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The effects of aggregation on memory and prediction of FIGARCH variances

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Abstract: In this work we focus on the effects of aggregation on parameters estimation and Value-at-Risk computation if the data generator follow a FIGARCH model. We present a Montecarlo experiment which shows that the memory structure is affected. In a second simulation study we compare Value-at-Risk estimates obtained by high frequency and aggregated data. We verify that aggregated data have a better performance on a loss function approach while on a statistical based test analysis high frequency data are preferred.

Keywords: aggregation, long-memory, FIGARCH

JEL classification: C13, C15, C22

In a previous work, Caporin (2002b), we analyzed the performances of Value-at-Risk measures obtained by different models on data generated by a FIGARCH process. The analysis was motivated by the findings of long memory on the variances of financial instrument prices coupled with the practice of using GARCH-based models to compute variance forecasts. Moreover we were interested in comparing our results with the one of Beltratti and Morana (1998), that dealt with this problem but on an applied framework. We were therefore trying to assess if a misspecified model could be used in forecasting variances and we showed that this were not allowed on the basis of an extensive Montecarlo study, involving a comparison of the different Value-at-Risk measures. We employed in the analysis the loss-function approach of Lopez (1998) and the statistical based testing approach of Kupiec (1995), Christoffersen (1998), Lopez (1998) and Christoffersen, Hahn and Inoue (2001). In this paper we extend our previous study analyzing the effects of aggregation on the memory structure of the series, on the parameter estimates of different models and on Value-at-Risk computation. We are now interested in verifying if high frequency data can improve the 1-day ahead forecast of the variance and we analyze if the memory structure is affected by the aggregation process. We will show that the long memory behavior is in general robust to the aggregation process and that aggregated data produce better volatility forecast but only on a loss function comparison approach. In the following section we briefly recall the model and the tools used in the VaR comparison analysis. Section 2 deal with the effects on aggregation on parameter estimates and VaR measures, presenting the Montecarlo results.

Section 3 will conclude.

1 FIGARCH models, estimation and VaR comparison

The long memory GARCH model has been introduced by Baillie, Bollerslev and Mikkelsen (1996) as a generalization of the IGARCH process. Assume that the mean process has the following representation

$$y_t = \mu_t + \varepsilon_t \tag{1}$$

where for simplicity $\mu_t = 0$, I^{t-1} represent the information set up to time $t-1$ and $\varepsilon_t | I^{t-1} \sim iid\,(0, \sigma_t^2)$, the error term has a time-dependent variance called FIGARCH(p,d,m) with the following parameterization:

$$\sigma_t^2 = \omega + \left\{1 - [1 - \beta(L)]^{-1} (1-L)^d \phi(L)\right\} \varepsilon_t^2$$

where $\beta(L) = \sum_{i=1}^{p} \beta_i L^i$, $\phi(L) = \sum_{i=1}^{m-1} \phi_i L^i$, $d \in [0,1]$ and $(1-L)^d = \sum_{i=0}^{\infty} \left(\prod_{0 \leq k \leq i} \frac{k-1-d}{k}\right) L^i$. To ensure the positivity of conditional variances the parameters have to satisfy a set of restrictions, for the FIGARCH(1,d,1) case these are

$$\begin{aligned} \beta - d &\leq \phi \leq \frac{2-d}{3} \\ d\left(\phi - \frac{1-d}{2}\right) &\leq \beta(d - \beta + \phi) \end{aligned} \tag{2}$$

The strict stationarity has been verified in Caporin (2002a) on the assumption of normality of the standardized residuals $z_t = \varepsilon_t / \sigma_t$. The estimation of the model is carried out by a quasi-maximum likelihood approach, however the consistency of parameter estimates has not yet been proven, for this point we rely on the Montecarlo simulation reported on Baillie et al. (1996) and Caporin (2002a). We report here the equation of the variance forecaster, its derivation together with a discussion on the estimation and stationarity issues can be found in Caporin (2002a-b-c).

$$
\begin{aligned}
E\left[\sigma_{t+s}^2 | I^{t-1}\right] &= \theta_s \omega + \sum_{i=0}^{\infty} \psi_{i+1} \varepsilon_{t-i}^2 \qquad (3) \\
\psi_k &= \sum_{i=1}^{s} \phi_i \lambda_{k+s-i} \qquad \phi_1 = 1 \qquad \phi_i = \sum_{j=1}^{i-1} \lambda_j \phi_{i-j} \qquad \theta_s = \sum_{i=1}^{s} \phi_i \\
\lambda(L) &= 1 - \left[1 - \beta(L)\right]^{-1} (1-L)^d \phi(L) = \sum_{i=0}^{\infty} \lambda_i L^i
\end{aligned}
$$

The Value-at-Risk is defined as the maximum amount of loss we can face from t to $t+h$ using a given model m and at a confidence level α. To evaluate and compare Value-at-Risk measures we focused on two different approaches, a loss function one, introduced by Lopez (1998), and a statistical based one, which refer to a group of tests, due to Kupiec (1995), Lopez (1998), Christoffersen (1998), and Christoffersen, Hahn and Inoue (2001). All the approaches focus on the exceptions of Value-at-Risk measures, that is when the VaR level is violated by market performances. The test of Kupiec focus on the Unconditional coverage, and under the null of correct unconditional coverage is distributed as a $\chi^2(1)$. Lopez (1998) introduced two tests based on the work of Christoffersen (1998), the test on Independence, again a $\chi^2(1)$ under the null of independence across VaR exceptions, and the test of Conditional Coverage, (the sum of the previous two test), distributed as a $\chi^2(2)$ under the null of correct coverage. Finally the tests of model specification and model comparison introduced by Christoffersen et al. (2000): the first detect if the model is correctly specified, while the latter compute a pairwise comparison among different models, both tests have an asymptotic normal distribution under the null hypothesis of correct specification. For a deeper discussion on the tests please refer to the cited papers. The loss function approach of Lopez (1998) is again based on the exceptions of VaR models, while our approach, introduced in Caporin (2002a-b) deal with the whole VaR forecasts. In both cases the best model is the one that minimize the loss. Again refer to the paper for a detailed presentation of the methods. The Value-at-Risk is used by regulators as one of the possible measures of capital requirement that banks must fulfill. This measure and the associated model are tested in a backtesting approach, that is observing their performances on a time window of 250 days (approximately one year). The results depend on the exceptions, between 0 and 4 the model is accepted without correction on the capital requirements, between 5 and 9, the model is accepted but capital margins are increased, with more than 9 exceptions regulators suggest a revision of the model.

2 VaR, FIGARCH and aggregation

A point raised up by the Beltratti and Morana (2000) paper was the following: using high frequency data could we get better estimates of our 1-day VaR? Their conclusion was that the simple GARCH(1,1), on high frequency data, will do the task even if there is an evident long memory in the data. We examine this relation in detail with a limited Montecarlo study dealing with a group of problems. We generate data as they were hourly returns and then aggregate them in order to obtain daily returns, assuming that a normal open market day last for eight hours. The data are generated with normal distributed standardized residuals. On the aggregated data we are at first interested in assessing if there are changes in parameter estimates, specially on the memory behavior, therefore we examine this point computing a group of information criteria on different models, a GARCH(1,1), an IGARCH(1,1) and three FIGARCH(p,d,m) with $p = m = 0$, $p = 1$ and $m = 0$ and $p = m = 1$. By this methods, given the results of Caporin (2002c), we will asses if the aggregation process change the structure of the series into an integrated GARCH, a short memory model or if the long memory behavior is robust against the aggregation.

All experiments consist of 1000 replications with series of 18000 non aggregated observations. We simulated log-returns series. We considered five different DGP with the following parameters combinations: $d = 0.8$, $\beta = 0.5$, $\psi = 0$; $d = 0.8$, $\beta = 0.5$, $\psi = 0.05$; $d = 0.8$, $\beta = 0.5$, $\psi = 0.3$; $d = 0.4$, $\beta = 0.3$, $\psi = 0$; $d = 0.4$, $\beta = 0.3$, $\psi = 0.2$. The identification analysis is limited to the first 2000 aggregated data (16000 non aggregated points) leaving the last 250 (2000 non aggregated) for a VaR backtesting evaluation. We consider this as a limited Montecarlo since we do not take into consideration the consistence of model selection based on information criteria and we restrict our attention to a limited range of models and parameter combinations. This choice strictly depend on CPU time needed to run a full experiment: to simulate 18000 observations (plus 2000 points to avoid dependance from initial values), run the identification tests and then the VaR evaluation, we need between 6 and 15 days, depending on DGP and "external" events (blackouts, computer failures, etc.). In all cases, on aggregated data, we estimated the following models: FIGARCH(1,d,1), FIGARCH(1,d,0), FIGARCH(0,d,0), GARCH(1,1) and IGARCH(1,1). We included a limited group of tables and graphs in this paper (the detailed results can be found in Caporin(2002a)), where we report the frequency of model selection based on the information criteria of Akaike (AIC), Hannan-Quinn (HQ), Schwarz (BIC) and Shibata (SH), together with the estimated parameters and standard errors. Finally we added a kernel density of the distribution of the quasi maximum likelihood estimator. We can summarize our results as follows:

- A first consideration on the memory parameter estimates: in general we can observe that the aggregation does not change the Montecarlo average of the long memory coefficient, d. This result is much stronger for the experiments conducted with d set equal to 0.8, rather than in the case where it assume the value 0.4. Compare table 1 with table 3, the discrepance between the non-aggregated true value and the Montecarlo average is less than 0.01 in the first while it is close to 0.1 in the second. Even with this evidence we are not sure that this can be interpreted as a true effect of aggregation. The picture can be clarified analyzing also the Montecarlo standard deviation, and comparing it with the one obtained on non-aggregated estimates: we can observe that it heavily increase for d=0.8 while the change for 0.4 is less evident. This may be much more evident comparing the kernel density estimates of aggregated data with the ones of pure FIGARCH processes (for the first see Caporin (2002a), while for the second refer also to Baillie et al. (1996)). From these observation we extrapolate the following picture: we believe that the effect of aggregation depends on the memory parameter level, we can distinguish between series with high memory (d=0.4) and intermediate memory (d=0.8), in the first case aggregation matter, memory properties increase (the distribution of the estimator has a stable variance across aggregated and non aggregated data), in the second case the aggregation does not affect the memory structure but lead to an increase in variation among parameter estimates.

- Consider now the estimates of the other FIGARCH parameters: these are much more affected from the aggregation process, as if this will change the short-memory structure of the underlying process. Here we must note that kernel densities evidence a problem in the consistence and in the biasedness of the QML estimator for the FIGARCH(1,d,1). This might be coupled with the algorithm convergence problem evidenced in Caporin (2002a-2002c), and can be interpreted as an effect of the aggregation, valid for all the cases considered even if in the series with intermediate memory this is much more evident. We believe that in these processes the aggregation scheme push the model to the critic region for the optimization process, therefore small variations can be sufficient to obtain different otptima from similar non-aggregated series.

- As we can expect the aggregation process highly affect the constant in the variance that highly increase, while the constant in the mean is not affected. This last effect is due to the fact that it was fixed to zero, with a different value the aggregation will affect it.

- Finally observe the parameters of the GARCH and IGARCH: in the case of high memory the two models appear to be different, the sum of the GARCH parameter, at least in average, is different from one, while in the models with d=0.8, GARCH and IGARCH are very close, as if the aggregation push the model to a new process with d=1.

- Take a look now at the identification: the memory property of the simulated series is identified by the information criteria with an error percentage of 20%, near the value recorded for non aggregated series. Again we can note that the identification is affect by the structure of the process and by parameter values. Moreover none of the criteria appear to prevail on the others.

We will now turn to our main point, the evaluation of 1-day-Value-at-Risk both with aggregated and non aggregated data. Given the structure of tests for Value-at-Risk comparison and the time requested to run a Montecarlo experiment on simulated high frequency data we decided to split this analysis in two branches, and on the first we compare the VaR computed on aggregated data with: the correct DGP, a GARCH(1,1) an IGARCH(1,1), the EWMA with smoothing parameter set to 0.97 and finally with the VaR computed on hourly data with the true DGP. In a second group of simulations we compare the VaR performances with the following models: again on aggregated data the true DGP and the EWMA(0.97) and on high frequency data with the true DGP and a GARCH(1,1). In all cases we estimate the different models and we compare the 1-day ahead VaR. A point arise, on daily data the computation of 1-day-ahead prediction intervals is a standard procedure, as in the previous Montecarlo, while on hourly data we use two different approaches: normal practice in this field to obtain a T-step-ahead forecast of the volatility (T=8 in our case) is to multiply the 1-step-ahead forecast by $\sqrt{T}$, a solution based on the independence and identically distribution hypothesis of the residuals, however in the GARCH type modelling this can be differently interpreted, the T-step-ahead forecast may be computed as the sum of 1 to T step forecasts. The T step return may be expressed as the sum of single step returns, postulating independence its expected value will be the sum of expected values and with a GARCH generator this will be zero:

$$\begin{aligned} r_T &= \sum_{j=1}^{T} r_{t+j} \qquad (4) \\ E_t\left[r_T\right] &= \sum_{j=1}^{T} E_t\left[r_T\right] = 0 \end{aligned}$$

The variance computed conditionally at time t, will be therefore

$$Var_t\left[r_T\right] = Var_t\left[\sum_{j=1}^{T} r_{t+j}\right] \qquad (5)$$

the law of iterated expectations allow us to set covariances between time dependent returns to zero obtaining

$$Var_t\left[r_T\right] = \sum_{j=1}^{T} Var_t\left[r_T\right] \quad (6)$$

that is the sum of the predictions from 1 to T step ahead variance made in time T. This will be the second VaR computation technique used with hourly data. In the following we will refer to the forecast obtained with the first methods as "square root forecasts", while the second will be labelled "sum forecasts". On these VaR measures we will compute all the tests and the loss functions as in the previous Montecarlo.

These two sets of Montecarlo experiments are run on the same generators used for aggregated data model identification analysis. The Value-at-Risk analysis is performed again on a backtesting approach using 250 observations to assess number of exceptions, compute tests and loss functions. The complete set of tables of these Montecarlo experiments can be found in Caporin (2002a), here we present the results of one single DGP. As in the previous analysis we summarize the tables with the following observations:

Average exceptions and MRA. Consider at first the comparison among the aggregated FIGARCH, the RiskMetrics and the high frequency FIGARCH and GARCH. In these cases aggregated models give the smaller percentage of exceptions for the 1-day VaR, while, among the high frequency models, the FIGARCH with square root forecasts produce the better results. This behavior indicate that even if the true generator is an high frequency process with long memory, in computing 1-day VaR better results are obtained by aggregated data. This result is confirmed in the second Montecarlo where we compare different aggregated specifications with the true high frequency generator. We restrict now our attention on the aggregated models, among these specifications two clearly dominates the other, the long memory GARCH and the RiskMetrics, with a prevalence of the latter at the 1% VaR while the FIGARCH is preferred at the 5% VaR level. A final comments on the MRA: here the models differently satisfy the requirements, leading to different zones, in most cases the green zone is reached by the long memory GARCH on aggregated data and by the RiskMetrics, while the other specifications switch between the green and the yellow zone. Again this indicate that aggregated data are preferred to high frequency specifications.

Tests of Conditional and Unconditional Coverage. Test results again cannot help in the choice of the best specification, however we must note that variation among different models is wider than in the previous analysis allowing to exclude, in some cases, one of the models employed. As an example we can consider the FIGARCH(0.5,0.8,0.3) case, the CC test allow to exclude at least on of the high frequency FIGARCH specifications, or again consider the FIGARCH(0.3,0.4,0), the Independence test at 5% allow to exclude all daily models. Unfortunately in all these cases we cannot reduce our choices to one model, leading to a small power of these test in discerning among the alternative models.

Loss functions. Now the situation change, while considering only the exeptions aggregated data are always preferred, turning to a loss function approach high frequency data are in some cases the best choice. Consider the Lopez loss function: the preferred models are the RiskMetrics and the high frequency FIGARCH with square root forecasts, the choice switch between this two models. However focusing on the extended loss functions analysis results are different, here the choice switch between the RiskMetrics and the high frequency GARCH with square root forecasts in the first Montecarlo while in the second the preferred models are again the RiskMetrics together with the high frequency FIGARCH with sum forecasts.

Model comparison test. If we consider the first Montecarlo, which include high frequency GARCH and FIGARCH specifications, this test allow the derivation of a preference ordering among the different models. This test compute a pairwise comparison among the models and report a frequency of preference of the first or of the second model. If we state that, given the test comparison of two models, one is preferred to the other when the frequency of preference is above 50%, we have a set of reference relations that may allow to construct an ordering. In the first Montecarlo this is possible with the full set of generators and all the ordering have a common point: the high frequency GARCH specification with square root forecast is always the preferred. The ordering of the remaining models change across the generators. This result allow to conjecture that in computing 1-day Value-at-Risk with high frequency data, even if in presence of long memory, a short memory model give a finer matching to the efficient moment condition. A similar result was obtained by Beltratti and Morana (1999) in an applied framework. Their conclusion was mainly driven by the closeness of forecasts obtained by the FIGARCH and GARCH specifications, while in this case we obtain this conclusion via a Montecarlo approach. Turn now the attention to the second Montecarlo, that report the comparison across daily specifications and the high frequency true generator. In this case the preference relation among the specifications do not exist, in most of the cases the relation is not transitive, however the high frequency FIGARCH specification with sum forecasts is the candidate to be the preferred solution. The preference ordering are reported in the Caporin (2002a) paper, whenever they exists. A couple of additional remarks is needed: first of all we stress on the fact that high frequency specifications are most of the times preferred to the daily ones, showing that, even if with a misspecified model, high frequency data matter; moreover the RiskMetrics model is most of the time the worst solution in the model comparison tests, this is due, to our advise, to the structure of the model, in the sense that any GARCH specification, even an highly misspecified one, long or short memory, has a greater flexibility that allow to adequately match the (simulated) data; finally, note that this result is not influenced by the true data generating process.

Model specification test. The results obtained by this last instruments are similar across the Montecarlo experiments and the different models, showing that the Value-at-Risks is not correctly specified. We conjecture that this is due to the limited number of points used in our analysis, 250 observations, that might influence test power.

3 Conclusions

In this paper and in a companion one, Caporin (2002b), we analyzed with a Montecarlo approach, the performances of misspecified models in the computation of Value-at-Risk. This work analyze in details the effects of aggregation of high frequency data on the memory structure of the series, showing that the memory behavior is affected, leading to an increase of memory, if the series has intermediate memory (d parameter around 0.8), while the memory structure remain unchanged with high memory series (d around 0.4). We also compare the performances of different models for the 1-day ahead variance forecast, on a backtesting Value-at-Risk approach, using a different set of tests and loss functions. We show that none of the methods used, loss functions or tests, allow to choose one model as the preferred. Focusing only on exceptions the best choice will be the RiskMetrics model, therefore on aggregated data, similar result is obtained with the various loss functions. The tests based on the correct coverage are no informative on this comparison, while the tests of Christoffersen et al. (2001) shows that high frequency data are preferred, however most of the times a misspecified GARCH(1,1) is the best solution.

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1 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	0.00165	0.00163	0.00188	0.00137	0.00153
	0.03302	0.03330	0.03449	0.03254	0.03310
	0.03305	0.03332	0.03452	0.03256	0.03312
ω	0.22918	0.27255	0.36017	0.23088	0.21694
	0.08860	0.09532	0.11183	0.09084	0.08614
	0.23640	0.27930	0.36757		
d	0.79122	0.77549	0.56146		
	0.14726	0.14049	0.09312		
	0.14745	0.14255	0.25606		
ϕ - α	0.11348			0.38744	0.57265
	0.12680			0.09584	0.10501
	0.17011				
β	0.43189	0.31831		0.57354	
	0.16916	0.18316		0.10390	
	0.18228	0.25792			

For each parameter the table reports in the order: Montecarlo average, Montecarlo standard deviation and Root Mean Squared Error. The DGP is a FIGARCH(1,d,1) - d=0.8 β=0.5 -models were estimated only on aggregated data.

2 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.283	0.444	0.071	0.150	0.053
Hannan-Quinn	0.163	0.470	0.120	0.138	0.110
Schwarz	0.458	0.348	0.025	0.154	0.016
Shibata	0.285	0.443	0.070	0.150	0.053
LL	0.700	0.149	0.000	0.152	0.000
4 IC	0.283	0.444	0.071	0.150	0.053

The table reports the frequency of model selection with each of the different fitted models. The DGP is a FIGARCH(1,d,1) - d=0.8 β=0.5 -models were estimated only on aggregated data.

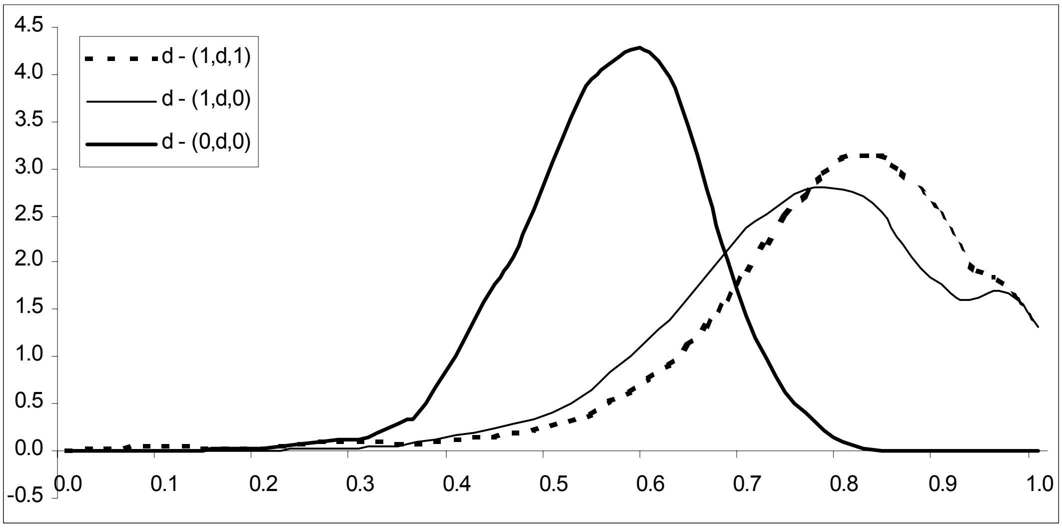

Figure 1: kernel density of the estimated long-memory parameter, in parenthesis the FIGARCH specification. The DGP is a FIGARCH(1,d,1) - d=0.8 β=0.5 -models were estimated only on aggregated data.

3 - QML estimates – 2000 aggregated data – 1000 replications – Mean (s.d.) [RMSE]					
	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
μ	-0.00099	-0.00091	-0.00071	-0.00089	-0.00088
	0.02985	0.02998	0.03008	0.02988	0.03029
	0.02985	0.02998	0.03007	0.02987	0.03029
ω	0.18721	0.32086	0.46797	0.10599	0.02968
	0.09307	0.10479	0.11640	0.07272	0.02271
	0.20014	0.32803	0.47251		
d	0.29960	0.25335	0.19294		
	0.08482	0.06894	0.03795		
	0.13141	0.16203	0.21050		
ϕ - α	0.24367			0.09594	0.90282
	0.12175			0.03142	0.03998
	0.27237				
β	0.41204	0.12287		0.85146	
	0.15374	0.07730		0.06428	
	0.19018	0.19325			

For each parameter the table reports in the order: Montecarlo average, Montecarlo standard deviation and Root Mean Squared Error. The DGP is a FIGARCH(1,d,1) - d=0.4 β=0.3 - models were estimated only on aggregated data

4 - Frequency of model selection – 2000 aggregated observations – 1000 replications					
Criteria	Fitted models				
	Figarch(1,d,1)	Figarch(1,d,0)	Figarch(0,d,0)	Garch(1,1)	Igarch(1,1)
Akaike	0.367	0.279	0.155	0.193	0.006
Hannan-Quinn	0.167	0.308	0.298	0.206	0.021
Schwarz	0.590	0.188	0.051	0.171	0.000
Shibata	0.367	0.280	0.154	0.193	0.006
LL	0.782	0.062	0.000	0.156	0.000
4 IC	0.367	0.279	0.155	0.193	0.006

The table reports the frequency of model selection with each of the different fitted models. The DGP is a FIGARCH(1,d,1) - d=0.4 β=0.3 -models were estimated only on aggregated data.

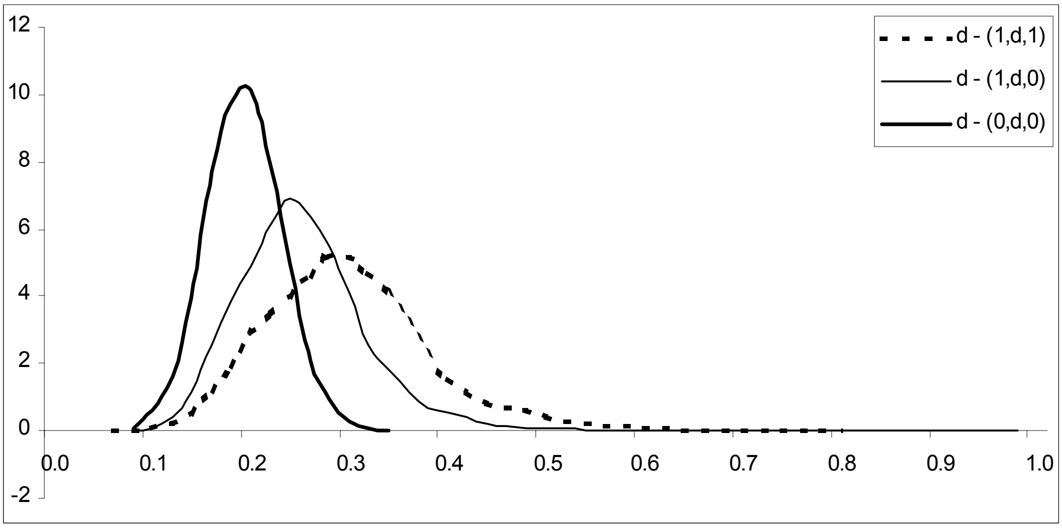

Figure 2: kernel density of the estimated long-memory parameter, in parenthesis the FIGARCH specification. The DGP is a FIGARCH(1,d,1) - d=0.4 β=0.3 -models were estimated only on aggregated data.

5 - Average number of exceptions (standard deviation) *mean percentage* 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	3.521 (1.954) *1.408*	2.387 (1.379) *0.955*	3.376 (1.868) *1.350*	3.751 (1.947) *1.500*	5.154 (2.689) *2.062*	3.309 (1.995) *1.324*
5% VaR	11.372 (3.647) *4.549*	11.461 (2.967) *4.584*	12.333 (3.469) *4.933*	13.202 (3.554) *5.281*	16.268 (4.872) *6.507*	12.342 (4.020) *4.937*

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum

6 - Frequency of less exceptions - 1000 replications – 250 daily forecast						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.248	0.690	0.280	0.201	0.069	0.364
5% VaR	0.362	0.579	0.239	0.133	0.064	0.203

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum

7 - TESTS – frequencies of accepting the null hypothesis - 1000 replications – 250 daily forecast							
	α	Fitted models					
		1	2	3	4	5	6
Test of unconditional coverage: Null							
1% VaR	1%	0.964	0.999	0.978	0.967	0.824	0.968
	5%	0.898	0.929	0.911	0.892	0.712	0.882
5% VaR	1%	0.971	0.992	0.992	0.987	0.896	0.974
	5%	0.900	0.958	0.939	0.930	0.742	0.880
Test of independence: Null							
1% VaR	1%	0.846	0.739	0.998	0.997	0.996	0.996
	5%	0.831	0.730	0.981	0.987	0.982	0.981
5% VaR	1%	0.958	0.938	0.994	0.995	0.994	0.997
	5%	0.888	0.842	0.975	0.979	0.956	0.979
Test of conditional coverage: Null							
1% VaR	1%	0.937	0.954	0.989	0.979	0.879	0.980
	5%	0.776	0.730	0.971	0.958	0.802	0.962
5% VaR	1%	0.943	0.957	0.989	0.990	0.917	0.979
	5%	0.831	0.845	0.951	0.945	0.797	0.923

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified, α is the test confidence level.

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum

8 - Lopez loss function – frequency of model selection - 1000 replications – 250 daily forecasts						
	Fitted models					
	1	2	3	4	5	6
1% VaR	0.031	0.258	0.226	0.106	0.010	0.465
5% VaR	0.031	0.240	0.213	0.083	0.002	0.431

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum

9 - LOSS FUNCTIONS – frequency of model selection (best is lower loss function) 1000 replications – 250 daily forecasts							
		Fitted models					
		1	2	3	4	5	6
Loss Function 1: absolute value of return VaR measure ratio							
1% VaR	E	0.076	0.637	0.098	0.062	0.010	0.213
	T	0.444	0.335	0.105	0.000	0.000	0.116
5% VaR	E	0.240	0.445	0.088	0.019	0.000	0.208
	T	0.444	0.335	0.105	0.000	0.000	0.116
Loss Function 2: square return-VaR normalized by absolute VaR measure							
1% VaR	E	0.052	0.721	0.081	0.054	0.010	0.178
	T	0.006	0.119	0.000	0.012	0.863	0.000
5% VaR	E	0.044	0.679	0.049	0.020	0.000	0.208
	T	0.002	0.160	0.000	0.005	0.833	0.000
Loss Function 3: absolute of return-VaR							
1% VaR	E	0.050	0.704	0.076	0.055	0.010	0.201
	T	0.011	0.108	0.000	0.013	0.868	0.000
5% VaR	E	0.105	0.556	0.062	0.020	0.000	0.257
	T	0.009	0.114	0.000	0.012	0.865	0.000
Loss function 1+2							
1% VaR	E	0.068	0.663	0.088	0.060	0.010	0.207
	T	0.007	0.131	0.000	0.010	0.852	0.000
5% VaR	E	0.158	0.558	0.071	0.014	0.000	0.199
	T	0.003	0.256	0.000	0.002	0.738	0.001
Loss function 1+3							
1% VaR	E	0.066	0.659	0.090	0.061	0.010	0.210
	T	0.012	0.114	0.000	0.012	0.862	0.000
5% VaR	E	0.178	0.499	0.076	0.022	0.000	0.225
	T	0.007	0.145	0.000	0.008	0.840	0.000
Loss function 2+3							
1% VaR	E	0.049	0.712	0.079	0.053	0.010	0.193
	T	0.009	0.113	0.000	0.013	0.865	0.000
5% VaR	E	0.070	0.628	0.053	0.024	0.000	0.225
	T	0.006	0.126	0.000	0.010	0.858	0.000
Loss function 1+2+3							
1% VaR	E	0.062	0.674	0.081	0.057	0.010	0.212
	T	0.007	0.116	0.000	0.012	0.865	0.000
5% VaR	E	0.144	0.562	0.070	0.018	0.000	0.206
	T	0.005	0.151	0.000	0.007	0.837	0.000

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified. E stands for exceptions only, while T for total sample

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum

10 - Test of VaR model specification (null: VaR(p) is correctly specified) Frequency of accepting H_0 – 1000 replications – 250 daily forecasts							
VaR p-value	Test α-value	Fitted models					
		1	2	3	4	5	6
1%	1%	0.023	0.002	0.013	0.017	0.029	0.009
	5%	0.012	0.000	0.007	0.008	0.014	0.005
	10%	0.007	0.000	0.006	0.006	0.008	0.003
5%	1%	0.144	0.085	0.199	0.195	0.215	0.192
	5%	0.068	0.037	0.118	0.110	0.099	0.093
	10%	0.039	0.023	0.078	0.066	0.058	0.058

The DGP is a FIGARCH(1,d,0) d=0.4 b=0.3 - % represent VaR p-level unless differently specified

Model reference:1 - Aggregated, FIGARCH(1,d,0); 2- Aggregated, EWMA(0.97); 3 - High Frequency (HF), FIGARCH(1,d,0) square root; 4 - HF, FIGARCH(1,d,0) sum; 5 - HF, GARCH(1,1) square root; 6 - HF, GARCH(1,1) sum