table 4.40

H_0 5%: DGP FIGARCH(9,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.795	0.798	0.705	0.796	0.790
JB	1000	0.830	0.831	0.820	0.806	0.774
	2000	0.847	0.849	0.823	0.773	0.752

Table 4.41

H_0 1%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.909	0.911	0.843	0.909	0.906
JB	1000	0.914	0.918	0.824	0.926	0.921
	2000	0.897	0.900	0.771	0.905	0.901

Table 4.42

H_0 5%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.819	0.818	0.740	0.835	0.819
JB	1000	0.806	0.811	0.695	0.810	0.805
	2000	0.801	0.802	0.620	0.801	0.802

Table 4.43

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.992	0.992	0.989	0.991	0.991
LM(2)	1000	0.989	0.990	0.989	0.989	0.988
	2000	0.992	0.992	0.992	0.989	0.989
	500	0.980	0.981	0.978	0.982	0.981
LM(5)	1000	0.984	0.983	0.978	0.985	0.985
	2000	0.988	0.988	0.985	0.987	0.987
	500	0.981	0.984	0.977	0.984	0.985
LM(10)	1000	0.983	0.983	0.981	0.982	0.982
	2000	0.988	0.988	0.986	0.988	0.988
	500	0.984	0.987	0.982	0.987	0.987
LM(20)	1000	0.985	0.985	0.979	0.983	0.983
	2000	0.989	0.989	0.986	0.990	0.990
	500	0.979	0.980	0.970	0.982	0.983
LM(50)	1000	0.984	0.985	0.985	0.985	0.985
	2000	0.993	0.994	0.990	0.993	0.993

Table 4.44

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.941	0.943	0.949	0.944	0.944
LM(2)	1000	0.939	0.939	0.936	0.938	0.941
	2000	0.947	0.948	0.954	0.946	0.947
	500	0.922	0.924	0.907	0.922	0.925
LM(5)	1000	0.934	0.935	0.919	0.940	0.942
	2000	0.949	0.948	0.943	0.948	0.949
	500	0.938	0.938	0.914	0.940	0.942
LM(10)	1000	0.946	0.949	0.929	0.950	0.953
	2000	0.956	0.955	0.946	0.959	0.959
	500	0.940	0.942	0.911	0.945	0.945
LM(20)	1000	0.942	0.943	0.928	0.943	0.947
	2000	0.951	0.951	0.939	0.951	0.952
	500	0.901	0.905	0.894	0.906	0.908
LM(50)	1000	0.935	0.935	0.917	0.935	0.937
	2000	0.955	0.955	0.929	0.958	0.957

Table 4.45

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.992	0.992	0.992	0.991	0.991
LM(2)	1000	0.989	0.990	0.992	0.989	0.989
	2000	0.992	0.991	0.991	0.990	0.991
	500	0.993	0.994	0.992	0.995	0.995
LM(5)	1000	0.983	0.986	0.984	0.985	0.985
	2000	0.992	0.992	0.982	0.992	0.992
	500	0.991	0.992	0.989	0.993	0.993
LM(10)	1000	0.988	0.990	0.986	0.991	0.991
	2000	0.988	0.988	0.987	0.990	0.991
	500	0.988	0.989	0.985	0.987	0.990
LM(20)	1000	0.988	0.990	0.986	0.990	0.991
	2000	0.991	0.990	0.989	0.991	0.991
	500	0.980	0.980	0.979	0.979	0.979
LM(50)	1000	0.986	0.989	0.986	0.987	0.986
	2000	0.989	0.991	0.984	0.990	0.990

Table 4.46

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.05$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.956	0.957	0.951	0.957	0.958
LM(2)	1000	0.963	0.964	0.960	0.955	0.955
	2000	0.956	0.959	0.959	0.954	0.957
	500	0.952	0.955	0.941	0.958	0.959
LM(5)	1000	0.950	0.952	0.942	0.952	0.955
	2000	0.950	0.953	0.938	0.952	0.955
	500	0.954	0.959	0.940	0.960	0.961
LM(10)	1000	0.953	0.955	0.946	0.960	0.960
	2000	0.949	0.952	0.932	0.953	0.954
	500	0.943	0.948	0.941	0.947	0.949
LM(20)	1000	0.953	0.953	0.941	0.951	0.952
	2000	0.947	0.947	0.929	0.947	0.949
	500	0.915	0.917	0.909	0.921	0.924
LM(50)	1000	0.944	0.944	0.930	0.938	0.942
	2000	0.945	0.946	0.928	0.941	0.941

Table 4.47

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.987	0.992	0.994	0.992	0.991
LM(2)	1000	0.988	0.991	0.992	0.993	0.992
	2000	0.990	0.990	0.989	0.990	0.990
	500	0.988	0.992	0.990	0.992	0.993
LM(5)	1000	0.983	0.988	0.985	0.990	0.991
	2000	0.983	0.988	0.988	0.988	0.988
	500	0.982	0.989	0.987	0.988	0.990
LM(10)	1000	0.981	0.987	0.983	0.987	0.987
	2000	0.986	0.988	0.987	0.989	0.990
	500	0.988	0.991	0.992	0.989	0.990
LM(20)	1000	0.985	0.987	0.989	0.987	0.987
	2000	0.985	0.989	0.989	0.991	0.991
	500	0.982	0.985	0.989	0.984	0.984
LM(50)	1000	0.979	0.984	0.984	0.976	0.977
	2000	0.981	0.987	0.985	0.985	0.985

Table 4.48

H_0 5%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0.5$ $\phi = 0.3$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.943	0.948	0.943	0.942	0.949
LM(2)	1000	0.943	0.952	0.951	0.954	0.954
	2000	0.944	0.950	0.950	0.947	0.947
	500	0.938	0.949	0.946	0.947	0.950
LM(5)	1000	0.932	0.945	0.938	0.949	0.954
	2000	0.943	0.950	0.945	0.949	0.953
	500	0.943	0.951	0.948	0.952	0.954
LM(10)	1000	0.927	0.937	0.935	0.950	0.947
	2000	0.932	0.943	0.943	0.948	0.950
	500	0.932	0.940	0.938	0.941	0.940
LM(20)	1000	0.931	0.944	0.944	0.938	0.940
	2000	0.931	0.943	0.940	0.941	0.941
	500	0.917	0.925	0.917	0.913	0.914
LM(50)	1000	0.917	0.930	0.928	0.925	0.927
	2000	0.925	0.932	0.933	0.923	0.924

Table 4.49

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.992	0.993	0.993	0.992	0.992
LM(2)	1000	0.987	0.993	0.992	0.994	0.995
	2000	0.974	0.984	0.984	0.986	0.987
	500	0.991	0.990	0.987	0.991	0.991
LM(5)	1000	0.981	0.991	0.990	0.991	0.991
	2000	0.980	0.992	0.991	0.991	0.991
	500	0.988	0.989	0.983	0.988	0.990
LM(10)	1000	0.979	0.992	0.992	0.990	0.990
	2000	0.974	0.992	0.991	0.991	0.993
	500	0.990	0.990	0.987	0.991	0.990
LM(20)	1000	0.983	0.990	0.990	0.987	0.987
	2000	0.981	0.988	0.988	0.985	0.985
	500	0.982	0.983	0.981	0.980	0.981
LM(50)	1000	0.984	0.986	0.987	0.984	0.984
	2000	0.976	0.987	0.987	0.982	0.982

Table 4.50

H_0 5%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.8$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.960	0.963	0.961	0.958	0.960
LM(2)	1000	0.941	0.959	0.955	0.962	0.964
	2000	0.927	0.946	0.946	0.940	0.942
	500	0.951	0.952	0.947	0.953	0.955
LM(5)	1000	0.927	0.954	0.953	0.959	0.960
	2000	0.911	0.945	0.944	0.946	0.946
	500	0.947	0.946	0.929	0.948	0.952
LM(10)	1000	0.919	0.950	0.951	0.939	0.941
	2000	0.914	0.951	0.951	0.939	0.943
	500	0.944	0.948	0.925	0.946	0.948
LM(20)	1000	0.927	0.949	0.948	0.938	0.940
	2000	0.924	0.950	0.948	0.947	0.948
	500	0.913	0.914	0.909	0.911	0.915
LM(50)	1000	0.927	0.933	0.935	0.925	0.925
	2000	0.927	0.951	0.949	0.941	0.941

Table 4.51

H_0 1%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.990	0.990	0.988	0.992	0.993
LM(2)	1000	0.987	0.987	0.991	0.989	0.987
	2000	0.987	0.987	0.988	0.987	0.989
	500	0.992	0.992	0.987	0.990	0.992
LM(5)	1000	0.994	0.994	0.993	0.994	0.995
	2000	0.989	0.992	0.980	0.986	0.988
	500	0.990	0.990	0.986	0.990	0.991
LM(10)	1000	0.986	0.987	0.982	0.985	0.987
	2000	0.988	0.988	0.984	0.990	0.992
	500	0.994	0.994	0.991	0.994	0.996
LM(20)	1000	0.981	0.982	0.979	0.978	0.979
	2000	0.988	0.988	0.986	0.989	0.988
	500	0.984	0.986	0.987	0.984	0.985
LM(50)	1000	0.983	0.984	0.978	0.985	0.986
	2000	0.993	0.993	0.992	0.991	0.994

Table 4.52

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.954	0.954	0.952	0.952	0.953
LM(2)	1000	0.940	0.940	0.945	0.940	0.943
	2000	0.945	0.946	0.950	0.946	0.943
	500	0.968	0.970	0.946	0.966	0.966
LM(5)	1000	0.953	0.953	0.946	0.954	0.952
	2000	0.936	0.936	0.932	0.938	0.942
	500	0.956	0.955	0.944	0.955	0.959
LM(10)	1000	0.941	0.943	0.939	0.944	0.949
	2000	0.941	0.942	0.935	0.938	0.942
	500	0.958	0.958	0.946	0.958	0.957
LM(20)	1000	0.933	0.932	0.919	0.933	0.940
	2000	0.956	0.954	0.944	0.959	0.958
	500	0.930	0.932	0.919	0.930	0.930
LM(50)	1000	0.924	0.922	0.921	0.923	0.931
	2000	0.961	0.960	0.954	0.966	0.972

Table 4.53

H_0 1%: DGP FIGARCH(1,d,1) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.990	0.991	0.989	0.987	0.988
LM(2)	1000	0.988	0.989	0.991	0.989	0.989
	2000	0.993	0.993	0.994	0.990	0.993
	500	0.992	0.992	0.983	0.992	0.992
LM(5)	1000	0.992	0.992	0.990	0.993	0.993
	2000	0.991	0.991	0.990	0.992	0.993
	500	0.987	0.987	0.984	0.987	0.987
LM(10)	1000	0.990	0.990	0.988	0.990	0.990
	2000	0.992	0.992	0.992	0.994	0.994
	500	0.989	0.990	0.987	0.990	0.990
LM(20)	1000	0.987	0.988	0.986	0.990	0.992
	2000	0.994	0.994	0.994	0.995	0.995
	500	0.979	0.980	0.977	0.979	0.980
LM(50)	1000	0.985	0.986	0.983	0.984	0.990
	2000	0.989	0.989	0.988	0.985	0.986

Table 4.54

H_0 5%: DGP FIGARCH(1,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0.3$ $\phi = 0.2$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.941	0.942	0.936	0.939	0.938
LM(2)	1000	0.940	0.946	0.946	0.934	0.937
	2000	0.956	0.956	0.959	0.950	0.949
	500	0.937	0.938	0.921	0.936	0.940
LM(5)	1000	0.951	0.948	0.944	0.948	0.954
	2000	0.954	0.955	0.950	0.949	0.955
	500	0.949	0.949	0.922	0.946	0.951
LM(10)	1000	0.945	0.941	0.934	0.954	0.959
	2000	0.966	0.966	0.961	0.963	0.968
	500	0.936	0.937	0.919	0.937	0.940
LM(20)	1000	0.939	0.940	0.934	0.940	0.950
	2000	0.957	0.958	0.955	0.956	0.965
	500	0.921	0.921	0.909	0.918	0.921
LM(50)	1000	0.929	0.928	0.925	0.927	0.940
	2000	0.941	0.939	0.933	0.935	0.948

Table 4.55

H_0 1%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.983	0.984	0.989	0.983	0.984
LM(2)	1000	0.991	0.992	0.992	0.993	0.993
	2000	0.992	0.992	0.993	0.991	0.991
	500	0.982	0.983	0.981	0.983	0.985
LM(5)	1000	0.994	0.994	0.993	0.993	0.993
	2000	0.989	0.990	0.987	0.986	0.988
	500	0.983	0.985	0.981	0.990	0.990
LM(10)	1000	0.989	0.989	0.988	0.987	0.988
	2000	0.991	0.990	0.989	0.989	0.993
	500	0.989	0.991	0.989	0.992	0.991
LM(20)	1000	0.991	0.991	0.991	0.990	0.991
	2000	0.985	0.985	0.984	0.985	0.987
	500	0.978	0.979	0.974	0.978	0.980
LM(50)	1000	0.988	0.988	0.989	0.984	0.987
	2000	0.986	0.985	0.984	0.985	0.987

Table 4.56

H_0 5%: DGP FIGARCH(0,d,0) $\mu = 0$ $\omega = 0.01$ $d = 0.4$ $\beta = 0$ $\phi = 0$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.944	0.946	0.939	0.945	0.945
LM(2)	1000	0.960	0.960	0.956	0.958	0.960
	2000	0.949	0.950	0.951	0.944	0.951
	500	0.939	0.941	0.922	0.933	0.937
LM(5)	1000	0.950	0.952	0.950	0.951	0.960
	2000	0.940	0.939	0.935	0.946	0.952
	500	0.937	0.939	0.931	0.940	0.942
LM(10)	1000	0.941	0.942	0.939	0.946	0.951
	2000	0.943	0.941	0.940	0.940	0.948
	500	0.935	0.936	0.928	0.940	0.940
LM(20)	1000	0.941	0.943	0.940	0.947	0.955
	2000	0.948	0.947	0.946	0.947	0.954
	500	0.920	0.921	0.910	0.910	0.915
LM(50)	1000	0.928	0.925	0.923	0.924	0.932
	2000	0.925	0.926	0.924	0.921	0.930

Table 4.57

H_0 1%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.988	0.989	0.986	0.988	0.989
LM(2)	1000	0.994	0.995	0.995	0.995	0.995
	2000	0.987	0.987	0.987	0.987	0.987
	500	0.990	0.990	0.984	0.989	0.991
LM(5)	1000	0.992	0.993	0.989	0.993	0.993
	2000	0.994	0.994	0.984	0.991	0.995
	500	0.994	0.994	0.989	0.995	0.995
LM(10)	1000	0.992	0.993	0.988	0.990	0.993
	2000	0.987	0.988	0.984	0.988	0.990
	500	0.993	0.993	0.984	0.990	0.994
LM(20)	1000	0.993	0.993	0.989	0.989	0.993
	2000	0.988	0.990	0.987	0.988	0.990
	500	0.989	0.989	0.982	0.988	0.991
LM(50)	1000	0.990	0.991	0.989	0.991	0.992
	2000	0.987	0.989	0.990	0.988	0.989

Table 4.58

H_0 5%: DGP GARCH(1,1) $\mu = 0$ $\omega = 0.01$ $\alpha = 0.3$ $\beta = 0.65$						
TEST	T	(1,d,1)	(1,d,0)	(0,d,0)	(1,1)	I(1,1)
	500	0.934	0.934	0.944	0.930	0.935
LM(2)	1000	0.963	0.962	0.959	0.956	0.963
	2000	0.946	0.945	0.936	0.937	0.946
	500	0.948	0.948	0.935	0.946	0.948
LM(5)	1000	0.959	0.959	0.945	0.956	0.960
	2000	0.949	0.949	0.938	0.944	0.953
	500	0.952	0.952	0.922	0.946	0.956
LM(10)	1000	0.961	0.963	0.942	0.963	0.968
	2000	0.947	0.950	0.934	0.946	0.952
	500	0.950	0.950	0.929	0.944	0.951
LM(20)	1000	0.947	0.948	0.938	0.941	0.950
	2000	0.946	0.950	0.936	0.943	0.950
	500	0.928	0.928	0.924	0.926	0.926
LM(50)	1000	0.937	0.942	0.931	0.933	0.939
	2000	0.945	0.948	0.940	0.941	0.947