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The Effects of Aggregation and Misspecification on Value-at-Risk Measures with Long Memory Conditional Variances

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The effects of misspecification and aggregation on Value-at-Risk measures with long memory conditional heteroskedasticity

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Abstract

In a Montecarlo setting, generating data with a FIGARCH process, we analyse the effects of a misspecification and data aggregation on Value-at-Risk measures. The analysis is performed on a backtesting approach comparing different GARCH-type models fitted on the simulated data. The alternative VaR measures are compared with a groups of tests and loss functions. We show that on daily data the generator is always preferred, while on aggregated data the loss function approach prefer the RiskMetrics model on daily data, while the tests choice is for a misspecified model on high frequency data.

In the last few years there has been a huge increment in analysis concerning Value-at-Risk (VaR), both from a theoretical point and from the empirical approach, in particular dealing with: the best methods to compute the risk exposure needed to satisy regulators requirements, the choice of the best model for VaR computation, the evaluation of performances of different VaR models. The literature is still growing and with this work we will add some extensions showing how VaR is affected by model misspecification when variance follow a long memory conditional heteroskedastic process. This is related with the numerous findings of persistence in financial markets, coupled with the use of high frequency data for VaR computation, see among other Christoffersen and Diebold (2000) and Beltratti and Morana (1999). In many VaR papers the long memory behavior of the series has not been taken yet into account; even if Beltratti and Morana introduced a first empirical analysis, some problems arise, as pointed out by Christoffersen and Diebold (2000): what does really mean having a long range forecast with high frequency data? in other words is it correct estimating 1-day VaR (or more) using intra-day observations? Belatratti and Morana (2000) solved that using the traditional $\sqrt{T}-$rule for computing s-step-ahead variance forecasts, but ending with a choice of a GARCH process for their foreign exchange data even if the observations showed a clear long memory behavior. They motivated the choice by the closeness of the results obtained by the long and short memory models, preferring then the simplest one, the GARCH. This is a particular effect, maybe due to the data used and is not yet proved in a general context. Apart these considerations the square root rule is not optimal as a scaling in a GARCH framework as Diebold et al. (1996) showed. With this work we will shed some light in a couple of situations, in the next section we will give a brief introduction on the Value-at-Risk, dealing with the Basel accord of 1996, the evaluation schemes of regulators, and the problems connected with the use of the VaR as a risk measure, in particular referring to its coherence. In section 2 we will focus our attention to a specific case, assuming that our world (that is our generators for the simulated return series) follow a FIGARCH scheme. We present the forecasting equations for GARCH and FIGARCH specifications, precisely the forecasting equations for the mean square error of the mean predictor, when the residulas follow a conditional heteroskedastic model, extending in such a way the results of Baillie and Bollerslev (1992). In this part we will focus on point forecasts, not on density forecasts, for such an extension, which is straightforward, refer again to Baillie and Bollerslev (1992). In section 3 we will present a survey on the usual methods applied by banks to evaluate VaR performances on their models, introducing a new loss function that will show the discrepancy between the best choice for the regulators and the best one for a bank, the regulator may push to the choice of a misspecified model; in section 4 we will run a first montecarlo experiment with GARCH(1,1) and FIGARCH data generating processes, estimating then, on both DGP, GARCH, IGARCH and FIGARCH models. For all model specified, even if uncorrectly, we will compute VaR for 1-day horizon, both assuming that the simulated series is a daily series, and also a high frequency series, comparing the different results, using backtesting procedures and the evaluation techniques of section 2. In section 5 we will investigate the effects of aggregation on quasi maximum likelihood estimators with a FIGARCH generator for high frequency data, and then test the ability of a forecast made with higher frequency data, comparing it to the ones obtained from daily data. In section 6 we will conclude.

1 The Value-at-Risk as a risk measure: a coherent need?

The use of risk measures to determine the market risk implicit in any portfolio, investment or financial instrument is a need for all banks, investors and any firms that operate with in financial markets. This need is particularly important for banks acting on both sides of the money market, investing with their funds and collecting savings, all banks have to fulfill requiremts that are there to prevent q default that will be particularly burdensome for the collectivity. In this view most of the banks started in the last decades, given the increased sofistication in the financial markets, to measure the risk of their positions and balance sheets (the whole bank can be viewed as a portfolio of credits and debts, including by this way direct investments and other credit positions) with adequate and therefore complicated instruments. This lead to the diffusion of many "internal" models whose ultimate purpose was the same: monitoring the risk and the losses of all positions. In this situation the Basle Commitee on Banking Supervision, gave a regulated framework, with minimal requirements in term of model choice, to measure and compare the ability of internal models in meeting some very basilar qualifications, giving also an alternative valuation method, that is the "standardised approach". These were included in the accord of 1996, the well known, Amendement to the Capital accord to incorporate market risk (MRA). With this document the Basle Committee, stated the formal rules that an internal model for market risk should meet, how to compute the exposure to this kind of risk, how to define the minimal capital requirements needed to cover this risk. The MRA require that each bank communicate daily the market exposure determined with any internal model or the standardized approach to the national regulator, this exposure has to be determined with a 99% one tail probability and with an holding period of 10 days. The measure of risk should represent the maximum loss with the 99% probability in the holding period. This is just the definition of the Value-at-Risk. Given these measures the regulator will verify if the internal model meet a minimal requirement: in the past year does this model gave a 1% of failures or more? The verification is conducted with a techniques described in the MRA accord, the backtesting approach, that is the regulator verify the performances of the internal model in the last 250 days, and simply counts the exceptions, how many time the internal model fails. Given this number of exceptions the regulator classify the internal model with a grid in the exceptions (0-4, 5-9, more then 9) matched with a colour (green, yellow and red!). The classification allow the regulator to impose some penalty, this because the MRA compute the correct VaR as the maximum between today's VaR and the average of last 60 VaR measures, multiplied by a scaling factor that depend on the classification.

This methodolgy however may be inefficient for banks, as may lead to the application of a model that fulfill the requirements of the Basel accord but translate in a bigger cost: the minimal capital requirement can be viewed as an immobilization of resources, of liquidity, and given the operativity of the banks this represent an opportunity cost of investing resources. This point will be discussed in the next section, here we want to stress that the VaR is now used to determine market risk exposure because is the methodology required by the Basle Committee, the one used by the regulators to verify the minimal capital requirements, however its characteristics are such that it is a "coherent" risk measure in limited cases. Let us clarify this point: the coherency of a risk measure was the object of a recent paper of .Artzner, Delbaen, Eber and Heath (1998). In that paper they considered market risks and presented a group of properties that a risk measure should fulfill. In this framework they called "coherent" a risk measure that satisfy all these axioms. Let us summarize the first part of the Artzer et al. (1998) work:

given Ω a set of all possible states of nature, X a random variable in this set, G the set of all possible risks, A the set of acceptable risks (for the regulators) and assuming that the acceptable risks include all strictly positive risks in G

and exclude all strictly negative risks, they state that a risk measure is just a mapping from G to R; given a reference instrument with rate of return r the risk measure is defined as

$$\rho_{A,r}(X) = \inf\{m | m \cdot r + X \in A\}$$

In this framework four axioms are defined (omitting subscripts)

Axiom 1 *Translation invariance: for all $X \in G$ and for all real α $\rho(X + \alpha \cdot r) = \rho(X) - \alpha$*

Axiom 2 *Positive homogeneity: for all $X \in G$ and for all $\lambda \geq 0$ $\rho(\lambda X) = \lambda \rho(X)$*

Axiom 3 *Monotonicity: for all $X,Y \in G$ with $X \leq Y$ $\rho(X) \geq \rho(Y)$*

Axiom 4 *Subadditivity: for all $X,Y \in G$ $\rho(X+Y) \leq \rho(X) + \rho(Y)$*

and as an outcome

Definition 5 *A risk measure that satisfy all the previous axioms is defined coherent*

Given this definition Artzner et al. (1998) show then that the Value-at-Risk does not fulfill the subadditivity axiom and therefore is a non-coherent risk measure, however in the particular case of normal and in general elliptical distributions this characteristic is recovered, see Embrechts et al. (1999) for a formal proof. Recently Cicchitelli (2002) allow anyway the application of VaR methodology even if distributions are non elliptical provided we are determining the VaR of a portfolio with an adequate number of components (read assets or instruments). In the following we will assume that the return of the hypothetical instrument we are analysing are extracted from a normal distribution, this to ensure the existence of the FIGARCH as well as its stationarity, this allow us to consider VaR as a coherent measure of risk. Therefore we restrict our attention to a special case: standardized returns follow a standardised normal distribution and volatility follow a FIGARCH structure. This because we are interested in analysing performances of Value-at-Risk in presence of long memory.

2 Prediction mean square errors with FIGARCH

In this section we will follow the approach of Baillie and Bollerslev (1992) who were considering prediction with dynamic models and conditional heteroskedasticity. Assume that the series object of our study follow a generic process in the mean

$$y_t = \mu_t + \varepsilon_t$$

and that the residuals are such that $\varepsilon_t|I^{t-1} \sim iid\left(0, \sigma_t^2\right)$, where with I^{t-1} we identify the information set up to time t-1. Assuming that the mean term is always zero, we are in the framework of a GARCH-type process, where the forecast for the mean process is always zero and the MSE depend on the s-step ahead prediction for the variance. The last will also depend nontrivially on the information set, an extensive discussion and numerous expression can be found in the above cited paper. For the simple GARCH(1,1) the s-step ahead predictor for the variance (the MSE of the s-step ahead predictior for the mean) is:

$$E\left[\varepsilon_{t+s}^2|I^{t-1}\right] = E\left[\sigma_{t+s}^2|I^{t-1}\right] = \omega\sum_{i=1}^{s-1}\left(\alpha_1+\beta_1\right)^i + \left(\alpha_1+\beta_1\right)^{s-1}\sigma_{t+1}^2 \tag{1}$$

If the DGP is a FIGARCH process the predictor depend nontrivially on all past values, and an expression like the previous one cannot be derived given the dependence on infinite past. The volatility structure induced by a FIGARCH can be defined as follows:

$$\sigma_t^2 = \omega + \beta\left(L\right)\sigma_t^2 + \left[1-\beta\left(L\right)-\Phi\left(L\right)\left(1-L\right)^d\right]\varepsilon_t^2$$

where $\beta\left(L\right) = \sum_{j=1}^{p}\beta_j L^j$, $\Phi\left(L\right) = \sum_{j=0}^{m}\phi_j L^j$ and $\left(1-L\right)^d$ is the fractional integration component. In our analysis we will use the following representation, which is derived after some boring algebra (see Appendix):

$$E\left[\sigma_{t+s}^2|I^{t-1}\right] = \theta_s\omega + \sum_{i=0}^{\infty}\psi_{i+1}\varepsilon_{t-i}^2 \tag{2}$$

$$\psi_k = \sum_{i=1}^{s}\phi_i\lambda_{k+s-i} \qquad \phi_1 = 1 \qquad \phi_i = \sum_{j=1}^{i-1}\lambda_j\phi_{i-j} \qquad \theta_s = \sum_{i=1}^{s}\phi_i$$

One question on the worthness of the previuos formula arise: why not using a recursion based on

$$\begin{aligned} E\left[\sigma_{t+s}^2|I^{t-1}\right] &= \theta_s\omega + \sum_{i=0}^{\infty}\lambda_{i+1}E\left[\varepsilon_{t+s-i}^2|I^{t-1}\right] \\ E\left[\varepsilon_{t+i}^2|I^{t-1}\right] &= \varepsilon_{t+i}^2 \qquad \text{if } s \le 0 \\ E\left[\varepsilon_{t+i}^2|I^{t-1}\right] &= E\left[\sigma_{t+i}^2|I^{t-1}\right] \qquad \text{if } s > 0 \end{aligned} \tag{3}$$

The main reasons is only based on computational advantages and rounding error that, implementing procedures with any software, arises: in every point or density forecast of the conditional variance we use the past value of the observed series or residuals, given the infinite past dependence of any conditional variances with the simple recursion formula we induce a greater rounding error than the one induced by aggregating coefficients. By our formula we just induce one rounding error not the sum of s rounding errors.

3 Comparing Value-at-Risk estimates

The main task of risk management is the evaluation of market risk implicit in the positions (on financial instruments or portfolios) helded. This risk is mainly measured by the Value-at-Risk, the maximum amount of loss we can incur in a given time interval and at a specific level of confidence. The exposure measured by the VaR depend crucially on the underlying model employed for the return series of thefinancial instrument of interest. A group of questions arise: how can we judge if the underlying model is correct? how should the Value-at-Risk perform under different approaches? What are the consequences of a misspecification? In this section we will try to give an answer to some of these question in a particular case: we assume the the true data generating process (DGP) follow a FIGARCH in the variance, and we will compare via tests and other approaches the true DGP with a gruop of misspecified models. The main works in this field are the ones of Kupiec (1995), Christoffersen (1998) and Lopez (1998) who proposed, respectively, a statistical based procedure and a loss function approach to test if the VaR estimates are correct and consistent with the data.

The reliability of VaR measures depend on the correct specification of the underlying models, this is necessary in providing an accurate measure of risk exposure. Considering the computation of Value-at-Risk using instruments (or portfolio) returns,indexing VaR estimates with time t, and model index m, assuming that the return follow a possibly time-dependent distribution f_t, the Value-at-Risk computed conditional on the information set on time t, for k-steps-ahead, is the α-quantile of the forecasted distribution f given for the model m. $\mathrm{VaR}_{m,t}(\alpha, k)$ is the solution of the following equation

$$\int_{-\infty}^{VaR_{m,t}(\alpha,k)} f_{m,t+k}(x)\,dx = \alpha \tag{4}$$

Two different approaches are actually available to evaluate the VaR estimates: statistical based procedures, and loss functions approaches. To the first group belong the Proportion Failure test (or Unconditional coverage test), the Time Until First Failure test of Kupiec (1995) and the Conditional coverage test of Christoffersen (1998) and Lopez (1998). To the second group belong the approach of Lopez (1998). The main difference bewteen the two is that with statistical procedure, inference analysis is available. The tests of Kupiec and Christoffersen are based on likelihood ratios, and on the assumption that VaR should exhibit a conditional or unconditional coverage equal to α.

The Unconditional Coverage test (UC) of Kupiec is based precisely on the first assumption: if VaR estimates are accurate, the exceptions x (the number of times return underperform VaR measures) can be modeled with a binomial distribution with probability of occurrence equal to α. In this case, comparing the required unconditional coverage α (usually set to 0.05 or 0.01), with the measured coverage $\widehat{\alpha} = x/T$, is possible to derive a likelihood ratio test under the null hypothesis $\alpha = \widehat{\alpha}$

$$LR_{UC} = 2\left[\ln\left(\widehat{\alpha}^{x}\left(1-\widehat{\alpha}^{T-x}\right)\right) - \ln\left(\alpha^{x}\left(1-\alpha^{T-x}\right)\right)\right] \tag{5}$$

Under the null huypothesis LR_{uc} is distributed as a $\chi^2(1)$. The UC test is also the statistica transposition of the procedure used by the regulator authority in judging if the internal model is accurate. As pointed out by Lopez (1998) this method does not show any power in distinguishing among different, but close, alternatives.

This test, as pointed out by Christoffersen (1998), consider only exceptions over the sample size, however in presence of conditional heteroskedasticity, also the conditional coverage is important. Ignoring this issue, the volatility dynamics, we could have forecasts (VaR estimates with a GARCH-type model, include the forecast of the conditional variance as we will see) with correct unconditional coverage and uncorrect conditional coverage,in this cases UC test is of limited accuracy. Lopez adapted the general approach of Christoffersen formulating the following Conditional Coverage (CC) test. First a dummy variable is setted to identify exceptions

$$D_{m,t+1} = \begin{cases} 1 \text{ if } \varepsilon_{t+1} < VaR_{m,t+1} \\ 0 \text{ if } \varepsilon_{t+1} \geq VaR_{m,t+1} \end{cases}$$

Under the null hypothesis that the VaR present correct conditional and unconditional coverage, this indicator variable should be independent. Thus the CC test is computed as the sum of the UC test and of a test of independence on $D_{m,t+1}$, against a first-order Markoc process. The independence test is constructed as follow: with $T_{i,j}$ we identify the number of observations in the sample T in state j after having been in state i, under the Markov process the likelihood function is

$$L_M = (1-\pi_{0,1})^{T_{0,0}}\,\pi_{0,1}^{T_{0,1}}\,(1-\pi_{1,1})^{T_{1,0}}\,\pi_{1,1}^{T_{1,1}} \tag{6}$$

where $\pi_{0,1} = T_{0,1}/(T_{0,0}+T_{0,1})$ and $\pi_{1,1} = T_{1,1}/(T_{1,0}+T_{1,1})$. Under serial independence the likelihood function is

$$L_I = (1-\pi)^{T_{0,0}+T_{1,0}}\,\pi^{T_{0,1}+T_{1,1}} \tag{7}$$

where $\pi = (T_{0,1}+T_{1,1})/T$. The test statistic is then

$$LR_{CC} = L_{UC} + 2\left[\ln(L_M) - \ln(L_I)\right] \tag{8}$$

and is distributed as a $\chi^2(2)$ under the null hypothesis of correct coverage (undel the null hypothesis of independence the dependence test is a likelihood ratio test, whose limiting distribution is a $\chi^2(1)$).

We turn now to another approach, the one of loss functions. The main work in this area is the one of Lopez, based on computing a loss function distinguishing between exception and not-exception. In the general form he propose the following formula

$$C_{m,t+1} = \begin{cases} f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) & \text{if } \varepsilon_{t+1} < VaR_{m,t+1} \\ g\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) & \text{if } \varepsilon_{t+1} \geq VaR_{m,t+1} \end{cases} \tag{9}$$

where $f(x,y)$ and $g(x,y)$ are such that $f(x,y) \geq g(x,y)$. In this formulation higher values of the functions are associated with exceptions, thus summing $C_{m,t+1}$ over the backtesting sample used by regulators we obtain

$$C_m = \sum_{i=1}^{T} C_{m,t+i} \tag{10}$$

and the best model is the one that minimise 10. The choice of the correct model can be done referring to a benchmark, once the functions heve been specified. Lopez proposed different functions: one derived from the dummy for exception, another using weight as for the regulator choices, and then the following, that take into account the exception and the discrepancy between the realization and the VaR forecasted measure.

$$C_{m,t+1} = \begin{cases} 1 + \left(\varepsilon_{t+1} - VaR_{m,t+1}\right)^2 & \text{if } \varepsilon_{t+1} < VaR_{m,t+1} \\ 0 \text{ if } \varepsilon_{t+1} \geq VaR_{m,t+1} \end{cases} \tag{11}$$

This function was suggested in order to take into account not only the risk but also the amount of the possible default in the position. This function was built mainly for regulatory purposes, helping the regulator in the evaluation of bank internal models. But there is an open point, with this function we may be tempted to reject a model only because, at parity of exceptions, it realize an higher loss function. In this case we may reject a correct model, a correctly specified and identified model for the series of returns, choosing an incorrect model. This may, up to some extent, observed in the work of Beltratti and Morana (2000) on FX data, when they end choosing a Garch process for computing VaR even if the data show a clear long memory proporty, because the number of exceptions of the Figarch was lower, too conservative (this is a loss function based on the dummy). This can be clarified with an example: assume that two different models are fitted to a real series, a GRACH(1,1) and an IGARCH(1,1); the forecast from both models differ only in the wideness of 1-sted-ahead prediction intervals for the mean, the one of the IGARCH is bigger; moreover assume that both models present exactly the same number of exceptions, then using the loss function suggested by Lopez we will choose the IGARCH model because its bands are wider and therefore the loss function is lower (the difference between VaR and the realisation in the market is lower given that bands are wider); this will be translated in an higher cost for the bank, they will have to fulfill higher capital margin to stick to the IGARCH bands, even if the exceptions of the two models are the same. To solve this point we suggest using different loss functions, dealing not only with the failure of the VaR measures but also taking into account the distance between the different forecasts and the past realisations. We suggest to check that the model fulfill the quantile requirements and also have to be stick to the realisation of the underlying process. We propose three different distance measures, adopting the same terminology of Lopez:

$$\begin{aligned} {}^1f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &= \left|\frac{\varepsilon_t}{VaR_{m,t+1}}\right| \\ {}^2f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &= \frac{\left(\varepsilon_t - VaR_{m,t+1}\right)^2}{\left|VaR_{m,t+1}\right|} \\ {}^3f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &= \left|\varepsilon_t - VaR_{m,t+1}\right| \end{aligned} \tag{12}$$

In all three cases the best choice is the model that minimize the loss function. Taking these as they are we can incurr in the same problems as using the loss functions of Lopez: we may be not able to correctly choose the right model, preferring a solution with narrower bands. For this reason we suggest also to apply these loss functions not only to the exceptions but to the whole sample:

$$\begin{aligned} {}^1f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &=^1 g\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) \\ {}^2f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &=^2 g\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) \\ {}^3f\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) &=^3 g\left(\varepsilon_{t+1}, VaR_{m,t+1}\right) \end{aligned} \tag{13}$$

The three functions suggested consider different approaches to testing the discrepancy between the identified model and the realisations: the first one consider the ratio between one step VaR and the realisation, the second is the squared error realised with the VaR, divided by the VaR itself to be standardised to the same quantity of the first function, to be able to build a fourth criteria addig the 2 measures, just a kind of first and second order loss; the third function take into consideration only the difference between VaR measure and the realisation. The effect of such different approaches will be presented in the following chapter with a limited Montecarlo experiment (we deal with FIGARCH DGP, an extensive Montecarlo dealing with different generators will be object of future reseaches).

With these functions we can apply at a first stage the usual analysis of Kupiec and Christoffersen and then use the loss function approach to compare the cost of different admissible choices. Clearly from a regulatory point of view this choice may not be worthwile because regulators objective is to reduce the risk of default in case of extreme events, position represented by the loss function of Lopez, but the function we propose represent the best choice for bank purposes, choosing model that fulfill regulatory requirements (compare with the Basel agreement...) and allow for a lower cost. Considering the system in a whole these functions may help in choosing a model that is closer to the data, just choose the "true" model, leaving aside only real extreme events, that, if the model is really the true, and will be used by all economic agents in the economy (or all financial intermediaries, read banks) will hit all of them, really an extreme event. With this choice we ensure both conditional and unconditional coverage, instead of choices of misspecified models that may lead to uncorrect conditional coverage.

3.1 A GMM-based testing approach

Recently Christoffersen, Hahn and Inoue (2001) introduced a new approach in the evaluation of Value-at-risk measures. In a general approach we can define the VaR via a quantile regression:

$$VaR_{m,t}(\alpha,\beta) = \beta_{1,m}(\alpha) + \beta_{2,m}(\alpha)\,\sigma_{t,m} \tag{14}$$

where the conditional volatility depend on the model we are using and parameters depend both on the model and on the significance level (coverage probability). Then we can state the following

Definition 6 *(CHI 2001 definition 1) The VaR is efficient with respect to the information set Ψ^{t-1} when*

$$E\left[I\left(\varepsilon_t < VaR_{m,t}(\alpha)\right) - p|\Psi^{t-1}\right] = 0$$

where I(.) is the indicator function

Using then this efficient condition we can test if VaR measure satisfy it, but also by this way we can compare different VaR even if misspecified. The methodology of the analysis require conditioning on some information set, and the choice among different models. The first point is achieved considering as the information set at time t, as the measure of volatility in time t-1 obtained with the different models we are comparing and wih a constant

$$E\left[\left(I\left(\varepsilon_t < VaR_{m,t}(\alpha,\beta)\right) - p\right) \times k\left(1, \sigma_{t-1,m1}, \sigma_{t-1,m2}, \sigma_{t-1,m3}...\right)\right] = 0 = E\left[f\left(\varepsilon_t,\beta\right)\right] \tag{15}$$

Specification testing is achieved using the test suggested by Kitamura and Stutzer (1997), the information theoretic alternative to a general method of moments (GMM) based test. Define the following quantity

$$M_T(\beta,\gamma) = \frac{1}{T}\sum_{i=1}^{T}\exp\left(\gamma' f\left(\varepsilon_t,\beta\right)\right)$$

and maximizing over the two parameter sets

$$\hat{M}_T\left(\hat{\beta}_T, \hat{\gamma}_T\right) = \max_{\beta}\min_{\gamma}\frac{1}{T}\sum_{i=1}^{T}\exp\left(\gamma' f\left(\varepsilon_t,\beta\right)\right)$$

then the Kitamura Stutzer test has the following equation

$$\kappa_T = -2T\log\left(\hat{M}_T\right) \rightarrow \chi^2(r-k) \tag{16}$$

where r are the number of conditioning information variables (constant included) and k are the estimated parameters (β) in the quantile regression. The null hypothesis of this test is that the VaR measure satisfy the efficiency condition, therefore accepting the null will mean that the VaR model is correctly specified. In this approach we have, however, a challenge: the funtion $f(\varepsilon_t, \beta)$ is non-differentiable due to the presence of the indicator function. This problem will cause the traditional optimization techniques to burn down, requiring simulation based methods to estimate parameters or to employ generalised algorithm such as the simplex method or simulated annealing. This problem can be easily avoided in our case: considering that we focus on GARCH-type models, the VaR measure depend only on the evaluated conditional variance and on the coverage probability

$$VaR_{m,t}(\alpha, \beta) = \Phi^{-1}(\alpha)\, \sigma_{t,m} \tag{17}$$

using the cumulative standard normal inverse and excluding the effect of a constant. By this way we exclude the optimization over the parameters in the quantile regression and the traditional optimization routines can be used without problems.

Christoffersen et al. (2001) introduced another testing approach that allow to compare directly two different VaR measures. This test is based on the difference between two KLIC distances. If we consider two different VaR measures $m1$ and $m2$, and we define the KLIC respectively as

$$\hat{M}_{T,m1}\left(\hat{\beta}_T, \hat{\gamma}_T\right) \text{ and } \hat{M}_{T,m2}\left(\hat{\beta}_T, \hat{\gamma}_T\right)$$

CHI generalising a result of Kitamura (1997) state the following:

Theorem 7 *(CHI theorem 1) Let*

$$\begin{aligned} M_{m1,T}(\beta_1^*, \gamma_1^*) &= \max_{\beta_1} \min_{\gamma_1} M_{m1,T}(\beta_1, \gamma_1) \\ M_{m2,T}(\beta_2^*, \gamma_2^*) &= \max_{\beta_2} \min_{\gamma_2} M_{m2,T}(\beta_2, \gamma_2) \end{aligned}$$

Under the null that $M_{m1}(\beta_1^*, \gamma_1^*) = M_{m2}(\beta_2^*, \gamma_2^*)$ *we have*

$$\sqrt{T}\left(\hat{M}_{T,m1}\left(\hat{\beta}_T, \hat{\gamma}_T\right) - \hat{M}_{T,m2}\left(\hat{\beta}_T, \hat{\gamma}_T\right)\right) \rightarrow N\left(0, \sigma_\infty^2\right)$$

where $\sigma_\infty^2 = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left(\exp\left(\gamma_1^{*\prime} f(\varepsilon_t, \beta_1^*)\right) - \exp\left(\gamma_2^{*\prime} f(\varepsilon_t, \beta_2^*)\right)\right)\right)$ *and the* T *subscript denote quantities computed with* T *observations instead of the infinite past.*

Proof. *See the appendix for a revised proof of this theorem and the application to GARCH-type models* ■

In this case the rejection of the null hypothesis will imply that the two measures do not match equally well the efficiency condition in favour of the model 2. When the null is accepted a positive measure imply the preference of model 1, a negative result the preference of model 2.

4 VaR and Long memory GARCH

In analyzing the performances of tests and loss functions in identifying and choosing the best model for VaR computation we run a Montecarlo experiment: we deal with a group of simulating DGP, eight FIGARCH with differents orders and parameter values and a GARCH(1,1) used as a comparative test for evaluating the ability of tests and measures on Value-at-Risk. The DGP are described in the following table.

DGP	μ	ω	d	β	ϕ
FIGARCH(1,d,1)	0	0.01	0.8	0.5	0.3
FIGARCH(1,d,1)	0	0.01	0.8	0.5	0.05
FIGARCH(1,d,0)	0	0.01	0.8	0.5	0
FIGARCH(0,d,0)	0	0.01	0.8	0	0
FIGARCH(1,d,1)	0	0.01	0.1	0.4	0.5
FIGARCH(1,d,1)	0	0.01	0.4	0.3	0.2
FIGARCH(1,d,0)	0	0.01	0.4	0.3	0
FIGARCH(0,d,0)	0	0.01	0.4	0	0
GARCH(1,1)	0	0.01	0	0.65	0.3 (α)

In this experiment we act as the simulated series were daily series, simulating 2250 observations, using the first 2000 to estimate the model and the last 250 to assess the validity of Value at risk measures in a backtesting approach. On all simulated series we estimate 4 different models: the true DGP (on the identification problem see the first part of this dissertation), a GARCH(1,1), an IGARCH(1,1) and an exponentially weighted moving average (EWMA, the well known RiskMetrics model), with smoothing parameter set to 0.97. Given the parameter and variance estimates, we use these to compute VaR and then we test the correctness of these risk measures. We use the tests and loss functions described in the previuos section. For all DGP we ran 1000 replications. The results are summarized in tables from (2) to (71). Tables are grouped with respect to the DGP and contain in the order (inside each group): the average number of exceptions across the 1000 replications, for each of the four fitted models, the standard deviation and the average percentage of exceptions; the frequency of less exception, that is, we count how many times each model is the one that give a lower number of exceptions, note that the cumulate frequency can be above one since different models can lead to the same number of exceptions; the frequency of accepting the null hypothesis for the test of Unconditional Coverage (UC), Independence (I) and Conditional Coverage (CC); the frequency of model selection using Lopez loss function, that is we count how many time each model minimize the loss function; the frequency of model selection with the alternative loss functions previously suggested, and their combinations, computed only on exceptions (E) or on the full sample (T); the results of the model comparison test of Christoffersen et al. (2000), we consider 4 different VaR p-level (1%, 5%, 10% and 25% to compare results with the cited paper), and we report the frequencies of having a significant test statistics and the frequency of choice of the first or of the second model, all at confidence levels of 1%, 5% and 10%; finally the results of the model specification test of Christoffersen et al. (2000), again computed at the previuos 4 VaR p-values and confidence levels. The tests developed by Christoffersen et al. (2000) presume a comparison of non-nested models; as we verified in the first part of this dissertation FIGARCH and GARCH (or IGARCH) most of the time are non-nested models, this let us compute the tests and perform the analysis. However GARCH, IGARCH and EWMA are nested models, therefore we expect significant results comparing long and short memory models, while we will have to take with care results among short memory specification. All tables are listed in the Annex. The following conclusions arise from the Montecarlo study:

Average exceptions and MRA. In most of the cases (excluding only the FIGARCH(0,d,0) with d=0.8) at 1% Value-at-Risk p-level, the RiskMetrics model is too conservative, leading to an average number (and percentage) of exceptions strictly below 2.5 (correspondent to 1%). This effect is, even if with less evident, present also at 5% level and is influenced by the memory property of the generator: with higher memory (lower d) the RiskMetrics is much more conservative. This is probably due to the different structure of the two processes: in the FIGARCH case a bigger weight is given to past innovations, so there is a greater sensitivity to market movements, this imply a variance forecast with abrupt changes without signals of convergence of variances to an unconditional level, while in the RiskMetrics, a particular IGARCH model, the parameter configuration give much more importance to movement in the variances (the β parameter is 0.97) leading to gradual movement and slower convergence to unconditional variance level. This effect remain also in GARCH and IGARCH specifications, since no constraints are imposed (apart the one for positivity of variances) on the parameters, and this lead to an estimated β much smaller than 0.9. Comparing then FIGARCH, GARCH and IGARCH results we can see that they are very close showing that a misspecified model, can be good enough for MRA requirements, however we must precise that the forecasts obtained with misspecified models lead to uncorrect conditional coverage. In all cases, on average, all models strongly satisfy the requirements of the amendement to Basle accord for market risks, leading to Green zone positioning (exceptions lower than 5). Considering now the frequencies of model selection in particular just the number of exceptions, the best choice is most of the time the EWMA, but this result is strictly related to the fact that this is the most conservative of the models, and is therefore of limited significance. As an example we reported in graphs from 1 to 8 (in the appendix) two experiments, showing two very different paths that can be generated by a long memory structure. In the graphs (1), (2), (5) and (6) are reported the simulated series and the simulated conditional variances, while in graphs (3) and (7) we show the estimated conditional variances in the backtesting period (or part of it). Finally in graphs (4) and (8) we report the Value-at-Risk bound computed on the two series with the true model and the RiskMetrics, evidencing the exceptions.

Tests of Conditional and Unconditional Coverage. As in the previuos work of Lopez (1999), we find that these test show no power in distinguishing among different models. All null hypothesis of correct unconditional or conditional coverage and of independence among exception are accepted with a percentage ranging from 75% to 100% at the 1% level of the test and for both 1% and 5% VaR. Results do not depend on parameter values. For the test at 5% significance level the null hypothesis is rejected with higer probability, especially for the Independence test, however this is true for all the 4 models, again we cannot infer on the best solution for our purposes.

Loss functions. We can observe that the Lopez loss function, given its formulation, depend crucially on the number of exceptions, this influence its value and therefore the model selection frequencies based on it. In all cases considered (again apart the FIGARCH(0,d,0) with d set to 0.8) the Lopez approach lead to the choice of the RiskMetrics as the best model for Value-at-Risk computation. This in the sense that the best model is the one that minimize the cost of an exception, is a choice based on the risk of default, a choice driven by regulators objectives. However this does not imply that the best model is the true generator or the one that minimize the cost for a private bank: as we can observe from figure (4) and (8) the EWMA has a smaller number of exceptions, sice its VaR bands are much widers compared to the bands of the true generator, this can be interpreted as an higher cost for the bank, in fact the VaR level represent a minimal capital requirement that banks must hold on to cover market risks. Immobilizing this capital translate to an opportunit cost of liquidity resources, and reduce the operativity for the bank. A VaR based on the true generator meet the Basle MRA requirements and give a correct conditional coverage for market risks, with narrower VaR bands. In spite of that none of the loss functions lead to a correct choice of the generator as the best model. All the functions considered, if applied only on the exceptions, select most of the times the EWMA, with percentange ranging fro 40% to 60%, second best choice switch between GARCH and IGARCH, in none of the cases the FIGARCH is chosen. Considering the whole sample the FIGARCH does not appear as the best model, even if its frequencies of selections increase. In this case the best choice switch between GARCH and EWMA, leading again a possible choice of a misspecified model. Now this solution can be considered on a different point of view: should we prefer a model that minimize the number of exceptions but impose a greter opportunitity cost, or will be better a choice of a model that is closer to the true generator, satisfy in the meantime regulators requirements and allow for narrower VaR bands? The answer depend on the subject whom is posed: a regulator will surely prefer the first solution, while private banks will chose the second one. A consideration on the GARCH generator case: the model is correctly chosen with our alternative loss functions, but only if we consider the whole sample, if not IGARCH or EWMA are preferred.

Model comparison test. Now choices change. A first group of observation on the tables: the test is labelled as "not significant" when the two models equally well match the efficient moment condition, therefor the label "significant" is given to the rejection of the null hypothesis; we can observe that the null is accepted with high percentage when we compare very close models, that is the case of GARCH and IGARCH, when the GARCH parameters are close to the constraint $\alpha + \beta < 1$; the frequencies of selection of the first or of the second model are computed as percentages on the "significant" tests, they always sum to one, moreover I can always choose between the two models, provided I rejected the null, depending on the sign of the test statistics. All tables show a similar behaviuor, the EWMA model is never preferred to the DGP with a percentage greater than 40%, and most of the time this is true also for GARCH and IGARCH. This can be interpreted as a result of our observations on the correct conditional coverage given by the true generator, a condition that is extracted form the information set (here this is represente by the forects obtained with the four models in the past) by the estimation procedure. Moreover the true FIGARCH generator is preferred also to the GARCH and IGARCH with frequencies always above 50% in all cases considered.

Model specification test. In this case the test show dependence on the VaR p-level, leading to very poor results, none of the model are correctly specifed for the simulated series, for the 1% case, while for the remaining the percentage of accepting the null (the model is correctly specified) increase with p, with a jump from 1% to 5%. This may be due to the very limited number of exceptions in the 1% case, not sufficient to extract an indication on the ability of the model in matching the efficient moment condition. This result will probably change extending the backtesting period, however we will not pursue this point since we focus on the selection process of a model that should be analysed by a regulator who use 250 period for backtesting (see MRA).

We conclude this section with a word of advise on the results we obtained compared to the ones of Christoffersen et al. (2001): we developed this Montecarlo on a backtesting approach, to verify the power of the VaR specification test and VaR comparison test in the framework used by regulators following the MRA, that is on 250 observations. In this setup the number of exception is very limited and the size and power of the two test is affected: the tests are built on an efficiency condition that depend on an indicator function selecting exceptions, lower the number of exceptions lower will be the number of significant points used in (15)and in the tests. Moreover we want to stress that once the number of exceptions are the same in two or more models, the VaR specification test will lead to the very same result and the VaR comparison test will show clustered results including one or more groups of zeros. We tried, in a limited Montecarlo, to compute tests on the whole sample, results seem not to differ from the ones here presented, however an additional analysis in this direction will be necessary and left for future researches, but we stress that it must be developed as a suggestion for an alternative framework that will allow regulators to test the reliability of internal models, otherwise, with the current MRA, the results of this work apply.

5 VaR, FIGARCH and aggregation

A point rised up by the Beltratti and Morana (2000) paper was the following: using high frequency data could we get better estimates of our 1-day VaR? Their conclusion was that the simple GARCH(1,1), on high frequency data, will do the task even if there is an evident long memory in the data. We examine this relation in detail with a limited Montecarlo study dealing with a group of questions. We generate data as they were hourly returns and then aggregate them in order to obtain daily returns, assuming that a normal open market day last for eight hours. The data are generated with normal distributed standardized residuals, to ensure stationarity. On the aggregated data we are at first interested in assessing if there are changes in parameter estimates, specially on the memory behaviour, therefore we examine this point computing kernel density estimates of the parameters of interes and a calculating group of information criteria on different models, a GARCH(1,1), an IGARCH(1,1) and three FIGARCH(p,d,m) with p=m=0, p=1 and m=0 and p=m=1, that will be used to identify the preferred model. By this methods, given the results of chapter 1, we will asses if the aggregation process change the structure of the series into an integrated GARCH, a short memory model or if the long memory behaviour is robust against the aggregation.

All experiments consist of 1000 replications with time series of 18000 non aggregated observations. We considered five different DGP with the following parameters combinations: $d = 0.8$, $\beta = 0.5$, $\psi = 0$; $d = 0.8$, $\beta = 0.5$, $\psi = 0.05$; $d = 0.8$, $\beta = 0.5$, $\psi = 0.3$; $d = 0.4$, $\beta = 0.3$, $\psi = 0$; $d = 0.4$, $\beta = 0.3$, $\psi = 0.2$. The identification analysis is limited to the first 2000 aggregated data (16000 non aggregated points) leaving the last 250 (2000 non aggregated) for a VaR backtesting evaluation. We consider this as a limited Montecarlo since we do not take into consideration the consistence of model selection based on information cirteria and we restrict our attention to a limited range of models and parameter combinations. This choice strictly depend on CPU time needed to run a full experiment: to simulate 18000 observations (plus 2000 points added to avoid dependance from initial values), run the identification tests and then the VaR evaluation, we need between 6 and 15 days, depending on DGP and "external" events (blackouts, computer failures etc.). In all cases on aggregated data we estimated the following models: FIGARCH(1,d,1), FIGARCH(1,d,0), FIGARCH(0,d,0), GARCH(1,1) and IGARCH(1,1). In the tables and graphs included in Annex 3 we report the frequency of model selection based on the information criteria of Akaike (AIC), Hannan-Quinn (HQ), Schwarz (BIC) and Shibata (SH), together with the frequency of accepting the null hypothesis of the following tests: Box-Pierce for residuals autocorrelation, computed also for squared residuals; Engle, lagrange multiplier for residuals ARCH effects; Jarque-Bera normality test for residuals. For all the different DGP we report also the estimated parameters and standard errors, together with a kernel density of the distribution of the quasi maximum likelihood estimator. The tables and graphs used on which the following observations are based can be found in the Annex 2. We can summarize our results as follows:

- A first consideration on the memory parameter estimates: in general we can observe that the aggregation does not change the Montecarlo average of the long memory coefficient, d. This result is much stronger for the expreriments conducted with d setted equal to 0.8, rather than in the case where it assume the value 0.4. Compare as an eample table 68 with table 72, the discrepance between the non-aggregated true value and the Montecarlo average is less than 0.01 in the first while it is close to 0.1 in the second. Even with this evidence we are not sure that this can be interpreted as a true effect of aggregation. The picture can be clarified analysing also the Montecarlo standard deviation, and comparing it with the one obtained on non-aggregated estimates: we can observe that it heavily increase for d=0.8 while the change for 0.4 is less evident. This may be much more evident comparing the kernel density estimates of this section (in the Annex 2) with the one on the first chapter. From these observation we extrapolate the following picture: we believe that the effect of aggregation depend on the memory parameter level, we can therefore distinguish between series with high memory (d=0.4) and intermediate memory (d=0.8): in the first case aggregation matter, memory properties increase (the distribution of the estimator has a stable variance across aggregated and non aggregated data), while in the second case the aggregation does not affect the memory structure but lead to an increase in variation among parameter estimates.

- Consider now the estimates of the other FIGARCH parameters: these are much more affected from the aggregation process, as if this will change the short-memory structure of the underlying process. Here we must note that kernel densityes evidence a problem in the consistence and in the biasedness of the QML estimator for the FIGARCH(1,d,1). This might be coupled with the algorithm convergence problem evidenced in the first chapter, and can be interpreted as an effect of the aggregation, valid for all the cases considered even if in the series with intermediate memory this is much more evident. We believe that in these processes the aggregation process push the model to the critic region for the optimization process, therefore small variations can be sufficients to obtain different otptima from similar non-aggregated series.

- As we can expect the aggregation process highly affect the constant in the variance that highly increase, while the constant in the mean is not affected. This last effect is due to the fact that it was fixed to zero, with a different value the aggregation will affect it.

- Finally observe the parameters of the GARCH and IGARCH: in these case with the high memory processes the two models appear to be different, the sum of the GARCH parameter, at least in average, is different from one, while in the models with d=0.8, GARCH and IGARCH are very close, as if the aggregation push the model to a new process with d=1.

- Take a look now at the identification: the memory property of the simulated series is identified by the information criteria with an error percentage of 20%, near the value recorded for non aggregated series. Again we can note that the identification is affect by the structure of the process and by parameter values. Moreover none of the criteria appear to prevail on the others.

We will now turn to our main point, the evaluation of 1-day-Value-at-Risk both with aggregated and non aggregated data. Given the structure of the test for Value-at-Risk comparison and the time requested to run a Montecarlo experiment on simulated high frequency data we decided to split this analysis in two parts: in the first we compare the VaR computed on aggregated data with the correct DGP, a GARCH(1,1) an IGARCH(1,1), the EWMA with smoothing parameter set to 0.97 and finally with the VaR computed on hourly data with the true DGP; in a second group of simulations we compare the VaR performances with the following models, again on aggregated data the true DGP and the EWMA(0.97) while on high frequency data with the true DGP and a GARCH(1,1). In all cases we estimate the different models and we compare the 1-day ahead VaR. However a point arise: on daily data the computation of 1-day-ahead prediction intervals is a standard procedure, as in the previous Montecarlo, while on hourly data we could in principle use two different approaches. A normal practice in this field, employed to obtain a T-step-ahead forecast of the volatility (T=8 in our case), is to multiply the 1-step-ahead forecast by $\sqrt{T}$, a solution based on the independence and identically distribution hypothesis of the residuals. However in the GARCH-type model framework this can be differently interpreted, the T-step-ahead forecast maybe computed as the sum of 1 to T step forecasts. The T step return could be expressed as the sum of single step returns and, postulating independence in the mean, its expected value will be the sum of expected values, therefore with a pure GARCH generator, without any dynamics in the mean, this will be zero:

$$\begin{aligned} r_T &= \sum_{j=1}^{T} r_{t+j} \\ E_t\left[r_T\right] &= \sum_{j=1}^{T} E_t\left[r_T\right] = 0 \end{aligned} \tag{18}$$

The variance computed conditionally at time t, will be therefore

$$Var_t\left[r_T\right] = Var_t\left[\sum_{j=1}^{T} r_{t+j}\right] \tag{19}$$

the law of iterated expectations allow us to set covariances between time-dependent returns to zero obtaining

$$Var_t\left[r_T\right] = \sum_{j=1}^{T} Var_t\left[r_T\right] \tag{20}$$

that is the sum of the predictions from 1 to T steps ahead variance made in time T. This will be the second computation technique used to forecast daily variance with hourly data. In the following we will refer to the forecast obtained with the first methods as "square root forecasts", while the second will be labelled "sum forecasts". On the VaR measures otained we will compute all the tests and the loss functions as in the previuos Montecarlo.

These two sets of Montecarlo experiments are run on the same generators used for aggregated data model identification analysis. The Value-at-Risk analysis is performed again on a backtesting approach using 250 daily observations to assess number of exceptions, compute tests and loss functions. The tables of these Montecarlo expreriments can be found in Annex 3. As in the previous analysis we summarize the tables with the following observations:

Average exceptions and MRA. Consider first the comparison among the aggregated FIGARCH, the RiskMetrics and the high frequency FIGARCH and GARCH. In these cases aggregated models give the smaller percentage of exceptions for the 1-day VaR, while, among the high frequency models, the FIGARCH with square root forecasts produce the better results. This behaviour indicate that even if the true generator is an high frequency process with long memory, in computing 1-day VaR better results are obtained by aggregated data. This result is confimed in the second Montecarlo where we compare different aggregated specifications with the true high frequency generator. We restrict now our attention on the aggregated models, among these specifications two cleraly dominates the other, the long memory GARCH and the RiskMetrics, with a prevalence of the latter at the 1% VaR while the FIGARCH is preferred at the 5% VaR level. A final comments on the MRA: here the models diferenlty satisfy the requirements, leading to different zones, in most cases the green zone is reached by the long memory GARCH on aggregated data and by the RiskMetrics, while the other sepcifications switch beteen the green and the yellow zone. Again this indicate that aggregated data are preferred to high frequency specifications.

Tests of Conditional and Unconditional Coverage. Test results again cannot help in the choice of the best specification, however we must note that variation in tests results among different models is wider than in the previous analysis allowing to exlude, in some cases, one of the models employed. As an example consider the FIGARCH(0.5,0.8,0.3) in table 90, the CC test allow to exclude at least the high frequency FIGARCH specifications, or again consider the FIGARCH(0.3,0.4,0) in table 125, the Independence test at 5% allow to exclude all daily models. Unfortunately in all these cases we cannot reduce our choices to one model, leading to a small power of these test in discerning among the alternative specifications.

Loss functions. Now the situation change: while considering only the eceptions aggregated data are always preferred, turning to a loss funcion approach high frequency data are in some cases the best choice. Consider the Lopez loss function: the preferred models are the RiskMetrics and the high frequency FIGARCH with square root forecasts, and the choice switch between this two models, a good example is in table 91 or 119. Focusing on the loss functions previuosly suggested, computed on the whole backtesting period and not only on the exceptions, results are different, here the model choice switch between the RiskMetrics and the GARCH with square root forecasts in the first Montecarlo while in the second the preferred models are again the RiskMetrics this time with the FIGARCH with sum forecasts. These observations, coupled with the ones on the numer of exceptions, allow to identify in the RiskMetrics model a ggod choice, it satisfy regulator requirements and is one of the best choices if we focus on loss functions.

Model comparison test. If we consider the first Montecarlo, which include high frequency GARCH and FIGARCH specifications, this test allow the derivation of a preference ordering among the different models. This test compute a pairwise comparison among the models and report a frequency of preference of the first or of the second model. If we state that, given the test comparison of two models, one is preferred to the other when the frequency of preference is above 50%, we have a set of reference relations that may allow to construct an ordering. In the first Montecarlo this is possible with the full set of generators and all the orderings have a common point: the high frequency GARCH specification with square root forecast is always the preferred. The ordering of the remaining models change across the generators. This result allow to conjecture that in computing 1-day Value-at-Risk with high frequency data, even if in presence of long memory, a short memory model give a finer matching to the efficient moment condition. A similar result was obtained by Beltratti and Morana (1999) in an applied framework. Their conclusion was mainly driven by the closeness of forecasts obtained by the FIGARCH and GARCH specifications, while in this case we obtain this conclusion via a Montecarlo approach. Turn now the attention to the second Montecarlo, that report the comparison across daily specifications and the high frequency true generator. In this case the preference raltion among the specifications do not exist, in most of the cases the relation is not transitive, however the high frequency FIGARCH specification with sum forecasts is the candidate to be the preferred solution. The preference ordering are reported in the appendix whenerer thy exists. A couple of additional remarks is needed: first of all we strees on the fact that high frequency specifications are most of the times preferred to the daily ones, showing that, even if with a misspecified model, high frequency data matter; moreover the RiskMetrics model is most of the time the worst solution in the model comparison tests, this is due, to our advise, to the structure of the model, in the sense that any GARCH specification, even an highly misspecified one, long or short memory, has a greater flexibility that allow to adequately match the (simulated) data; finally, note that this result is not influenced by the true data generating process.

Model specification test. The results obtained by this last instruments are similar across the Montecarlo experiments and the different models, showing that the Value-at-Risks is not correctly specified. We conjecture that this is due to the limited number of points used in our analysis, 250 observations, that might influence test power.

6 Conclusions

In this chapter we derived the equations for the mean squared error when the underlying noise has a long memory GARCH structure. We then used these formulae to asses the effects of misspecification in the Value-at-Risk framework. Our analysis was conducted comparing different VaR measures with a group of tests and loss functions. The results shows that the tests of Kupiec (1995) and Christoffersen (1998), together with the loss functions approach of Lopez (1999) have limited power in distinguishing among a group of alternative VaR models, a similar result was obtained by Lopez (1999) with a restricted set of loss functions and tests. In this framework we extended the current literature including in the Montecarlo analysis two recent tests of Christoffersen et al. (2001), tests that are based on a moment condition. By this way a pairwise comparison among different models is possible and our Montecarlo showed that: the RiskMetrics model is never preferred to any other GARCH specification; the true generator is the best solution. In the second part of this chapter we focused on a slightly different problem, trying to asses the effects of aggregation on long memory and Value-at-Risk computation. We showed that the memory properties are influenced by the aggregation process if the variance is highly persistent (memory parameter around 0.4) while the effects are lower when we have intermediate memory (parameter around 0.8). We compared then in two different Montecarlo the VaR measures computed by aggregated and non-aggregated data, showing that for 1-day VaR computation high frequency data allow a finer matching with the moment condition used by Christofferesen et. al. (2001) in their tests. However different results are obtained by the loss function approach where aggregated estimates of the VaR are preferred. If we consider the problem of VaR model selection on the basis of the regulators requirements the best choice seem the RiskMetrics approach if we use daily data obtained from the aggregation of hourly FIGARCH data. We must evidence that this result strictly depend on the setup we considered and is not valid in general.

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7 Appendix

7.1 Forecasting with FIGARCH

Turn now to the analysis of forecasting the mean of a series when the error component is heteroskedastic, and in particular we will describe the behaviuor of prediction in presence of a FIGARCH structure. This chapter represent the extension to FIGARCH case of the study of Baillie and Bollerslev (1992). Given the assumptions introduced on chapter 1, that the mean process $\mu_t = 0$, we can derive the following relations:

$$E_t\left[y_{t+s}\right] = E_t\left[\varepsilon_{t+s}\right] = 0 \qquad s \geq 1$$

$$e_{t,s} = y_{t+s} - E_t\left[y_{t+s}\right] = \varepsilon_{t+s}$$

Conditional on the information set up to time t, the s-step-ahead predictor for the mean is zero, indipendently from s, and the prediction error, again for s-step ahead, is equal to the innovation in time t+s. We are interested now in computing the Prediction Mean Square Error (PMSE), whose expression is simply

$$E_t\left[e_{t,s}^2\right] \tag{21}$$

again conditional on time t information set. Assume also that the conditional variance follow a long memory GARCH model

$$\varepsilon_t | I^{t-1} \sim id\left(0, \sigma_t^2\right)$$

$$\sigma_t^2 \sim FIGARCH(p, d, m)$$

with the parameters satisfying all usual restrictions that ensure the conditional variance to be positive. We are now interested in computing the PMSE for the mean forecast given the FIGARCH structure on residuals. For the PMSE for a general model for the mean ($\mu_t \neq 0$) see the cited paper of Baillie-Bollerslev (1992). We make use of the following relations, derived by the application of the law of iterated expectations:

$$\begin{cases} E_t\left[\varepsilon_{t+j}^2\right] = E_t\left[\sigma_{t+j}^2\right] & j > 0 \\ E_t\left[\varepsilon_{t+j}^2\right] = \varepsilon_{t+j}^2 & j \leq 0 \\ E_t\left[\sigma_{t+j}^2\right] = \sigma_{t+j}^2 & j \leq 0 \end{cases} \tag{22}$$

Now we can express the PMSE as

$$E_t\left[e_{t,s}^2\right] = E_t\left[\varepsilon_{t+s}^2\right] = E_t\left[\sigma_{t+s}^2\right]$$

We are then interested in computing the s-step-ahead predictor of the conditional variance, conditional on time t information set. Consider the standard FIGARCH(m,d,q) formulation for the conditional variance process

$$\left[1 - \beta\left(L\right)\right]\sigma_t^2 = \tilde{\omega} + \left[1 - \beta\left(L\right) - \left(1 - L\right)^d \phi\left(L\right)\right]\varepsilon_t^2$$

and using $\omega = \tilde{\omega}/[1-\beta(1)]$ and $\left[1-\beta(L)-(1-L)^d\phi(L)\right][1-\beta(1)]^{-1} = \lambda(L) = \sum_{i=1}^{\infty}\lambda_i L^i$ we can write

$$\sigma_t^2 = \omega + \sum_{i=1}^{\infty}\lambda_i\varepsilon_{t-i}^2$$

Our objective is the computation of the following quantity (using 22):

$$E_t\left[\sigma_{t+s}^2\right] = \omega + \sum_{i=1}^{\infty}\lambda_i\varepsilon_{t-i}^2 = \omega + \sum_{i=1}^{s-1}\lambda_i E_t\left[\sigma_{t-i}^2\right] + \sum_{i=s}^{\infty}\lambda_i\varepsilon_{t-i}^2 \qquad (23)$$

The best s-step ahead predictor for the conditional variance depend on all past history of the error term, and on the forecast made for 1,2..to s-1 step ahead (all made conditional to the information set in time t). This directly give an important information: the computation of the forecast s-step ahead with real data will obviously require a truncation in formula 23, given the limited dimension of sample for time series. This will introduce an error in the estimated MSE and, given the structure of equation 23, we will underestimate the forecast of the conditional variance (all terms in 23 are positives). We will give now a more compact formulation of 23 expressing it only in term of the infinite past history of the error term, and we will add another representation that will be used later.

Define the following quantity:

$$A_j = \sum_{i=j}^{\infty}\lambda_i\varepsilon_{t+j-i}^2 \qquad (24)$$

Substituting recursively $E_t\left[\sigma_{t-i}^2\right]$ in 23 and using 24 we obtain

$$E_t\left[\sigma_{t+s}^2\right] = \omega\theta_s + \sum_{i=1}^{s}\phi_i A_{s+1-i} \qquad (25)$$

$$\phi_i = \sum_{j=1}^{i-1}\lambda_j\phi_{i-j} \qquad \theta_s = \sum_{i=1}^{s}\phi_i$$

exploiting the relation implicit in 24 we can finally rewrite the predictor for the conditional variance as

$$E_t\left[\sigma_{t+s}^2\right] = \theta_s\omega + \sum_{i=0}^{\infty}\psi_{i+1}\varepsilon_{t-i}^2 \qquad (26)$$

$$\psi_k = \sum_{i=1}^{s}\phi_i\lambda_{k+s-i} \qquad \phi_1 = 1$$

Via 23 come out an interesting observation: given that the coefficients in 24 are constrained to be positive to ensure the conditional variances to be strictly positive, we see that the predictor for the conditional variance diverge increasing the forecasting horizon. Figarch processes share this characteristic with IGARCH, the predictor diverge to infinity, even if the process is ergodic and stationary and the impact of shocks (or news) decay to zero at an hyperbolic rate, laying in between GARCH and IGARCH which present respectively exponential decaying and constant effect. This behaviour make Figarch usage for long range forecasting very difficult, but a correct approach must take into consideration also the impact of short memory parameters. At the moment we know that the predictor diverge, but will diverge so quickly in the IGARCH case or slowly? This can be assessed analysing the behaviour of the MA coefficients and of the coefficients of formula 26.

(insert analysis on coefficients)

Using a one-step-ahead strategy, predicting for t+s with information set up to t+s-1, Figarch processes should give better results, specially when the DGP is correctly identified and the parameters consistently and correctly estimate. We will deal with these problems in a next section.

We define also another alternative formulation, not compact as the previous, but that will be useful in the following. This representation has a recursive structure and avoid the computation of the θ, ψ and ϕ coefficients:

Note at first that: $E_t\left[\sigma^2_{t+1}\right] = \sigma^2_{t+1}$(known in t)

Consider now the following equality and definitions:

$$\sigma^2_{t+1} = \tilde{A}_1 = B_1$$

$$\tilde{A}_j = \omega + \sum_{i=j}^{\infty} \lambda_i \varepsilon^2_{t+j-i} \tag{27}$$

$$E_t\left[\sigma^2_{t+j}\right] = B_j \tag{28}$$

we can now write

$$E_t\left[\sigma^2_{t+s}\right] = \tilde{A}_s + \sum_{i=1}^{s-1} \lambda_i B_{s-i} \tag{29}$$

With s=2, this will depend on B_1, known, and on the past: we get B_2. With these two we can compute recursively B_3, and then all easily follows.

Using indifferently 25, 26 or 29 we are now able to compute the MSE of the mean-predictor, in the Baillie-Bollerslev framework, for the case in which the error term has a conditional long memory structure.

Another important issue in forecasting with long memory behaviour is connected directly with volatility. As example in the Value-at-Risk framework is of direct interest the forecast of the conditional volatility, and then will come into role also the computation of the MSE of this quantity. The MSE of the volatility predictor will be also useful in computing density prediction instead of point prediction as we will se later on.

The best predictor for the conditional volatility was previously computed, In this section we focus on the computation of the MSE for the conditional volatility predictor. Define the forecast error for s-step ahead prediction of the conditional variance as :

$$e^{v}_{t,s} = \sigma^2_{t+s} - E_t\left[\sigma^2_{t+s}\right] \tag{30}$$

Note that in this case $e^{v}_{t,1} = 0$ given that we are not dealing with estimated models or correct specification. This will be true only in theory, applying this methodologies we will have to take into account also some additional stochastic term involved in the estimated parameters distributions.

Rearranging using 29 and noting that

$$\sigma^2_{t+j} = \tilde{A}_j + \sum_{i=1}^{j-1} \lambda_i \varepsilon^2_{t+j-i} \tag{31}$$

we can write

$$e^{v}_{t,s} = \sigma^2_{t+s} - \tilde{A}_s - \sum_{i=1}^{s-1} \lambda_i B_{s-i} = \sum_{i=1}^{s-1} \lambda_i \varepsilon^2_{t+s-i} - \sum_{i=1}^{s-1} \lambda_i B_{s-i} \tag{32}$$

substituting then recursively B$_{s-i}$ with its expression and using 27 and 28, then substituting the conditional variance with 30 and 31 and rearranging we obtain this nice expression:

$$e^{v}_{t,s} = \sum_{i=1}^{s-1} \lambda_i \left(\upsilon_{t+s-i} - e^{v}_{t,s-i}\right) \tag{33}$$

where$\upsilon_t = \varepsilon^2_t - \sigma^2_t$.

Working again with iterated substitutions for j=2,...s-1 and reorganizing formulae we find this final representation :

$$e^{v}_{t,s} = \sum_{i=1}^{s-1} \phi_{s-i+1} \upsilon_{t+i} \tag{34}$$

where the coefficients are the same of formula 25. This was only the formula for the prediction error, to evaluate the PMSE we have to derive an expression for $E_t\left[\left(e^{v}_{t,s}\right)^2\right]$. In doing that we will make use of the following relations:

$$E_t\left[\upsilon_{t+j}\upsilon_{t+i}\right] = 0 \qquad 1 \leq j < i < s \tag{35}$$

$$E_t\left[\upsilon^2_{t+j}\right] = \left(\kappa_2 - 1\right) E_t\left[\sigma^4_{t+j}\right] \tag{36}$$

where in the last equation κ_2 is the second order cumulant for the conditional distribution of the error term. Using 34, 35 and 36 we can write

$$E_t\left[\left(e^v_{t,s}\right)^2\right] = E_t\left[\left(\sum_{i=1}^{s-1}\phi_{s-i+1}\upsilon_{t+i}\right)^2\right] = E_t\left[\sum_{i=1}^{s-1}\phi^2_{s-i+1}\upsilon^2_{t+i}\right] = \sum_{i=1}^{s-1}\phi^2_{s-i+1}E_t\left[\upsilon^2_{t+i}\right] \tag{37}$$

$$= (\kappa_2 - 1)\sum_{i=1}^{s-1}\phi^2_{s-i+1}E_t\left[\sigma^4_{t+j}\right]$$

To evaluate the MSE of the variance forecast we need to know the 4th order conditional moment of the distribution of ε_t, we state the following

Lemma 8 *The 4th order conditional moment of ε_t when σ^2_t follow a FIGARCH(p,d,m) process is equal to:*

$$E_t\left[\sigma^4_{t+j}\right] = \omega^2 + \kappa_2\sum_{i=1}^{j-1}\lambda_i^2 E_t\left[\sigma^4_{t+j-i}\right] + 2\omega\sum_{i=1}^{j-1}\lambda_i E_t\left[\sigma^2_{t+j-i}\right] +$$

$$+\sum_{i=j}^{\infty}\lambda_i^2\varepsilon^4_{t+j-i} + 2\omega\sum_{i=j}^{\infty}\lambda_i\varepsilon^2_{t+j-i} + 2\sum_{i=j}^{\infty}\sum_{h>i}^{\infty}\lambda_i\lambda_h\varepsilon^2_{t+j-i}\varepsilon^2_{t+j-h} +$$

$$+2\sum_{i=1}^{j-1}\sum_{h=j}^{\infty}\lambda_i\lambda_h\varepsilon^2_{t+j-h}E_t\left[\sigma^2_{t+j-i}\right] + 2\sum_{i=1}^{j-1}\sum_{h>1}^{j-1}\lambda_i\lambda_h E_t\left[\varepsilon^2_{t+j-i}\varepsilon^2_{t+j-h}\right]$$

where

$$E_t\left[\varepsilon^2_{t+j-i}\varepsilon^2_{t+j-h}\right] = \omega E_t\left[\sigma^2_{t+j-h}\right] + \sum_{l=1}^{j-i-1}\lambda_l E_t\left[\varepsilon^2_{t+j-i-l}\varepsilon^2_{t+j-h}\right] + E_t\left[\sigma^2_{t+j-i}\right]\sum_{l=j-i}^{\infty}\lambda_l\varepsilon^2_{j+j-i-l} \tag{38}$$

Proof. Just square the process out and with some tedious algebra

$$E_t\left[\sigma^4_{t+j}\right] = E_t\left[\left(\omega + \sum_{i=1}^{\infty}\lambda_i\varepsilon^2_{t+j-i}\right)^2\right] =$$

$$= E_t\left[\omega^2 + \sum_{i=1}^{\infty}\lambda_i^2\varepsilon^4_{t+j-i} + 2\omega\sum_{i=1}^{\infty}\lambda_i\varepsilon^2_{t+j-i} + 2\sum_{i=1}^{\infty}\sum_{h>1}^{\infty}\lambda_i\lambda_h\varepsilon^2_{t+j-i}\varepsilon^2_{t+j-h}\right] =$$

$$= \omega^2 + \sum_{i=1}^{\infty}\lambda_i^2 E_t\left[\varepsilon^4_{t+j-i}\right] + 2\omega\sum_{i=1}^{\infty}\lambda_i E_t\left[\varepsilon^2_{t+j-i}\right] + 2\sum_{i=1}^{\infty}\sum_{h>1}^{\infty}\lambda_i\lambda_h E_t\left[\varepsilon^2_{t+j-i}\varepsilon^2_{t+j-h}\right]$$

by law of iterated expectations we can extend Baillie and Bollerslev (1992) theorem 1, p 102, to

$$E_t\left[\varepsilon_{t+j}^4\right]=\kappa_2 E_t\left[\sigma_{t+j}^4\right] \qquad \text{for } j\leq 0$$

therefore

$$\begin{aligned}E_t\left[\sigma_{t+j}^4\right]=\omega^2+\kappa_2\sum_{i=1}^{j-1}\lambda_i^2 E_t\left[\sigma_{t+j-i}^4\right]+\sum_{i=j}^{\infty}\lambda_i^2 E_t\left[\varepsilon_{t+j-i}^4\right]+\\+2\omega\sum_{i=1}^{j-1}\lambda_i E_t\left[\sigma_{t+j-i}^2\right]+2\omega\sum_{i=j}^{\infty}\lambda_i\varepsilon_{t+j-i}^2+\\+2\sum_{i=1}^{\infty}\sum_{h>1}^{\infty}\lambda_i\lambda_h E_t\left[\varepsilon_{t+j-i}^2\varepsilon_{t+j-h}^2\right]\end{aligned}$$

last summation can be rewritten as

$$\begin{aligned}\sum_{i=1}^{\infty}\sum_{h>1}^{\infty}\lambda_i\lambda_h E_t\left[\varepsilon_{t+j-i}^2\varepsilon_{t+j-h}^2\right]=\sum_{i=j}^{\infty}\sum_{h>i}^{\infty}\lambda_i\lambda_h\varepsilon_{t+j-i}^2\varepsilon_{t+j-h}^2+\sum_{i=1}^{j-1}\sum_{h=j}^{\infty}\lambda_i\lambda_h\varepsilon_{t+j-h}^2 E_t\left[\sigma_{t+j-i}^2\right]+\\+\sum_{i=1}^{j-1}\sum_{h>1}^{j-1}\lambda_i\lambda_h E_t\left[\varepsilon_{t+j-i}^2\varepsilon_{t+j-h}^2\right]\end{aligned}$$

$$\sum_{i=1}^{j-1}\sum_{h>1}^{j-1}\lambda_i\lambda_h E_t\left[\varepsilon_{t+j-i}^2\varepsilon_{t+j-h}^2\right]=\sum_{i=1}^{j-1}\sum_{h>1}^{j-1}\lambda_i\lambda_h\left(\begin{array}{c}\omega E_t\left[\sigma_{t+j-h}^2\right]+\sum_{l=1}^{j-i-1}\lambda_l E_t\left[\varepsilon_{t+j-i-l}^2\varepsilon_{t+j-h}^2\right]+\\E_t\left[\sigma_{t+j-i}^2\right]\sum_{l=j-i}^{\infty}\lambda_l\varepsilon_{j+j-i-l}^2\end{array}\right)$$

■

In the first expansion the first term has all known elements in time t, the second 1 element is known and then is straightforward compute the expectation, substituting with the predictor of the conditional variance and for the third we have to evaluate an s(s-1)/2 matrix of unknown elements, whose final expansion is given in the second formula. Combining these two terms we obtain the expression for the 4th order conditional moment. Since expressions for higher order conditional moments are not needed for the purpose of this work (up to this moment) their expression is not computed.

Given formulae for the FIGARCH formulation of Baillie-Bollerslev-Mikkelsen, is easy to derive the correspondent expressions for the reparametrisation proposed by Chung. His model can be written as:

$$\left[1-\beta\left(L\right)\right]\sigma_t^2=\left[1-\beta\left(L\right)\right]\varepsilon_t^2-\left[\left(1-L\right)^d\phi\left(L\right)\right]\left(\varepsilon_t^2-\sigma^2\right) \tag{39}$$

rearranging and noting that the infinite summation of the long memory operator coefficients is identically equal to zero, we get that the model can be simply written also as:

$$\sigma_t^2=\sum_{i=1}^{\infty}\lambda_i\varepsilon_{t-i}^2 \tag{40}$$

$$\lambda\left(L\right)=1-\left(1-L\right)^d\phi\left(L\right)\left[1-\beta\left(L\right)\right]^{-1}$$

From this formulation we can derive formulae for predictor and MSE from the previous case just substituting a constant equal to zero. In this case we will loose the relation between constant and the other parameters; given that we will use the following formulation, equivalent to 40:

$$\sigma_t^2 = \sigma^2 + \sum_{i=1}^{\infty} \lambda_i \left(\varepsilon_{t-i}^2 - \sigma^2 \right) \tag{41}$$

From 40 we can see directly that the main changes are due to the cross products between observations of ε_t^2 in **??**, since

$$E_t \left[\varepsilon_{t-i}^2 - \sigma^2 \right] = \begin{cases} E_t \left[\sigma_{t-i}^2 \right] - \sigma^2 & i \geq 1 \\ \varepsilon_{t-i}^2 - \sigma^2 & i \leq 0 \end{cases}$$

we give the correspondent expression of the predictor. Define this quantity:

$$\hat{A}_j = \sum_{i=j}^{\infty} \lambda_i \left[\varepsilon_{t+j-i}^2 - \sigma^2 \right] \tag{42}$$

and using it and substituting recursively we get

$$\begin{aligned} E_t \left[\sigma_{t+s}^2 \right] &= \sigma^2 + \sum_{i=1}^{\infty} \lambda_i E_t \left[\varepsilon_{t+s-i}^2 - \sigma^2 \right] = \\ &= \sigma^2 + \sum_{i=1}^{s-1} \lambda_i E_t \left[\varepsilon_{t+s-i}^2 - \sigma^2 \right] + \sum_{i=s}^{\infty} \lambda_i \left[\varepsilon_{t+s-i}^2 - \sigma^2 \right] \end{aligned}$$

$$E_t \left[\sigma_{t+s}^2 - \sigma^2 \right] = \sum_{i=1}^{s-1} \lambda_i E_t \left[\varepsilon_{t+s-i}^2 - \sigma^2 \right] + \sum_{i=s}^{\infty} \lambda_i \left[\varepsilon_{t+s-i}^2 - \sigma^2 \right] = \sum_{i=1}^{s-1} \phi_i \hat{A}_i \tag{43}$$

with the same coefficients of 25. Then the constant can be easily moved on the right side of the equation. By a similar argument is straightforward obtaining, the recursive formulation of (12b), the correspondent of (16), that is used for the computation of the MSE of the conditional volatility predictor:

$$E_t \left[\sigma_{t+s}^2 - \sigma^2 \right] = B_s = \hat{A}_s + \sum_{j=1}^{s-1} \lambda_j B_{s-j} \tag{44}$$

again we make use of 42 and we define everything in deviation from the constant term. Making use of expected values in deviation from the constant and noting that:

$$\sigma_{t+j}^2 - \sigma^2 = \hat{A}_j + \sum_{i=1}^{j-1} \lambda_i \varepsilon_{t+j-i} \tag{45}$$

is possible to verify that the expressions 37 and 34 are valid also for the alternative parametrisation of FIGARCH models. By the way there are changes in the 4th order conditional moment. Reconsidering the proof of the revious lemma:

Proof.

$$E_t\left[\sigma^4_{t+j}\right] = E_t\left[\left(\sigma^2 + \sum_{i=1}^{\infty}\lambda_i\left(\varepsilon^2_{t+j-i} - \sigma^2\right)\right)^2\right] =$$

$$= E_t\left[\begin{array}{c}\sigma^4 + \sum_{i=1}^{\infty}\lambda_i^2\left(\varepsilon^2_{t+j-i} - \sigma^2\right)^2 + 2\sigma^2\sum_{i=1}^{\infty}\lambda_i\left(\varepsilon^2_{t+j-i} - \sigma^2\right) + \\ +2\sum_{i=1}^{\infty}\sum_{h>1}^{\infty}\lambda_i\lambda_h\left(\varepsilon^2_{t+j-i} - \sigma^2\right)\left(\varepsilon^2_{t+j-h} - \sigma^2\right)\end{array}\right]$$

■

we can observe that the terms increase, given the innovation deviation from the mean. The formula can be simplified noting that

$$\sigma^2 + \sum_{i=1}^{\infty}\lambda_i\left(-\sigma^2\right) = 0$$

burning down to

$$E_t\left[\left(\sum_{i=1}^{\infty}\lambda_i\varepsilon^2_{t+j-i}\right)^2\right]$$

and from this last expression we can compute the 4th order conditional moment of the error component under the Chung parametrisation, using the previously derived equation for the FIGARCH I. In last formula the constant term does not appear directly, but it influence the moment through its effect in the innovation.

The previous section we were dealing with point prediction of the mean process and of the computation of its MSE. The same approach can be used also to compute the predictor and the MSE for the conditional variance. In the following we will focus on density forecasting, we will extend the approach of Baillie and Bollerslev to the FIGARCH case, giving an expression for the Cornish-Fischer expansion under a FIGARCH DGP.

In order to compute prediction interval for a FIGARCH model, we have to compute

$$e_{t,s} = y_{t+s} - E_t\left[y_{t+s}\right] = y_{t+s}$$

and the conditional mean square error

$$E_t\left[e^2_{t,s}\right] = E_t\left[y^2_{t+s}\right] = E_t\left[\sigma^2_{t+s}\right]$$

However in presence of ARCH-type effect, the unconditional distribution of the observations (or residuals for the mean model) have fatter tails than the conditional one. Moreover the prediction error distribution depend on the information set available at time t. In these cases the usual computation of prediction intervals, based on (assuming that the model is a pure FIGARCH):

$$\left\{-\Phi^{-1}(p)\,E_t\left[\sigma_{t+s}^2\right]^{1/2},\Phi^{-1}(p)\,E_t\left[\sigma_{t+s}^2\right]^{1/2}\right\}$$

where $\Phi^{-1}(p)$ is the p-quantile of the standardized normal, is no more valid. Following Baillie and Bolloerslev we will use in this case the Cornish-Fisher expansion for a correction up to the fourth moment. This expansion allow to compute the p-quantile for the conditional distribution for the s-step ahead prediction error. The Cornish-Fischer approximation for the s-step-ahead time varying p-quantile is defined as

$$\begin{aligned} z_{t,s}(p) &= \rho_{t,s}(p)\,E_t\left[e_{t,s}^2\right]^{1/2} \\ \rho_{t,s}(p) &= \Phi^{-1}(p)+\rho_2\left(\Phi^{-1}(p)\right)\gamma_{2,t,s} \\ \rho_2(z) &= \left(z^3-3z\right)/24 \end{aligned}$$

for a correction up to the fourth order moment. In this expression $\gamma_{2,t,s}$ represent the conditional excess curtosis for the s-step-ahead prediction error. Letting $\rho_2(z)=0$ we get back to the usual interval definition. Under the FIGARCH model we just have to define and derive an expression for the excess kurtosis. Consider that

$$\gamma_{2,t,s}=\frac{E_t\left[\varepsilon_{t+s}^4\right]-3\left(E_t\left[\varepsilon_{t+s}^2\right]\right)^2}{\left(E_t\left[\varepsilon_{t+s}^2\right]\right)^2}$$

and using previous results we have only to compute $\left(E_t\left[\varepsilon_{t+s}^2\right]\right)^2$ recalling that $E_t\left[\varepsilon_{t+s}^4\right]=\kappa_2 E_t\left[\sigma_{t+s}^4\right]$.

7.2 Implementing the VaR comparison tests

In this Montecarlo we used an adapted version of the Christoffersen at. el. (2001) GMM based test. The formulae for the asymptotic variance on which it is evaluated is computed as follows:

$$Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)-\exp\left(\hat{\gamma}_2' f\left(\varepsilon_t,\beta_2^*\right)\right)\right)\right)=Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right)+$$

$$Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\exp\left(\hat{\gamma}_2' f\left(\varepsilon_t,\beta_2^*\right)\right)\right)-2Cov\left[\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right)\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\exp\left(\hat{\gamma}_2' f\left(\varepsilon_t,\beta_2^*\right)\right)\right)\right]$$

under the hypothesis that the two VaR measures are independent the covariance is null given that they equally match the efficient moment condition

$$Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right) = \frac{1}{T}\sum_{t=1}^{T} Var\left[\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right] + \frac{2}{T}\sum_{t=1}^{T}\sum_{j=t+1}^{T} Cov\left[\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\exp\left(\hat{\gamma}_1' f\left(\varepsilon_j,\beta_1^*\right)\right)\right]$$

under the hypothesis that the moment condition is satisfaid in a non-time-dependent fashion, otherwise we will find a property of a Markov process, for which we tested previously, again the covariance is null

$$Var\left[\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right] = E\left[\left(\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right) - E\left[\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right]\right)^2\right]$$

recalling the moment generating function of a multinormal variable

$$E\left[\exp\hat{\gamma}_1' f\right] = E_\gamma\left[\exp\hat{\gamma}_1' f\right] = \exp(\gamma_1^{*\prime} f + 0.5 f^{'}\Omega f/T)$$

$$\sqrt{T}\left(\hat{\gamma}-\gamma^*\right) \to N\left(0,\Omega\right)$$

then

$$\begin{aligned} Var\left[\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right)\right] &= E\left[\left(\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right) - \exp(\gamma_1^{*\prime} f + 0.5 f^{'}\Omega f/T)\right)^2\right] = \\ &= E\left[\begin{array}{c} \exp\left(2\hat{\gamma}_1' f\right) + \exp(2\gamma_1^{*\prime} f + f^{'}\Omega f/T) \\ -2\exp\left(\hat{\gamma}_1' f + \hat{\gamma}_1' f + 0.5 f^{'}\Omega f/T\right) \end{array}\right] \\ &= \exp(2\gamma_1^{*\prime} f + 2 f^{'}\Omega f/T) + \exp(2\gamma_1^{*\prime} f + f^{'}\Omega f/T) - \\ &\quad -2\exp(\gamma_1^{*\prime} f + 0.5 f^{'}\Omega f/T)\exp(\gamma_1^{*\prime} f + 0.5 f^{'}\Omega f/T) \\ &= \exp(2\gamma_1^{*\prime} f + 2 f^{'}\Omega f/T) - \exp(2\gamma_1^{*\prime} f + f^{'}\Omega f/T) = V_{1,t} \end{aligned}$$

therefore

$$Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\exp\left(\hat{\gamma}_1' f\left(\varepsilon_t,\beta_1^*\right)\right) - \exp\left(\hat{\gamma}_2' f\left(\varepsilon_t,\beta_2^*\right)\right)\right)\right) = \frac{1}{T}\sum_{t=1}^{T}\left(V_{1,t}+V_{2,t}\right)$$

Here we consider three different FIGARCH DGP: in two cases a FIGARCH(1,d,0) and a FIGARCH(1,d,1). For all the models considered we estimate the true

DGP, therefore assuming a correct identification of the model and of its orders, and two "short" memory formulations, a IGARCH(1,1) and a GARCH(1,1). The evaluation of VaR measures is carried out both with backtesting. The lenght of simulated series is of 2250 observations, the first 2000 points are used for model estimation, the other 250 for out of sample forecasting of volatility, only 1-step-ahead. In all cases we use also 500 observations, from 1501 to 2000, to check VaR performances with backtesting.

7.3 Proof Christoffersen Inoue and Hahn theorem

This is a partially revised proof of this theorem. The author prooved at a first stage the stochastic equicontinuity of $M_{m1}(\beta_1^*, \gamma_1^*)$ and then used that to derive a relation between this quantity and its correspondant with estimated parameters. These derivation are a bit unclear and partially unnecessary. A direct application of the ergodic theorem allow us to write for model 1

$$\left| M_{m1}(\beta_1^*, \gamma_1^*) - M_{m1,T}\left(\hat{\beta}_1, \hat{\gamma}_1\right) \right| = o_p(1)$$

and similarly for model 2

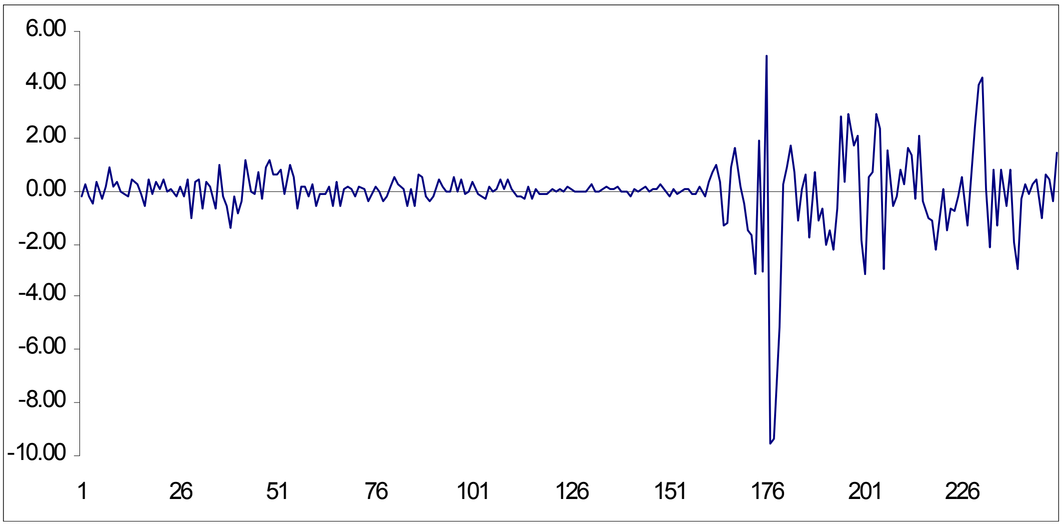
$$\left| M_{m2}(\beta_2^*, \gamma_2^*) - M_{m2,T}\left(\hat{\beta}_2, \hat{\gamma}_2\right) \right| = o_p(1)$$

using this equations we ca rewrite the test as

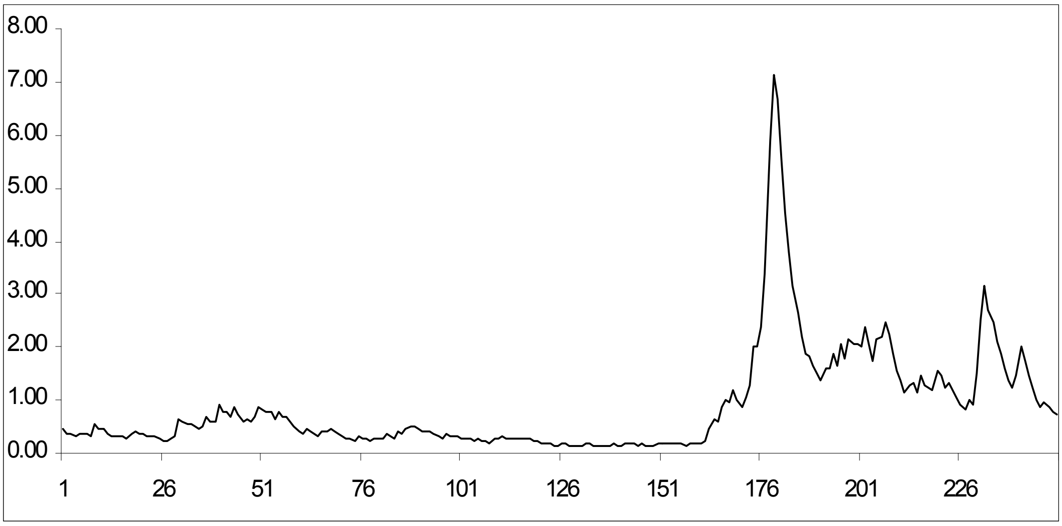
$$\sqrt{T}\left[M_{m1,T}\left(\hat{\beta}_1, \hat{\gamma}_1\right) - M_{m2,T}\left(\hat{\beta}_2, \hat{\gamma}_2\right)\right] = \sqrt{T}\left[M_{m1,T}\left(\hat{\beta}_2, \hat{\gamma}_2\right) - M_{m2,T}(\beta_2^*, \gamma_2^*) + o_p(1)\right]$$

then using the null hypothesis the asymptotic distribution we get the asymptotic relation.

Graphs of two simulations

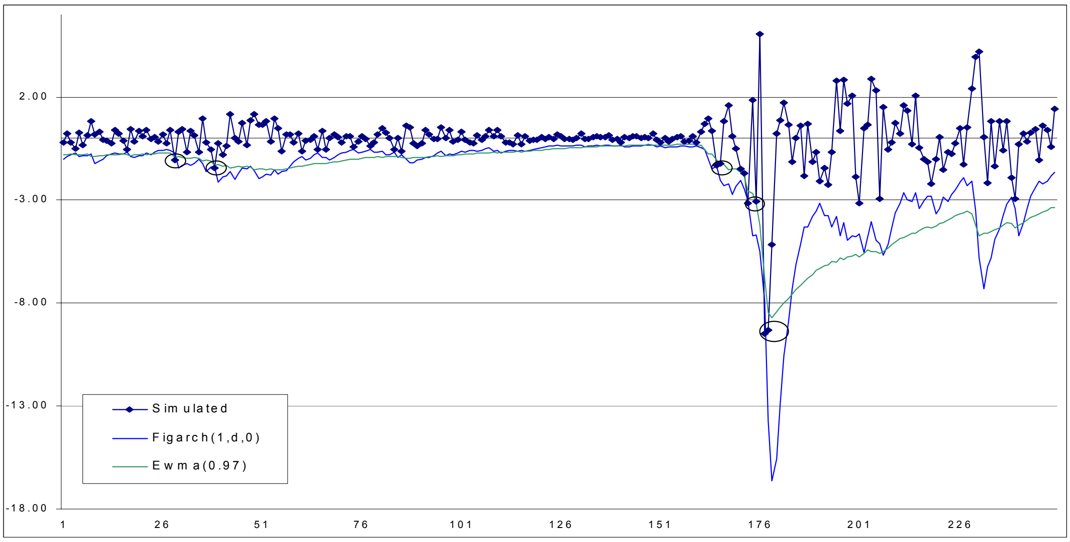


Graph 1: Simulated series FIGARCH(0.5,0.8,0)

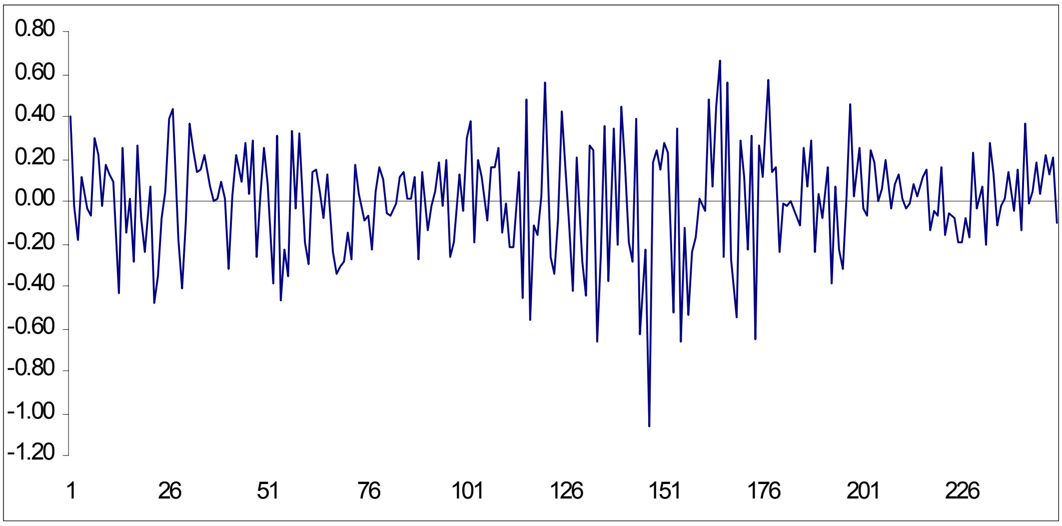


Graph 2: simulated conditional variance, series of Graph 1

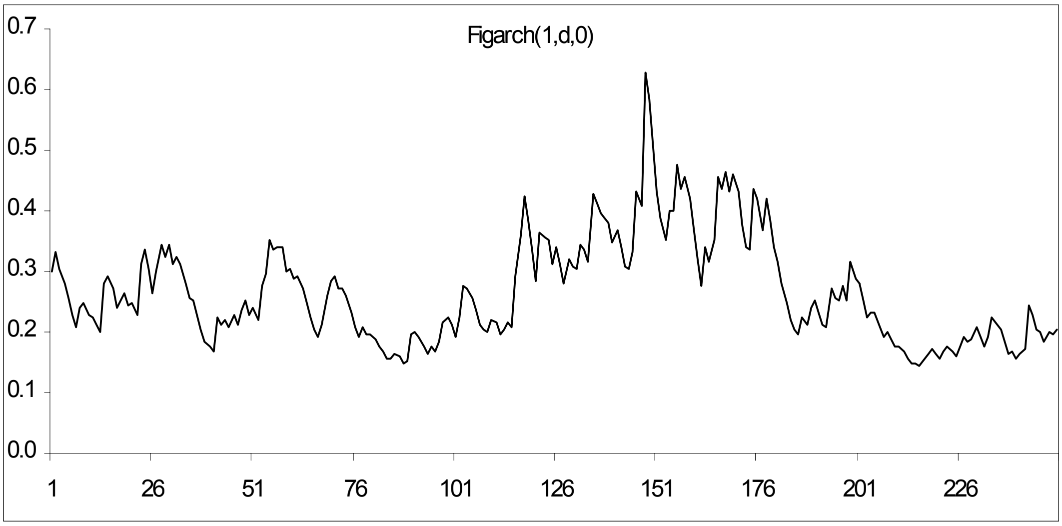
Graph 3: estimated conditional variance, last 100 observations of Graph 1



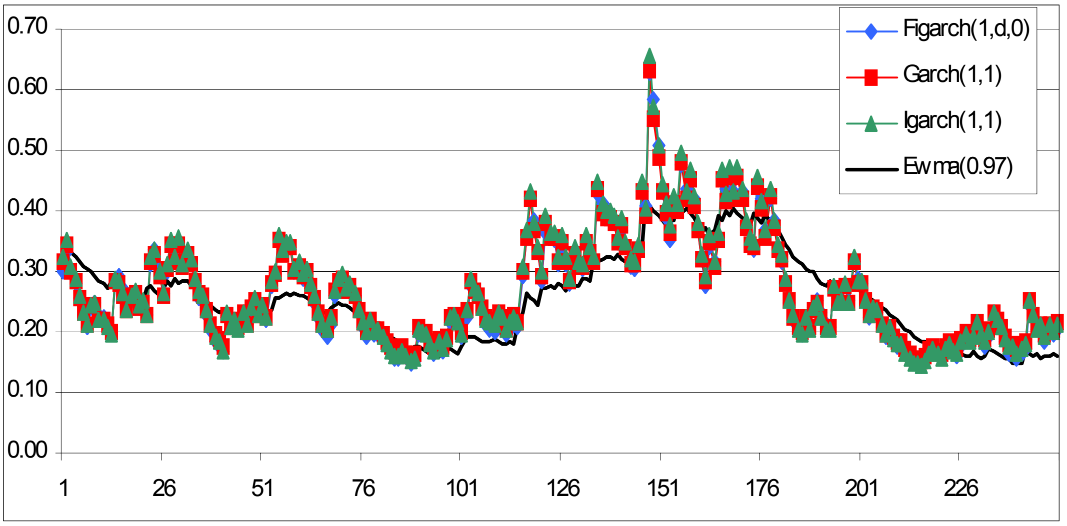
Graph 4: Value-at-Risk bands, series of Graph 1



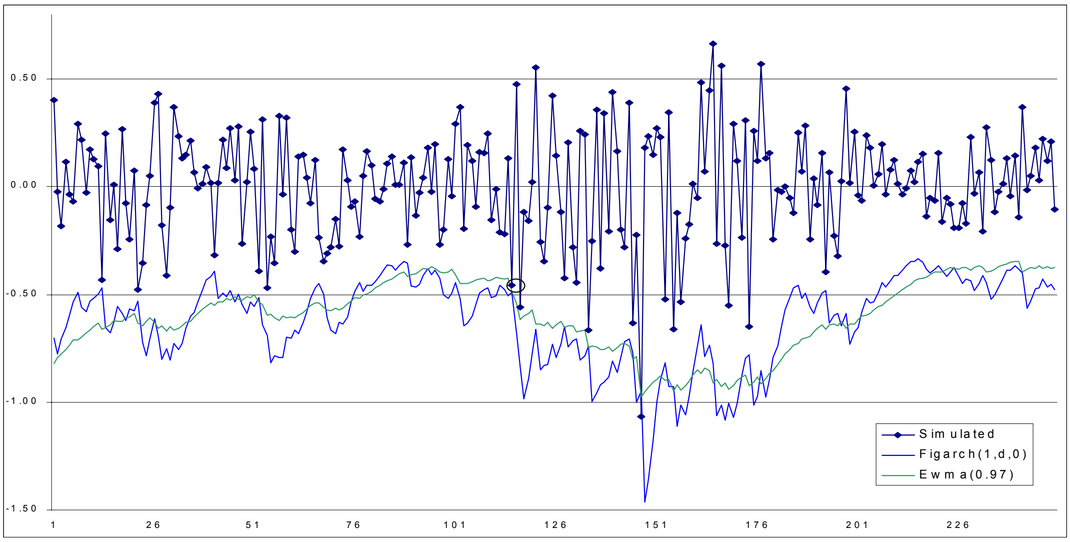
Graph 5: Simulated series FIGARCH(0.5,0.8,0)



Graph 6: simulated conditional variance, series of Graph 5



Graph 7: estimated conditional variance, series of Graph 5



Graph 8: Value-at-Risk bands, series of Graph 5

8 Annexes

8.1 Tables of Montecarlo on non-aggregated data

In the following pages you will find the tables for the Montecarlo described in section 5. The tables are grouped by DGP, listed in the first row at the beginnig of each group. In the next rows we just describe table contents:

- Tables 1, 8, 15, 22, 29, 36, 43, 50, 57: the tables list for each of the four model considered and two level of Value-at-Risk coverage (1% and 5%) the average number of exceptions, its standard deviation and the average percentage of exceptions for an experiment conducted on 1000 replications and for a sample of 250 1-day-ahead forecasts, using the backtesting approach.

- Tables 2, 9, 16, 23, 30, 37, 44, 51, 58: in this case for the models and VaR coverage levels we report frequency of model selction based on counting exceptions, a model is preferred to the others when its number of exceptions is lower. Given that the exceptions are integer numbers the frequencies sum may be higher than 1.

- Tables 3, 10, 17, 24, 31, 38, 45, 52, 59: these tables report the frequencies of accepting the null hypothesis of the tests of unconditional coverage of Kupiec (1995 - null is correct coverage), the test of independence of Christoffersen-Lopez (1998 - null is independence) and the test of conditional coverage of Christoffersen-Lopez (1998 - null is again correct coverage).

- Tables 4, 11, 18, 25, 32, 39, 46, 53, 60: these are the first tables on the loss funcions results, they report the frequency of model selection based on the application of the loss function suggested by Lopez (1999) that focus only on exceptions. Given that the parameters of GARCH(1,1) and IGARCH(1,1) are often very close this cause an identical loss function for the two models, same exceptions and same forecast, therefore the frequencies sum may be higher than 1.

- Tables 5, 12, 19, 26, 33, 40, 47, 54, 61: in these tables we report the frequencies of selection based on our alternative loss functions, that focus on exceptions (rows labelled with an E) and on the whole backtesting sample, 250 observations (rows labelled with a T). Again the closeness of GARCH and IGARCH may cause a sum of frequencies over 1. The results are grouped by loss functions and combination of loss functions as described in the italics rows.

- Tables 6, 13, 20, 27, 34, 41, 48, 55, 62: in these tables and in the next group we deal with the test of Christoffersen et al. (2001). These tables report the result of the test of model comparison and consider four different

Value-at-Risk coverage. For each one of these levels of confidence the tables report the test results for a pairwise comparison between models, using the legend at the bottom of the table. For each level and comparison we reported the frequence of accepting the test (null hypothesis is the the two models do not equally match the efficiency moment condition of Christoffersen et al. 2001, this is implied by a significat test statistic) and then usign the sign of the test statistic we report the percentage of preference of the first or of the second model. The percentage is computed using only the cases when the test null hypothesis is accepted. In all cases we considered three level of confidence for the test statistics, the percentage indicated with test α-value.

- Tables 7, 14, 21, 28, 35, 42, 49, 56, 63: in these last group of tables we report the second test suggested by Christoffersen et al. (2001) the test on Value-at-Risk specification. In these tables we report for the different model considered at the four level of VaR confidence used in the previuos tables the frequency of accepting the null hypothesis of the test (null is that the VaR is correctly specified). As in the previuos case we report three level of confidence for the test statistic.

DGP FIGARCH(1,d,1) d=0.4 β=0.3 ϕ=0.2 - % represent VaR p-level unless differently specified

1 - Average number of exceptions – (standard deviation) - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.561 (1.628) *1.024*	2.758 (1.943) *1.103*	2.384 (1.622) *0.954*	1.436 (1.144) *0.574*
5% VaR	12.749 (3.393) *5.100*	12.852 (4.354) *5.141*	11.665 (3.494) *4.666*	11.590 (3.031) *4.636*

2 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.271	0.321	0.382	0.814
5% VaR	0.380	0.283	0.283	0.539

3 - Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.992	0.981	0.995	1.000
	5%	0.913	0.848	0.881	0.764
5% VaR	1%	0.994	0.964	0.991	0.994
	5%	0.940	0.873	0.925	0.951
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.748	0.781	0.742	0.630
	5%	0.313	0.404	0.312	0.273
5% VaR	1%	0.981	0.968	0.969	0.852
	5%	0.924	0.890	0.892	0.693
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.967	0.955	0.966	0.945
	5%	0.725	0.731	0.722	0.623
5% VaR	1%	0.986	0.958	0.970	0.905
	5%	0.906	0.833	0.868	0.723

4 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.120	0.156	0.224	0.727
5% VaR	0.120	0.107	0.138	0.635

5 - Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.115	0.158	0.213	0.741
	T	0.033	0.143	0.414	0.410
5% VaR	E	0.058	0.169	0.266	0.507
	T	0.033	0.143	0.414	0.410
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.115	0.140	0.202	0.770
	T	0.080	0.366	0.003	0.551
5% VaR	E	0.038	0.088	0.181	0.693
	T	0.065	0.307	0.001	0.627
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.109	0.139	0.216	0.763
	T	0.100	0.381	0.004	0.515
5% VaR	E	0.037	0.106	0.283	0.574
	T	0.089	0.365	0.004	0.542
Loss Function 1 + Loss Function 2					
1% VaR	E	0.115	0.157	0.213	0.742
	T	0.060	0.258	0.001	0.681
5% VaR	E	0.053	0.162	0.259	0.526
	T	0.052	0.070	0.000	0.878
Loss Function 1 + Loss Function 3					
1% VaR	E	0.117	0.158	0.215	0.737
	T	0.075	0.314	0.003	0.608
5% VaR	E	0.050	0.163	0.274	0.513
	T	0.051	0.127	0.000	0.822
Loss Function 2 + Loss Function 3					
1% VaR	E	0.112	0.140	0.214	0.761
	T	0.093	0.376	0.004	0.527
5% VaR	E	0.033	0.102	0.235	0.630
	T	0.083	0.346	0.003	0.568
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.117	0.158	0.214	0.738
	T	0.083	0.344	0.003	0.570
5% VaR	E	0.044	0.160	0.266	0.530
	T	0.060	0.221	0.000	0.719

6 - Test of model comparison –1000 replications – 250 forecasts							
frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.676	0.680	0.874	0.509	0.843	0.846
	5%	0.677	0.682	0.874	0.511	0.845	0.849
	10%	0.678	0.682	0.874	0.513	0.846	0.850
Prefer 1st model	1%	0.533	0.699	0.811	0.729	0.807	0.734
	5%	0.533	0.698	0.811	0.726	0.805	0.731
	10%	0.534	0.698	0.811	0.723	0.804	0.732
Prefer 2nd model	1%	0.467	0.301	0.189	0.271	0.193	0.266
	5%	0.467	0.302	0.189	0.274	0.195	0.269
	10%	0.466	0.302	0.189	0.277	0.196	0.268
VaR(5%)							
Test is significant	1%	0.913	0.922	0.976	0.733	0.981	0.978
	5%	0.916	0.926	0.983	0.741	0.986	0.985
	10%	0.918	0.927	0.984	0.743	0.989	0.987
Prefer 1st model	1%	0.645	0.767	0.662	0.673	0.556	0.466
	5%	0.644	0.767	0.659	0.671	0.554	0.465
	10%	0.644	0.767	0.659	0.672	0.554	0.465
Prefer 2nd model	1%	0.355	0.233	0.338	0.327	0.444	0.534
	5%	0.356	0.233	0.341	0.329	0.446	0.535
	10%	0.356	0.233	0.341	0.328	0.446	0.535
VaR(10%)							
Test is significant	1%	0.961	0.975	0.984	0.815	0.987	0.988
	5%	0.963	0.977	0.986	0.821	0.989	0.989
	10%	0.965	0.977	0.990	0.823	0.989	0.992
Prefer 1st model	1%	0.670	0.760	0.678	0.640	0.555	0.470
	5%	0.670	0.759	0.676	0.638	0.554	0.469
	10%	0.669	0.759	0.676	0.639	0.554	0.470
Prefer 2nd model	1%	0.330	0.240	0.322	0.360	0.445	0.530
	5%	0.330	0.241	0.324	0.362	0.446	0.531
	10%	0.331	0.241	0.324	0.361	0.446	0.530
VaR(25%)							
Test is significant	1%	0.953	0.969	0.989	0.842	0.990	0.991
	5%	0.954	0.971	0.991	0.846	0.992	0.993
	10%	0.957	0.972	0.992	0.849	0.996	0.996
Prefer 1st model	1%	0.592	0.665	0.627	0.594	0.563	0.503
	5%	0.591	0.664	0.626	0.593	0.563	0.503
	10%	0.591	0.664	0.625	0.594	0.562	0.503
Prefer 2nd model	1%	0.408	0.335	0.373	0.406	0.437	0.497
	5%	0.409	0.336	0.374	0.407	0.438	0.497
	10%	0.409	0.336	0.375	0.406	0.438	0.497

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

7 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.027	0.017	0.011	0.005
	5%	0.020	0.012	0.009	0.004
	10%	0.017	0.009	0.007	0.003
5%	1%	0.392	0.314	0.209	0.240
	5%	0.270	0.210	0.143	0.175
	10%	0.211	0.164	0.116	0.141
10%	1%	0.586	0.480	0.389	0.425
	5%	0.442	0.333	0.267	0.303
	10%	0.360	0.249	0.209	0.235
25%	1%	0.725	0.683	0.625	0.648
	5%	0.573	0.523	0.469	0.466
	10%	0.486	0.424	0.381	0.358

DGP FIGARCH(1,d,0) d=0.4 β=0.3 - % represent VaR p-level unless differently specified

8 - Average number of exceptions – (standard deviation) - *average percentage of exception* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.606 (1.641) *1.042*	2.704 (1.848) *1.082*	2.270 (1.516) *0.908*	1.152 (1.086) *0.461*
5% VaR	12.771 (3.548) *5.108*	12.867 (4.191) *5.147*	11.671 (3.221) *4.668*	11.449 (2.967) *4.580*

9 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.243	0.278	0.319	0.946
5% VaR	0.352	0.229	0.274	0.634

10 - Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.995	0.989	0.997	1.000
	5%	0.900	0.869	0.892	0.684
5% VaR	1%	0.987	0.968	0.990	0.994
	5%	0.935	0.881	0.942	0.961
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.779	0.780	0.744	0.623
	5%	0.313	0.370	0.286	0.336
5% VaR	1%	0.973	0.976	0.981	0.951
	5%	0.909	0.924	0.923	0.852
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.964	0.968	0.980	0.987
	5%	0.756	0.737	0.734	0.621
5% VaR	1%	0.970	0.960	0.977	0.966
	5%	0.895	0.859	0.897	0.857

11 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.094	0.123	0.115	0.923
5% VaR	0.094	0.065	0.038	0.803

12 - Loss - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.092	0.125	0.115	0.923
	T	0.052	0.193	0.472	0.283
5% VaR	E	0.043	0.170	0.189	0.598
	T	0.052	0.193	0.472	0.283
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.090	0.118	0.118	0.929
	T	0.066	0.339	0.000	0.595
5% VaR	E	0.009	0.081	0.021	0.889
	T	0.034	0.234	0.000	0.732
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.089	0.119	0.115	0.932
	T	0.094	0.375	0.000	0.531
5% VaR	E	0.010	0.105	0.068	0.817
	T	0.073	0.331	0.000	0.596
Loss Function 1 + Loss Function 2					
1% VaR	E	0.093	0.124	0.115	0.923
	T	0.031	0.208	0.000	0.761
5% VaR	E	0.036	0.163	0.164	0.637
	T	0.004	0.036	0.000	0.960
Loss Function 1 + Loss Function 3					
1% VaR	E	0.092	0.125	0.115	0.923
	T	0.059	0.307	0.000	0.634
5% VaR	E	0.034	0.163	0.167	0.636
	T	0.021	0.096	0.000	0.883
Loss Function 2 + Loss Function 3					
1% VaR	E	0.089	0.118	0.114	0.934
	T	0.082	0.361	0.000	0.557
5% VaR	E	0.009	0.093	0.049	0.849
	T	0.051	0.302	0.000	0.647
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.091	0.124	0.115	0.925
	T	0.065	0.324	0.000	0.611
5% VaR	E	0.030	0.154	0.141	0.675
	T	0.020	0.166	0.000	0.814

13 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.552	0.606	0.785	0.510	0.778	0.724
	5%	0.553	0.607	0.787	0.510	0.778	0.725
	10%	0.554	0.608	0.787	0.510	0.778	0.726
Prefer 1st model	1%	0.478	0.711	0.925	0.747	0.919	0.870
	5%	0.479	0.712	0.925	0.747	0.919	0.870
	10%	0.478	0.712	0.925	0.747	0.919	0.869
Prefer 2nd model	1%	0.522	0.289	0.075	0.253	0.081	0.130
	5%	0.521	0.288	0.075	0.253	0.081	0.130
	10%	0.522	0.288	0.075	0.253	0.081	0.131
VaR(5%)							
Test is significant	1%	0.848	0.877	0.955	0.778	0.948	0.914
	5%	0.851	0.882	0.958	0.782	0.954	0.917
	10%	0.852	0.888	0.958	0.785	0.956	0.920
Prefer 1st model	1%	0.560	0.673	0.711	0.622	0.679	0.592
	5%	0.559	0.670	0.709	0.620	0.676	0.592
	10%	0.560	0.668	0.709	0.619	0.677	0.591
Prefer 2nd model	1%	0.440	0.327	0.289	0.378	0.321	0.408
	5%	0.441	0.330	0.291	0.380	0.324	0.408
	10%	0.440	0.332	0.291	0.381	0.323	0.409
VaR(10%)							
Test is significant	1%	0.910	0.939	0.983	0.860	0.977	0.970
	5%	0.916	0.944	0.985	0.864	0.980	0.973
	10%	0.919	0.948	0.986	0.864	0.980	0.974
Prefer 1st model	1%	0.575	0.649	0.635	0.610	0.598	0.513
	5%	0.575	0.648	0.636	0.611	0.598	0.513
	10%	0.575	0.649	0.636	0.611	0.598	0.513
Prefer 2nd model	1%	0.425	0.351	0.365	0.390	0.402	0.487
	5%	0.425	0.352	0.364	0.389	0.402	0.487
	10%	0.425	0.351	0.364	0.389	0.402	0.487
VaR(25%)							
Test is significant	1%	0.893	0.953	0.979	0.854	0.987	0.983
	5%	0.894	0.953	0.983	0.855	0.988	0.985
	10%	0.897	0.953	0.984	0.855	0.989	0.987
Prefer 1st model	1%	0.560	0.594	0.612	0.546	0.579	0.534
	5%	0.559	0.594	0.610	0.546	0.578	0.534
	10%	0.561	0.594	0.610	0.546	0.577	0.534
Prefer 2nd model	1%	0.440	0.406	0.388	0.454	0.421	0.466
	5%	0.441	0.406	0.390	0.454	0.422	0.466
	10%	0.439	0.406	0.390	0.454	0.423	0.466

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

14 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.030	0.035	0.015	0.002
	5%	0.022	0.018	0.009	0.001
	10%	0.020	0.017	0.008	0.001
5%	1%	0.456	0.415	0.312	0.279
	5%	0.326	0.300	0.235	0.180
	10%	0.268	0.227	0.184	0.137
10%	1%	0.588	0.565	0.487	0.490
	5%	0.446	0.410	0.340	0.331
	10%	0.347	0.329	0.271	0.258
25%	1%	0.750	0.722	0.690	0.698
	5%	0.581	0.543	0.525	0.528
	10%	0.463	0.440	0.423	0.412

DGP FIGARCH(0,d,0) d=0.4 - % represent VaR p-level unless differently specified

15 - Average number of exceptions – (standard deviation) - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.571 (1.665) *1.028*	2.893 (2.178) *1.157*	2.493 (1.867) *0.997*	1.712 (1.287) *0.685*
5% VaR	12.593 (3.505) *5.037*	12.642 (4.648) *5.057*	11.557 (3.809) *4.623*	11.394 (3.004) *4.558*

16 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.318	0.329	0.435	0.702
5% VaR	0.418	0.282	0.261	0.485

17 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.988	0.963	0.989	1.000
	5%	0.898	0.828	0.844	0.818
5% VaR	1%	0.989	0.946	0.976	0.996
	5%	0.935	0.832	0.882	0.952
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.774	0.767	0.749	0.597
	5%	0.309	0.389	0.368	0.226
5% VaR	1%	0.977	0.957	0.965	0.785
	5%	0.910	0.853	0.873	0.592
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.955	0.926	0.962	0.902
	5%	0.749	0.695	0.720	0.590
5% VaR	1%	0.978	0.930	0.953	0.855
	5%	0.900	0.780	0.816	0.624

18 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.166	0.151	0.282	0.611
5% VaR	0.166	0.101	0.188	0.545

19 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.161	0.148	0.272	0.629
	T	0.025	0.113	0.405	0.457
5% VaR	E	0.072	0.155	0.290	0.483
	T	0.025	0.113	0.405	0.457
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.182	0.144	0.264	0.620
	T	0.188	0.361	0.005	0.446
5% VaR	E	0.088	0.073	0.310	0.529
	T	0.159	0.333	0.003	0.505
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.171	0.141	0.286	0.612
	T	0.208	0.372	0.007	0.413
5% VaR	E	0.077	0.081	0.390	0.452
	T	0.194	0.357	0.005	0.444
Loss Function 1 + Loss Function 2					
1% VaR	E	0.161	0.149	0.274	0.626
	T	0.140	0.267	0.002	0.591
5% VaR	E	0.072	0.145	0.290	0.493
	T	0.132	0.115	0.001	0.752
Loss Function 1 + Loss Function 3					
1% VaR	E	0.164	0.148	0.277	0.621
	T	0.147	0.313	0.004	0.536
5% VaR	E	0.071	0.147	0.304	0.478
	T	0.119	0.126	0.000	0.755
Loss Function 2 + Loss Function 3					
1% VaR	E	0.172	0.144	0.284	0.610
	T	0.195	0.369	0.005	0.431
5% VaR	E	0.076	0.071	0.361	0.492
	T	0.180	0.348	0.004	0.468
Loss Function 1 + Loss Function 2 + Loss Function 3					
		Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	E	0.162	0.149	0.276	0.623
	T	0.157	0.340	0.005	0.498
5% VaR	E	0.067	0.142	0.304	0.487
	T	0.138	0.241	0.001	0.620

20 - Test of model comparison –description1000 replications – 250 forecasts							
frequncies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.733	0.752	0.900	0.485	0.892	0.895
	5%	0.734	0.754	0.902	0.487	0.893	0.896
	10%	0.737	0.755	0.902	0.490	0.895	0.897
Prefer 1st model	1%	0.536	0.670	0.697	0.742	0.679	0.606
	5%	0.537	0.671	0.696	0.743	0.679	0.606
	10%	0.536	0.672	0.696	0.743	0.679	0.605
Prefer 2nd model	1%	0.464	0.330	0.303	0.258	0.321	0.394
	5%	0.463	0.329	0.304	0.257	0.321	0.394
	10%	0.464	0.328	0.304	0.257	0.321	0.395
VaR(5%)							
Test is significant	1%	0.939	0.950	0.989	0.717	0.993	0.991
	5%	0.947	0.954	0.992	0.723	0.995	0.993
	10%	0.949	0.956	0.993	0.724	0.996	0.994
Prefer 1st model	1%	0.661	0.800	0.638	0.710	0.546	0.425
	5%	0.660	0.797	0.639	0.710	0.546	0.424
	10%	0.661	0.796	0.639	0.709	0.545	0.425
Prefer 2nd model	1%	0.339	0.200	0.362	0.290	0.454	0.575
	5%	0.340	0.203	0.361	0.290	0.454	0.576
	10%	0.339	0.204	0.361	0.291	0.455	0.575
VaR(10%)							
Test is significant	1%	0.974	0.982	0.994	0.808	0.991	0.995
	5%	0.976	0.986	0.996	0.809	0.992	0.995
	10%	0.978	0.987	0.997	0.811	0.994	0.996
Prefer 1st model	1%	0.699	0.774	0.649	0.635	0.505	0.421
	5%	0.699	0.774	0.650	0.635	0.504	0.421
	10%	0.698	0.774	0.650	0.635	0.505	0.422
Prefer 2nd model	1%	0.301	0.226	0.351	0.365	0.495	0.579
	5%	0.301	0.226	0.350	0.365	0.496	0.579
	10%	0.302	0.226	0.350	0.365	0.495	0.578
VaR(25%)							
Test is significant	1%	0.969	0.981	0.995	0.805	0.991	0.992
	5%	0.973	0.983	0.995	0.809	0.994	0.995
	10%	0.976	0.984	0.995	0.812	0.995	0.996
Prefer 1st model	1%	0.658	0.660	0.629	0.532	0.501	0.473
	5%	0.657	0.659	0.629	0.532	0.501	0.472
	10%	0.657	0.660	0.629	0.532	0.502	0.473
Prefer 2nd model	1%	0.342	0.340	0.371	0.468	0.499	0.527
	5%	0.343	0.341	0.371	0.468	0.499	0.528
	10%	0.343	0.340	0.371	0.468	0.498	0.527

Model reference:1 - Figarch(0,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

21 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.037	0.023	0.016	0.010
	5%	0.024	0.014	0.009	0.005
	10%	0.018	0.011	0.007	0.004
5%	1%	0.375	0.256	0.170	0.230
	5%	0.266	0.178	0.116	0.164
	10%	0.220	0.140	0.094	0.134
10%	1%	0.543	0.396	0.293	0.407
	5%	0.410	0.277	0.200	0.269
	10%	0.322	0.205	0.139	0.209
25%	1%	0.692	0.602	0.582	0.613
	5%	0.518	0.445	0.433	0.444
	10%	0.427	0.356	0.338	0.365

DGP FIGARCH(1,d,1) d=0.5 β=0.8 ϕ=0.3 - % represent VaR p-level unless differently specified

22 - Average number of exceptions – (standard deviation) - *average percentage of exception* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.375 (1.680) *0.950*	2.245 (1.670) *0.898*	2.167 (1.618) *0.867*	2.099 (1.458) *0.840*
5% VaR	11.933 (3.699) *4.773*	11.517 (3.893) *4.607*	11.305 (3.782) *4.522*	11.383 (3.273) *4.553*

23 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.477	0.555	0.595	0.602
5% VaR	0.573	0.385	0.386	0.355

24 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.992	0.993	0.995	0.999
	5%	0.882	0.858	0.854	0.867
5% VaR	1%	0.979	0.974	0.974	0.985
	5%	0.919	0.888	0.887	0.926
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.741	0.709	0.706	0.595
	5%	0.303	0.310	0.300	0.211
5% VaR	1%	0.977	0.958	0.959	0.714
	5%	0.895	0.871	0.873	0.541
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.965	0.962	0.966	0.842
	5%	0.720	0.690	0.688	0.587
5% VaR	1%	0.969	0.953	0.953	0.778
	5%	0.852	0.806	0.806	0.556

25 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.245	0.212	0.334	0.470
5% VaR	0.245	0.134	0.206	0.415

26 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.229	0.214	0.324	0.494
	T	0.021	0.136	0.317	0.526
5% VaR	E	0.102	0.158	0.285	0.455
	T	0.021	0.136	0.317	0.526
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.252	0.202	0.319	0.488
	T	0.282	0.241	0.037	0.440
5% VaR	E	0.164	0.127	0.357	0.352
	T	0.313	0.224	0.034	0.429
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.247	0.212	0.336	0.466
	T	0.253	0.239	0.040	0.468
5% VaR	E	0.143	0.142	0.404	0.311
	T	0.278	0.234	0.036	0.452
Loss Function 1 + Loss Function 2					
1% VaR	E	0.227	0.214	0.325	0.495
	T	0.271	0.187	0.021	0.521
5% VaR	E	0.106	0.155	0.291	0.448
	T	0.213	0.224	0.067	0.496
Loss Function 1 + Loss Function 3					
1% VaR	E	0.235	0.213	0.329	0.484
	T	0.247	0.192	0.025	0.536
5% VaR	E	0.111	0.151	0.301	0.437
	T	0.199	0.192	0.024	0.585
Loss Function 2 + Loss Function 3					
1% VaR	E	0.244	0.212	0.337	0.468
	T	0.268	0.238	0.037	0.457
5% VaR	E	0.157	0.136	0.397	0.310
	T	0.290	0.234	0.036	0.440
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.234	0.212	0.329	0.486
	T	0.268	0.217	0.031	0.484
5% VaR	E	0.113	0.154	0.309	0.424
	T	0.289	0.177	0.020	0.514

27 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.572	0.571	0.894	0.199	0.885	0.886
	5%	0.576	0.573	0.896	0.206	0.887	0.888
	10%	0.579	0.577	0.897	0.211	0.889	0.890
Prefer 1st model	1%	0.568	0.623	0.556	0.603	0.501	0.484
	5%	0.566	0.621	0.556	0.597	0.502	0.485
	10%	0.566	0.622	0.555	0.592	0.502	0.485
Prefer 2nd model	1%	0.432	0.377	0.444	0.397	0.499	0.516
	5%	0.434	0.379	0.444	0.403	0.498	0.515
	10%	0.434	0.378	0.445	0.408	0.498	0.515
VaR(5%)							
Test is significant	1%	0.785	0.768	0.993	0.221	0.990	0.989
	5%	0.791	0.773	0.995	0.223	0.993	0.992
	10%	0.793	0.774	0.995	0.223	0.995	0.994
Prefer 1st model	1%	0.576	0.608	0.655	0.588	0.602	0.586
	5%	0.575	0.609	0.655	0.587	0.602	0.587
	10%	0.574	0.609	0.655	0.587	0.602	0.587
Prefer 2nd model	1%	0.424	0.392	0.345	0.412	0.398	0.414
	5%	0.425	0.391	0.345	0.413	0.398	0.413
	10%	0.426	0.391	0.345	0.413	0.398	0.413
VaR(10%)							
Test is significant	1%	0.867	0.857	0.994	0.247	0.992	0.990
	5%	0.870	0.860	0.995	0.247	0.993	0.991
	10%	0.873	0.862	0.995	0.248	0.993	0.992
Prefer 1st model	1%	0.612	0.634	0.678	0.575	0.625	0.610
	5%	0.613	0.634	0.677	0.575	0.625	0.610
	10%	0.614	0.635	0.677	0.573	0.625	0.610
Prefer 2nd model	1%	0.388	0.366	0.322	0.425	0.375	0.390
	5%	0.387	0.366	0.323	0.425	0.375	0.390
	10%	0.386	0.365	0.323	0.427	0.375	0.390
VaR(25%)							
Test is significant	1%	0.873	0.874	0.996	0.278	0.994	0.995
	5%	0.878	0.878	0.997	0.280	0.996	0.996
	10%	0.881	0.879	0.999	0.281	0.998	0.998
Prefer 1st model	1%	0.576	0.598	0.678	0.507	0.634	0.625
	5%	0.576	0.599	0.678	0.507	0.635	0.626
	10%	0.575	0.598	0.677	0.505	0.633	0.624
Prefer 2nd model	1%	0.424	0.402	0.322	0.493	0.366	0.375
	5%	0.424	0.401	0.322	0.493	0.365	0.374
	10%	0.425	0.402	0.323	0.495	0.367	0.376

Model reference:1 - Figarch(.d.); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

28 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.055	0.045	0.037	0.033
	5%	0.043	0.031	0.029	0.027
	10%	0.038	0.027	0.024	0.021
5%	1%	0.395	0.340	0.330	0.229
	5%	0.297	0.254	0.248	0.154
	10%	0.256	0.207	0.200	0.116
10%	1%	0.562	0.493	0.477	0.333
	5%	0.409	0.375	0.360	0.220
	10%	0.340	0.309	0.299	0.173
25%	1%	0.741	0.708	0.704	0.540
	5%	0.579	0.558	0.555	0.385
	10%	0.486	0.466	0.467	0.303

DGP FIGARCH(1,d,1) d=0.8 β=0.5 ϕ=0.05- % represent VaR p-level unless differently specified

29 - Average number of exceptions – (standard deviation) - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.309 (1.767) *0.924*	2.362 (1.725) *0.945*	2.295 (1.672) *0.918*	1.954 (1.481) *0.782*
5% VaR	11.792 (3.811) *4.717*	11.570 (4.084) *4.628*	11.368 (3.985) *4.547*	11.575 (3.390) *4.630*

30 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.493	0.508	0.526	0.656
5% VaR	0.575	0.324	0.312	0.440

31 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.998	0.993	0.993	0.998
	5%	0.889	0.859	0.863	0.846
5% VaR	1%	0.983	0.970	0.968	0.980
	5%	0.923	0.876	0.880	0.929
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.727	0.736	0.730	0.607
	5%	0.293	0.331	0.318	0.233
5% VaR	1%	0.968	0.963	0.962	0.781
	5%	0.903	0.883	0.883	0.606
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.971	0.960	0.965	0.893
	5%	0.713	0.714	0.713	0.598
5% VaR	1%	0.966	0.954	0.951	0.842
	5%	0.870	0.818	0.819	0.628

32 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.227	0.192	0.291	0.547
5% VaR	0.227	0.104	0.172	0.497

33 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.213	0.189	0.289	0.566
	T	0.017	0.141	0.344	0.498
5% VaR	E	0.100	0.156	0.301	0.443
	T	0.017	0.141	0.344	0.498
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.214	0.178	0.274	0.591
	T	0.174	0.260	0.049	0.517
5% VaR	E	0.131	0.105	0.319	0.445
	T	0.180	0.249	0.046	0.525
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.231	0.179	0.294	0.553
	T	0.167	0.260	0.049	0.524
5% VaR	E	0.120	0.130	0.391	0.359
	T	0.169	0.262	0.047	0.522
Loss Function 1 + Loss Function 2					
1% VaR	E	0.214	0.189	0.288	0.566
	T	0.164	0.205	0.035	0.596
5% VaR	E	0.097	0.152	0.311	0.440
	T	0.182	0.223	0.056	0.539
Loss Function 1 + Loss Function 3					
1% VaR	E	0.220	0.187	0.291	0.559
	T	0.141	0.217	0.044	0.598
5% VaR	E	0.106	0.149	0.318	0.427
	T	0.162	0.181	0.032	0.625
Loss Function 2 + Loss Function 3					
1% VaR	E	0.224	0.177	0.293	0.563
	T	0.173	0.260	0.048	0.519
5% VaR	E	0.131	0.125	0.356	0.388
	T	0.177	0.258	0.048	0.517
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.219	0.187	0.290	0.561
	T	0.156	0.238	0.048	0.558
5% VaR	E	0.108	0.147	0.321	0.424
	T	0.169	0.212	0.028	0.591

34 - Test of model comparison - 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.458	0.445	0.879	0.211	0.882	0.884
	5%	0.460	0.446	0.879	0.217	0.882	0.885
	10%	0.462	0.452	0.880	0.222	0.883	0.887
Prefer 1st model	1%	0.561	0.600	0.580	0.592	0.586	0.569
	5%	0.561	0.601	0.580	0.590	0.586	0.569
	10%	0.561	0.602	0.581	0.590	0.586	0.570
Prefer 2nd model	1%	0.439	0.400	0.420	0.408	0.414	0.431
	5%	0.439	0.399	0.420	0.410	0.414	0.431
	10%	0.439	0.398	0.419	0.410	0.414	0.430
VaR(5%)							
Test is significant	1%	0.715	0.712	0.985	0.222	0.984	0.982
	5%	0.725	0.721	0.989	0.223	0.986	0.984
	10%	0.729	0.723	0.991	0.224	0.987	0.986
Prefer 1st model	1%	0.571	0.604	0.655	0.586	0.624	0.603
	5%	0.571	0.605	0.655	0.587	0.624	0.603
	10%	0.568	0.603	0.654	0.585	0.623	0.601
Prefer 2nd model	1%	0.429	0.396	0.345	0.414	0.376	0.397
	5%	0.429	0.395	0.345	0.413	0.376	0.397
	10%	0.432	0.397	0.346	0.415	0.377	0.399
VaR(10%)							
Test is significant	1%	0.819	0.801	0.995	0.285	0.993	0.994
	5%	0.826	0.809	0.996	0.285	0.995	0.996
	10%	0.828	0.811	0.996	0.288	0.996	0.996
Prefer 1st model	1%	0.598	0.617	0.686	0.540	0.658	0.645
	5%	0.599	0.617	0.686	0.540	0.658	0.646
	10%	0.598	0.615	0.686	0.542	0.658	0.646
Prefer 2nd model	1%	0.402	0.383	0.314	0.460	0.342	0.355
	5%	0.401	0.383	0.314	0.460	0.342	0.354
	10%	0.402	0.385	0.314	0.458	0.342	0.354
VaR(25%)							
Test is significant	1%	0.809	0.776	0.995	0.256	0.994	0.993
	5%	0.813	0.778	0.997	0.258	0.994	0.993
	10%	0.818	0.782	0.997	0.260	0.994	0.993
Prefer 1st model	1%	0.544	0.555	0.658	0.484	0.619	0.625
	5%	0.544	0.555	0.658	0.488	0.619	0.625
	10%	0.544	0.555	0.658	0.485	0.619	0.625
Prefer 2nd model	1%	0.456	0.445	0.342	0.516	0.381	0.375
	5%	0.456	0.445	0.342	0.512	0.381	0.375
	10%	0.456	0.445	0.342	0.515	0.381	0.375

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

35 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.058	0.048	0.044	0.026
	5%	0.044	0.036	0.034	0.023
	10%	0.038	0.029	0.027	0.020
5%	1%	0.388	0.360	0.339	0.244
	5%	0.297	0.257	0.242	0.158
	10%	0.245	0.213	0.201	0.125
10%	1%	0.551	0.511	0.497	0.308
	5%	0.426	0.386	0.370	0.217
	10%	0.355	0.326	0.306	0.171
25%	1%	0.719	0.697	0.695	0.566
	5%	0.563	0.554	0.555	0.410
	10%	0.473	0.458	0.463	0.327

DGP FIGARCH(1,d,0) d=0.8 β=0.5 - % represent VaR p-level unless differently specified

36 - Average number of exceptions – (standard deviation) - *average percentage of exception* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.331 (1.553) *0.932*	2.296 (1.709) *0.918*	2.200 (1.631) *0.880*	1.833 (1.344) *0.733*
5% VaR	11.799 (3.329) *4.720*	11.537 (3.822) *4.615*	11.309 (3.661) *4.524*	11.353 (3.332) *4.541*

37 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.433	0.512	0.546	0.691
5% VaR	0.511	0.354	0.347	0.474

38 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.996	0.993	0.994	0.998
	5%	0.888	0.848	0.847	0.848
5% VaR	1%	0.992	0.977	0.977	0.981
	5%	0.935	0.883	0.894	0.924
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.751	0.749	0.744	0.625
	5%	0.307	0.345	0.331	0.221
5% VaR	1%	0.982	0.983	0.983	0.828
	5%	0.930	0.924	0.925	0.661
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.981	0.979	0.982	0.927
	5%	0.737	0.727	0.727	0.622
5% VaR	1%	0.984	0.967	0.968	0.865
	5%	0.893	0.851	0.854	0.665

39 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.183	0.210	0.304	0.586
5% VaR	0.183	0.120	0.170	0.527

40 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.179	0.214	0.290	0.600
	T	0.015	0.151	0.325	0.509
5% VaR	E	0.087	0.153	0.299	0.461
	T	0.015	0.151	0.325	0.509
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.188	0.198	0.298	0.599
	T	0.182	0.225	0.054	0.539
5% VaR	E	0.096	0.107	0.314	0.483
	T	0.185	0.218	0.052	0.545
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.193	0.204	0.306	0.580
	T	0.179	0.229	0.056	0.536
5% VaR	E	0.096	0.133	0.396	0.375
	T	0.180	0.224	0.054	0.542
Loss Function 1 + Loss Function 2					
1% VaR	E	0.178	0.213	0.292	0.600
	T	0.153	0.176	0.041	0.630
5% VaR	E	0.090	0.148	0.303	0.459
	T	0.137	0.202	0.060	0.601
Loss Function 1 + Loss Function 3					
1% VaR	E	0.180	0.212	0.297	0.594
	T	0.146	0.187	0.042	0.625
5% VaR	E	0.094	0.140	0.317	0.449
	T	0.134	0.161	0.027	0.678
Loss Function 2 + Loss Function 3					
1% VaR	E	0.189	0.201	0.305	0.588
	T	0.177	0.229	0.055	0.539
5% VaR	E	0.094	0.117	0.364	0.425
	T	0.183	0.217	0.053	0.547
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.178	0.212	0.295	0.598
	T	0.160	0.212	0.049	0.579
5% VaR	E	0.093	0.140	0.319	0.448
	T	0.155	0.173	0.039	0.633

41 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.467	0.461	0.870	0.198	0.863	0.863
	5%	0.474	0.467	0.872	0.213	0.863	0.866
	10%	0.475	0.471	0.875	0.219	0.868	0.866
Prefer 1st model	1%	0.576	0.633	0.643	0.611	0.606	0.579
	5%	0.574	0.632	0.641	0.601	0.606	0.579
	10%	0.575	0.633	0.641	0.607	0.607	0.579
Prefer 2nd model	1%	0.424	0.367	0.357	0.389	0.394	0.421
	5%	0.426	0.368	0.359	0.399	0.394	0.421
	10%	0.425	0.367	0.359	0.393	0.393	0.421
VaR(5%)							
Test is significant	1%	0.715	0.709	0.987	0.215	0.984	0.985
	5%	0.722	0.715	0.989	0.218	0.984	0.985
	10%	0.723	0.717	0.991	0.218	0.988	0.988
Prefer 1st model	1%	0.520	0.574	0.655	0.633	0.641	0.614
	5%	0.519	0.572	0.654	0.633	0.641	0.614
	10%	0.519	0.570	0.655	0.633	0.642	0.615
Prefer 2nd model	1%	0.480	0.426	0.345	0.367	0.359	0.386
	5%	0.481	0.428	0.346	0.367	0.359	0.386
	10%	0.481	0.430	0.345	0.367	0.358	0.385
VaR(10%)							
Test is significant	1%	0.790	0.767	0.991	0.264	0.993	0.993
	5%	0.793	0.771	0.993	0.265	0.996	0.996
	10%	0.794	0.773	0.994	0.266	0.996	0.996
Prefer 1st model	1%	0.571	0.581	0.701	0.557	0.683	0.672
	5%	0.571	0.582	0.702	0.558	0.684	0.673
	10%	0.572	0.582	0.701	0.560	0.684	0.673
Prefer 2nd model	1%	0.429	0.419	0.299	0.443	0.317	0.328
	5%	0.429	0.418	0.298	0.442	0.316	0.327
	10%	0.428	0.418	0.299	0.440	0.316	0.327
VaR(25%)							
Test is significant	1%	0.780	0.769	0.998	0.251	0.997	0.997
	5%	0.784	0.772	0.998	0.252	0.997	0.997
	10%	0.785	0.773	0.998	0.253	0.998	0.997
Prefer 1st model	1%	0.537	0.541	0.680	0.522	0.655	0.659
	5%	0.536	0.539	0.680	0.524	0.655	0.659
	10%	0.536	0.539	0.680	0.522	0.655	0.659
Prefer 2nd model	1%	0.463	0.459	0.320	0.478	0.345	0.341
	5%	0.464	0.461	0.320	0.476	0.345	0.341
	10%	0.464	0.461	0.320	0.478	0.345	0.341

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

42 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.044	0.042	0.037	0.017
	5%	0.030	0.030	0.025	0.014
	10%	0.025	0.027	0.021	0.012
5%	1%	0.396	0.378	0.356	0.219
	5%	0.307	0.294	0.275	0.152
	10%	0.260	0.237	0.226	0.119
10%	1%	0.564	0.544	0.542	0.331
	5%	0.423	0.411	0.394	0.229
	10%	0.359	0.334	0.327	0.164
25%	1%	0.725	0.718	0.715	0.579
	5%	0.587	0.575	0.573	0.400
	10%	0.484	0.479	0.472	0.322

DGP FIGARCH(0,d,0) d=0.8 - % represent VaR p-level unless differently specified

43 - Average number of exceptions – (standard deviation) - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.272 (1.496) *0.909*	2.380 (1.781) *0.952*	2.280 (1.727) *0.912*	3.036 (1.664) *1.214*
5% VaR	11.918 (3.368) *4.767*	11.582 (3.995) *4.633*	11.278 (3.782) *4.511*	11.449 (3.156) *4.580*

44 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.584	0.589	0.637	0.365
5% VaR	0.648	0.367	0.353	0.250

45 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.998	0.989	0.990	0.990
	5%	0.889	0.852	0.846	0.927
5% VaR	1%	0.987	0.966	0.970	0.990
	5%	0.935	0.888	0.895	0.940
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.743	0.741	0.739	0.568
	5%	0.278	0.327	0.319	0.179
5% VaR	1%	0.980	0.970	0.972	0.577
	5%	0.917	0.908	0.908	0.377
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.976	0.970	0.971	0.699
	5%	0.723	0.707	0.711	0.551
5% VaR	1%	0.976	0.954	0.956	0.655
	5%	0.893	0.843	0.842	0.409

46 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.369	0.232	0.375	0.237
5% VaR	0.369	0.163	0.249	0.219

47 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.350	0.236	0.368	0.259
	T	0.001	0.056	0.241	0.702
5% VaR	E	0.167	0.118	0.344	0.371
	T	0.001	0.056	0.241	0.702
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.354	0.221	0.381	0.257
	T	0.487	0.284	0.016	0.213
5% VaR	E	0.318	0.126	0.416	0.140
	T	0.553	0.265	0.014	0.168
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.376	0.235	0.403	0.199
	T	0.443	0.288	0.019	0.250
5% VaR	E	0.288	0.121	0.474	0.117
	T	0.482	0.280	0.018	0.220
Loss Function 1 + Loss Function 2					
1% VaR	E	0.351	0.237	0.369	0.256
	T	0.438	0.224	0.009	0.329
5% VaR	E	0.187	0.120	0.350	0.343
	T	0.272	0.258	0.132	0.338
Loss Function 1 + Loss Function 3					
1% VaR	E	0.355	0.235	0.377	0.246
	T	0.379	0.235	0.013	0.373
5% VaR	E	0.186	0.120	0.362	0.332
	T	0.259	0.228	0.068	0.445
Loss Function 2 + Loss Function 3					
1% VaR	E	0.376	0.227	0.391	0.219
	T	0.465	0.288	0.017	0.230
5% VaR	E	0.294	0.121	0.468	0.117
	T	0.513	0.275	0.015	0.197
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.357	0.236	0.376	0.244
	T	0.445	0.274	0.013	0.268
5% VaR	E	0.202	0.123	0.368	0.307
	T	0.483	0.224	0.005	0.288

48 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.620	0.620	0.957	0.183	0.954	0.954
	5%	0.625	0.624	0.959	0.191	0.956	0.956
	10%	0.626	0.624	0.962	0.196	0.957	0.958
Prefer 1st model	1%	0.513	0.552	0.369	0.628	0.360	0.343
	5%	0.515	0.553	0.368	0.618	0.360	0.343
	10%	0.514	0.553	0.368	0.617	0.361	0.344
Prefer 2nd model	1%	0.487	0.448	0.631	0.372	0.640	0.657
	5%	0.485	0.447	0.632	0.382	0.640	0.657
	10%	0.486	0.447	0.632	0.383	0.639	0.656
VaR(5%)							
Test is significant	1%	0.886	0.884	0.980	0.250	0.978	0.978
	5%	0.891	0.887	0.982	0.252	0.982	0.983
	10%	0.894	0.891	0.984	0.255	0.985	0.986
Prefer 1st model	1%	0.573	0.610	0.610	0.628	0.549	0.531
	5%	0.574	0.609	0.610	0.631	0.550	0.531
	10%	0.572	0.606	0.611	0.624	0.549	0.530
Prefer 2nd model	1%	0.427	0.390	0.390	0.372	0.451	0.469
	5%	0.426	0.391	0.390	0.369	0.450	0.469
	10%	0.428	0.394	0.389	0.376	0.451	0.470
VaR(10%)							
Test is significant	1%	0.943	0.944	0.991	0.268	0.990	0.989
	5%	0.946	0.946	0.991	0.269	0.992	0.992
	10%	0.948	0.948	0.992	0.269	0.992	0.992
Prefer 1st model	1%	0.650	0.665	0.752	0.578	0.675	0.670
	5%	0.649	0.665	0.752	0.580	0.674	0.670
	10%	0.648	0.664	0.751	0.580	0.674	0.670
Prefer 2nd model	1%	0.350	0.335	0.248	0.422	0.325	0.330
	5%	0.351	0.335	0.248	0.420	0.326	0.330
	10%	0.352	0.336	0.249	0.420	0.326	0.330
VaR(25%)							
Test is significant	1%	0.947	0.949	0.992	0.275	0.993	0.992
	5%	0.950	0.952	0.993	0.276	0.995	0.994
	10%	0.952	0.954	0.993	0.276	0.995	0.994
Prefer 1st model	1%	0.603	0.614	0.800	0.527	0.744	0.739
	5%	0.603	0.614	0.800	0.529	0.745	0.739
	10%	0.603	0.614	0.800	0.529	0.745	0.739
Prefer 2nd model	1%	0.397	0.386	0.200	0.473	0.256	0.261
	5%	0.397	0.386	0.200	0.471	0.255	0.261
	10%	0.397	0.386	0.200	0.471	0.255	0.261

Model reference:1 - Figarch(0,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

49 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.048	0.041	0.039	0.081
	5%	0.040	0.031	0.029	0.059
	10%	0.036	0.026	0.024	0.045
5%	1%	0.370	0.286	0.272	0.220
	5%	0.291	0.221	0.204	0.131
	10%	0.249	0.174	0.160	0.109
10%	1%	0.511	0.424	0.411	0.218
	5%	0.391	0.295	0.277	0.146
	10%	0.321	0.239	0.233	0.114
25%	1%	0.699	0.656	0.654	0.366
	5%	0.557	0.507	0.507	0.227
	10%	0.462	0.409	0.418	0.169

DGP FIGARCH(1,d,1) d=0.1 β=0.4 ϕ=0.5 - % represent VaR p-level unless differently specified

50 - Average number of exceptions – (standard deviation) - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.694 (1.749) *1.078*	2.498 (1.988) *0.999*	2.435 (1.470) *0.974*	1.228 (1.033) *0.491*
5% VaR	12.787 (3.683) *5.115*	11.742 (5.214) *4.697*	11.273 (3.083) *4.509*	11.184 (2.845) *4.474*

51 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.257	0.368	0.254	0.848
5% VaR	0.345	0.321	0.324	0.541

52 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.988	0.981	0.999	1.000
	5%	0.896	0.790	0.917	0.735
5% VaR	1%	0.990	0.877	0.995	0.996
	5%	0.925	0.804	0.943	0.949
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.796	0.802	0.727	0.586
	5%	0.324	0.413	0.255	0.285
5% VaR	1%	0.973	0.931	0.912	0.909
	5%	0.919	0.862	0.805	0.781
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.963	0.960	0.930	0.964
	5%	0.765	0.764	0.714	0.580
5% VaR	1%	0.976	0.871	0.940	0.931
	5%	0.896	0.788	0.798	0.791

53 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.096	0.229	0.101	0.811
5% VaR	0.096	0.158	0.057	0.689

54 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.097	0.228	0.099	0.813
	T	0.007	0.242	0.676	0.075
5% VaR	E	0.033	0.258	0.180	0.529
	T	0.007	0.242	0.676	0.075
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.097	0.228	0.095	0.817
	T	0.101	0.370	0.002	0.527
5% VaR	E	0.009	0.174	0.029	0.788
	T	0.047	0.314	0.001	0.638
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.094	0.216	0.094	0.833
	T	0.123	0.389	0.006	0.482
5% VaR	E	0.010	0.211	0.076	0.703
	T	0.099	0.370	0.003	0.528
Loss Function 1 + Loss Function 2					
1% VaR	E	0.097	0.227	0.099	0.814
	T	0.016	0.015	0.000	0.969
5% VaR	E	0.033	0.251	0.165	0.551
	T	0.013	0.276	0.129	0.582
Loss Function 1 + Loss Function 3					
1% VaR	E	0.097	0.228	0.099	0.813
	T	0.028	0.069	0.001	0.902
5% VaR	E	0.032	0.254	0.172	0.542
	T	0.024	0.258	0.058	0.660
Loss Function 2 + Loss Function 3					
1% VaR	E	0.094	0.216	0.093	0.834
	T	0.111	0.384	0.004	0.501
5% VaR	E	0.010	0.187	0.056	0.747
	T	0.069	0.351	0.002	0.578
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.096	0.228	0.099	0.814
	T	0.050	0.300	0.002	0.648
5% VaR	E	0.031	0.250	0.154	0.565
	T	0.013	0.012	0.000	0.975

55 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.396	0.810	0.821	0.835	0.813	0.756
	5%	0.396	0.811	0.822	0.835	0.814	0.761
	10%	0.397	0.813	0.822	0.837	0.814	0.762
Prefer 1st model	1%	0.624	0.669	0.892	0.593	0.812	0.856
	5%	0.624	0.670	0.892	0.593	0.812	0.854
	10%	0.622	0.668	0.892	0.593	0.812	0.854
Prefer 2nd model	1%	0.376	0.331	0.108	0.407	0.188	0.144
	5%	0.376	0.330	0.108	0.407	0.188	0.146
	10%	0.378	0.332	0.108	0.407	0.188	0.146
VaR(5%)							
Test is significant	1%	0.644	0.974	0.976	0.984	0.990	0.812
	5%	0.653	0.976	0.977	0.988	0.993	0.819
	10%	0.654	0.976	0.978	0.988	0.994	0.819
Prefer 1st model	1%	0.620	0.781	0.764	0.685	0.668	0.483
	5%	0.617	0.780	0.765	0.683	0.669	0.485
	10%	0.616	0.780	0.764	0.683	0.669	0.485
Prefer 2nd model	1%	0.380	0.219	0.236	0.315	0.332	0.517
	5%	0.383	0.220	0.235	0.317	0.331	0.515
	10%	0.384	0.220	0.236	0.317	0.331	0.515
VaR(10%)							
Test is significant	1%	0.746	0.993	0.991	0.988	0.993	0.929
	5%	0.748	0.996	0.994	0.992	0.994	0.937
	10%	0.751	0.996	0.995	0.993	0.996	0.938
Prefer 1st model	1%	0.618	0.790	0.725	0.685	0.624	0.405
	5%	0.616	0.787	0.722	0.684	0.624	0.406
	10%	0.617	0.787	0.722	0.684	0.623	0.405
Prefer 2nd model	1%	0.382	0.210	0.275	0.315	0.376	0.595
	5%	0.384	0.213	0.278	0.316	0.376	0.594
	10%	0.383	0.213	0.278	0.316	0.377	0.595
VaR(25%)							
Test is significant	1%	0.707	0.996	0.987	0.995	0.990	0.980
	5%	0.711	0.996	0.989	0.996	0.992	0.981
	10%	0.711	0.996	0.989	0.997	0.992	0.981
Prefer 1st model	1%	0.605	0.691	0.655	0.597	0.557	0.427
	5%	0.603	0.691	0.654	0.597	0.555	0.426
	10%	0.603	0.691	0.654	0.597	0.555	0.426
Prefer 2nd model	1%	0.395	0.309	0.345	0.403	0.443	0.573
	5%	0.397	0.309	0.346	0.403	0.445	0.574
	10%	0.397	0.309	0.346	0.403	0.445	0.574

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

56 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(1,d,1)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.049	0.052	0.014	0.001
	5%	0.032	0.033	0.008	0.001
	10%	0.022	0.021	0.005	0.001
5%	1%	0.471	0.417	0.217	0.243
	5%	0.350	0.304	0.136	0.155
	10%	0.281	0.234	0.108	0.132
10%	1%	0.644	0.572	0.349	0.437
	5%	0.468	0.416	0.232	0.280
	10%	0.369	0.319	0.171	0.216
25%	1%	0.747	0.663	0.584	0.667
	5%	0.560	0.512	0.425	0.483
	10%	0.473	0.423	0.342	0.379

DGP GARCH(1.)1 β=0.65 α=0.3 - % represent VaR p-level unless differently specified

57 - Average number of exceptions - standard deviation - *average percentage of exceptions* - 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	2.270 (1.473) *0.908*	2.606 (1.652) *1.042*	2.231 (1.480) *0.892*	1.886 (1.286) *0.754*
5% VaR	11.656 (3.168) *4.662*	12.556 (3.539) *5.022*	11.528 (3.208) *4.611*	11.598 (3.028) *4.639*

58 - Frequency of less exceptions – 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.498	0.355	0.520	0.722
5% VaR	0.599	0.279	0.454	0.494

59 – Tests - Frequency of accepting H_0 – 1000 replications – 250 forecasts					
	α	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Test of Unconditional Coverage of Kupiec					
1% VaR	1%	0.997	0.993	0.997	1.000
	5%	0.882	0.893	0.876	0.862
5% VaR	1%	0.994	0.989	0.993	0.995
	5%	0.950	0.935	0.945	0.948
Test of Independence of Christoffersen-Lopez					
1% VaR	1%	0.769	0.791	0.755	0.634
	5%	0.303	0.343	0.303	0.215
5% VaR	1%	0.973	0.975	0.973	0.820
	5%	0.912	0.917	0.918	0.647
Test of Conditional Coverage of Christoffersen-Lopez					
1% VaR	1%	0.982	0.972	0.982	0.920
	5%	0.757	0.768	0.744	0.627
5% VaR	1%	0.980	0.978	0.979	0.864
	5%	0.898	0.896	0.899	0.684

60 - Lopez loss function – frequency of model selection 1000 replications – 250 forecasts				
	Fitted models			
	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1% VaR	0.208	0.130	0.277	0.617
5% VaR	0.208	0.053	0.167	0.572

61 – Loss functions - Frequency of model selection – 1000 replications – 250 forecasts					
		Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
Loss Function 1: absolute value of return VaR measure ratio					
1% VaR	E	0.205	0.125	0.255	0.647
	T	0.127	0.017	0.331	0.525
5% VaR	E	0.184	0.078	0.257	0.481
	T	0.127	0.017	0.331	0.525
Loss Function 2: square return-VaR normalized by absolute VaR measure					
1% VaR	E	0.199	0.122	0.258	0.653
	T	0.001	0.536	0.000	0.463
5% VaR	E	0.165	0.017	0.301	0.517
	T	0.001	0.527	0.000	0.472
Loss Function 3: absolute of return-VaR					
1% VaR	E	0.211	0.120	0.282	0.619
	T	0.001	0.545	0.000	0.454
5% VaR	E	0.224	0.017	0.377	0.382
	T	0.001	0.534	0.000	0.465
Loss Function 1 + Loss Function 2					
1% VaR	E	0.203	0.126	0.258	0.645
	T	0.000	0.431	0.001	0.568
5% VaR	E	0.186	0.074	0.256	0.484
	T	0.011	0.270	0.000	0.719
Loss Function 1 + Loss Function 3					
1% VaR	E	0.205	0.130	0.265	0.632
	T	0.001	0.453	0.000	0.546
5% VaR	E	0.197	0.071	0.270	0.462
	T	0.000	0.288	0.000	0.712
Loss Function 2 + Loss Function 3					
1% VaR	E	0.207	0.119	0.279	0.627
	T	0.001	0.538	0.000	0.461
5% VaR	E	0.193	0.014	0.356	0.437
	T	0.001	0.533	0.000	0.466
Loss Function 1 + Loss Function 2 + Loss Function 3					
1% VaR	E	0.207	0.129	0.264	0.632
	T	0.001	0.506	0.000	0.493
5% VaR	E	0.195	0.066	0.268	0.471
	T	0.000	0.420	0.001	0.579

62 - Test of model comparison – 1000 replications – 250 forecasts							
Frequencies of	α	Model comparison					
		1-2	1-3	1-4	2-3	2-4	3-4
VaR(1%)							
Test is significant	1%	0.484	0.269	0.864	0.491	0.867	0.872
	5%	0.493	0.274	0.867	0.495	0.871	0.877
	10%	0.495	0.277	0.868	0.496	0.875	0.879
Prefer 1st model	1%	0.273	0.524	0.604	0.741	0.696	0.599
	5%	0.274	0.522	0.606	0.741	0.697	0.600
	10%	0.275	0.523	0.606	0.740	0.695	0.600
Prefer 2nd model	1%	0.727	0.476	0.396	0.259	0.304	0.401
	5%	0.726	0.478	0.394	0.259	0.303	0.400
	10%	0.725	0.477	0.394	0.260	0.305	0.400
VaR(5%)							
Test is significant	1%	0.738	0.348	0.992	0.748	0.992	0.988
	5%	0.741	0.348	0.993	0.754	0.995	0.992
	10%	0.744	0.349	0.995	0.757	0.995	0.994
Prefer 1st model	1%	0.348	0.540	0.637	0.678	0.727	0.635
	5%	0.350	0.540	0.637	0.675	0.726	0.634
	10%	0.349	0.542	0.637	0.674	0.726	0.634
Prefer 2nd model	1%	0.652	0.460	0.363	0.322	0.273	0.365
	5%	0.650	0.460	0.363	0.325	0.274	0.366
	10%	0.651	0.458	0.363	0.326	0.274	0.366
VaR(10%)							
Test is significant	1%	0.813	0.368	0.995	0.812	0.991	0.993
	5%	0.821	0.376	0.995	0.817	0.994	0.993
	10%	0.825	0.378	0.995	0.821	0.994	0.993
Prefer 1st model	1%	0.391	0.563	0.701	0.632	0.744	0.677
	5%	0.391	0.561	0.701	0.630	0.743	0.677
	10%	0.390	0.558	0.701	0.631	0.743	0.677
Prefer 2nd model	1%	0.609	0.438	0.299	0.368	0.256	0.323
	5%	0.609	0.439	0.299	0.370	0.257	0.323
	10%	0.610	0.442	0.299	0.369	0.257	0.323
VaR(25%)							
Test is significant	1%	0.794	0.378	0.997	0.797	0.995	0.997
	5%	0.798	0.382	0.999	0.800	0.998	0.998
	10%	0.805	0.384	0.999	0.804	0.998	0.998
Prefer 1st model	1%	0.458	0.540	0.703	0.548	0.720	0.686
	5%	0.461	0.542	0.702	0.546	0.719	0.685
	10%	0.461	0.542	0.702	0.546	0.719	0.685
Prefer 2nd model	1%	0.542	0.460	0.297	0.452	0.280	0.314
	5%	0.539	0.458	0.298	0.454	0.281	0.315
	10%	0.539	0.458	0.298	0.454	0.281	0.315

Model reference:1 - Figarch(.d.); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97)

63 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts					
VaR p-value	Test α-value	Fitted models			
		Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)	EWMA(0.97)
1%	1%	0.052	0.092	0.049	0.029
	5%	0.041	0.073	0.039	0.025
	10%	0.034	0.065	0.033	0.022
5%	1%	0.350	0.450	0.335	0.213
	5%	0.266	0.359	0.243	0.150
	10%	0.213	0.303	0.207	0.114
10%	1%	0.531	0.578	0.517	0.300
	5%	0.395	0.464	0.383	0.198
	10%	0.317	0.384	0.306	0.143
25%	1%	0.707	0.726	0.698	0.502
	5%	0.554	0.570	0.546	0.342
	10%	0.446	0.473	0.449	0.257

8.2 Tables and Graphs on Estimation and Identification of aggregated data

We report here the tables of parameter estimates and model identification based on information criteria for the aggregated data series. We also present the kernel density estimates of the parameters.

For each of the five data generating processes, indicated at the bottom of the page, tables 64, 66, 68, 70 and 72 include the Quasi Maximum Likelohood estimates of the five estimated models listed in the first rows. For each parameter we report the Montecarlo average, the standard deviation and the Root Mean Squared Error.

Tables 65, 67, 69, 71 and 73 report the frequency of model selection based on the information critria of Akaike, Hannan-Quinn, Schwarz and Shibata, together with the log-likelihoo and the four information criteria in the meantime.

The graphs are also grouped by DGP and report the Kernel density estimates of the different parameters.

DGP FIGARCH(1,d,1) – d=0.8 β=0.5 ϕ=0.05 – estimates only on aggregated data

64 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	0.00010	0.00016	-0.00031	-0.00008	-0.00010
	0.03203	0.03214	0.03471	0.03175	0.03215
	0.03201	0.03212	0.03469	0.03173	0.03213
ω	0.25777	0.28008	0.36281	0.23645	0.22373
	0.09034	0.09812	0.11246	0.09283	0.09106
	0.26371	0.28733	0.37029		
d	0.77591	0.77020	0.56290		
	0.12076	0.14085	0.09158		
	0.12308	0.14390	0.25415		
ϕ - α	0.05871			0.40185	0.56070
	0.06050			0.10245	0.10502
	0.06109				
β	0.36289	0.30630		0.56186	
	0.14277	0.16989		0.10380	
	0.19789	0.25759			

65 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.190	0.498	0.084	0.166	0.062
Hannan-Quinn	0.071	0.497	0.134	0.147	0.151
Schwarz	0.389	0.399	0.030	0.168	0.014
Shibata	0.190	0.498	0.084	0.167	0.061
LL	0.662	0.173	0.000	0.164	0.001
4 IC	0.190	0.498	0.084	0.166	0.062

DGP FIGARCH(1,d,1) – d=0.8 β=0.5 ϕ=0.3 – estimates only on aggregated data

66 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	0.00100	0.00120	0.00153	0.00171	0.00150
	0.03962	0.03994	0.04140	0.03891	0.04026
	0.03962	0.03994	0.04141	0.03893	0.04026
ω	0.25804	0.33523	0.42857	0.31402	0.29518
	0.13560	0.17301	0.17444	0.17180	0.17163
	0.28265	0.36835	0.45343		
d	0.79462	0.81233	0.63521		
	0.26084	0.16972	0.14274		
	0.26076	0.17008	0.21797		
ϕ - α	0.20869			0.46606	0.45021
	0.18782			0.15529	0.16284
	0.20876				
β	0.43239	0.25356		0.45422	
	0.19726	0.22189		0.15912	
	0.20843	0.33154			

67 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.378	0.225	0.102	0.222	0.073
Hannan-Quinn	0.256	0.182	0.202	0.211	0.149
Schwarz	0.536	0.184	0.032	0.218	0.030
Shibata	0.378	0.225	0.102	0.222	0.073
LL	0.676	0.095	0.003	0.223	0.003
4 IC	0.378	0.225	0.102	0.222	0.073

DGP FIGARCH(1,d,1) – d=0.8 β=0.5 ϕ=0 – estimates only on aggregated data

68 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	0.00165	0.00163	0.00188	0.00137	0.00153
	0.03302	0.03330	0.03449	0.03254	0.03310
	0.03305	0.03332	0.03452	0.03256	0.03312
ω	0.22918	0.27255	0.36017	0.23088	0.21694
	0.08860	0.09532	0.11183	0.09084	0.08614
	0.23640	0.27930	0.36757		
d	0.79122	0.77549	0.56146		
	0.14726	0.14049	0.09312		
	0.14745	0.14255	0.25606		
ϕ - α	0.11348			0.38744	0.57265
	0.12680			0.09584	0.10501
	0.17011				
β	0.43189	0.31831		0.57354	
	0.16916	0.18316		0.10390	
	0.18228	0.25792			

69 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.283	0.444	0.071	0.150	0.053
Hannan-Quinn	0.163	0.470	0.120	0.138	0.110
Schwarz	0.458	0.348	0.025	0.154	0.016
Shibata	0.285	0.443	0.070	0.150	0.053
LL	0.700	0.149	0.000	0.152	0.000
4 IC	0.283	0.444	0.071	0.150	0.053

DGP FIGARCH(1,d,1) – d=0.4 β=0.3 ϕ=0.2 – estimates only on aggregated data

70 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	0.00097	0.00098	0.00104	0.00104	0.00105
	0.03327	0.03336	0.03340	0.03334	0.03404
	0.03327	0.03336	0.03340	0.03334	0.03404
ω	0.20273	0.31184	0.43336	0.13105	0.05639
	0.10371	0.11526	0.12932	0.09154	0.04883
	0.21884	0.32307	0.44265		
d	0.32851	0.29031	0.22327		
	0.10450	0.09729	0.04623		
	0.12657	0.14659	0.18267		
ϕ - α	0.22204			0.11977	0.86859
	0.12536			0.04315	0.05674
	0.12722				
β	0.38664	0.12814		0.82572	
	0.15867	0.10463		0.07622	
	0.18072	0.20118			

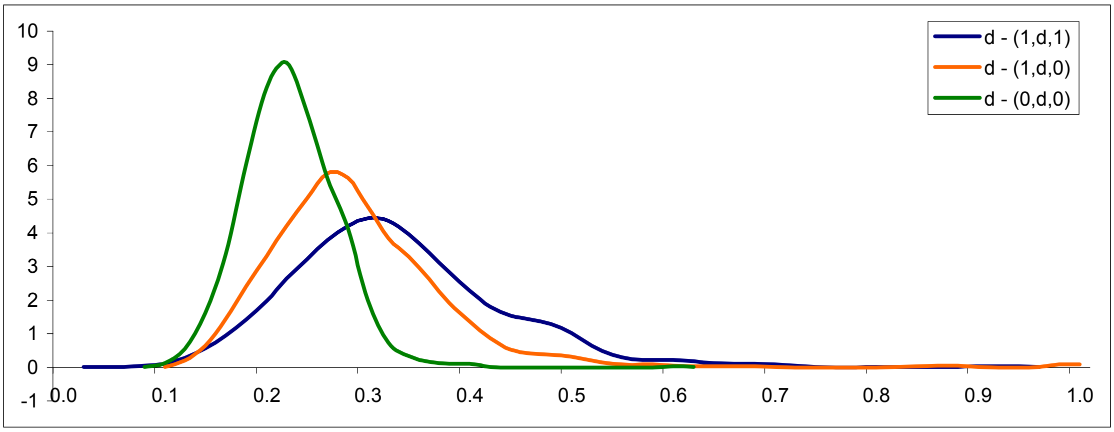
71 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.341	0.314	0.197	0.139	0.009
Hannan-Quinn	0.173	0.324	0.339	0.148	0.016
Schwarz	0.584	0.213	0.079	0.120	0.004
Shibata	0.341	0.314	0.197	0.139	0.009
LL	0.818	0.060	0.000	0.120	0.002
4 IC	0.341	0.314	0.197	0.139	0.009

DGP FIGARCH(1,d,1) – d=0.4 β=0.3 ϕ=0 – estimates only on aggregated data

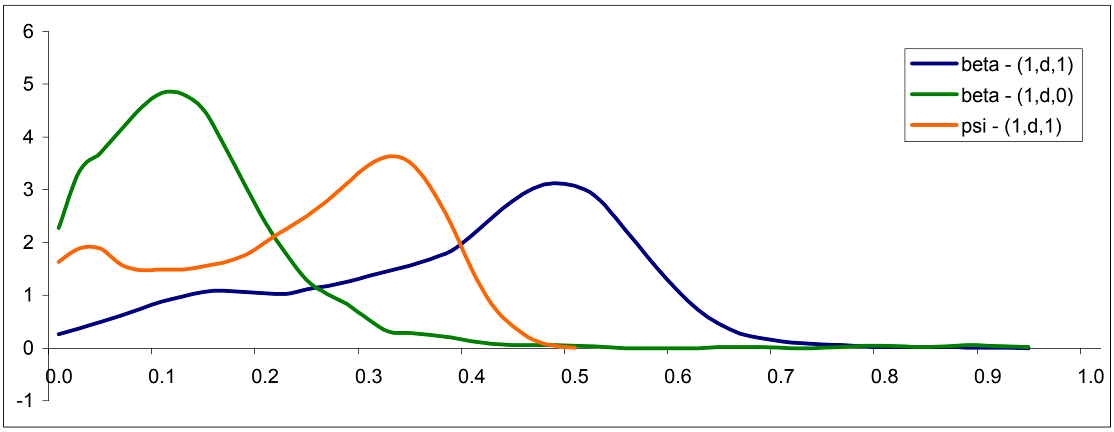
72 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	-0.00099	-0.00091	-0.00071	-0.00089	-0.00088
	0.02985	0.02998	0.03008	0.02988	0.03029
	0.02985	0.02998	0.03007	0.02987	0.03029
ω	0.18721	0.32086	0.46797	0.10599	0.02968
	0.09307	0.10479	0.11640	0.07272	0.02271
	0.20014	0.32803	0.47251		
d	0.29960	0.25335	0.19294		
	0.08482	0.06894	0.03795		
	0.13141	0.16203	0.21050		
ϕ - α	0.24367			0.09594	0.90282
	0.12175			0.03142	0.03998
	0.27237				
β	0.41204	0.12287		0.85146	
	0.15374	0.07730		0.06428	
	0.19018	0.19325			

73 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.367	0.279	0.155	0.193	0.006
Hannan-Quinn	0.167	0.308	0.298	0.206	0.021
Schwarz	0.590	0.188	0.051	0.171	0.000
Shibata	0.367	0.280	0.154	0.193	0.006
LL	0.782	0.062	0.000	0.156	0.000
4 IC	0.367	0.279	0.155	0.193	0.006

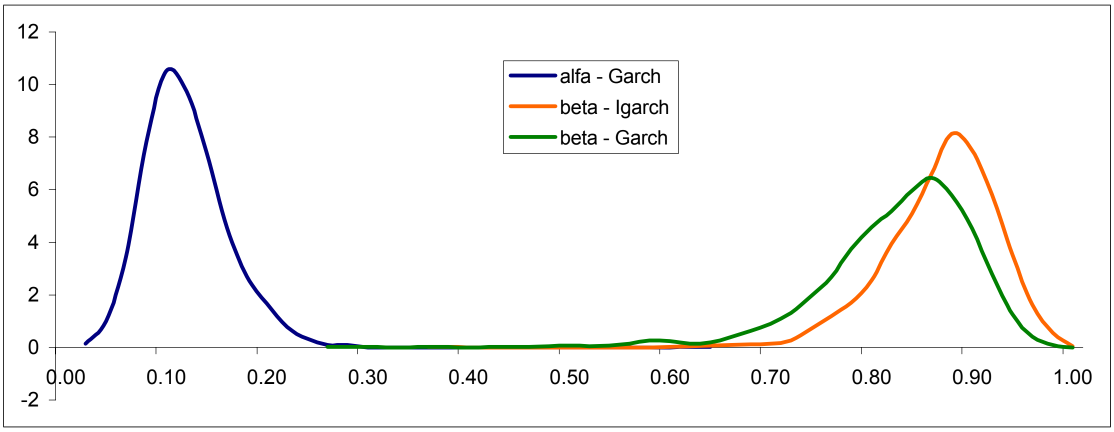
DGP - FIGARCH(0.3,0.4,0.2)



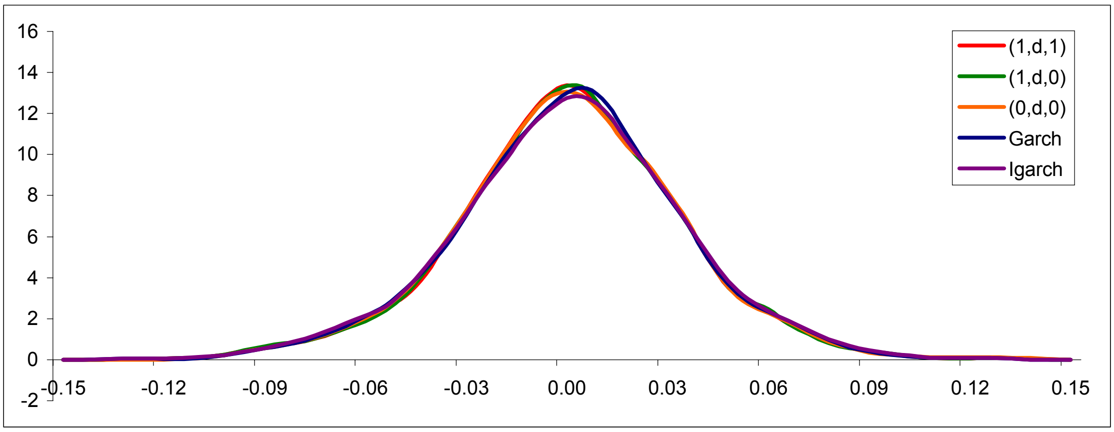
Graph : long memory parameter estimates



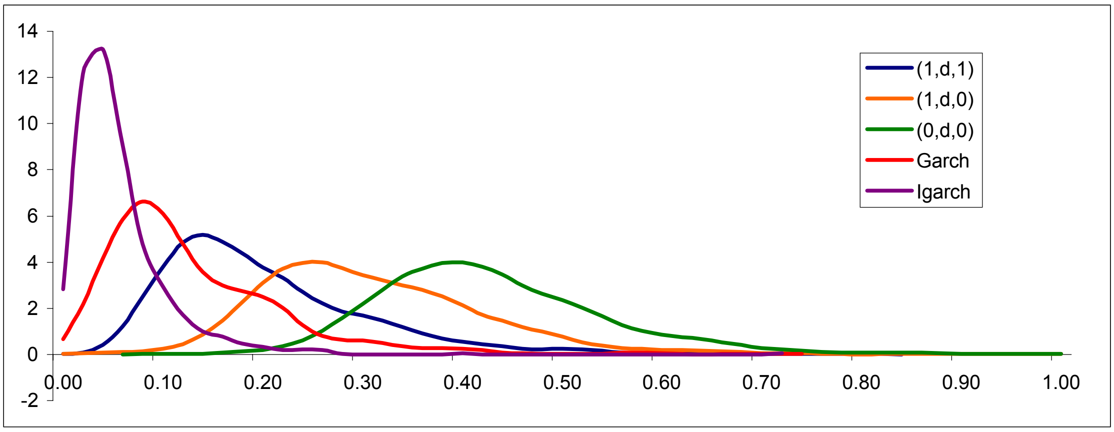
Graph : estimates of the FIGARCH short memory parameters



Graph : parameters of GARCH and IGARCH

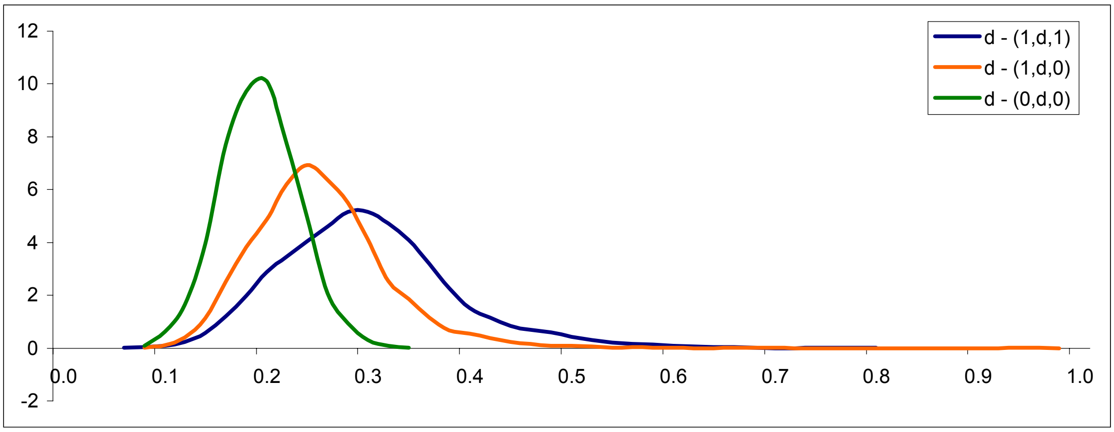


Graph : in mean constant

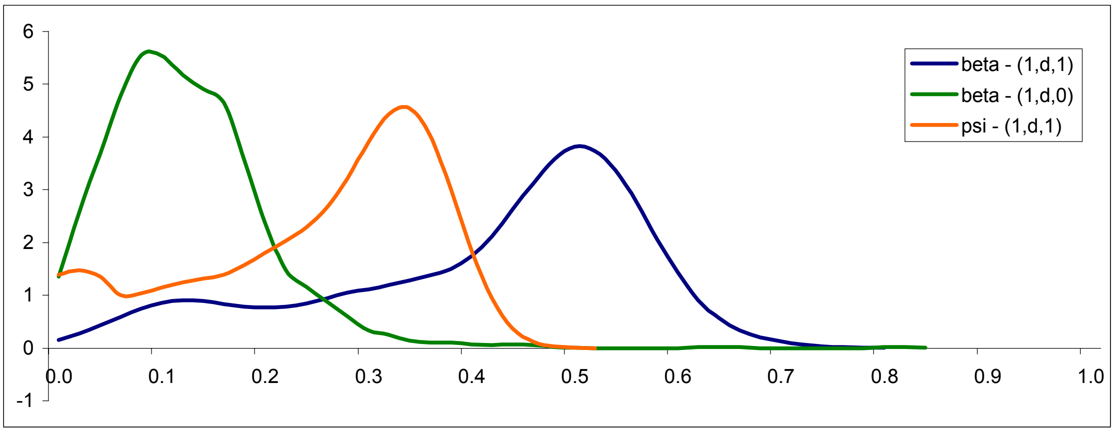


Graph : constant in variance

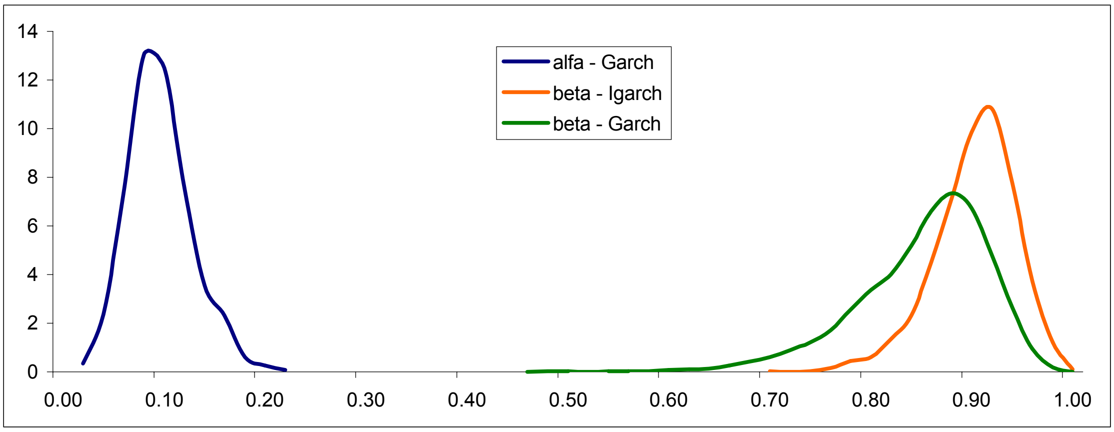
DGP - FIGARCH(0.3,0.4,0)



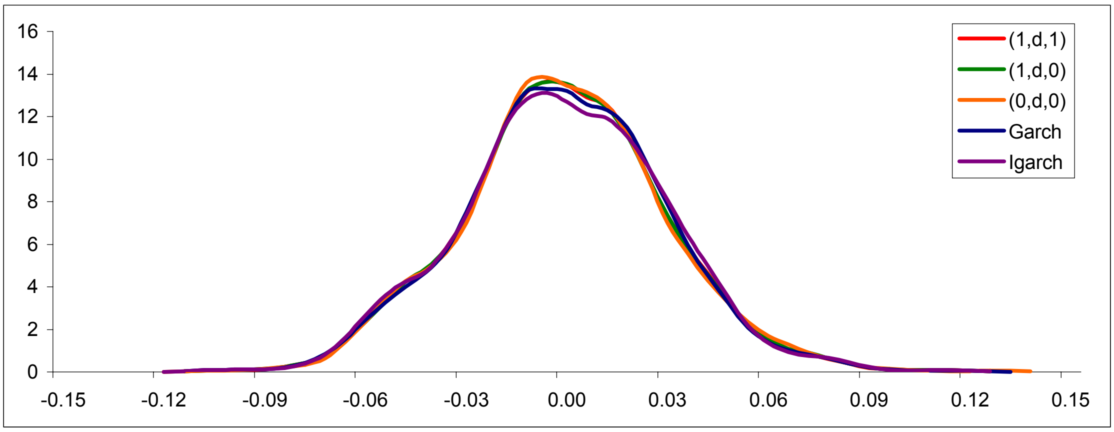
Graph : long memory parameter estimates



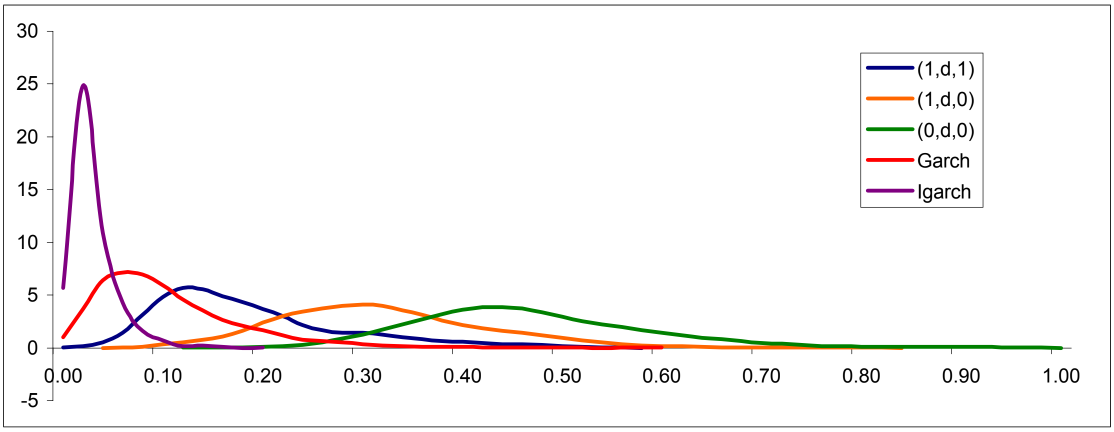
Graph : estimates of the FIGARCH short memory parameters



Graph : parameters of GARCH and IGARCH

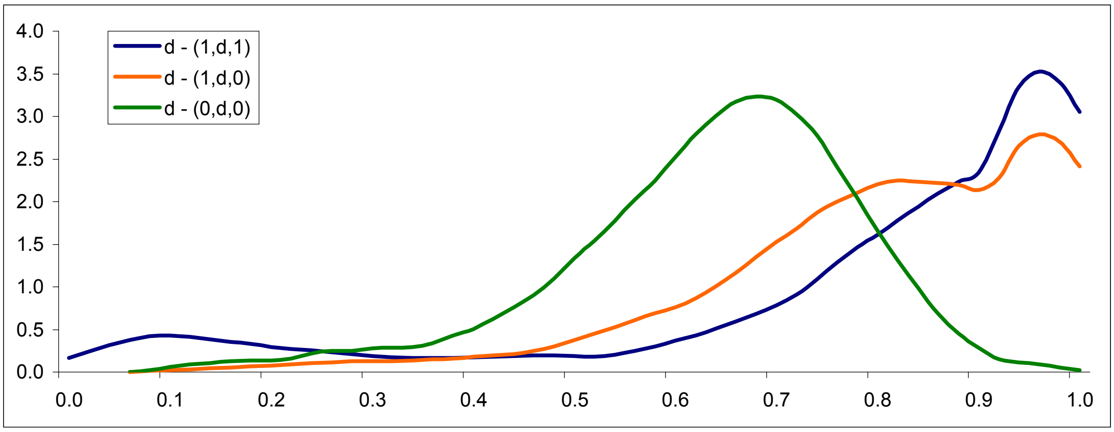


Graph : in mean constant

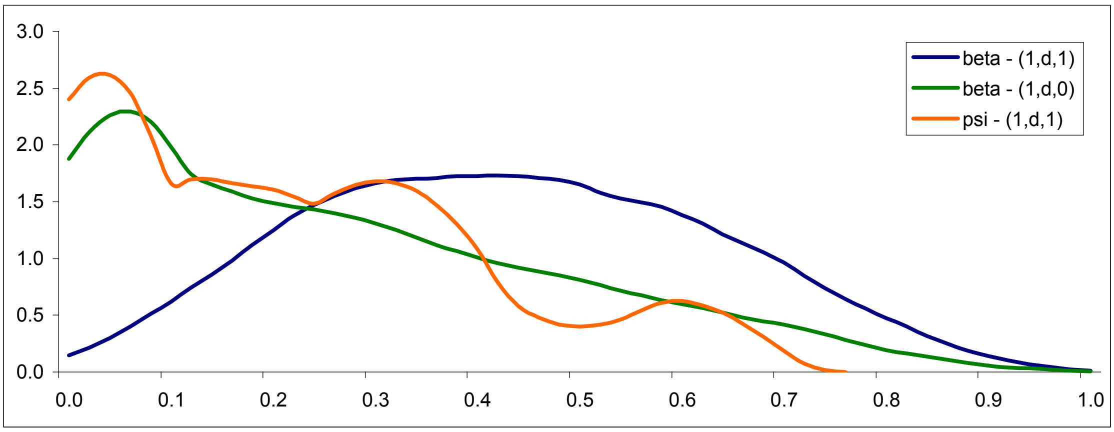


Graph : constant in variance

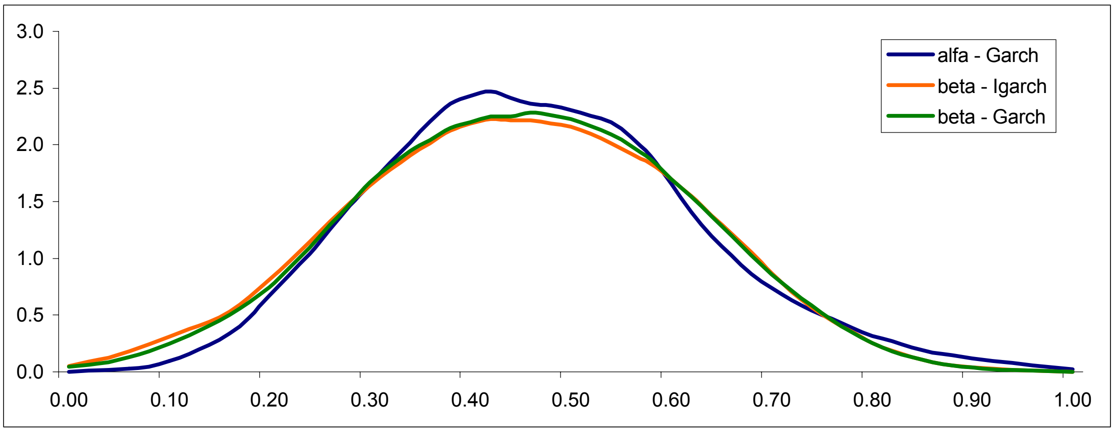
DGP - FIGARCH(0.5,0.8,0.3)



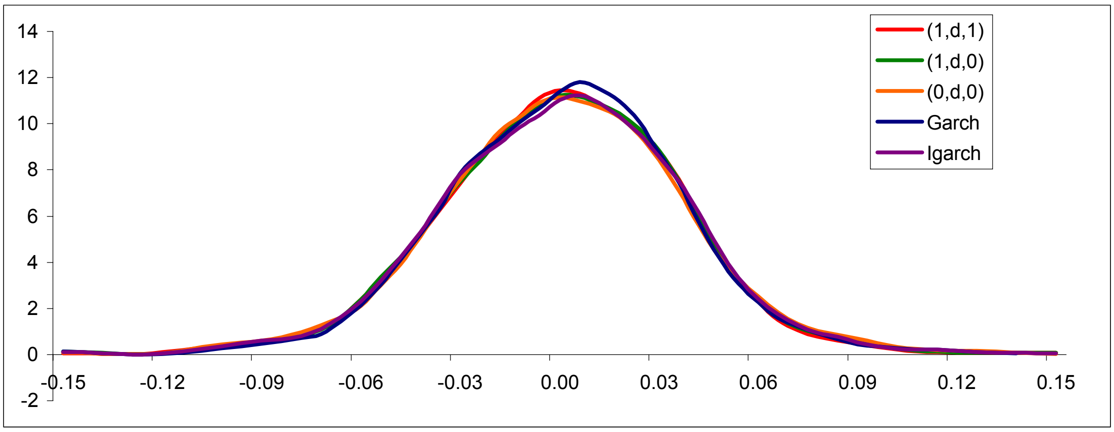
Graph : long memory parameter estimates



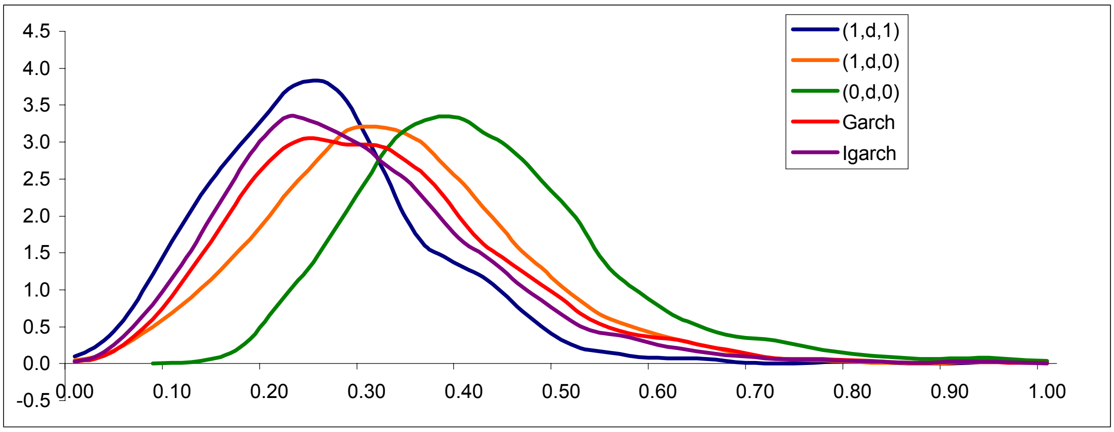
Graph : estimates of the FIGARCH short memory parameters



Graph : parameters of GARCH and IGARCH

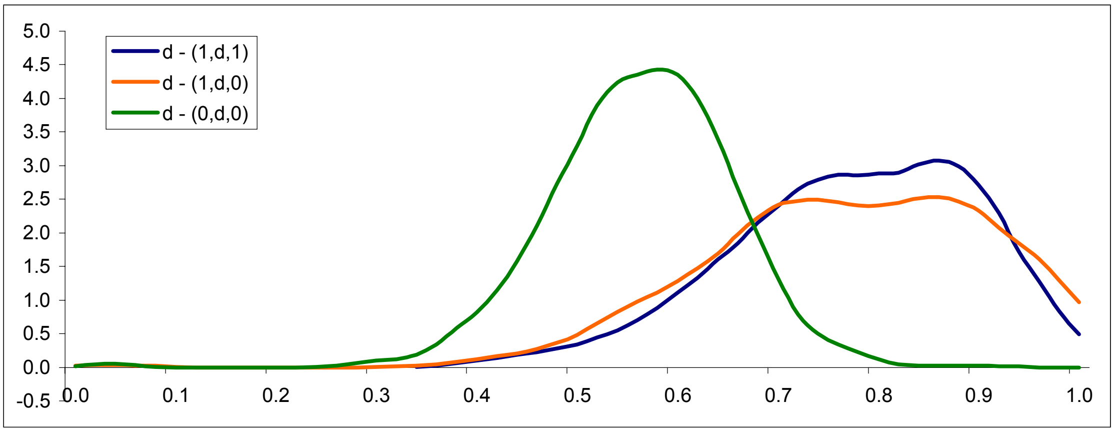


Graph : in mean constant

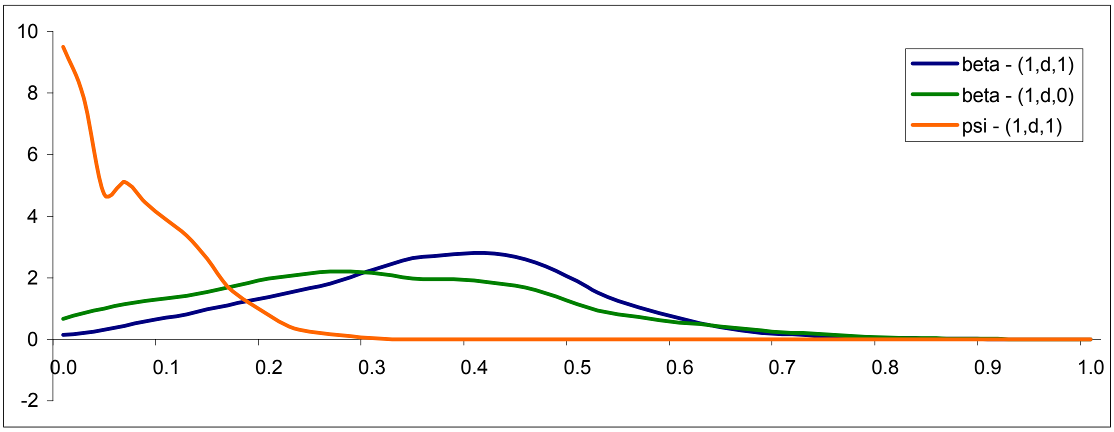


Graph : constant in variance

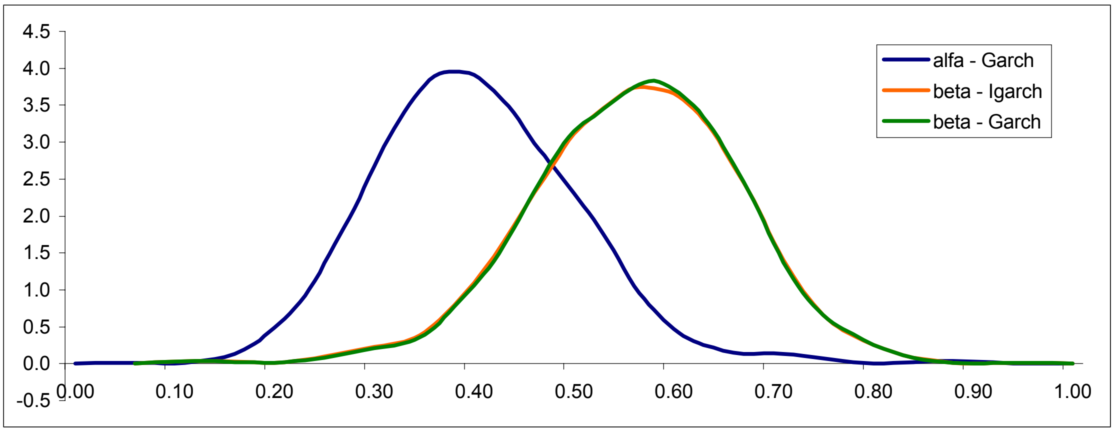
DGP - FIGARCH(0.5,0.8,0.05)



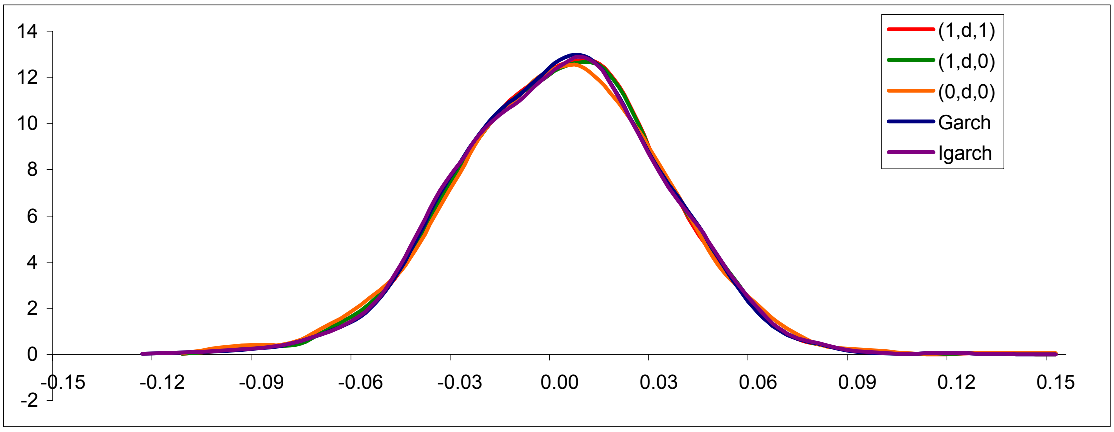
Graph : long memory parameter estimates



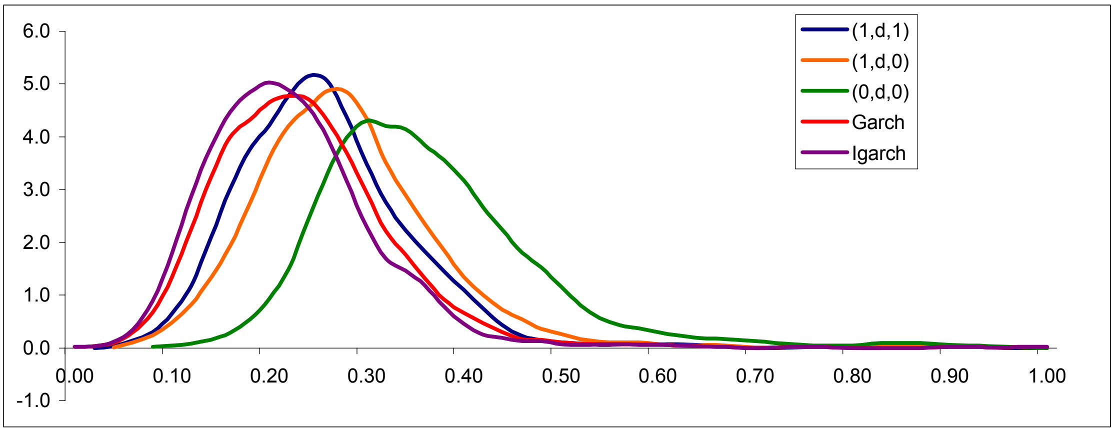
Graph : estimates of the FIGARCH short memory parameters



Graph : parameters of GARCH and IGARCH

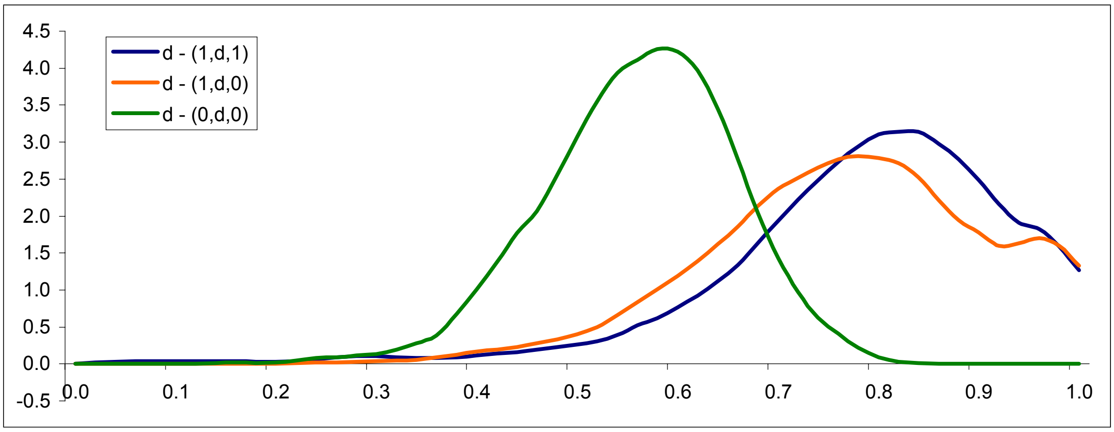


Graph : in mean constant

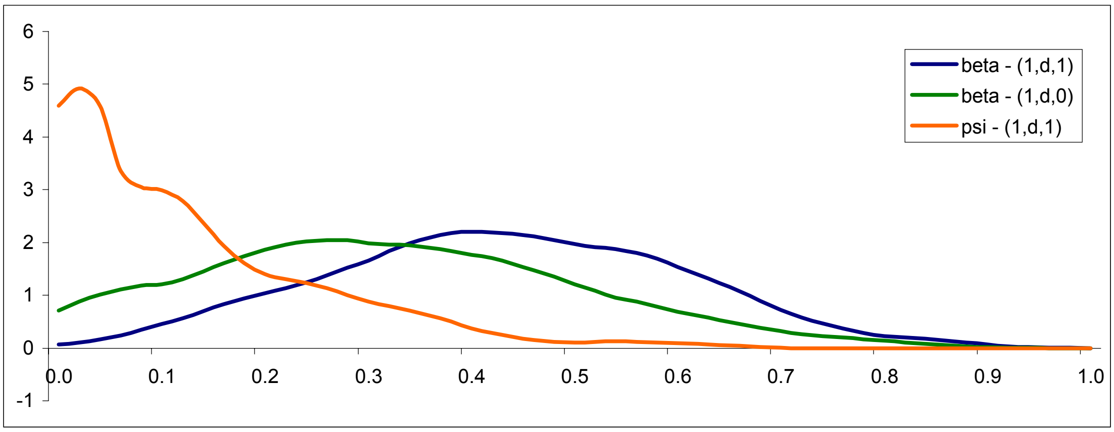


Graph : constant in variance

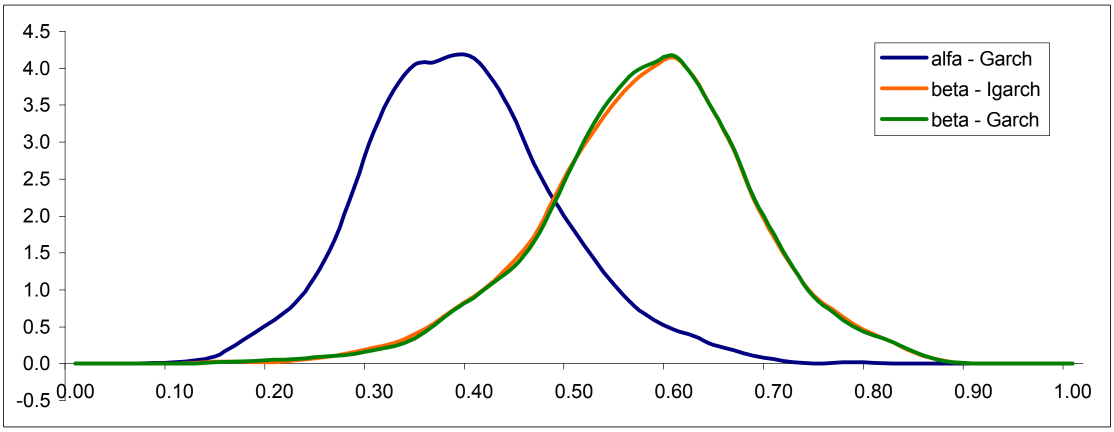
DGP - FIGARCH(0.5,0.8,0)



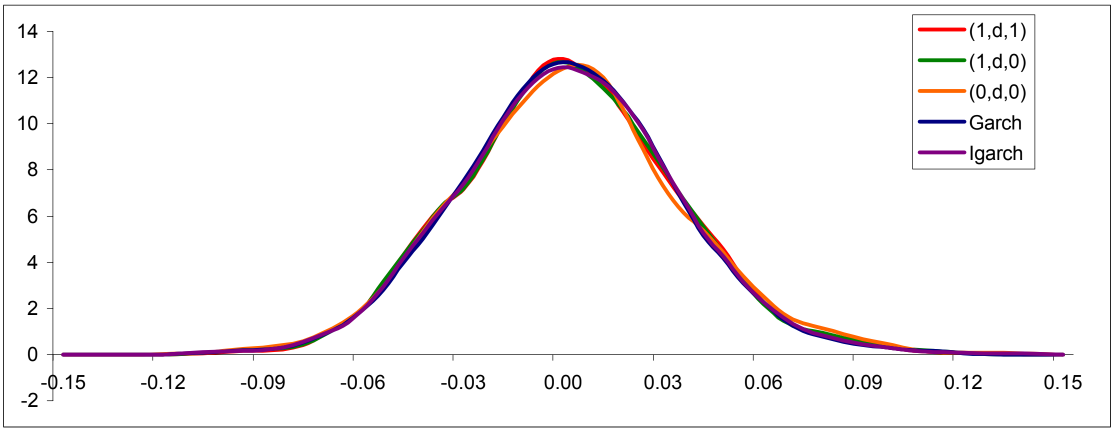
Graph : long memory parameter estimates



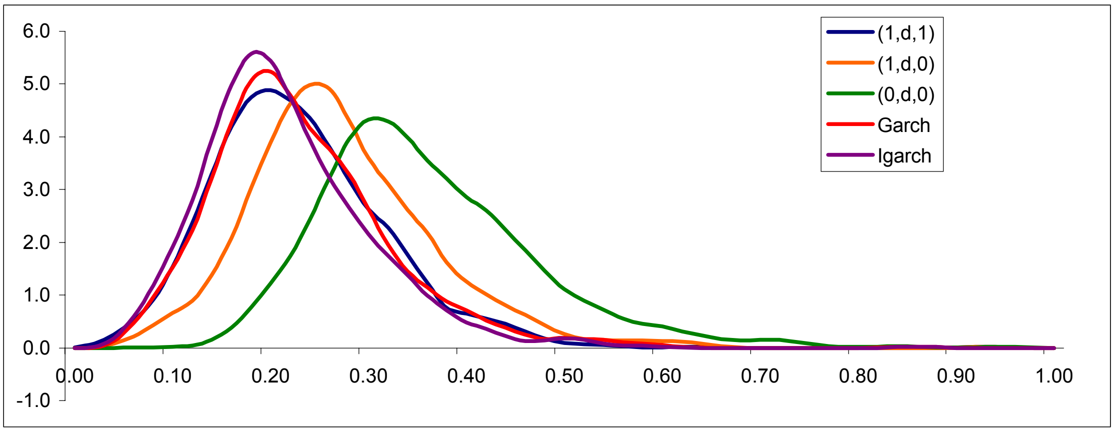
Graph : estimates of the FIGARCH short memory parameters



Graph : parameters of GARCH and IGARCH



Graph : in mean constant



Graph : constant in variance

8.3 Tables on Value-at-Risk comparison of aggregated data

In the following pages you will find the tables for the Montecarlo described in section 5. The tables are grouped by DGP, listed in the first row at the beginnig of each group. In the next rows we just describe table contents:

- Tables 74, 81, 88, 95, 102, 109, 116, 123, 130, 137: the tables list for each of the six model considered and two level of Value-at-Risk coverage (1% and 5%) the average number of exceptions, its standard deviation and the average percentage of exceptions for an experiment conducted on 1000 replications and for a sample of 250 1-day-ahead forecasts, using the backtesting approach.

- Tables 75, 82, 89, 96, 103, 110, 117, 124, 131, 138: in this case for the models and VaR coverage levels we report frequency of model selction based on counting axceptions, a model is preferred to the others when its number of exceptions is lower. Given that the exceptions are integer numbers the frequencies sum may be higher than 1.

- Tables 76, 83, 90, 97, 104, 111, 118, 125, 132, 139: these tables report the frequencies of accepting the null hypothesis of the tests of unconditional coverage of Kupiec (1995 - null is correct coverage), the test of independence of Christoffersen-Lopez (1998 - null is independence) and the test of conditional coverage of Christoffersen-Lopez (1998 - null is again correct coverage).

- Tables 77, 84, 91, 98, 105, 112, 119, 126, 133, 140: these are the first tables on the loss funcions results, they report the frequency of model selection based on the application of the loss function suggested by Lopez (1999) that focus only on exceptions. Given that the parameters of GARCH(1,1) and IGARCH(1,1) are often very close this cause an identical loss function for the two models, same exceptions and same forecast, therefore the frequencies sum may be higher than 1.

- Tables 78, 85, 92, 99, 106, 113, 120, 127, 134, 141: in these tables we report the frequencies of selection based on our alternative loss functions, that focus on exceptions (rows labelled with an E) and on the whole backtesting sample, 250 observations (rows labelled with a T). Again the closeness of GARCH and IGARCH may cause a sum of frequencies over 1. The results are grouped by loss functions and combination of loss functions as described in the italics rows. Models are identified by a number, the leged is at the bottom of the table.

- Tables 79, 86, 93, 100, 107, 114, 121, 128, 135, 142: in these tables and in the next group we deal with the test of Christoffersen et al. (2001). These tables report the result of the test of model comparison and consider four different Value-at-Risk coverage. For each one of these levels of confidence the tables report the test results for a pairwise comparison between models, using the legend at the bottom of the table. For each level and comparison we reported the frequence of accepting the test (null hypothesis is the the two models do not equally match the efficiency moment condition of Christoffersen et al. 2001, this is implied by a significat test statistic) and then usign the sign of the test statistic we report the percentage of preference of the first or of the second model. The percentage is computed using only the cases when the test null hypothesis is accepted. In all cases we considered three level of confidence for the test statistics, the percentage indicated with test α-value. Models are identified by a number, the leged is at the bottom of the previuos tables.

- Tables 80, 87, 94, 101, 108, 115, 122, 129, 136, 143: in these last group of tables we report the second test suggested by Christoffersen et al. (2001) the test on Value-at-Risk specification. In these tables we report for the different model considered at the four level of VaR confidence used in the previuos tables the frequency of accepting the null hypothesis of the test (null is that the VaR is correctly specified). As in the previuos case we report three level of confidence for the test statistic. Models are identified by a number, the leged is at the bottom of the previuos tables.

- After the last table of each group we report the preference ordering among the different models (if it exist) derived from the result of the model comparison test.

AGGREGATED ESTIMATES NON-AGGREGATED COMPARISON

DGP FIGARCH(1,d,0) d=0.4 β=0.3 - % represent VaR p-level unless differently specified

74 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.521 (1.954) *1.408*	2.387 (1.379) *0.955*	3.376 (1.868) *1.350*	3.751 (1.947) *1.500*	5.154 (2.689) *2.062*	3.309 (1.995) *1.324*
5% VaR	11.372 (3.647) *4.549*	11.461 (2.967) *4.584*	12.333 (3.469) *4.933*	13.202 (3.554) *5.281*	16.268 (4.872) *6.507*	12.342 (4.020) *4.937*

75 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.248	0.690	0.280	0.201	0.069	0.364
5% VaR	0.362	0.579	0.239	0.133	0.064	0.203

76 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.964	0.999	0.978	0.967	0.824	0.968
	5%	0.898	0.929	0.911	0.892	0.712	0.882
5% VaR	1%	0.971	0.992	0.992	0.987	0.896	0.974
	5%	0.900	0.958	0.939	0.930	0.742	0.880
Test of independence: Null							
1% VaR	1%	0.846	0.739	0.998	0.997	0.996	0.996
	5%	0.831	0.730	0.981	0.987	0.982	0.981
5% VaR	1%	0.958	0.938	0.994	0.995	0.994	0.997
	5%	0.888	0.842	0.975	0.979	0.956	0.979
Test of conditional coverage: Null							
1% VaR	1%	0.937	0.954	0.989	0.979	0.879	0.980
	5%	0.776	0.730	0.971	0.958	0.802	0.962
5% VaR	1%	0.943	0.957	0.989	0.990	0.917	0.979
	5%	0.831	0.845	0.951	0.945	0.797	0.923

77 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.031	0.258	0.226	0.106	0.010	0.465
5% VaR	0.031	0.240	0.213	0.083	0.002	0.431

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

78 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.076	0.637	0.098	0.062	0.010	0.213
	T	0.444	0.335	0.105	0.000	0.000	0.116
5% VaR	E	0.240	0.445	0.088	0.019	0.000	0.208
	T	0.444	0.335	0.105	0.000	0.000	0.116
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.052	0.721	0.081	0.054	0.010	0.178
	T	0.006	0.119	0.000	0.012	0.863	0.000
5% VaR	E	0.044	0.679	0.049	0.020	0.000	0.208
	T	0.002	0.160	0.000	0.005	0.833	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.050	0.704	0.076	0.055	0.010	0.201
	T	0.011	0.108	0.000	0.013	0.868	0.000
5% VaR	E	0.105	0.556	0.062	0.020	0.000	0.257
	T	0.009	0.114	0.000	0.012	0.865	0.000
Loss function 1+2							
1% VaR	E	0.068	0.663	0.088	0.060	0.010	0.207
	T	0.007	0.131	0.000	0.010	0.852	0.000
5% VaR	E	0.158	0.558	0.071	0.014	0.000	0.199
	T	0.003	0.256	0.000	0.002	0.738	0.001
Loss function 1+3							
1% VaR	E	0.066	0.659	0.090	0.061	0.010	0.210
	T	0.012	0.114	0.000	0.012	0.862	0.000
5% VaR	E	0.178	0.499	0.076	0.022	0.000	0.225
	T	0.007	0.145	0.000	0.008	0.840	0.000
Loss function 2+3							
1% VaR	E	0.049	0.712	0.079	0.053	0.010	0.193
	T	0.009	0.113	0.000	0.013	0.865	0.000
5% VaR	E	0.070	0.628	0.053	0.024	0.000	0.225
	T	0.006	0.126	0.000	0.010	0.858	0.000
Loss function 1+2+3							
1% VaR	E	0.062	0.674	0.081	0.057	0.010	0.212
	T	0.007	0.116	0.000	0.012	0.865	0.000
5% VaR	E	0.144	0.562	0.070	0.018	0.000	0.206
	T	0.005	0.151	0.000	0.007	0.837	0.000

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

79 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.023	0.002	0.013	0.017	0.029	0.009
	5%	0.012	0.000	0.007	0.008	0.014	0.005
	10%	0.007	0.000	0.006	0.006	0.008	0.003
5%	1%	0.144	0.085	0.199	0.195	0.215	0.192
	5%	0.068	0.037	0.118	0.110	0.099	0.093
	10%	0.039	0.023	0.078	0.066	0.058	0.058

Preference relation among the models as inferred from table 80

4,5 1 2,3,6 + 3,4,5,6 2 + 4,5,6 3 +5 4 6 + 5 6 5 4 1 6 3 2

that is

HF Garch(1,1) square root HF Figarch(1,d,0) sum Figarch(1,d,0) HF Garch(1,1) sum HF Figarch(1,d,0) square root EWMA(0.97)

80 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.796	0.914	0.949	0.957	0.945	0.912	0.949	0.967	0.945	0.557	0.551	0.858	0.860	0.794	0.827
	5%	0.799	0.915	0.951	0.957	0.946	0.912	0.951	0.967	0.946	0.565	0.554	0.859	0.863	0.798	0.829
	10%	0.802	0.916	0.952	0.957	0.947	0.913	0.951	0.967	0.946	0.572	0.558	0.859	0.866	0.800	0.833
Prefer 1st model	1%	0.665	0.493	0.459	0.294	0.520	0.261	0.241	0.130	0.293	0.218	0.216	0.206	0.276	0.492	0.673
	5%	0.666	0.494	0.461	0.294	0.521	0.261	0.241	0.130	0.294	0.221	0.219	0.207	0.278	0.496	0.673
	10%	0.667	0.494	0.461	0.294	0.522	0.261	0.241	0.130	0.294	0.222	0.219	0.207	0.280	0.498	0.675
Prefer 2nd model	1%	0.131	0.421	0.490	0.663	0.425	0.651	0.708	0.837	0.652	0.339	0.335	0.652	0.584	0.302	0.154
	5%	0.133	0.421	0.490	0.663	0.425	0.651	0.710	0.837	0.652	0.344	0.335	0.652	0.585	0.302	0.156
	10%	0.135	0.422	0.491	0.663	0.425	0.652	0.710	0.837	0.652	0.350	0.339	0.652	0.586	0.302	0.158
VaR 5%																
Test is signif.	1%	0.933	0.976	0.984	0.990	0.979	0.989	0.990	0.993	0.988	0.803	0.804	0.966	0.958	0.932	0.953
	5%	0.937	0.978	0.986	0.992	0.986	0.994	0.992	0.994	0.989	0.803	0.807	0.970	0.960	0.938	0.956
	10%	0.940	0.981	0.986	0.993	0.989	0.995	0.993	0.994	0.990	0.806	0.809	0.971	0.962	0.941	0.957
Prefer 1st model	1%	0.555	0.396	0.409	0.356	0.453	0.331	0.334	0.291	0.384	0.429	0.429	0.409	0.431	0.508	0.553
	5%	0.556	0.396	0.409	0.356	0.455	0.332	0.334	0.292	0.385	0.429	0.431	0.411	0.433	0.512	0.555
	10%	0.558	0.396	0.409	0.356	0.457	0.333	0.334	0.292	0.386	0.431	0.432	0.411	0.435	0.514	0.556
Prefer 2nd model	1%	0.378	0.580	0.575	0.634	0.526	0.658	0.656	0.702	0.604	0.374	0.375	0.557	0.527	0.424	0.400
	5%	0.381	0.582	0.577	0.636	0.531	0.662	0.658	0.702	0.604	0.374	0.376	0.559	0.527	0.426	0.401
	10%	0.382	0.585	0.577	0.637	0.532	0.662	0.659	0.702	0.604	0.375	0.377	0.560	0.527	0.427	0.401

DGP FIGARCH(1,d,0) d=0.8 β=0.5 ϕ=0 - % represent VaR p-level unless differently specified

81 - Average number of exceptions (standard deviation) [mean percentage] 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.883 (1.971) *1.553*	3.678 (1.747) *1.471*	4.918 (2.207) *1.967*	6.345 (2.479) *2.538*	6.465 (2.785) *2.586*	5.575 (2.480) *2.230*
5% VaR	9.572 (3.183) *3.829*	10.789 (2.915) *4.316*	13.358 (3.626) *5.343*	16.044 (3.839) *6.418*	16.312 (4.443) *6.525*	14.605 (3.962) *5.842*

82 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.484	0.553	0.270	0.057	0.066	0.151
5% VaR	0.560	0.442	0.192	0.033	0.029	0.071

83 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.964	0.974	0.875	0.691	0.682	0.795
	5%	0.869	0.920	0.762	0.544	0.540	0.685
5% VaR	1%	0.958	0.990	0.989	0.951	0.901	0.965
	5%	0.836	0.938	0.929	0.813	0.782	0.873
Test of independence: Null							
1% VaR	1%	0.874	0.756	0.997	0.997	0.997	0.997
	5%	0.538	0.384	0.987	0.987	0.989	0.988
5% VaR	1%	0.965	0.844	0.997	0.997	0.996	0.997
	5%	0.887	0.692	0.975	0.974	0.968	0.981
Test of conditional coverage: Null							
1% VaR	1%	0.937	0.813	0.940	0.801	0.777	0.871
	5%	0.764	0.704	0.859	0.677	0.666	0.784
5% VaR	1%	0.934	0.875	0.989	0.967	0.930	0.979
	5%	0.771	0.705	0.945	0.864	0.828	0.914

84 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.063	0.256	0.416	0.036	0.009	0.240
5% VaR	0.063	0.251	0.415	0.032	0.004	0.235

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

85 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.252	0.546	0.161	0.007	0.007	0.047
	T	0.553	0.444	0.002	0.000	0.000	0.001
5% VaR	E	0.467	0.442	0.071	0.001	0.000	0.019
	T	0.553	0.444	0.002	0.000	0.000	0.001
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.133	0.604	0.184	0.016	0.006	0.077
	T	0.001	0.051	0.000	0.146	0.802	0.000
5% VaR	E	0.162	0.537	0.212	0.004	0.001	0.084
	T	0.001	0.062	0.000	0.117	0.820	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.156	0.503	0.247	0.012	0.006	0.096
	T	0.003	0.044	0.000	0.162	0.791	0.000
5% VaR	E	0.280	0.350	0.272	0.004	0.000	0.094
	T	0.003	0.044	0.000	0.160	0.793	0.000
Loss function 1+2							
1% VaR	E	0.188	0.612	0.151	0.011	0.006	0.052
	T	0.001	0.080	0.000	0.072	0.847	0.000
5% VaR	E	0.308	0.539	0.125	0.000	0.000	0.028
	T	0.002	0.196	0.008	0.012	0.780	0.002
Loss function 1+3							
1% VaR	E	0.197	0.544	0.196	0.012	0.006	0.065
	T	0.002	0.071	0.000	0.096	0.831	0.000
5% VaR	E	0.405	0.417	0.147	0.000	0.000	0.031
	T	0.002	0.100	0.000	0.035	0.863	0.000
Loss function 2+3							
1% VaR	E	0.140	0.558	0.221	0.010	0.006	0.085
	T	0.001	0.048	0.000	0.155	0.796	0.000
5% VaR	E	0.201	0.479	0.231	0.005	0.000	0.084
	T	0.000	0.051	0.000	0.138	0.811	0.000
Loss function 1+2+3							
1% VaR	E	0.176	0.586	0.179	0.012	0.006	0.061
	T	0.001	0.057	0.000	0.120	0.822	0.000
5% VaR	E	0.312	0.496	0.159	0.000	0.000	0.033
	T	0.001	0.085	0.000	0.065	0.849	0.000

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

86 - Test of VaR model specification (null: VaR(p) is correctly specified)
Frequency of accepting H_0 – 1000 replications – 250 daily forecasts

VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.021	0.011	0.025	0.015	0.023	0.020
	5%	0.005	0.006	0.008	0.005	0.012	0.007
	10%	0.004	0.005	0.006	0.004	0.008	0.006
5%	1%	0.060	0.062	0.115	0.107	0.146	0.132
	5%	0.038	0.044	0.059	0.055	0.076	0.067
	10%	0.034	0.034	0.037	0.031	0.046	0.039

Preference relation among the models as inferred from table 87

3,4,5,6 1 2 + 3,4,5,6 2 + 4,5,6 3 +5,6 4 + 5 6   5 6 4 3 1 2

that is

HF Garch(1,1) square root   HF Garch(1,1) sum   HF Figarch(1,d,0) sum   HF Figarch(1,d,0) square root   Figarch(1,d,0)   EWMA(0.97)

87 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.896	0.981	0.992	0.992	0.988	0.980	0.993	0.984	0.983	0.809	0.807	0.780	0.823	0.778	0.671
	5%	0.897	0.982	0.994	0.993	0.989	0.983	0.996	0.986	0.986	0.813	0.809	0.785	0.832	0.784	0.678
	10%	0.899	0.982	0.994	0.993	0.989	0.985	0.996	0.988	0.987	0.816	0.815	0.786	0.832	0.785	0.683
Prefer 1st model	1%	0.590	0.413	0.321	0.278	0.341	0.340	0.263	0.231	0.274	0.366	0.366	0.285	0.399	0.479	0.633
	5%	0.590	0.412	0.321	0.278	0.341	0.339	0.262	0.230	0.273	0.367	0.366	0.287	0.399	0.480	0.633
	10%	0.590	0.412	0.321	0.278	0.341	0.340	0.262	0.232	0.274	0.368	0.368	0.288	0.399	0.479	0.631
Prefer 2nd model	1%	0.410	0.587	0.679	0.722	0.659	0.660	0.737	0.769	0.726	0.634	0.634	0.715	0.601	0.521	0.367
	5%	0.410	0.588	0.679	0.722	0.659	0.661	0.738	0.770	0.727	0.633	0.634	0.713	0.601	0.520	0.367
	10%	0.410	0.588	0.679	0.722	0.659	0.660	0.738	0.768	0.726	0.632	0.632	0.712	0.601	0.521	0.369
VaR 5%																
Test is signif.	1%	0.973	0.994	0.997	0.995	0.995	0.989	0.993	0.992	0.994	0.922	0.923	0.909	0.922	0.888	0.795
	5%	0.974	0.994	0.997	0.995	0.995	0.991	0.994	0.993	0.996	0.926	0.925	0.909	0.923	0.889	0.800
	10%	0.978	0.995	0.997	0.995	0.996	0.992	0.994	0.993	0.996	0.927	0.925	0.910	0.924	0.892	0.803
Prefer 1st model	1%	0.506	0.359	0.316	0.264	0.315	0.350	0.310	0.267	0.305	0.531	0.533	0.366	0.333	0.341	0.509
	5%	0.506	0.359	0.316	0.264	0.315	0.350	0.310	0.267	0.304	0.532	0.533	0.366	0.333	0.342	0.510
	10%	0.507	0.360	0.316	0.264	0.315	0.351	0.310	0.267	0.304	0.532	0.533	0.366	0.333	0.341	0.512
Prefer 2nd model	1%	0.494	0.641	0.684	0.736	0.685	0.650	0.690	0.733	0.695	0.469	0.467	0.634	0.667	0.659	0.491
	5%	0.494	0.641	0.684	0.736	0.685	0.650	0.690	0.733	0.696	0.468	0.467	0.634	0.667	0.658	0.490
	10%	0.493	0.640	0.684	0.736	0.685	0.649	0.690	0.733	0.696	0.468	0.467	0.634	0.667	0.659	0.488

DGP FIGARCH(1,d,0) d=0.8 b=0.5 f=0.3 - % represent VaR p-level unless differently specified

88 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.978 (2.203) *1.591*	3.824 (1.667) *1.530*	5.317 (2.337) *2.127*	8.682 (2.949) *3.473*	7.465 (2.986) *2.986*	7.245 (2.831) *2.898*
5% VaR	9.034 (3.261) *3.614*	10.382 (2.856) *4.153*	12.886 (3.710) *5.154*	18.828 (4.030) *7.531*	16.750 (4.544) *6.700*	16.406 (4.091) *6.562*

89 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.523	0.552	0.235	0.012	0.040	0.046
5% VaR	0.585	0.421	0.165	0.005	0.019	0.015

90 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.933	0.978	0.835	0.349	0.556	0.570
	5%	0.845	0.926	0.707	0.229	0.391	0.419
5% VaR	1%	0.933	0.990	0.985	0.829	0.891	0.925
	5%	0.789	0.924	0.928	0.563	0.738	0.777
Test of independence: Null							
1% VaR	1%	0.876	0.769	0.999	0.998	0.999	0.999
	5%	0.534	0.376	0.988	0.994	0.993	0.992
5% VaR	1%	0.967	0.840	0.997	0.998	0.997	1.000
	5%	0.880	0.684	0.983	0.929	0.962	0.965
Test of conditional coverage: Null							
1% VaR	1%	0.920	0.797	0.898	0.500	0.672	0.691
	5%	0.751	0.706	0.821	0.337	0.535	0.556
5% VaR	1%	0.910	0.881	0.991	0.874	0.916	0.953
	5%	0.725	0.687	0.949	0.660	0.793	0.828

91 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.111	0.455	0.333	0.030	0.014	0.068
5% VaR	0.111	0.454	0.329	0.029	0.011	0.066

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

92 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.269	0.600	0.128	0.002	0.004	0.008
	T	0.574	0.420	0.006	0.000	0.000	0.000
5% VaR	E	0.468	0.460	0.070	0.000	0.001	0.001
	T	0.574	0.420	0.006	0.000	0.000	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.133	0.692	0.148	0.005	0.010	0.023
	T	0.003	0.038	0.000	0.400	0.559	0.000
5% VaR	E	0.173	0.613	0.194	0.000	0.004	0.016
	T	0.003	0.111	0.006	0.295	0.585	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.169	0.586	0.214	0.003	0.008	0.031
	T	0.005	0.026	0.000	0.449	0.520	0.000
5% VaR	E	0.300	0.409	0.264	0.000	0.003	0.024
	T	0.005	0.027	0.000	0.435	0.533	0.000
Loss function 1+2							
1% VaR	E	0.186	0.688	0.122	0.001	0.007	0.007
	T	0.004	0.069	0.002	0.303	0.622	0.000
5% VaR	E	0.273	0.591	0.132	0.000	0.002	0.002
	T	0.003	0.267	0.046	0.063	0.618	0.003
Loss function 1+3							
1% VaR	E	0.205	0.611	0.175	0.002	0.007	0.011
	T	0.007	0.041	0.000	0.381	0.571	0.000
5% VaR	E	0.407	0.437	0.151	0.000	0.001	0.004
	T	0.007	0.071	0.000	0.250	0.672	0.000
Loss function 2+3							
1% VaR	E	0.148	0.645	0.182	0.003	0.009	0.024
	T	0.004	0.032	0.000	0.426	0.538	0.000
5% VaR	E	0.213	0.546	0.224	0.000	0.001	0.016
	T	0.002	0.050	0.002	0.377	0.569	0.000
Loss function 1+2+3							
1% VaR	E	0.179	0.648	0.167	0.001	0.006	0.010
	T	0.005	0.036	0.000	0.387	0.572	0.000
5% VaR	E	0.275	0.550	0.168	0.000	0.001	0.006
	T	0.003	0.085	0.002	0.261	0.649	0.000

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

93 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.020	0.015	0.015	0.012	0.036	0.027
	5%	0.011	0.013	0.010	0.007	0.018	0.012
	10%	0.011	0.012	0.007	0.005	0.011	0.008
5%	1%	0.044	0.049	0.069	0.053	0.115	0.109
	5%	0.023	0.034	0.035	0.022	0.056	0.051
	10%	0.017	0.027	0.019	0.015	0.039	0.036

Preference relation among the models as inferred from table 94

3,4,5,6 1 2 + 3,4,5,6 2 + 4,5,6 3 +5,6 4 + 5 6   5 6 4 3 1 2

that is

HF Garch(1,1) square root   HF Garch(1,1) sum   HF Figarch(1,d,1) sum   HF Figarch(1,d,1) square root   Figarch(1,d,1)   EWMA(0.97)

94 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.907	0.969	0.994	0.988	0.988	0.985	0.996	0.992	0.992	0.970	0.967	0.862	0.938	0.908	0.607
	5%	0.910	0.972	0.994	0.989	0.989	0.987	0.997	0.994	0.992	0.972	0.968	0.863	0.941	0.909	0.608
	10%	0.911	0.973	0.994	0.989	0.989	0.987	0.997	0.994	0.992	0.972	0.969	0.864	0.941	0.912	0.613
Prefer 1st model	1%	0.557	0.426	0.269	0.238	0.259	0.342	0.194	0.181	0.198	0.291	0.292	0.208	0.354	0.365	0.562
	5%	0.556	0.425	0.269	0.238	0.259	0.342	0.195	0.182	0.198	0.291	0.291	0.207	0.355	0.365	0.561
	10%	0.555	0.425	0.269	0.238	0.259	0.342	0.195	0.182	0.198	0.291	0.292	0.207	0.355	0.366	0.561
Prefer 2nd model	1%	0.443	0.574	0.731	0.762	0.741	0.658	0.806	0.819	0.802	0.709	0.708	0.792	0.646	0.635	0.438
	5%	0.444	0.575	0.731	0.762	0.741	0.658	0.805	0.818	0.802	0.709	0.709	0.793	0.645	0.635	0.439
	10%	0.445	0.575	0.731	0.762	0.741	0.658	0.805	0.818	0.802	0.709	0.708	0.793	0.645	0.634	0.439
VaR 5%																
Test is signif.	1%	0.974	0.986	0.983	0.987	0.987	0.987	0.985	0.987	0.987	0.989	0.985	0.950	0.985	0.961	0.767
	5%	0.976	0.989	0.986	0.987	0.989	0.988	0.988	0.987	0.989	0.991	0.988	0.950	0.986	0.964	0.773
	10%	0.977	0.990	0.988	0.988	0.989	0.989	0.989	0.987	0.989	0.991	0.988	0.951	0.986	0.964	0.776
Prefer 1st model	1%	0.488	0.379	0.307	0.241	0.261	0.360	0.293	0.238	0.248	0.437	0.437	0.303	0.284	0.282	0.546
	5%	0.489	0.379	0.309	0.241	0.263	0.360	0.293	0.238	0.248	0.438	0.438	0.303	0.285	0.282	0.547
	10%	0.488	0.380	0.310	0.241	0.263	0.361	0.293	0.238	0.248	0.438	0.438	0.303	0.285	0.282	0.548
Prefer 2nd model	1%	0.512	0.621	0.693	0.759	0.739	0.640	0.707	0.762	0.752	0.563	0.563	0.697	0.716	0.718	0.454
	5%	0.511	0.621	0.691	0.759	0.737	0.640	0.707	0.762	0.752	0.562	0.562	0.697	0.715	0.718	0.453
	10%	0.512	0.620	0.690	0.759	0.737	0.639	0.707	0.762	0.752	0.562	0.562	0.697	0.715	0.718	0.452

DGP FIGARCH(1,d,1) d=0.8 β=0.5 ϕ=0.05 - % represent VaR p-level unless differently specified

95 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.990 (2.110) *1.596*	3.729 (1.715) *1.492*	4.997 (2.186) *1.999*	6.600 (2.470) *2.640*	6.215 (2.758) *2.486*	5.380 (2.426) *2.152*
5% VaR	9.627 (3.266) *3.851*	10.766 (2.981) *4.306*	13.355 (3.609) *5.342*	16.374 (3.891) *6.550*	15.790 (4.373) *6.316*	14.215 (3.976) *5.686*

96 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.472	0.541	0.236	0.050	0.093	0.187
5% VaR	0.533	0.419	0.175	0.020	0.060	0.093

97 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.943	0.982	0.872	0.658	0.693	0.825
	5%	0.865	0.924	0.754	0.507	0.574	0.704
5% VaR	1%	0.953	0.988	0.990	0.934	0.936	0.972
	5%	0.826	0.922	0.942	0.794	0.785	0.891
Test of independence: Null							
1% VaR	1%	0.876	0.753	1.000	0.999	1.000	1.000
	5%	0.558	0.375	0.991	0.993	0.995	0.994
5% VaR	1%	0.966	0.840	0.994	0.993	0.996	0.996
	5%	0.887	0.695	0.977	0.960	0.970	0.978
Test of conditional coverage: Null							
1% VaR	1%	0.928	0.798	0.934	0.784	0.812	0.894
	5%	0.770	0.707	0.861	0.640	0.673	0.811
5% VaR	1%	0.928	0.870	0.990	0.953	0.954	0.982
	5%	0.766	0.690	0.954	0.851	0.844	0.926

98 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.094	0.348	0.273	0.033	0.009	0.253
5% VaR	0.094	0.341	0.272	0.033	0.009	0.251

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

99 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.240	0.542	0.123	0.002	0.002	0.101
	T	0.563	0.436	0.001	0.000	0.000	0.000
5% VaR	E	0.454	0.445	0.065	0.000	0.002	0.034
	T	0.563	0.436	0.001	0.000	0.000	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.118	0.606	0.140	0.014	0.006	0.126
	T	0.002	0.042	0.000	0.181	0.775	0.000
5% VaR	E	0.163	0.497	0.177	0.007	0.000	0.156
	T	0.001	0.057	0.000	0.139	0.803	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.145	0.487	0.188	0.015	0.003	0.172
	T	0.003	0.042	0.000	0.224	0.731	0.000
5% VaR	E	0.283	0.302	0.238	0.004	0.001	0.172
	T	0.003	0.041	0.000	0.205	0.751	0.000
Loss function 1+2							
1% VaR	E	0.181	0.590	0.127	0.005	0.003	0.104
	T	0.002	0.058	0.000	0.131	0.809	0.000
5% VaR	E	0.296	0.485	0.146	0.001	0.001	0.071
	T	0.001	0.118	0.003	0.042	0.835	0.001
Loss function 1+3							
1% VaR	E	0.194	0.514	0.163	0.007	0.003	0.129
	T	0.004	0.054	0.000	0.174	0.768	0.000
5% VaR	E	0.397	0.353	0.164	0.000	0.001	0.085
	T	0.003	0.066	0.000	0.100	0.831	0.000
Loss function 2+3							
1% VaR	E	0.128	0.554	0.155	0.011	0.005	0.157
	T	0.002	0.043	0.000	0.203	0.752	0.000
5% VaR	E	0.211	0.426	0.193	0.007	0.000	0.163
	T	0.002	0.047	0.000	0.170	0.781	0.000
Loss function 1+2+3							
1% VaR	E	0.161	0.563	0.147	0.007	0.004	0.128
	T	0.002	0.047	0.000	0.180	0.771	0.000
5% VaR	E	0.284	0.431	0.177	0.001	0.000	0.107
	T	0.002	0.066	0.000	0.118	0.814	0.000

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

100 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.029	0.021	0.019	0.022	0.024	0.023
	5%	0.018	0.017	0.012	0.013	0.012	0.012
	10%	0.014	0.016	0.009	0.010	0.010	0.008
5%	1%	0.078	0.062	0.083	0.073	0.109	0.106
	5%	0.046	0.042	0.040	0.035	0.055	0.047
	10%	0.036	0.033	0.028	0.027	0.037	0.033

Preference relation among the models as inferred from table 101

3,4,5,6 1 2 + 3,4,5,6 2 + 4,5,6 3 +5,6 4 + 5 6 5 6 4 3 1 2

that is

HF Garch(1,1) square root HF Garch(1,1) sum HF Figarch(1,d,1) sum HF Figarch(1,d,1) square root Figarch(1,d,1) EWMA(0.97)

101 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.905	0.988	0.995	0.994	0.989	0.989	0.997	0.995	0.995	0.834	0.830	0.845	0.883	0.847	0.612
	5%	0.910	0.989	0.995	0.995	0.991	0.990	0.997	0.996	0.996	0.837	0.833	0.847	0.884	0.849	0.616
	10%	0.912	0.989	0.995	0.995	0.991	0.990	0.997	0.996	0.996	0.839	0.835	0.847	0.884	0.851	0.618
Prefer 1st model	1%	0.591	0.418	0.329	0.312	0.373	0.348	0.263	0.249	0.302	0.372	0.371	0.299	0.414	0.536	0.655
	5%	0.589	0.418	0.329	0.312	0.372	0.347	0.263	0.249	0.301	0.373	0.371	0.301	0.414	0.536	0.651
	10%	0.588	0.418	0.329	0.312	0.372	0.347	0.263	0.249	0.301	0.372	0.371	0.301	0.414	0.536	0.650
Prefer 2nd model	1%	0.409	0.582	0.671	0.688	0.627	0.652	0.737	0.751	0.698	0.628	0.629	0.701	0.586	0.464	0.345
	5%	0.411	0.582	0.671	0.688	0.628	0.653	0.737	0.751	0.699	0.627	0.629	0.699	0.586	0.464	0.349
	10%	0.412	0.582	0.671	0.688	0.628	0.653	0.737	0.751	0.699	0.628	0.629	0.699	0.586	0.464	0.350
VaR 5%																
Test is signif.	1%	0.980	0.986	0.987	0.991	0.992	0.992	0.989	0.991	0.988	0.935	0.935	0.916	0.952	0.933	0.782
	5%	0.985	0.989	0.988	0.995	0.994	0.992	0.989	0.991	0.989	0.935	0.936	0.921	0.953	0.937	0.782
	10%	0.986	0.990	0.988	0.995	0.994	0.992	0.989	0.992	0.989	0.941	0.941	0.923	0.957	0.940	0.785
Prefer 1st model	1%	0.536	0.392	0.360	0.308	0.352	0.393	0.356	0.306	0.344	0.545	0.542	0.332	0.321	0.388	0.561
	5%	0.536	0.393	0.359	0.310	0.353	0.393	0.356	0.306	0.345	0.545	0.543	0.331	0.321	0.388	0.561
	10%	0.537	0.393	0.359	0.310	0.353	0.393	0.356	0.306	0.345	0.545	0.543	0.332	0.323	0.389	0.561
Prefer 2nd model	1%	0.464	0.608	0.640	0.692	0.648	0.607	0.644	0.694	0.656	0.455	0.458	0.668	0.679	0.612	0.439
	5%	0.464	0.607	0.641	0.690	0.647	0.607	0.644	0.694	0.655	0.455	0.457	0.669	0.679	0.612	0.439
	10%	0.463	0.607	0.641	0.690	0.647	0.607	0.644	0.694	0.655	0.455	0.457	0.668	0.677	0.611	0.439

DGP FIGARCH(1,d,1) d=0.4 β=0.3 ϕ=0.2 - % represent VaR p-level unless differently specified

102 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.714 (1.952) *1.486*	2.734 (1.498) *1.094*	3.610 (1.892) *1.444*	4.638 (2.120) *1.855*	6.055 (3.167) *2.422*	4.269 (2.450) *1.708*
5% VaR	10.894 (3.377) *4.358*	11.151 (2.845) *4.460*	11.688 (3.352) *4.675*	13.848 (3.514) *5.539*	16.725 (5.472) *6.690*	12.988 (4.339) *5.195*

103 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.272	0.660	0.342	0.131	0.060	0.264
5% VaR	0.395	0.555	0.251	0.109	0.056	0.166

104 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.963	0.998	0.969	0.895	0.712	0.900
	5%	0.889	0.931	0.900	0.811	0.586	0.792
5% VaR	1%	0.983	0.995	0.992	0.989	0.853	0.968
	5%	0.898	0.956	0.941	0.930	0.707	0.881
Test of independence: Null							
1% VaR	1%	0.875	0.788	0.998	1.000	1.000	1.000
	5%	0.516	0.296	0.986	0.990	0.992	0.991
5% VaR	1%	0.960	0.933	0.998	0.998	0.998	0.999
	5%	0.893	0.806	0.985	0.984	0.964	0.987
Test of conditional coverage: Null							
1% VaR	1%	0.936	0.936	0.985	0.958	0.796	0.941
	5%	0.805	0.767	0.961	0.889	0.691	0.888
5% VaR	1%	0.964	0.958	0.994	0.993	0.883	0.974
	5%	0.836	0.825	0.964	0.954	0.771	0.903

105 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.050	0.418	0.235	0.066	0.010	0.287
5% VaR	0.050	0.396	0.223	0.060	0.003	0.268

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

106 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.101	0.616	0.153	0.033	0.010	0.153
	T	0.447	0.365	0.112	0.000	0.000	0.076
5% VaR	E	0.238	0.449	0.175	0.009	0.000	0.129
	T	0.447	0.365	0.112	0.000	0.000	0.076
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.074	0.701	0.097	0.030	0.010	0.154
	T	0.001	0.115	0.000	0.051	0.833	0.000
5% VaR	E	0.067	0.656	0.089	0.020	0.001	0.167
	T	0.001	0.172	0.000	0.036	0.791	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.074	0.674	0.107	0.038	0.010	0.163
	T	0.002	0.095	0.000	0.063	0.840	0.000
5% VaR	E	0.117	0.535	0.133	0.015	0.000	0.200
	T	0.002	0.111	0.000	0.055	0.832	0.000
Loss function 1+2							
1% VaR	E	0.085	0.691	0.116	0.030	0.010	0.134
	T	0.002	0.129	0.000	0.040	0.829	0.000
5% VaR	E	0.143	0.575	0.128	0.009	0.001	0.144
	T	0.001	0.265	0.000	0.021	0.711	0.002
Loss function 1+3							
1% VaR	E	0.090	0.660	0.125	0.033	0.010	0.148
	T	0.002	0.111	0.000	0.052	0.835	0.000
5% VaR	E	0.185	0.495	0.141	0.011	0.000	0.168
	T	0.002	0.139	0.000	0.041	0.818	0.000
Loss function 2+3							
1% VaR	E	0.074	0.691	0.096	0.034	0.010	0.161
	T	0.002	0.104	0.000	0.058	0.836	0.000
5% VaR	E	0.087	0.614	0.109	0.020	0.001	0.169
	T	0.001	0.127	0.000	0.044	0.828	0.000
Loss function 1+2+3							
1% VaR	E	0.083	0.683	0.116	0.032	0.010	0.142
	T	0.002	0.110	0.000	0.053	0.835	0.000
5% VaR	E	0.134	0.568	0.127	0.011	0.001	0.159
	T	0.001	0.151	0.000	0.036	0.812	0.000

Model reference:1 - Figarch(1,d,0); 2- EWMA(0.97); 3 - HF Figarch(1,d,0) square root; 4 - HF Figarch(1,d,0) sum; 5 - HF Garch(1,1) square root; 6 - HF Garch(1,1) sum

107 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.019	0.001	0.015	0.029	0.030	0.024
	5%	0.010	0.001	0.009	0.012	0.013	0.012
	10%	0.007	0.001	0.006	0.007	0.006	0.007
5%	1%	0.100	0.082	0.135	0.140	0.156	0.136
	5%	0.037	0.030	0.049	0.059	0.059	0.056
	10%	0.014	0.015	0.023	0.027	0.027	0.030

Preference relation among the models as inferred from table 108

4,5,6 1 2,3 + 3,4,5,6 2 + 4,5,6 3 +5 4 6 + 5 6 5 4 6 1 3 2

that is

HF Garch(1,1) square root HF Figarch(1,d,1) sum HF Garch(1,1) sum Figarch(1,d,1) HF Figarch(1,d,1) square root EWMA(0.97)

108 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.789	0.915	0.972	0.976	0.968	0.902	0.972	0.974	0.960	0.767	0.764	0.919	0.920	0.841	0.799
	5%	0.791	0.917	0.973	0.976	0.968	0.903	0.972	0.974	0.961	0.770	0.765	0.919	0.920	0.842	0.801
	10%	0.793	0.918	0.974	0.976	0.968	0.903	0.973	0.974	0.961	0.771	0.766	0.919	0.920	0.843	0.802
Prefer 1st model	1%	0.776	0.519	0.390	0.280	0.439	0.288	0.208	0.143	0.254	0.297	0.296	0.221	0.357	0.561	0.741
	5%	0.774	0.520	0.390	0.280	0.439	0.288	0.208	0.143	0.255	0.299	0.297	0.221	0.357	0.561	0.740
	10%	0.774	0.521	0.390	0.280	0.439	0.288	0.208	0.143	0.255	0.300	0.296	0.221	0.357	0.560	0.739
Prefer 2nd model	1%	0.224	0.481	0.610	0.720	0.561	0.712	0.792	0.857	0.746	0.703	0.704	0.779	0.643	0.439	0.259
	5%	0.226	0.480	0.610	0.720	0.561	0.712	0.792	0.857	0.745	0.701	0.703	0.779	0.643	0.439	0.260
	10%	0.226	0.479	0.610	0.720	0.561	0.712	0.792	0.857	0.745	0.700	0.704	0.779	0.643	0.440	0.261
VaR 5%																
Test is signif.	1%	0.919	0.982	0.989	0.990	0.991	0.985	0.990	0.997	0.986	0.933	0.936	0.970	0.978	0.945	0.932
	5%	0.927	0.984	0.993	0.995	0.994	0.989	0.992	0.998	0.989	0.939	0.940	0.972	0.979	0.948	0.933
	10%	0.929	0.985	0.993	0.995	0.994	0.990	0.993	0.998	0.991	0.941	0.941	0.973	0.980	0.949	0.935
Prefer 1st model	1%	0.589	0.441	0.387	0.333	0.411	0.411	0.335	0.288	0.356	0.445	0.443	0.353	0.436	0.510	0.550
	5%	0.588	0.440	0.390	0.334	0.410	0.411	0.337	0.288	0.357	0.445	0.444	0.353	0.435	0.511	0.551
	10%	0.588	0.440	0.390	0.334	0.410	0.411	0.336	0.288	0.357	0.444	0.444	0.354	0.436	0.511	0.551
Prefer 2nd model	1%	0.411	0.559	0.613	0.667	0.589	0.589	0.665	0.712	0.644	0.555	0.557	0.647	0.564	0.490	0.450
	5%	0.412	0.560	0.610	0.666	0.590	0.589	0.663	0.712	0.643	0.555	0.556	0.647	0.565	0.489	0.449
	10%	0.412	0.560	0.610	0.666	0.590	0.589	0.664	0.712	0.643	0.556	0.556	0.646	0.564	0.489	0.449

AGGREGATED ESTIMATES AGGREGATED COMPARISON

DGP FIGARCH(1,d,1) d=0.4 b=0.3 f=0.2 - % represent VaR p-level unless differently specified

109 - Average number of exceptions (standard deviation) [mean percentage] 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.671 (1.958) *1.468*	3.995 (2.293) *1.598*	3.556 (1.841) *1.422*	2.670 (1.472) *1.068*	3.596 (1.888) *1.438*	4.638 (2.137) *1.855*
5% VaR	10.730 (3.484) *4.292*	11.406 (4.164) *4.562*	10.473 (3.142) *4.189*	11.099 (2.858) *4.440*	11.656 (3.463) *4.662*	13.849 (3.698) *5.540*

110 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	253	259	273	682	348	154
5% VaR	409	262	306	488	259	91

111 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	965	927	979	999	971	898
	5%	888	826	911	939	903	809
5% VaR	1%	969	954	975	995	987	987
	5%	879	853	903	947	932	916
Test of independence: Null							
1% VaR	1%	865	845	833	770	998	1000
	5%	483	514	444	275	981	992
5% VaR	1%	967	962	957	933	995	999
	5%	899	863	860	828	978	988
Test of conditional coverage: Null							
1% VaR	1%	928	886	918	934	985	953
	5%	780	708	773	758	961	891
5% VaR	1%	951	937	942	953	990	993
	5%	825	782	819	835	947	944

112 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.033	0.039	0.038	0.388	0.463	0.139
5% VaR	0.033	0.014	0.015	0.364	0.448	0.126

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

113 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.066	0.090	0.061	0.651	0.186	0.046
	T	0.147	0.188	0.333	0.221	0.111	0.000
5% VaR	E	0.124	0.160	0.121	0.382	0.201	0.012
	T	0.147	0.188	0.333	0.221	0.111	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.041	0.058	0.042	0.761	0.138	0.060
	T	0.004	0.133	0.000	0.325	0.003	0.535
5% VaR	E	0.024	0.053	0.030	0.710	0.146	0.037
	T	0.002	0.067	0.000	0.471	0.015	0.445
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.042	0.061	0.043	0.741	0.148	0.065
	T	0.010	0.164	0.000	0.270	0.002	0.554
5% VaR	E	0.046	0.106	0.064	0.528	0.221	0.035
	T	0.008	0.149	0.000	0.286	0.002	0.555
Loss function 1+2							
1% VaR	E	0.058	0.076	0.055	0.709	0.160	0.042
	T	0.005	0.125	0.000	0.389	0.007	0.474
5% VaR	E	0.072	0.131	0.074	0.521	0.192	0.010
	T	0.004	0.043	0.000	0.642	0.044	0.267
Loss function 1+3							
1% VaR	E	0.056	0.072	0.056	0.692	0.172	0.052
	T	0.009	0.159	0.000	0.307	0.004	0.521
5% VaR	E	0.088	0.144	0.097	0.441	0.215	0.015
	T	0.007	0.135	0.000	0.416	0.018	0.424
Loss function 2+3							
1% VaR	E	0.042	0.057	0.044	0.754	0.143	0.060
	T	0.006	0.146	0.000	0.297	0.002	0.549
5% VaR	E	0.037	0.087	0.037	0.631	0.175	0.033
	T	0.003	0.115	0.000	0.372	0.006	0.504
Loss function 1+2+3							
1% VaR	E	0.052	0.070	0.050	0.718	0.160	0.050
	T	0.006	0.146	0.000	0.319	0.004	0.525
5% VaR	E	0.068	0.123	0.067	0.530	0.198	0.014
	T	0.003	0.110	0.000	0.438	0.012	0.437

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

114 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.005	0.006	0.003	0.001	0.007	0.015
	5%	0.001	0.001	0.002	0.001	0.004	0.006
	10%	0.001	0.001	0.001	0.001	0.002	0.003
5%	1%	0.089	0.083	0.051	0.055	0.143	0.162
	5%	0.042	0.044	0.022	0.031	0.092	0.099
	10%	0.029	0.028	0.014	0.014	0.061	0.059

Preference relation among the models as inferred from table 115

2,5,6 1 3,4 + 6 2 3,4,5 + 6 3 4,5 +5,6 4 + 6 5 no order

115 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.542	0.616	0.798	0.895	0.967	0.559	0.810	0.913	0.968	0.730	0.718	0.910	0.927	0.969	0.739
	5%	0.545	0.616	0.799	0.895	0.967	0.562	0.811	0.913	0.969	0.733	0.718	0.911	0.927	0.969	0.739
	10%	0.547	0.618	0.800	0.895	0.967	0.563	0.813	0.913	0.969	0.733	0.718	0.911	0.927	0.969	0.739
Prefer 1^{st} model	1%	0.450	0.597	0.786	0.498	0.331	0.640	0.810	0.516	0.356	0.779	0.783	0.431	0.268	0.181	0.286
	5%	0.453	0.597	0.785	0.498	0.331	0.639	0.809	0.516	0.356	0.776	0.783	0.430	0.268	0.181	0.286
	10%	0.455	0.599	0.784	0.498	0.331	0.638	0.808	0.516	0.356	0.776	0.783	0.430	0.268	0.181	0.286
Prefer 2^{nd} model	1%	0.550	0.403	0.214	0.502	0.669	0.360	0.190	0.484	0.644	0.221	0.217	0.569	0.732	0.819	0.714
	5%	0.547	0.403	0.215	0.502	0.669	0.361	0.191	0.484	0.644	0.224	0.217	0.570	0.732	0.819	0.714
	10%	0.545	0.401	0.216	0.502	0.669	0.362	0.192	0.484	0.644	0.224	0.217	0.570	0.732	0.819	0.714
VaR 5%																
Test is signif.	1%	0.739	0.837	0.944	0.983	0.995	0.700	0.921	0.985	0.997	0.856	0.862	0.991	0.986	0.995	0.935
	5%	0.744	0.842	0.948	0.986	0.998	0.707	0.928	0.987	0.999	0.865	0.868	0.993	0.986	0.996	0.940
	10%	0.746	0.844	0.950	0.986	0.998	0.708	0.931	0.988	0.999	0.870	0.871	0.995	0.989	0.997	0.942
Prefer 1^{st} model	1%	0.532	0.695	0.612	0.413	0.327	0.699	0.613	0.421	0.329	0.464	0.464	0.322	0.350	0.269	0.425
	5%	0.530	0.694	0.611	0.415	0.327	0.694	0.612	0.422	0.330	0.462	0.462	0.322	0.350	0.270	0.424
	10%	0.529	0.693	0.611	0.415	0.327	0.694	0.613	0.423	0.330	0.462	0.462	0.323	0.350	0.270	0.425
Prefer 2^{nd} model	1%	0.468	0.305	0.388	0.587	0.673	0.301	0.387	0.579	0.671	0.536	0.536	0.678	0.650	0.731	0.575
	5%	0.470	0.306	0.389	0.585	0.673	0.306	0.388	0.578	0.670	0.538	0.538	0.678	0.650	0.730	0.576
	10%	0.471	0.307	0.389	0.585	0.673	0.306	0.387	0.577	0.670	0.538	0.538	0.677	0.650	0.730	0.575

DGP FIGARCH(1,d,0) d=0.8 b=0.5 - % represent VaR p-level unless differently specified

116 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.985 (2.021) *1.594*	4.144 (2.247) *1.657*	4.007 (2.131) *1.602*	3.738 (1.726) *1.495*	4.923 (2.197) *1.969*	6.332 (2.427) *2.532*
5% VaR	9.767 (3.248) *3.906*	9.979 (3.726) *3.991*	9.697 (3.494) *3.879*	10.766 (2.996) *4.306*	13.213 (3.577) *5.285*	15.917 (3.730) *6.366*

117 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.425	0.422	0.457	0.528	0.274	0.058
5% VaR	0.531	0.353	0.370	0.391	0.197	0.034

118 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.949	0.927	0.941	0.976	0.869	0.708
	5%	0.870	0.842	0.862	0.923	0.769	0.558
5% VaR	1%	0.966	0.942	0.943	0.987	0.991	0.955
	5%	0.832	0.812	0.803	0.923	0.933	0.824
Test of independence: Null							
1% VaR	1%	0.893	0.867	0.866	0.749	0.998	0.999
	5%	0.531	0.534	0.521	0.347	0.980	0.992
5% VaR	1%	0.963	0.951	0.953	0.817	0.996	1.000
	5%	0.882	0.868	0.872	0.670	0.989	0.976
Test of conditional coverage: Null							
1% VaR	1%	0.922	0.892	0.907	0.786	0.928	0.817
	5%	0.781	0.729	0.745	0.694	0.852	0.685
5% VaR	1%	0.935	0.912	0.911	0.869	0.991	0.970
	5%	0.767	0.739	0.735	0.669	0.962	0.887

119 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.036	0.034	0.040	0.255	0.600	0.076
5% VaR	0.036	0.017	0.023	0.253	0.597	0.074

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,0) square root; 6 - HF Figarch(1,d,0) sum

120 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.081	0.091	0.160	0.522	0.180	0.007
	T	0.070	0.119	0.403	0.404	0.004	0.000
5% VaR	E	0.103	0.144	0.255	0.395	0.102	0.001
	T	0.070	0.119	0.403	0.404	0.004	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.045	0.071	0.086	0.593	0.225	0.021
	T	0.001	0.017	0.001	0.081	0.000	0.900
5% VaR	E	0.039	0.059	0.105	0.503	0.287	0.007
	T	0.001	0.009	0.000	0.110	0.000	0.880
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.059	0.064	0.100	0.504	0.292	0.022
	T	0.001	0.032	0.003	0.066	0.000	0.898
5% VaR	E	0.051	0.086	0.214	0.310	0.335	0.004
	T	0.001	0.030	0.003	0.070	0.000	0.896
Loss function 1+2							
1% VaR	E	0.062	0.078	0.124	0.582	0.186	0.009
	T	0.001	0.017	0.001	0.137	0.000	0.844
5% VaR	E	0.060	0.106	0.180	0.486	0.166	0.002
	T	0.001	0.011	0.000	0.270	0.307	0.411
Loss function 1+3							
1% VaR	E	0.070	0.080	0.134	0.520	0.225	0.012
	T	0.001	0.035	0.004	0.119	0.000	0.841
5% VaR	E	0.082	0.126	0.237	0.374	0.180	0.001
	T	0.001	0.033	0.003	0.184	0.000	0.779
Loss function 2+3							
1% VaR	E	0.051	0.067	0.095	0.534	0.277	0.017
	T	0.001	0.024	0.001	0.074	0.000	0.900
5% VaR	E	0.046	0.073	0.143	0.428	0.304	0.006
	T	0.001	0.016	0.001	0.086	0.000	0.896
Loss function 1+2+3							
1% VaR	E	0.059	0.077	0.120	0.564	0.210	0.011
	T	0.001	0.024	0.001	0.097	0.000	0.877
5% VaR	E	0.053	0.106	0.187	0.441	0.211	0.002
	T	0.001	0.017	0.001	0.143	0.000	0.838

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,0) square root; 6 - HF Figarch(1,d,0) sum

121 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.018	0.010	0.010	0.011	0.014	0.013
	5%	0.010	0.008	0.008	0.007	0.008	0.007
	10%	0.005	0.006	0.006	0.007	0.006	0.004
5%	1%	0.054	0.051	0.041	0.063	0.093	0.090
	5%	0.032	0.033	0.025	0.038	0.049	0.047
	10%	0.023	0.026	0.021	0.030	0.032	0.029

Preference relation among the models as inferred from table 115

2,5,6 1 3,4 + 6 2 3,4,5 + 6 3 4,5 +5,6 4 + 6 5   no order

122 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.500	0.464	0.905	0.967	0.979	0.262	0.909	0.967	0.981	0.904	0.902	0.968	0.968	0.975	0.784
	5%	0.503	0.467	0.906	0.967	0.979	0.265	0.910	0.967	0.981	0.907	0.902	0.968	0.969	0.978	0.786
	10%	0.504	0.473	0.908	0.968	0.979	0.269	0.911	0.968	0.982	0.910	0.904	0.968	0.969	0.978	0.787
Prefer 1st model	1%	0.480	0.545	0.592	0.429	0.323	0.653	0.607	0.444	0.333	0.582	0.574	0.419	0.349	0.259	0.388
	5%	0.481	0.546	0.592	0.429	0.323	0.645	0.607	0.444	0.333	0.580	0.574	0.419	0.350	0.262	0.389
	10%	0.482	0.543	0.593	0.429	0.323	0.647	0.607	0.443	0.334	0.581	0.574	0.419	0.350	0.262	0.389
Prefer 2nd model	1%	0.520	0.455	0.408	0.571	0.677	0.347	0.393	0.556	0.667	0.418	0.426	0.581	0.651	0.741	0.612
	5%	0.519	0.454	0.408	0.571	0.677	0.355	0.393	0.556	0.667	0.420	0.426	0.581	0.650	0.738	0.611
	10%	0.518	0.457	0.407	0.571	0.677	0.353	0.393	0.557	0.666	0.419	0.426	0.581	0.650	0.738	0.611
VaR 5%																
Test is signif.	1%	0.587	0.559	0.924	0.939	0.945	0.272	0.934	0.941	0.943	0.928	0.942	0.942	0.947	0.946	0.877
	5%	0.594	0.566	0.931	0.946	0.949	0.278	0.936	0.944	0.945	0.933	0.943	0.945	0.947	0.949	0.879
	10%	0.599	0.569	0.933	0.946	0.949	0.281	0.939	0.946	0.945	0.937	0.945	0.947	0.948	0.949	0.884
Prefer 1st model	1%	0.518	0.617	0.545	0.358	0.325	0.658	0.539	0.358	0.329	0.498	0.496	0.322	0.327	0.307	0.520
	5%	0.517	0.615	0.545	0.359	0.326	0.658	0.538	0.359	0.328	0.497	0.495	0.322	0.327	0.308	0.521
	10%	0.519	0.617	0.544	0.359	0.326	0.658	0.538	0.359	0.328	0.496	0.495	0.322	0.328	0.308	0.523
Prefer 2nd model	1%	0.482	0.383	0.455	0.642	0.675	0.342	0.461	0.642	0.671	0.502	0.504	0.678	0.673	0.693	0.480
	5%	0.483	0.385	0.455	0.641	0.674	0.342	0.462	0.641	0.672	0.503	0.505	0.678	0.673	0.692	0.479
	10%	0.481	0.383	0.456	0.641	0.674	0.342	0.462	0.641	0.672	0.504	0.505	0.678	0.672	0.692	0.477

DGP FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified

123 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast]						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.719 (2.028) *1.488*	3.780 (2.227) *1.512*	3.401 (1.730) *1.360*	2.314 (1.369) *0.926*	3.365 (1.856) *1.346*	3.786 (1.957) *1.514*
5% VaR	11.866 (3.683) *4.746*	11.828 (4.099) *4.731*	10.870 (2.959) *4.348*	11.273 (2.778) *4.509*	12.011 (3.453) *4.804*	12.822 (3.502) *5.129*

124 - Frequency of less exceptions – 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.215	0.254	0.244	0.756	0.327	0.219
5% VaR	0.347	0.265	0.327	0.534	0.245	0.159

125 - TESTS – frequencies of accepting the null hypothesis – 1000 replications – 250 daily forecasts							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.954	0.937	0.988	1.000	0.976	0.955
	5%	0.880	0.857	0.927	0.914	0.895	0.885
5% VaR	1%	0.991	0.973	0.994	1.000	0.993	0.995
	5%	0.913	0.881	0.944	0.969	0.938	0.943
Test of independence: Null							
1% VaR	1%	0.870	0.843	0.834	0.756	1.000	1.000
	5%	0.513	0.501	0.439	0.252	0.982	0.988
5% VaR	1%	0.979	0.966	0.959	0.960	0.998	0.999
	5%	0.927	0.897	0.882	0.867	0.993	0.988
Test of conditional coverage: Null							
1% VaR	1%	0.937	0.909	0.935	0.962	0.990	0.977
	5%	0.777	0.735	0.790	0.748	0.968	0.946
5% VaR	1%	0.978	0.953	0.977	0.980	0.994	0.994
	5%	0.884	0.829	0.861	0.879	0.958	0.962

126 - Lopez loss function – frequency of model selection – 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.029	0.039	0.030	0.356	0.462	0.205
5% VaR	0.029	0.017	0.009	0.317	0.445	0.183

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,0) square root; 6 - HF Figarch(1,d,0) sum

127 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.048	0.087	0.037	0.714	0.157	0.078
	T	0.083	0.236	0.393	0.172	0.115	0.001
5% VaR	E	0.076	0.185	0.104	0.432	0.165	0.038
	T	0.083	0.236	0.393	0.172	0.115	0.001
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.031	0.051	0.034	0.812	0.112	0.081
	T	0.045	0.144	0.000	0.401	0.031	0.379
5% VaR	E	0.012	0.058	0.007	0.782	0.098	0.043
	T	0.024	0.086	0.000	0.523	0.029	0.338
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.031	0.057	0.032	0.811	0.113	0.077
	T	0.055	0.176	0.000	0.349	0.026	0.394
5% VaR	E	0.034	0.110	0.023	0.637	0.145	0.051
	T	0.052	0.160	0.000	0.378	0.035	0.375
Loss function 1+2							
1% VaR	E	0.042	0.079	0.036	0.755	0.137	0.072
	T	0.038	0.138	0.000	0.444	0.045	0.335
5% VaR	E	0.047	0.156	0.047	0.578	0.142	0.030
	T	0.010	0.034	0.000	0.700	0.063	0.193
Loss function 1+3							
1% VaR	E	0.042	0.078	0.036	0.752	0.136	0.077
	T	0.053	0.168	0.000	0.381	0.041	0.357
5% VaR	E	0.055	0.167	0.061	0.529	0.152	0.036
	T	0.040	0.138	0.000	0.471	0.057	0.294
Loss function 2+3							
1% VaR	E	0.030	0.054	0.032	0.817	0.113	0.075
	T	0.054	0.160	0.000	0.372	0.028	0.386
5% VaR	E	0.022	0.087	0.010	0.727	0.118	0.036
	T	0.035	0.128	0.000	0.444	0.031	0.362
Loss function 1+2+3							
1% VaR	E	0.035	0.071	0.036	0.775	0.129	0.075
	T	0.051	0.157	0.000	0.387	0.035	0.370
5% VaR	E	0.044	0.147	0.035	0.595	0.147	0.032
	T	0.035	0.114	0.000	0.490	0.046	0.315

Model reference:1 - Figarch(1,d,0); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,0) square root; 6 - HF Figarch(1,d,0) sum

128 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.003	0.004	0.000	0.000	0.011	0.009
	5%	0.002	0.002	0.000	0.000	0.003	0.005
	10%	0.001	0.001	0.000	0.000	0.003	0.004
5%	1%	0.121	0.100	0.067	0.079	0.159	0.179
	5%	0.061	0.053	0.038	0.040	0.095	0.097
	10%	0.039	0.032	0.026	0.028	0.065	0.062

Preference relation among the models as inferred from table 115

6 1 2,3,4,5 + 6 2 3,4,5 + 6 3 4,5 +5,6 4 + 6 5   6 1 2 3 4 5

that is

HF Figarch(1,d,0) sum   Figarch(1,d,0)   Garch(1,1)   Igarch(1,1)   EWMA(0.97)   HF Figarch(1,d,0) square root

129 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.578	0.696	0.813	0.899	0.939	0.572	0.785	0.907	0.949	0.718	0.699	0.910	0.911	0.955	0.578
	5%	0.581	0.697	0.813	0.899	0.939	0.573	0.787	0.907	0.949	0.719	0.703	0.910	0.911	0.955	0.580
	10%	0.582	0.698	0.814	0.899	0.939	0.574	0.789	0.908	0.949	0.719	0.706	0.911	0.912	0.955	0.581
Prefer 1st model	1%	0.517	0.649	0.866	0.543	0.460	0.631	0.842	0.514	0.453	0.845	0.854	0.447	0.239	0.203	0.343
	5%	0.518	0.650	0.866	0.543	0.460	0.630	0.841	0.514	0.453	0.844	0.852	0.447	0.239	0.203	0.345
	10%	0.517	0.649	0.865	0.543	0.460	0.631	0.840	0.514	0.453	0.844	0.848	0.447	0.240	0.203	0.344
Prefer 2nd model	1%	0.483	0.351	0.134	0.457	0.540	0.369	0.158	0.486	0.547	0.155	0.146	0.553	0.761	0.797	0.657
	5%	0.482	0.350	0.134	0.457	0.540	0.370	0.159	0.486	0.547	0.156	0.148	0.553	0.761	0.797	0.655
	10%	0.483	0.351	0.135	0.457	0.540	0.369	0.160	0.486	0.547	0.156	0.152	0.553	0.760	0.797	0.656
VaR 5%																
Test is signif.	1%	0.790	0.891	0.943	0.979	0.986	0.773	0.904	0.975	0.989	0.761	0.765	0.977	0.978	0.988	0.793
	5%	0.794	0.895	0.949	0.983	0.991	0.776	0.906	0.982	0.992	0.767	0.768	0.979	0.983	0.989	0.799
	10%	0.797	0.898	0.949	0.985	0.993	0.781	0.908	0.982	0.992	0.773	0.769	0.982	0.984	0.990	0.800
Prefer 1st model	1%	0.567	0.676	0.657	0.440	0.399	0.651	0.616	0.392	0.364	0.511	0.512	0.308	0.317	0.295	0.463
	5%	0.565	0.675	0.655	0.442	0.400	0.649	0.616	0.393	0.364	0.510	0.513	0.308	0.318	0.294	0.461
	10%	0.565	0.674	0.655	0.442	0.400	0.649	0.616	0.393	0.364	0.512	0.512	0.309	0.319	0.294	0.461
Prefer 2nd model	1%	0.433	0.324	0.343	0.560	0.601	0.349	0.384	0.608	0.636	0.489	0.488	0.692	0.683	0.705	0.537
	5%	0.435	0.325	0.345	0.558	0.600	0.351	0.384	0.607	0.636	0.490	0.487	0.692	0.682	0.706	0.539
	10%	0.435	0.326	0.345	0.558	0.600	0.351	0.384	0.607	0.636	0.488	0.488	0.691	0.681	0.706	0.539

DGP FIGARCH(1,d,0) d=0.8 b=0.5 f=0.3 - % represent VaR p-level unless differently specified

130 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast]						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.883 (2.058) *1.553*	4.038 (2.253) *1.615*	3.880 (2.142) *1.552*	3.875 (1.740) *1.550*	5.249 (2.405) *2.100*	8.664 (2.886) *3.466*
5% VaR	8.682 (3.176) *3.473*	8.896 (3.418) *3.558*	8.693 (3.276) *3.477*	10.291 (2.960) *4.116*	12.871 (3.700) *5.148*	18.719 (4.166) *7.488*

131 - Frequency of less exceptions – 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.482	0.454	0.507	0.502	0.219	0.008
5% VaR	0.598	0.390	0.422	0.332	0.163	0.007

132 - TESTS – frequencies of accepting the null hypothesis – 1000 replications – 250 daily forecasts							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.938	0.916	0.932	0.978	0.824	0.368
	5%	0.867	0.839	0.857	0.923	0.717	0.231
5% VaR	1%	0.916	0.915	0.912	0.982	0.981	0.817
	5%	0.760	0.753	0.745	0.905	0.930	0.582
Test of independence: Null							
1% VaR	1%	0.883	0.871	0.869	0.733	0.999	0.999
	5%	0.512	0.504	0.491	0.390	0.986	0.992
5% VaR	1%	0.960	0.940	0.945	0.820	0.998	0.997
	5%	0.848	0.824	0.844	0.650	0.984	0.944
Test of conditional coverage: Null							
1% VaR	1%	0.922	0.895	0.911	0.774	0.884	0.502
	5%	0.775	0.737	0.754	0.681	0.813	0.348
5% VaR	1%	0.884	0.869	0.869	0.853	0.994	0.871
	5%	0.682	0.665	0.670	0.625	0.947	0.661

133 - Lopez loss function – frequency of model selection – 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.056	0.047	0.063	0.312	0.538	0.039
5% VaR	0.056	0.028	0.039	0.306	0.537	0.038

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

134 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.116	0.121	0.164	0.519	0.134	0.001
	T	0.107	0.118	0.423	0.352	0.000	0.000
5% VaR	E	0.184	0.154	0.242	0.367	0.057	0.000
	T	0.107	0.118	0.423	0.352	0.000	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.071	0.074	0.077	0.637	0.186	0.010
	T	0.001	0.009	0.001	0.064	0.001	0.924
5% VaR	E	0.061	0.063	0.111	0.557	0.204	0.008
	T	0.001	0.004	0.000	0.143	0.022	0.830
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.079	0.086	0.113	0.521	0.244	0.012
	T	0.001	0.015	0.002	0.034	0.000	0.948
5% VaR	E	0.129	0.082	0.193	0.336	0.259	0.005
	T	0.001	0.014	0.003	0.041	0.000	0.941
Loss function 1+2							
1% VaR	E	0.091	0.097	0.128	0.594	0.144	0.001
	T	0.001	0.011	0.002	0.145	0.005	0.836
5% VaR	E	0.116	0.109	0.176	0.478	0.125	0.000
	T	0.002	0.003	0.001	0.447	0.402	0.145
Loss function 1+3							
1% VaR	E	0.105	0.099	0.154	0.513	0.182	0.002
	T	0.001	0.018	0.003	0.090	0.000	0.888
5% VaR	E	0.166	0.126	0.238	0.354	0.119	0.001
	T	0.001	0.023	0.004	0.230	0.018	0.724
Loss function 2+3							
1% VaR	E	0.074	0.081	0.094	0.590	0.206	0.010
	T	0.001	0.010	0.002	0.045	0.000	0.942
5% VaR	E	0.086	0.071	0.136	0.482	0.224	0.005
	T	0.001	0.008	0.001	0.075	0.001	0.914
Loss function 1+2+3							
1% VaR	E	0.086	0.090	0.132	0.571	0.174	0.002
	T	0.001	0.011	0.002	0.073	0.000	0.913
5% VaR	E	0.122	0.109	0.172	0.451	0.149	0.001
	T	0.001	0.009	0.002	0.174	0.014	0.800

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

135 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.013	0.010	0.010	0.011	0.012	0.011
	5%	0.006	0.004	0.004	0.008	0.005	0.003
	10%	0.006	0.004	0.004	0.006	0.004	0.003
5%	1%	0.053	0.054	0.039	0.055	0.086	0.084
	5%	0.035	0.036	0.022	0.032	0.053	0.042
	10%	0.025	0.027	0.020	0.023	0.034	0.025

Preference relation among the models as inferred from table 115

2,5,6 1 3,4 + 6 2 3,4,5 + 6 3 4,5 +5,6 4 + 6 5 no order

136 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test	Model comparison														
	(α)	1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.501	0.470	0.896	0.977	0.996	0.324	0.904	0.981	0.994	0.901	0.893	0.984	0.991	0.998	0.966
	5%	0.502	0.474	0.896	0.977	0.996	0.326	0.904	0.981	0.994	0.901	0.893	0.984	0.991	0.998	0.967
	10%	0.503	0.477	0.898	0.978	0.996	0.328	0.904	0.982	0.994	0.901	0.894	0.984	0.992	0.999	0.968
Prefer 1st model	1%	0.240	0.273	0.501	0.405	0.229	0.216	0.503	0.418	0.246	0.472	0.468	0.371	0.356	0.195	0.254
	5%	0.240	0.273	0.501	0.405	0.229	0.216	0.503	0.418	0.246	0.472	0.468	0.371	0.356	0.195	0.254
	10%	0.241	0.274	0.502	0.406	0.229	0.217	0.503	0.419	0.246	0.472	0.468	0.371	0.357	0.195	0.254
Prefer 2nd model	1%	0.261	0.197	0.395	0.572	0.767	0.108	0.401	0.563	0.748	0.429	0.425	0.613	0.635	0.803	0.712
	5%	0.262	0.201	0.395	0.572	0.767	0.110	0.401	0.563	0.748	0.429	0.425	0.613	0.635	0.803	0.713
	10%	0.262	0.203	0.396	0.572	0.767	0.111	0.401	0.563	0.748	0.429	0.426	0.613	0.635	0.804	0.714
VaR 5%																
Test is signif.	1%	0.583	0.566	0.970	0.993	0.996	0.350	0.973	0.992	0.997	0.976	0.981	0.992	0.993	0.993	0.985
	5%	0.588	0.569	0.973	0.995	0.996	0.354	0.976	0.992	0.997	0.978	0.986	0.994	0.995	0.995	0.987
	10%	0.591	0.569	0.976	0.996	0.996	0.354	0.978	0.993	0.997	0.981	0.988	0.994	0.997	0.995	0.988
Prefer 1st model	1%	0.309	0.355	0.501	0.340	0.292	0.239	0.494	0.343	0.288	0.457	0.461	0.291	0.328	0.250	0.441
	5%	0.311	0.357	0.502	0.341	0.292	0.241	0.496	0.343	0.288	0.458	0.464	0.293	0.329	0.251	0.442
	10%	0.313	0.357	0.504	0.342	0.292	0.241	0.498	0.344	0.288	0.461	0.466	0.293	0.331	0.251	0.442
Prefer 2nd model	1%	0.274	0.211	0.469	0.653	0.704	0.111	0.479	0.649	0.709	0.519	0.520	0.701	0.665	0.743	0.544
	5%	0.277	0.212	0.471	0.654	0.704	0.113	0.480	0.649	0.709	0.520	0.522	0.701	0.666	0.744	0.545
	10%	0.278	0.212	0.472	0.654	0.704	0.113	0.480	0.649	0.709	0.520	0.522	0.701	0.666	0.744	0.546

DGP FIGARCH(1,d,1) d=0.8 b=0.5 f=0.05 - % represent VaR p-level unless differently specified

137 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	4.086 (1.992) *1.634*	4.142 (2.253) *1.657*	4.021 (2.176) *1.608*	3.745 (1.768) *1.498*	4.864 (2.128) *1.946*	6.277 (2.378) *2.511*
5% VaR	9.692 (3.171) *3.877*	9.838 (3.594) *3.935*	9.631 (3.413) *3.852*	10.692 (3.001) *4.277*	13.047 (3.590) *5.219*	15.659 (3.772) *6.264*

138 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.367	0.431	0.470	0.537	0.279	0.069
5% VaR	0.500	0.380	0.386	0.401	0.187	0.033

139 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.941	0.907	0.921	0.975	0.893	0.719
	5%	0.868	0.839	0.854	0.908	0.779	0.581
5% VaR	1%	0.959	0.943	0.943	0.984	0.983	0.954
	5%	0.849	0.818	0.824	0.919	0.937	0.840
Test of independence: Null							
1% VaR	1%	0.902	0.884	0.876	0.768	0.995	0.998
	5%	0.544	0.516	0.511	0.386	0.985	0.992
5% VaR	1%	0.973	0.968	0.968	0.853	0.996	0.994
	5%	0.885	0.871	0.880	0.683	0.974	0.976
Test of conditional coverage: Null							
1% VaR	1%	0.919	0.887	0.898	0.803	0.943	0.816
	5%	0.783	0.735	0.744	0.702	0.875	0.707
5% VaR	1%	0.943	0.922	0.923	0.883	0.980	0.960
	5%	0.777	0.745	0.759	0.694	0.947	0.872

140 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.025	0.032	0.045	0.286	0.598	0.055
5% VaR	0.025	0.018	0.028	0.281	0.595	0.053

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

141 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.064	0.113	0.153	0.536	0.169	0.007
	T	0.055	0.147	0.389	0.407	0.002	0.000
5% VaR	E	0.095	0.162	0.247	0.418	0.077	0.001
	T	0.055	0.147	0.389	0.407	0.002	0.000
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.038	0.055	0.075	0.621	0.236	0.016
	T	0.000	0.012	0.002	0.094	0.000	0.892
5% VaR	E	0.022	0.065	0.097	0.518	0.287	0.011
	T	0.001	0.007	0.001	0.124	0.006	0.861
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.047	0.067	0.094	0.517	0.298	0.018
	T	0.001	0.025	0.003	0.085	0.000	0.886
5% VaR	E	0.051	0.106	0.167	0.312	0.356	0.009
	T	0.002	0.022	0.001	0.084	0.000	0.891
Loss function 1+2							
1% VaR	E	0.041	0.087	0.122	0.602	0.181	0.008
	T	0.000	0.014	0.002	0.136	0.001	0.846
5% VaR	E	0.055	0.125	0.165	0.484	0.170	0.002
	T	0.001	0.008	0.001	0.289	0.356	0.345
Loss function 1+3							
1% VaR	E	0.052	0.092	0.131	0.522	0.237	0.007
	T	0.001	0.030	0.003	0.117	0.000	0.848
5% VaR	E	0.082	0.143	0.218	0.383	0.173	0.001
	T	0.001	0.033	0.002	0.183	0.007	0.774
Loss function 2+3							
1% VaR	E	0.041	0.060	0.084	0.563	0.276	0.018
	T	0.000	0.013	0.002	0.090	0.000	0.895
5% VaR	E	0.033	0.082	0.125	0.424	0.329	0.006
	T	0.001	0.010	0.001	0.095	0.000	0.893
Loss function 1+2+3							
1% VaR	E	0.043	0.082	0.113	0.570	0.226	0.007
	T	0.000	0.014	0.002	0.109	0.000	0.875
5% VaR	E	0.051	0.123	0.167	0.439	0.217	0.003
	T	0.001	0.011	0.001	0.142	0.004	0.840

Model reference:1 - Figarch(1,d,1); 2 - Garch(1,1); 3 - Igarch(1,1); 4 - EWMA(0.97); 5 - HF Figarch(1,d,1) square root; 6 - HF Figarch(1,d,1) sum

142 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.013	0.015	0.012	0.012	0.011	0.019
	5%	0.006	0.007	0.007	0.007	0.006	0.009
	10%	0.005	0.005	0.005	0.007	0.005	0.005
5%	1%	0.051	0.041	0.036	0.048	0.086	0.089
	5%	0.035	0.025	0.024	0.029	0.051	0.055
	10%	0.024	0.019	0.019	0.027	0.040	0.039

Preference relation among the models as inferred from table 115

2,5,6 1 3,4 + 6 2 3,4,5 + 6 3 4,5 +5,6 4 + 6 5   no order

143 - Test of VaR model comparison - 1000 replications – 250 daily forecasts																
Freq. of	Test (α)	Model comparison														
		1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6	3-4	3-5	3-6	4-5	4-6	5-6
VaR 1%																
Test is signif.	1%	0.506	0.478	0.910	0.983	0.993	0.230	0.914	0.982	0.993	0.914	0.912	0.981	0.988	0.994	0.796
	5%	0.511	0.482	0.912	0.983	0.993	0.235	0.914	0.983	0.993	0.914	0.912	0.981	0.989	0.995	0.798
	10%	0.515	0.486	0.912	0.983	0.993	0.237	0.914	0.983	0.994	0.914	0.912	0.982	0.989	0.995	0.798
Prefer 1st model	1%	0.477	0.545	0.618	0.439	0.354	0.671	0.627	0.440	0.364	0.591	0.590	0.414	0.369	0.276	0.350
	5%	0.478	0.545	0.618	0.439	0.354	0.665	0.627	0.440	0.364	0.591	0.590	0.414	0.370	0.277	0.351
	10%	0.478	0.545	0.618	0.439	0.354	0.664	0.627	0.440	0.364	0.591	0.590	0.414	0.370	0.277	0.351
Prefer 2nd model	1%	0.523	0.455	0.382	0.561	0.646	0.329	0.373	0.560	0.636	0.409	0.410	0.586	0.631	0.724	0.650
	5%	0.522	0.455	0.382	0.561	0.646	0.335	0.373	0.560	0.636	0.409	0.410	0.586	0.630	0.723	0.649
	10%	0.522	0.455	0.382	0.561	0.646	0.336	0.373	0.560	0.636	0.409	0.410	0.586	0.630	0.723	0.649
VaR 5%																
Test is signif.	1%	0.615	0.553	0.985	0.987	0.988	0.247	0.984	0.988	0.991	0.983	0.991	0.989	0.998	0.997	0.927
	5%	0.620	0.557	0.988	0.992	0.992	0.254	0.990	0.992	0.993	0.990	0.993	0.993	0.998	0.997	0.929
	10%	0.623	0.561	0.989	0.994	0.994	0.257	0.990	0.992	0.995	0.992	0.994	0.994	0.999	0.997	0.931
Prefer 1st model	1%	0.535	0.594	0.533	0.365	0.325	0.608	0.532	0.367	0.321	0.506	0.505	0.335	0.327	0.294	0.528
	5%	0.536	0.592	0.533	0.367	0.326	0.602	0.530	0.368	0.320	0.503	0.504	0.336	0.327	0.294	0.529
	10%	0.535	0.589	0.533	0.368	0.327	0.594	0.530	0.368	0.321	0.502	0.503	0.335	0.327	0.294	0.530
Prefer 2nd model	1%	0.465	0.406	0.467	0.635	0.675	0.392	0.468	0.633	0.679	0.494	0.495	0.665	0.673	0.706	0.472
	5%	0.464	0.408	0.467	0.633	0.674	0.398	0.470	0.632	0.680	0.497	0.496	0.664	0.673	0.706	0.471
	10%	0.465	0.411	0.467	0.632	0.673	0.406	0.470	0.632	0.679	0.498	0.497	0.665	0.673	0.706	0.470